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► **To cite this version:**

Àngels Aragonès, Laurent Maxit, Oriol Guasch. A graph theory approach to identify resonant and non-resonant transmission paths in statistical modal energy distribution analysis. *Journal of Sound and Vibration*, 2015, pp.91-110. 10.1016/j.jsv.2015.04.001 . hal-01162151

**HAL Id: hal-01162151**

**<https://hal.science/hal-01162151v1>**

Submitted on 9 Jun 2015

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# A graph theory approach to identify resonant and non-resonant transmission paths in statistical modal energy distribution analysis

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## Abstract

Statistical modal energy distribution analysis (SmEdA) extends classical statistical energy analysis (SEA) to the mid frequency range by establishing power balance equations between modes in different subsystems. This circumvents the SEA requirement of modal energy equipartition and enables applying SmEdA to the cases of low modal overlap, locally excited subsystems and to deal with complex heterogeneous subsystems as well. Yet, widening the range of application of SEA is done at a price with large models because the number of modes per subsystem can become considerable when the frequency increases. Therefore, it would be worthwhile to have at one's disposal tools for a quick identification and ranking of the resonant and non-resonant paths involved in modal energy transmission between subsystems. It will be shown that previously developed graph theory algorithms for transmission path analysis (TPA) in SEA can be adapted to SmEdA and prove useful for that purpose. The case of airborne transmission between two cavities separated apart by homogeneous and ribbed plates will be first addressed to illustrate the potential of the graph approach. A more complex case representing transmission between non-contiguous cavities in a shipbuilding structure will be also presented.

*Keywords:* Statistical modal energy distribution analysis, Graph theory, Transmission path analysis

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## 1. Introduction

The mid-frequency problem in the vibration analysis of built-up mechanical structures appears when neither low frequency deterministic approaches (e.g., finite elements, semi-analytical modal expansions) nor high frequency statistical energy methods (e.g., statistical energy analysis, SEA) can be used for the correct description of the system response. Inconveniences with the former are due to the increase in computational cost with frequency and to the fact that the vibrational behavior becomes strongly sensitive to the variability in the structure properties. Yet, energy methods based on averaged power balance equations like SEA usually require the assumption of some strong hypotheses concerning, for instance, the modal overlap and modal number of the subsystems into which the structure has been divided (see e.g., [1]).

In recent years, several strategies have been followed to tackle the mid-frequency problem. Some approaches have attempted at extending the range of applicability of deterministic methods to higher frequencies. For instance, this is the case of wave based methods (WBM) which adopt exact solutions of the governing partial differential equations as basis functions, instead of the polynomials bases of finite elements (FEM), see e.g., [2] for a review. Another option could be the scaling procedure proposed in the asymptotical scaled modal analysis (ASMA) in [3]. Other methods tried combining deterministic and statistical methods. In [4], a scale separation approach was proposed in which large scales were resolved using a deterministic modal approach whereas the influence of small scales was taken into account using SEA. The coexistence of deterministic and statistical subsystems for a given frequency band, typical of aircraft and submarine hulls, has been addressed by means of hybrid approaches that couple FEM with SEA [5–7]. In [8, 9], a hybrid approach was also proposed using power flow analysis and SEA.

30 SEA has also been derived as limiting cases of wave intensity analysis and travelling wave packets [10–  
31 12], and recently presented as a low resolution ray-tracing technique [13]. Besides, SEA can alternatively be  
32 viewed as a particular case of the more general energy distribution (ED) models (also referred to as energy  
33 flow models), which are likewise based on averaged power flow equations and where energy transmission is  
34 characterized by means of the so called energy influence coefficients [14]. What conditions an ED model  
35 should satisfy to become an SEA model has been a matter of research [15] leading to the definition of quasi  
36 and proper SEA systems [16, 17].

37 Statistical modal energy distribution analysis (SmEdA), originally proposed in [18–20], can be envisaged  
38 in this last group of ED methods, in which SEA hypotheses are relaxed to extend SEA’s range of applicabil-  
39 ity to mid frequencies. However, SmEdA has a clear distinctive feature with respect to most ED methods  
40 in the sense that power balance equations are not established between subsystems but rather between the  
41 resonant modes of different subsystems. These modes can be extracted from the modal bases of uncoupled  
42 subsystems, which can be computed using FEM, thanks to the dual modal formulation (DMF) [19]. This  
43 offers the possibility of considering subsystems with complex geometries and varying properties. Moreover,  
44 circumventing SEA energy equipartition allows one to deal with locally excited subsystems with low modal  
45 overlap [20], as well as to evaluate the spatial distribution of energy density within subsystems [21]. Recently,  
46 SmEdA has been extended to incorporate the contribution of non-resonant transmission through condensa-  
47 tion of the DMF equations. This has resulted in the appearance of indirect coupling between modes in  
48 non-physically connected subsystems, standard non-resonant paths in SEA being recovered as a particular  
49 case [22].

50 Though SmEdA may offer several advantages when compared to SEA, the price to be paid is that  
51 of dealing with large matrix systems (yet much smaller than those encountered in deterministic methods  
52 like FEM, which can involve millions of degrees of freedom). This makes the analysis of the obtained  
53 results difficult. For instance, determining which modes play a predominant role in the energy transmission  
54 between subsystems for even simple cases, such as two cavities separated by a homogeneous wall, may  
55 implicate hundreds of modes at mid-frequencies. A thorough analysis of the interaction of modal works and  
56 involved modal coupling loss factors then becomes necessary to find the dominant modal energy transmission  
57 paths between the excited cavity and the receiver one [22]. It is the goal of this work to try to lighten this  
58 process by resorting to an alternative approach. In particular, it will be shown that graph theory may prove  
59 useful for that purpose. The idea of using graph theory tools to solve vibroacoustics problems was first  
60 introduced in [23], where it was pointed out that it was possible to define a SEA graph, whose weighting  
61 matrix was identified with the generating matrix of the SEA system solution geometric series expansion.  
62 In [24], an adaptation of the MSP algorithm (see [25, 26]) for the efficient computation and ranking of  
63 energy transmission paths from sources to receivers in SEA systems was presented, the inclusion of variance  
64 recently being taken into account in [27]. Graph cut algorithms were also used in [28] in an optimizing  
65 process to diminish energy transmission between subsystems in SEA. The prospect of defining SEA graphs  
66 for the more general ED models has recently been deemed viable in [29]. In this work, it will be shown that  
67 this is also feasible for SmEdA models, so that path graph algorithms can be applied to identify and rank  
68 the relevant modes governing the energy transmission between subsystems.

