

On the renormalizability of the theory Lagrangian (Extract).

The following correspondence exists within the theory framework:

$$\text{case } \phi_i = 0 \mid C_{jlk}^i = 0 \qquad \text{case } \phi_i \neq 0 \mid C_{jlk}^i \neq 0 \qquad (\text{A 2.1})$$

$$i\Pi_k = \frac{\partial}{\partial x_k}$$

$$\text{contr}(A_{ik}) = t_{ii}\Xi$$

$$C = \nu + \xi$$

$$i\Pi_k = \frac{\partial}{\partial x_k} - ie\phi_k$$

$$A_{ik} = ih_{ik}$$

$$C = \nu + \xi + i\xi'$$

where,

a wave-field A_{ik} is a symmetrical tensor of the second rank;

C is an auxiliary scalar field;

C_{jlk}^i is Weyl curvature tensor;

ϕ_i is electromagnetic field.

The following formula includes scalar field Φ :

$$\Phi(x) = \nu + \xi(x) + i\eta(x) \qquad (1.1)$$

where,

$\xi(x)$ is vacuum state (coincides with Goldstone field);

ξ field assumes a certain value ν , i.e. a value of the ξ field viz., the potential energy is zero but the value of the ξ field is nonzero;

ν - shift ^{note} in field $\Phi(x)$ corresponds to the vacuum state ξ which contains no particles:

vacuum state (a vacuum expectation value of a scalar field) \mapsto

ν expectation value which breaks the internal symmetry of a vacuum, i.e. spontaneous symmetry breaking occurs \mapsto

spontaneous symmetry breaking of the underlying global symmetry leads to the conversion of components of this ξ field to Goldstone bosons;

note:

have to differentiate the postulated Higgs field from a vacuum state, so ν is due to a vacuum state and not the Higgs field.

end_note

$\eta(x)$ is gravitational –dark field. It is complex due to the interaction with a Goldstone field;

$$\eta(x) = h(x) - i\Xi(x) \quad (1.1 \text{ a})$$

$h(x)$ is tensor-gravitational field $h\left\{\left(\equiv \langle g^{ik} \rangle h_{ik}\right), \langle \quad \rangle$ - vacuum value $\right\}$ which is scalar;

h_{ik} is gravitational field part, i.e. massless tensor spin 2 field;

$\Xi(x)$ is dark field part, i.e. massless scalar spin 0 field (dark boson);

The negative sign before the second term is important, which points out that dark energy is negative.

Eq. (1.4.1 c) describes the interaction of the gravitational field and the Goldstone field with the electromagnetic field ϕ_i

$$\begin{aligned} L_{FP} = & \frac{\partial A_{ik}}{\partial x_l} \frac{\partial A_{ik}^*}{\partial x_l} + e^2 \phi_l^2 A_{ik} A_{ik}^* - 2 \frac{\partial A_{rk}}{\partial x_r} \frac{\partial A_{sk}^*}{\partial x_s} - 2e^2 \phi_r \phi_s A_{rk} A_{sk}^* + \frac{1}{2} \frac{\partial A_{rk}}{\partial x_r} \frac{\partial C^*}{\partial x_k} + \frac{1}{2} e^2 \phi_r \phi_k A_{rk} C^* + \\ & \frac{1}{2} \frac{\partial A_{rk}^*}{\partial x_r} \frac{\partial C}{\partial x_k} + \frac{1}{2} e^2 \phi_r \phi_k A_{rk}^* C - \frac{3}{8} \frac{\partial C}{\partial x_l} \frac{\partial C^*}{\partial x_l} - \frac{3}{8} e^2 \phi_l^2 C C^* + \\ & i \left\{ e\phi_l A_{ik}^* \frac{\partial A_{ik}}{\partial x_l} - e\phi_l A_{ik} \frac{\partial A_{ik}^*}{\partial x_l} - 2e\phi_s A_{sk}^* \frac{\partial A_{rk}}{\partial x_r} + 2e\phi_r A_{rk} \frac{\partial A_{sk}^*}{\partial x_s} + A_{rk} A_{ik}^* \frac{\partial \phi_r}{\partial x_i} - A_{rk} A_{ik}^* \frac{\partial \phi_i}{\partial x_r} + \frac{1}{2} e\phi_k C^* \frac{\partial A_{rk}}{\partial x_r} - \right. \\ & \left. \frac{1}{2} e\phi_r A_{rk} \frac{\partial C^*}{\partial x_k} - \frac{1}{2} e\phi_k C \frac{\partial A_{rk}^*}{\partial x_r} + \frac{1}{2} e\phi_r A_{rk}^* \frac{\partial C}{\partial x_k} - \frac{3}{8} e\phi_l C^* \frac{\partial C}{\partial x_l} + \frac{3}{8} e\phi_l C \frac{\partial C^*}{\partial x_l} \right\} \end{aligned} \quad (1.4.1 \text{ c})$$

Eq. (A 2.0) describes the gravitational field and the Goldstone field without interaction with the electromagnetic field ϕ_i

$$L_{FP} = k^2 A_{ik} A_{ik} + \frac{\partial A_{ik}}{\partial x_l} \frac{\partial A_{ik}}{\partial x_l} - 2 \frac{\partial A_{rk}}{\partial x_r} \frac{\partial A_{sk}}{\partial x_s} - \frac{3}{4} k^2 C^2 - \frac{3}{8} \frac{\partial C}{\partial x_l} \frac{\partial C}{\partial x_l} + \frac{\partial A_{rk}}{\partial x_r} \frac{\partial C}{\partial x_k} \quad (\text{A } 2.0)$$