69 The paper is organized as follows. In section 2 a brief overview on the basics of SmEdA is presented. In  
70 section 3 it is exposed how to define a SmEdA graph, and the notion of modal energy transmission paths  
71 becomes introduced. Two benchmark examples involving resonant and non-resonant energy transmission  
72 between two adjacent cavities separated apart by homogeneous and ribbed plates are presented in section 4.  
73 It is shown how the graph theory approach can provide very valuable information in a quicker and more  
74 efficient way than when attempting a conventional analysis of the SmEdA results. A more complex case  
75 dealing with vibrational and acoustic energy transmission in a shipbuilding built-up structure is addressed  
76 in section 5. Conclusions close the paper in section 6.

## 2. An overview of SmEdA

### 2.1. Resonant transmission in a two-subsystem SmEdA model

An introduction to SmEdA will be next presented to describe its general performance, as well as to highlight some of its features which will allow one to establish a connection with graph theory. For the sake of simplicity, the SmEdA modal energy equations for a system simply made of two coupled subsystems will be first addressed.

As for SEA, the prerequisites of weak coupling and of dominant reverberant field will be assumed, so that the vibratory behavior of the two coupled subsystems can be solely described in terms of the energy transmission between resonant modes (i.e., modes having their modal frequencies within the frequency band of excitation). Numerical tests [19, 30] have shown that this assumption is compliant if the substructuring into subsystems is done well along mechanical impedance mismatch and if the boundary conditions at the coupling surfaces are correctly imposed to determine the subsystem modes. This can be achieved by describing the stiff subsystem by its uncoupled free modes (i.e., assuming free displacements on the coupling surface) while describing the soft one by its uncoupled blocked modes (i.e., imposition of null displacements on the coupling surface).

For illustrative purposes, consider that the two subsystems consist of an acoustic cavity (subsystem 1) coupled to a vibrating structure (subsystem 2). The cavity is therefore the soft subsystem and becomes characterized by means of its blocked modes (i.e., its boundary is assumed to be composed of rigid walls) whereas the vibrating structure is the stiff subsystem and its free modes (i.e. in vacuo modes) should be used instead. According to the dual modal formulation (see [18] for the case of two coupled mechanical subsystems and [22] for a cavity-structure problem), the acoustic pressure field (stress field in a general case) is the appropriate one to describe subsystem 1, whereas the displacement field is adequate for subsystem 2. Denoting by  $\hat{P}$  and  $\hat{Q}$  the sets of resonant modes in subsystems 1 and 2, respectively containing  $N_p$  and  $N_q$  modes, the acoustic pressure at point  $M$  in the cavity and the displacement at point  $N$  on the structure can be estimated using the modal expansions

$$p(M, t) = \sum_{p \in \hat{P}} \xi_p(t) \tilde{p}_p(M), \quad (1a)$$

$$u(N, t) = \sum_{q \in \hat{Q}} \zeta_q(t) \tilde{u}_q(N), \quad (1b)$$

where  $\xi_p$  and  $\zeta_q$  denote modal amplitudes,  $\tilde{p}_p$  stands for the spatial acoustic pressure distribution of the  $p$ -th cavity mode, and  $\tilde{u}_q$  represents the displacement spatial shape of the  $q$ -th structure mode.

Following the DMF approach, expressions (1a)-(1b) are to be introduced in the weak formulation of the coupled problem. Taking advantage of the orthogonality of the uncoupled modes, presuming viscous damping and making the change of variables  $\xi_p = \chi'_p$ ,  $\forall p \in \hat{P}$  (with the prime symbol indicating time derivative), the following modal equations of motion can be derived (see [18] for details)

$$\chi_p''(t) + \omega_p \eta_p \chi_p'(t) + \omega_p^2 \chi_p(t) - \sum_{q \in \hat{Q}} W_{pq} \zeta_q'(t) = Q_p(t), \quad \forall p \in \hat{P}, \quad (2a)$$

$$\zeta_q''(t) + \zeta_q \eta_q \zeta_q'(t) + \omega_q^2 \zeta_q(t) - \sum_{p \in \hat{P}} W_{pq} \chi_p'(t) = 0, \quad \forall q \in \hat{Q}. \quad (2b)$$

In (2a)-(2b),  $\omega_p$ ,  $\omega_q$  denote the modal angular frequencies and  $\eta_p$ ,  $\eta_q$  the modal damping loss factors. The mode shapes are supposed to be normalized to a unit modal mass for the free subsystem and to a unit modal stiffness for the blocked subsystem.  $Q_p$  represents the modal source strength at mode  $p$  due to external excitation and  $W_{pq}$  corresponds to the modal interaction work between  $p$  and  $q$ . For each pair of modes, the latter is defined as the integral over the coupling surface,  $S_C$ , of the product between a displacement mode shape of the free subsystem and a stress mode shape of the blocked subsystem. For the cavity-structure example the modal interaction work would be  $W_{pq} = \int_{S_C} \tilde{p}_p \tilde{u}_q dS$ .

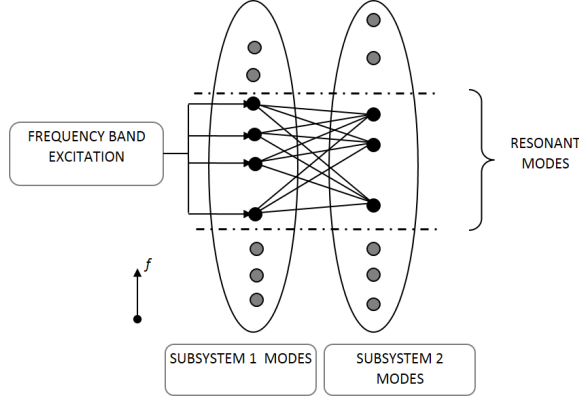


Figure 1: Sketch for the the modal coupling between two coupled subsystems.

101 The form of equations (2a)-(2b) allows one to interpret mode interactions as oscillators coupled by gyro-  
 102 scopic elements (which introduce opposite sign coupling forces proportional to the oscillator velocities [18]).  
 103 A schematic representation of this modal coupling is proposed in Fig. 1. Note that a mode in one subsystem  
 104 is coupled to all modes in the other subsystem but it is not coupled with the modes in its own subsystem.  
 105 The number of back and forth modal energy direct couplings (the former symbolized with black lines in  
 106 Fig. 1) is  $2N_p N_q$ , which strongly increases with the number of modes considered in each subsystem. More-  
 107 over, energy may obviously go back and forth between subsystem modes resulting in transmission paths of  
 108 high order, the total number of them being infinite. In Section 3 it will be shown how to identify the ones  
 109 dominating energy transmission in an efficient way.

110 SmEdA equations describe the power balance between the modes in different subsystems and are obtained  
 111 from the principle of energy conservation for each mode in a subsystem. For the  $p$ -th mode in subsystem 1,  
 112 they result in

$$\Pi_{inj}^p = \Pi_{diss}^p + \sum_{q \in \hat{Q}} \Pi_{pq}, \quad \forall p \in \hat{P}, \quad (3)$$

113 where  $\Pi_{inj}^p$  is the time-averaged injected power by the generalized force  $Q_p$ ,  $\Pi_{diss}^p$  is the time-averaged  
 114 power dissipated by the internal damping of mode  $p$  and  $\Pi_{pq}$  is the time-averaged power flow exchanged  
 115 between the resonant mode  $p$  of subsystem 1 and the resonant mode  $q$  of subsystem 2. The various powers  
 116 appearing in (3) can be evaluated from already established relations for one single oscillator and/or two  
 117 coupled oscillators, using the same assumptions as in SEA (e.g., white noise force spectra and uncorrelated  
 118 modal interaction forces [31]). It follows that

$$\Pi_{inj}^p \approx \frac{\pi}{4} \bar{S}_{Q_p}, \quad \Pi_{diss}^p \approx \omega_p \eta_p E_p, \quad \Pi_{pq} \approx \beta_{pq} (E_p - E_q) \quad (4)$$

119 where  $\bar{S}_{Q_p}$  is the power spectral density of the modal source strength,  $E_p$  is the time averaged energy of  
 120 mode  $p$ , and  $\beta_{pq}$  is called the modal coupling factor given by (see [20]),

$$\beta_{pq} = W_{pq}^2 \left[ \frac{\omega_p \eta_p \omega_q^2 + \omega_q \eta_q \omega_p^2}{(\omega_p^2 - \omega_q^2)^2 + (\omega_p \eta_p + \omega_q \eta_q) (\omega_p \eta_p \omega_q^2 + \omega_q \eta_q \omega_p^2)} \right]. \quad (5)$$

121 Note that  $\beta_{pq} \geq 0$  which will be crucial for the proper definition of modal energy transmission paths and  
 122 SmEdA graphs in the following section.

123 The power balance equation for any resonant mode of subsystem 2 will be analogous to that of subsys-  
 124 tem 1 in (3) (though for simplicity it will be supposed that no external force is acting on subsystem 2). The  
 125  $N_p + N_q$  equations for all modes in subsystems 1 and 2 can be combined in the linear matrix system

$$\begin{pmatrix} \beta_{11} & -\beta_{12} \\ -\beta_{12}^\top & \beta_{22} \end{pmatrix} \begin{pmatrix} \mathbf{E}_1 \\ \mathbf{E}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{\Pi}_1 \\ \mathbf{0} \end{pmatrix}, \quad (6)$$

126 with unknown modal energies  $\mathbf{E}_1 = (E_p)_{N_P \times 1}$  and  $\mathbf{E}_2 = (E_q)_{N_Q \times 1}$ , external input power at subsystem 1  
 127 modes  $\mathbf{\Pi}_1 = (\Pi_{inj}^p)_{N_P \times 1}$ , coupling loss factor matrix  $\beta_{12} = (\beta_{pq})_{N_P \times N_Q}$  and diagonal loss factor matrices  
 128  $\beta_{11} = \text{diag} \left( \omega_p \eta_p + \sum_{q \in \hat{Q}} \beta_{pq} \right)_{N_P \times N_P}$  and  $\beta_{22} = \text{diag} \left( \omega_q \eta_q + \sum_{p \in \hat{P}} \beta_{pq} \right)_{N_Q \times N_Q}$ . Inversion of (6) results  
 129 in the SmEdA system modal energies. From the summation of all modal energies in a subsystem its overall  
 130 energy can be obtained and then related to a spatial mean square velocity in the case of a vibrating plate,  
 131 or to a mean square acoustic pressure in the case of a cavity, following standard SEA formulations [31–33].