When $\phi_i = 0$:

Eq. (0.1.1)

$$L(\Phi) = |\partial\Phi|^2 - \lambda(|\Phi|^2 - v^2)^2 \quad (0.1.1)$$

is compared to eq. (A 2.0);

By virtue of Goldstone theorem ($k_\xi = 0$) and ref. eq. (1.3 a) below

$$L = \frac{1}{2}(\partial_i \zeta)^2 + \frac{1}{2}(\partial_i h)^2 - \frac{1}{2}\lambda v^2 \zeta^2 - \frac{1}{2}\lambda v \zeta (\zeta^2 + h^2) - \frac{1}{8}\lambda (\zeta^2 + h^2)^2 \quad (1.3 a)$$

The following is obtained:

$$2\lambda \Xi^2 v^2 = k_\Xi^2 \left(\langle g^{ik} \rangle \right)^2 A_{ik}^2 \quad (A 2.2.1)$$

i.e., $k_\Xi \sim \sqrt{\lambda} v$ (if quartic interaction terms are omitted and the constant terms are contracted), masses for dark field (ref. eq. (1.3 b)) are obtained.

In eq. 1.3 a, shifted Goldstone-dark field is defined as

$$\zeta(x) = v + \xi(x) + \Xi(x) \quad (1.3 b)$$

$v < 0$ corresponds to dark energy; $v > 0$ corresponds to gravitational energy;

massless (k_h) for h and mass ($k_\zeta \equiv$) $\sqrt{\lambda} v$ for ζ .

k_Ξ appears as a result of obtaining mass by a dark field, i.e. $k_\Xi = k_\zeta$; ansatz Goldstone boson does not disappear.

When $\phi_i \neq 0$:

Eq. (0.1.2)

$$L(\Phi, \phi) = -\frac{1}{4} F^{ik} F_{ik} + |D\Phi|^2 - \lambda(|\Phi|^2 - v^2)^2 \quad (0.1.2)$$

where,

$$F_{ik} = \partial_i \phi_k - \partial_k \phi_i;$$

ϕ_i is a covariant four-potential of the electromagnetic field;

the interaction with photons via minimal substitution (ref. 4.8 Problems; (M. Kachelrieß, 2010)

$$D_i \Phi = \partial_i \Phi + ie\phi_i \Phi;$$

is compared to eq. (1.4.1 c).

By virtue of dark field, which does not interact with electromagnetic field, therefore Goldstone-dark field can be omitted altogether and ref. eq. (1.5) below

$$L = -\frac{1}{4} F_{ik} F^{ik} + \frac{1}{2} (\partial_i \zeta)^2 + \frac{1}{2} (\partial_i h + e v \phi_i)^2 - \frac{1}{2} \lambda v^2 \zeta^2 - \frac{1}{2} \lambda v \zeta (\zeta^2 + h^2) - \frac{1}{8} \lambda (\zeta^2 + h^2)^2 \quad (1.5)$$

The following is obtained

$$\frac{1}{2} (\partial h_{ik} + e A_{ik} v)^2 = k_h^2 A_{ik}^* A_{ik} \quad (\text{A 2.2.2})$$

where

k_h ($k_h \equiv$) $e v$ for h ^{note1} is a massive gravitational field;

k_ζ ^{note2} ($k_\zeta \equiv$) $\sqrt{\lambda} v$ for ζ is a massive Goldstone-dark field.

note1:

$$h \mapsto \phi_i + \frac{1}{e\nu} \partial_i h;$$

when Ricci theorem is applied for tensor analysis the following is obtained: $\phi_i + \frac{1}{e\nu \langle g_{ik} \rangle} \partial_i h_{ik}$

note2:

the k_ζ is not related to ϕ_i , i.e., it does not contain charge e since dark field does not interact with an electromagnetic field;

k_ξ appears as a result of obtaining mass by Goldstone boson, i.e., $k_\xi = k_\zeta$.

end_note

Thus, it is confirmed that mass terms in eqs. (A 2.0) and (1.4.1 c) correspond to mass terms in eqs. (1.3 a) and (1.5) and have the same values respectively.

This correspondence is a proof that current theory Lagrangian (1.4.1 c) (\leftrightarrow (1.5)) is of Φ^4 type^{note}. Renormalizability of the interacting Lagrangians in four space-time dimensions, which contains a symmetric tensor field (1.4.1 c), is proved in five dimensions.

The existence of the fifth dimension appears as a required mathematical condition for the Lagrangian of the theory to be renormalizable.

note

The replacement of the original Lagrangian by one for scalars only is an approximation.

end_note

In the framework of the specified mechanism:

i. dark field represents the fifth dimension possessing a dark-like Killing field, which breaks the symmetry into four dimensions. As follows from the notes (ref. Witten, 2014), this dimension can be introduced in a non-contradictory way that is still referenced from a physical standpoint to four dimensions.

The physics of the fifth dimension appears as a Lorentz violation and the mechanism of cancellation of this violation is presented in Appendix C, paragraph 1.

ii. dark-like Killing field corresponds to the conserved quantity, i.e. dark energy.

Thus, the renormalizability of the theory is formulated in the five-dimensional space.

E. Witten, arXiv: hist-ph/1401.8048 v1 (2014).