## 132 2.2. Resonant and non-resonant transmission in a three-subsystem SmEdA model

133 A slightly more involved case is next considered which consists of a system made of three subsystems.  
 134 This will allow one to inspect how non-resonant transmission can be accounted for in SmEdA, a topic that  
 135 has only been addressed very recently [22]. For the ease of exposition and without loss of generality, suppose  
 136 that the system is made of two cavities separated by a panel. Subsystems 1 and 3 are identified with the  
 137 cavities and subsystem 2 with the panel (this system will be re-encountered in the benchmark examples of  
 138 Section 4). The sets of resonant modes for the cavities in the frequency range of interest will be denoted by  
 139  $\hat{P}$  and  $\hat{R}, \hat{Q}$  standing for the set of panel resonant modes. The SmEdA matrix formulation analogous to (6)  
 140 if external input power is supplied to the first cavity, will be given by

$$\begin{pmatrix} \beta_{11} & -\beta_{12} & \mathbf{0} \\ -\beta_{12}^\top & \beta'_{22} & -\beta_{23} \\ \mathbf{0} & -\beta_{23}^\top & \beta_{33} \end{pmatrix} \begin{pmatrix} \mathbf{E}_1 \\ \mathbf{E}_2 \\ \mathbf{E}_3 \end{pmatrix} = \begin{pmatrix} \mathbf{\Pi}_1 \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix}, \quad (7)$$

141 with  $\mathbf{E}_1, \mathbf{E}_2$  and  $\mathbf{E}_3$  respectively standing for the modal energy vectors of the first cavity, panel and second  
 142 cavity.

143 Two remarks should be made with regard to (7). First, the diagonal matrix  $\beta'_{22}$  is different from  $\beta_{22}$   
 144 in (6) because it incorporates the coupling with subsystem 3. Its expression becomes

$$\beta'_{22} = \text{diag} \left( \omega_q \eta_q + \sum_{p \in \hat{P}} \beta_{pq} + \sum_{r \in \hat{R}} \beta_{qr} \right)_{N_Q \times N_Q}. \quad (8)$$

145 Second, the null blocks  $\mathbf{0}$  in (7) indicate that there is no direct coupling between the resonant modes  
 146 of both cavities, given that they are not physically connected. However, it is well known that resonant  
 147 transmission, as described by the standard SmEdA approach (7), cannot correctly represent the whole  
 148 acoustic transmission through the panel below the critical frequency, which is governed by the mass law. To  
 149 tackle with this problem it was proposed in [22] to include the panel non-resonant modes in the analysis.  
 150 Although the frequencies of these non-resonant panel modes do not coincide with those of the cavity resonant  
 151 modes, the modes are strongly coupled one to another because of spatial matching. If one incorporates non-  
 152 resonant transmission in the DMF equations, it turns out that after matrix condensation the former can  
 153 be accounted for by establishing a direct coupling between the modes of both cavities (even though not  
 154 being physically connected). The coupling factors  $\beta_{pr}$  between modes in  $\hat{P}$  and  $\hat{R}$  are characterized by  
 155 spring connections rather than gyroscopic ones. The stiffness of the spring connection can be related to  
 156 the intermodal works between the resonant cavity modes and the non-resonant panel modes (see [22] for  
 157 details).

158 As a consequence, when non-resonant paths are considered (7) transforms to

$$\begin{pmatrix} \beta''_{11} & -\beta_{12} & -\beta_{13} \\ -\beta_{12}^\top & \beta'_{22} & -\beta_{23} \\ -\beta_{13}^\top & -\beta_{23}^\top & \beta''_{33} \end{pmatrix} \begin{pmatrix} \mathbf{E}_1 \\ \mathbf{E}_2 \\ \mathbf{E}_3 \end{pmatrix} = \begin{pmatrix} \mathbf{\Pi}_1 \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix}, \quad (9)$$

159 where the block  $\beta_{13} = (\beta_{pr})_{N_P \times N_R}$  is no longer zero. Note that the matrices  $\beta''_{11}$  and  $\beta''_{33}$  differ from  
 160  $\beta_{11}$  and  $\beta_{33}$  in (9) because they include the terms corresponding to the direct connections between cavities,

161 characterized by  $\beta_{13}$ . Numerical validations on test cases [22] had shown the ability of this SmEdA approach  
 162 to describe the non-resonant transmission through the panel, and in particular, the mass law behaviour.  
 163 However, determining which cavity and panel modes play a relevant role in the transmission through spatial  
 164 matching may be rather lengthy and intricate. A fast way to do so by resorting to graph theory will be  
 165 presented for the cavity-panel-cavity system in Section 4.

### 166 2.3. General matrix formulation for an $N$ -subsystem SmEdA model

167 Generalization of the above SmEdA matrix system formulation for a model consisting of  $N$  subsystems  
 168 that includes both, resonant and non-resonant transmission, is straightforward and yields

$$\begin{pmatrix} \beta_{11} & -\beta_{12} & -\beta_{13} & \cdots & -\beta_{1N} \\ -\beta_{12}^\top & \beta_{22} & -\beta_{23} & \cdots & -\beta_{2N} \\ -\beta_{13}^\top & -\beta_{23}^\top & \beta_{33} & \cdots & -\beta_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\beta_{1N}^\top & -\beta_{2N}^\top & -\beta_{3N}^\top & \cdots & \beta_{NN} \end{pmatrix} \begin{pmatrix} \mathbf{E}_1 \\ \mathbf{E}_2 \\ \mathbf{E}_3 \\ \vdots \\ \mathbf{E}_N \end{pmatrix} = \begin{pmatrix} \mathbf{\Pi}_1 \\ \mathbf{\Pi}_2 \\ \mathbf{\Pi}_3 \\ \vdots \\ \mathbf{\Pi}_N \end{pmatrix}, \quad (10)$$

169 with  $\mathbf{E}_i = (E_i)_{I \times 1}$  standing for the vector of modal energies of the  $i$ -th subsystem where an external power  
 170  $\mathbf{\Pi}_i = (\Pi_{inj}^i)_{I \times 1}$  is being input. In order to lighten forthcoming expressions, system (10) will be simply  
 171 rewritten as

$$\beta \mathbf{E} = \mathbf{\Pi}. \quad (11)$$

## 172 3. Modal energy transmission paths and SmEdA graphs

### 173 3.1. Modal energy transmission paths in SmEdA

174 The definition of energy transmission paths between subsystems in SEA models was introduced by Craik  
 175 in [32, 34]. As quoted in [35], the notion of transmission paths between two adjacent subsystems relies on  
 176 the concept of blocked transmissibility [36–38]; the energy transmitted from an arbitrary subsystem  $i$  to a  
 177 neighboring subsystem  $j$  is given by the quotient of energies  $E_j/E_i$  when  $i$  is excited and all energies in the  
 178 SEA model but  $i$  and  $j$  are set to zero. This results in  $E_j/E_i = \eta_{ij}/\eta_j$ ,  $\eta_{ij}$  standing for the SEA coupling loss  
 179 factor between  $i$  and  $j$ , and  $\eta_j$  for the total loss factor of subsystem  $j$ . Therefore  $\eta_{ij}/\eta_j$  can be interpreted  
 180 as the fraction of energy at  $j$  that directly comes from  $i$ , and thus identified with the weight of a first order  
 181 transmission path connecting subsystems  $i$  and  $j$  i.e.,  $w(p_{ij}^1) = \eta_{ij}/\eta_j$ . A second order path  $p_{st}^2$  linking e.g.,  
 182 subsystem  $s$  with  $t$  through a third subsystem  $j$  could be built concatenating the first order paths  $p_{sj}^1$  and  
 183  $p_{jt}^1$ , with weight  $w(p_{st}^2) = (\eta_{sj}/\eta_j)(\eta_{jt}/\eta_t)$ . Following this procedure arbitrary  $n$ -order paths linking  $n + 1$   
 184 subsystems can be built having weights  $w(p_{st}^n) = \prod_{h_i=1}^{n-1} (\eta_{h_i h_{i+1}}/\eta_{h_i})$  ( $s$  being identified with  $h_1$  and  $t$  with  
 185  $h_{n-1}$ ), see [23, 24] for details.

186 The same line of reasoning can be pursued to define energy transmission in SmEdA, from an arbitrary  
 187 mode  $p$  in a subsystem  $P$  to a mode  $q$  in subsystem  $Q$ . The weight of the first order order transmission  
 188 path,  $p_{pq}^1$ , from mode  $p \in \hat{P}$  to  $q \in \hat{Q}$ , in the case of subsystems  $P$  and  $Q$  being adjacent, will be given by

$$w(p_{pq}^1) = \frac{\beta_{pq}}{\beta_q}. \quad (12)$$

189 A general  $n$ -th order path between a mode  $s \in \hat{S}$  and a mode  $t \in \hat{T}$  in the SmEdA system can then be built  
 190 by concatenation as

$$w(p_{st}^n) = \frac{\beta_{sh_1}}{\beta_{h_1}} \frac{\beta_{h_1 h_2}}{\beta_{h_2}} \cdots \frac{\beta_{h_{n-1} t}}{\beta_t}. \quad (13)$$

191 Note from the considerations in Section 2.1 that (13) makes sense given that  $\beta_{h_i h_{i+1}} \geq 0$  and  $\beta_{h_i} > 0$ ,  
 192  $\forall h_i \in \hat{P}$ ,  $h_{i+1} \in \hat{Q}$ . Note also that, as specified, no transmission is allowed from a mode in one subsystem

193 to another mode in the same subsystem. However, a path going from one mode, say  $p_1 \in \hat{P}$  to  $q \in \hat{Q}$  and  
 194 then back to  $p_2 \in \hat{P}$ , or even  $p_1 \in \hat{P}$ , is perfectly feasible.

195 A transmission path, as defined in (13), allows one to know the energy that has been transmitted from  
 196 a particular source mode, where the external energy is input, to a target mode in a different subsystem,  
 197 involving a particular set of intermediate modes. As it will be shown in Section 4, the fast and efficient  
 198 computation of such paths will prove very useful to determine which modes play a significant role in reso-  
 199 nant and non-resonant transmission in SmEdA models. However, it should be remarked that in order for  
 200 transmission path analysis to be a well-posed problem from a mathematical and physical point of view, an  
 201 additional condition has to be satisfied. It has to be possible to recover the overall energy at any target mode  
 202 as the summation of the energy contributions of all transmission paths linking the source mode with the  
 203 target one (see [24] for a detailed explanation in the case of SEA). Assuming fulfillment of that condition,  
 204 a transmission path analysis makes sense from a practical point of view whenever the energy transmission  
 205 is justified by a small set of dominant paths, so that one could act on their constituent modes/subsystems  
 206 for remedial action.

207 Let us next see that transmission path analysis in SmEdA is in fact a well-posed problem. Define the  
 208 matrix  $\mathbf{B} := \mathbf{I} - \text{diag}(1/\beta_{ii})\boldsymbol{\beta}$  whose diagonal is null and its off-diagonal entries non-negative. This matrix  
 209 is also irreducible, i.e. it cannot be made similar to a block upper triangular matrix via a permutation. This  
 210 amounts to saying that the adjacency matrix associated to  $\mathbf{B}$  is that of a strongly connected directed graph  
 211 (this is actually the case, see next section). Next one can make use of the Perron-Frobenius theorem (see  
 212 e.g., [39]) and easily show that the spectral radius of  $\mathbf{B}$ ,  $\rho(\mathbf{B}) < 1$ . As a consequence  $\mathbf{B}$  is convergent, i.e.,  
 213 the series of matrices  $\mathbf{B}^m$ ,  $m = 0, 1, 2 \dots$ , converge to the null matrix. Defining  $\boldsymbol{\Pi}' = \text{diag}(1/\beta_{ii}) \boldsymbol{\Pi}$  permits  
 214 rewriting (11) as

$$(1) \quad (\mathbf{I} - \mathbf{B})\mathbf{E} = \boldsymbol{\Pi}' \quad (14)$$

215 so that

$$\mathbf{E} = (\mathbf{I} - \mathbf{B})^{-1} \boldsymbol{\Pi}' = \left( \sum_{n=0}^{\infty} \mathbf{B}^n \right) \boldsymbol{\Pi}'. \quad (15)$$

216 This proves that the energy at any mode can effectively be recovered as the infinite summation of the energy  
 217 contributions of the transmission paths linking the SmEdA system source modes with the target ones. The  
 218 entries in  $\mathbf{B}^T$  correspond to the energy contribution of the first order paths between pairs of modes; note  
 219 that  $B_{pq} = \beta_{pq}/\beta_p$ ,  $p \neq q$ , which according to (12) corresponds to a path from  $q$  to  $p$  (v.  $\beta_{pq} = \beta_{qp}$ ). Thus  
 220  $B_{qp}$  will be the entry for a path from  $p$  to  $q$ . In the same way, the transposed of  $\mathbf{B}^n$  includes the total  
 221 contribution of the  $n$ -th order paths linking modes in the SmEdA system. As the paths get longer their  
 222 contribution decreases because as said  $n \rightarrow \infty \Rightarrow \mathbf{B}^n \rightarrow \mathbf{0}$ . A series like (15) was first proposed for SEA  
 223 systems in [40], see also [13, 23, 32, 40]. As will be shown next, (15) constitutes the key to link SmEdA with  
 224 graph theory.

### 225 3.2. The SmEdA graph

226 Succinctly, a graph  $G = (U, E)$  consists of two sets of elements,  $U$  being the set of nodes and  $E$  the set of  
 227 arcs, or edges, connecting the nodes. The arcs in  $E$  can be assigned numerical values which can be gathered  
 228 in the so called weighting matrix of the graph [41]. A SmEdA graph can be built by defining all subsystem  
 229 modes as the nodes of the graph and identifying the arcs with the first order paths connecting them. The  
 230 arcs are then assigned the weights of these first order paths. Note that this would come down to identifying  
 231 the transpose of the generating matrix in (13),  $\mathbf{B}^T$ , as the weighting matrix of a strongly connected directed  
 232 SmEdA graph, analogously to what is done in SEA [23, 24]. The graph is strongly connected because for  
 233 any two arbitrary nodes in the graph there always exists a path connecting them.

234 For the correct definition of a SmEdA graph an additional subtle point has to be considered. In practice  
 235 it is not possible to excite a particular mode of a given subsystem; standard mechanisms excite the whole  
 236 subsystem at once. Moreover, one is usually interested in knowing the influence of a given transmission  
 237 path to the whole target subsystem energy, not to one of its modes. To take into account these two facts,  
 238 the SmEdA graph has to be enlarged with two fictitious nodes (not corresponding to system modes), one



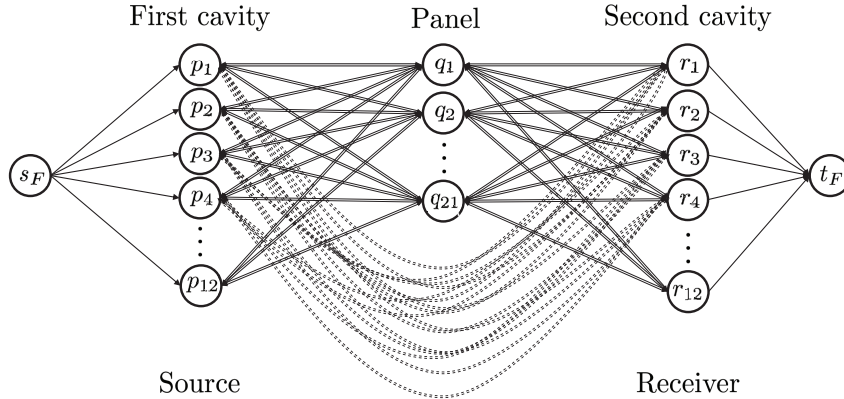


Figure 2: SmEdA graph corresponding to the cavity-panel-cavity model in Fig. 3

239 connected to the source subsystem modes, and the other one to the target subsystem modes. With regard  
 240 to the former, one of the advantages of SmEdA is precisely that it can deal with both, localized and rain  
 241 on the roof excitations. Let us denote the source subsystem as  $P$  with modes  $p \in \hat{P}$ . A fictitious source  
 242 node  $s_F$  will be included in the graph with one-direction connections to every  $p \in \hat{P}$ . The weights of these  
 243 connections will depend on the excitation being point-like or distributed. The power balance equation for a  
 244 mode in the source subsystem  $P$ , when setting all remaining SmEdA system modal energies to zero, provides

$$\Pi_{inj}^p = \beta_p E_p. \quad (16)$$

245 In the case of a rain on the roof excitation  $\Pi_{inj}^p$  will be constant for all  $p \in \hat{P}$ . As opposes to this, in the  
 246 case e.g. of a monopole source term,  $\Pi_{inj}^p \approx (\pi/4)\bar{S}_{Q_p}$ , with  $\bar{S}_{Q_p}$  standing for the power spectral density of  
 247 the monopole, as stated in (4). The following weights will be thus assigned to the edges linking  $s_F$  to every  
 248  $p \in \hat{P}$ ,

$$w_{s_F p} = \frac{\Pi_{inj}^p}{\beta_p}. \quad (17)$$

249 With regard to the target or receiver subsystem, say  $R$ , another fictitious node  $t_F$  is included in the  
 250 graph. All modes  $r \in \hat{R}$  become connected to it through arcs with unitary weights  $w_{rt_F} = 1$ . The energy at  
 251 the target subsystem can be recovered from the summation of the energy of all its modes,  $E_T = \sum_{r \in \hat{R}} E_r$ .

252 To summarize, a SmEdA graph will consist of  $(N + 2)$  nodes, where  $N$  is the total number of the SmEdA  
 253 system modes. The two additional nodes are a fictitious source node  $s_F$ , which is connected to all the modes  
 254 in the source subsystem with the weight in (17), and a fictitious target node to which all target subsystem  
 255 modes become connected with unit weight. As an example of SmEdA graph, in Fig. 2 that corresponding to  
 256 the cavity-panel-cavity example in Section 2.2 is presented, which will also be referred to in the forthcoming  
 257 Section 4 (see also Fig. 3). The arcs plotted in solid lines correspond to resonant first order paths while the  
 258 dashed line arcs correspond to non-resonant first order paths. The source is placed in the first cavity and  
 259 the second cavity corresponds to the target subsystem. Therefore, the node  $s_F$  is connected to all nodes in  
 260 the first cavity, whereas all nodes in the second cavity become connected to  $t_F$ .

261 Once a SmEdA graph has been generated, use can be made of previously developed algorithms in  
 262 graph theory for the computation of transmission paths linking the source and the target subsystems. In  
 263 particular, it suffices to apply the adaptation of the MPS algorithm [25, 26] to SEA that was done in [24],  
 264 to compute a list of dominant transmission paths in SmEdA systems. With a brief post-process, it will be  
 265 also possible to automatically identify the modes more often appearing in the dominant paths. This will  
 266 considerably facilitate the cumbersome task of detecting those modes regulating the energy transmission  
 267 between subsystems.

268 The MPS algorithm in [25, 26] was designed to solve the so called K-shortest loopless path problem in a  
 269 graph. To do so, it resorts to the Bellman's optimality principle which states that there is a shortest path

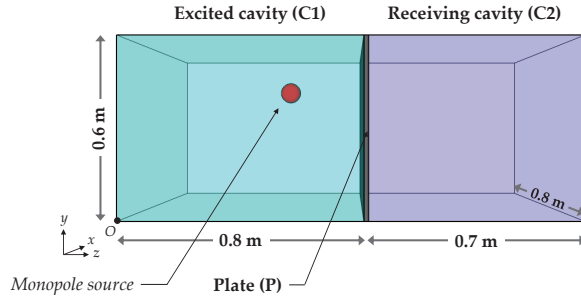


Figure 3: Cavity-panel-cavity system.

270 in a graph formed by shortest sub-paths. Once the shortest path between the source and target nodes has  
 271 been computed, one can obtain the second-shortest path in the graph, the third-shortest path, the fourth  
 272 and so on, from the analysis of their corresponding deviation paths [42]. The MPS algorithm was modified  
 273 in [24] to allow one to compute maximum energy transmission paths instead of minimum ones, as well as to  
 274 deal with the presence of loops.

#### 275 4. Application to cavity-panel-cavity models

276 In this section, the above developments will be applied to cavity-panel-cavity models. Most conclusions  
 277 in [22] with regard to modal energy transmission between cavities will be validated, but now following a  
 278 totally different and faster approach. The models under analysis consist of two cavities separated apart  
 279 by a panel, as shown in Fig. 3. It will be assumed that energy transmission only takes place through the  
 280 dividing panel, the remaining cavity walls being totally rigid. In a first example a homogeneous panel will  
 281 be considered. This comprises of a steel plate with dimensions  $0.8 \text{ m} \times 0.6 \text{ m} \times 0.001 \text{ m}$ , mass density  
 282  $\rho = 7800 \text{ kg/m}^3$ , Young modulus  $E = 2 \times 10^{11} \text{ Pa}$  and damping loss factor  $\eta = 0.01$ . The panel is supposed  
 283 to be simply-supported on its four edges. In the second example, the steel plate will be stiffened with the  
 284 addition of some ribs, as detailed in Section 4.2.

285 The dimensions of the source cavity C1, are  $0.8 \text{ m} \times 0.6 \text{ m} \times 0.8 \text{ m}$ , and those of the receiver cavity  
 286 C2,  $0.8 \text{ m} \times 0.6 \text{ m} \times 0.7 \text{ m}$ . The cavities are filled with air (mass density  $\rho_0 = 1.29 \text{ kg/m}^3$ , speed of sound  
 287  $c_0 = 340 \text{ m/s}$  and damping loss factors  $\eta_{C1} = \eta_{C2} = 0.01$ ). C1 will be excited with a monopole source of unit  
 288 strength located at the point  $(0.24, 0.42, 0.54) \text{ m}$ , according to the coordinate system  $[O; x, y, z]$  in Fig. 3.  
 289 In forthcoming explanations, the particular  $i$ -th mode belonging to the source cavity C1 will be referred to  
 290 as  $p_i \in \hat{P}$ , the  $i$ -th mode belonging to the plate as  $q_i \in \hat{Q}$  and the  $i$ -th mode in the receiver cavity C2 as  
 291  $r_i \in \hat{R}$ . For both examples, the most dominant modal energy transmission paths have been computed for  
 292 the one third octave bands ranging from 400 Hz to 4000 Hz central frequencies.

##### 293 4.1. Cavity – bare plate – cavity

294 In this case, the subsystem modal information required in SmEdA can be calculated analytically for  
 295 both, the natural frequencies and the interaction modal works (i.e. integral of the product between the  
 296 pressure cavity modes and the displacement panel modes). In Table 1, a first overview on how modal energy  
 297 transmission takes place for every analyzed 1/3 octave band is presented (first column in the table). The  
 298 results are built from the outputs of the the MPS algorithm as implemented in [24]. The second column in  
 299 the table contains the overall energy at the receiver cavity C2. The third one indicates the total number  
 300 of paths that have been computed for each band and the fourth and fifth columns show the contributions  
 301 of the computed paths, respectively in percentage and decibels, to the energy at C2. Next, the type of  
 302 paths that participate in the energy transmission from C1 to C2 are indicated (tri-block sixth column). The  
 303 acronym PNR stands for purely non-resonant paths, herein identified with paths that only involve the bare  
 304 plate non-resonant modes, whereas PR stands for purely resonant paths, i.e. paths only involving the plate

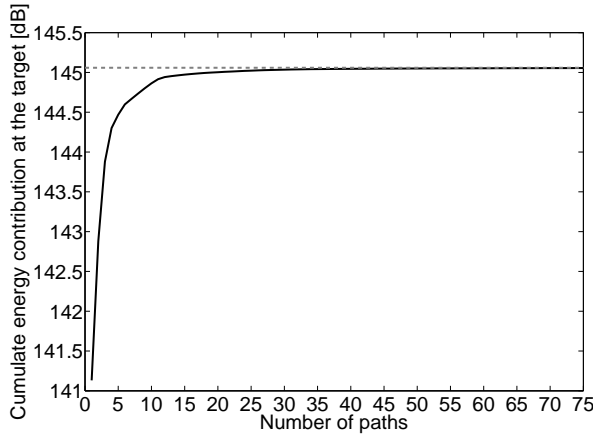


Figure 4: Cumulative path energy contribution at the receiver cavity C2 versus number of considered dominant paths, for the cavity - bare plate - cavity example at 630 Hz. Continuous black line: cumulative energy contribution, dashed grey line: total energy at the receiver C2.

305 resonant modes. M denotes mixed transmission paths that contain both, resonant and non-resonant plate  
 306 modes. The contributions in percentage and decibels of the various types of paths is exposed in the tri-block  
 307 columns seven and eight.

308 As observed in the table, though the number of resonant paths is large and for several bands surpass  
 309 that of non-resonant paths, their contribution to the energy level at C2 becomes negligible when compared  
 310 to the non-resonant one, as one would expect for the behavior of a homogeneous plate well below the  
 311 critical frequency  $f_c$  ( $f_c \sim 11$  kHz in the present example). Let us next focus, for the ease of exposition,  
 312 on the results of the 630 Hz one-third octave band. There are 12 resonant modes for the source cavity  
 313 C1 and for the receiver cavity C2 in this band. In what concerns the plate, it has 21 resonant modes and  
 314 75 non-resonant modes in the band. As explained, the latter do not result in extra nodes in the SmEdA  
 315 graph but in additional edges directly connecting the modes of the two cavities. The SmEdA graph for the  
 316 problem at hand will thus contain 47 nodes, 45 arising from all resonant modes in the system, plus two  
 317 nodes corresponding to the fictitious source and target nodes (see Fig. 2).

318 In Fig. 4, the cumulative contribution of the first 75 paths has been plotted. The dashed line represents  
 319 the total energy at the receiver cavity which has a value of 145.06 dB ref.  $10^{-12}$  J, whereas the black solid  
 320 line corresponds to the cumulative energy contribution when increasing the number of considered paths. It

Band [Hz]	Cavity C2 Energy	Number of Paths	Accum. Contr. [%]	Accum. Contr. [dB]	Type of paths [%]			Accum. Contr. [%]			Accum. Contr. [dB]		
					PNR	PR	M	PNR	PR	M	PNR	PR	M
400	148.06	100	100.00	148.06	16.00	33.00	51.00	93.96	5.31	0.73	147.79	135.32	126.69
500	136.57	100	100.00	136.57	10.00	50.00	40.00	80.16	18.89	0.96	135.61	129.33	116.37
630	145.06	100	99.95	145.06	29.00	69.00	2.00	95.91	4.02	0.02	144.88	131.11	107.39
800	143.72	100	99.38	143.69	41.00	57.00	2.00	84.57	14.76	0.05	142.99	135.41	110.86
1000	142.08	200	98.69	142.02	39.50	60.50	0.00	87.57	11.12	0.00	141.50	132.54	$-\infty$
1250	141.90	500	98.57	141.84	39.00	60.20	0.80	86.15	12.40	0.02	141.25	132.83	105.14
1600	140.87	1000	97.79	140.77	39.90	60.10	0.00	91.41	6.38	0.00	140.48	128.92	$-\infty$
2000	140.87	1000	95.83	140.68	65.10	34.90	0.00	92.56	3.26	0.00	140.53	126.00	$-\infty$
2500	140.33	2000	93.72	140.04	69.75	30.25	0.00	90.02	3.69	0.00	139.87	126.00	$-\infty$
3150	140.21	5000	93.10	139.90	67.10	32.90	0.00	89.97	3.13	0.00	139.75	125.16	$-\infty$
4000	139.29	10000	91.39	138.90	71.58	28.42	0.00	88.76	2.64	0.00	138.77	123.50	$-\infty$

Table 1: Cavity - bare plate - cavity example. Contributions of resonant, non-resonant and mixed transmission paths for the considered 1/3 octave frequency bands.

321 can readily be checked, for instance, that the first 5 paths had an overall contribution of 144.47 dB whereas  
 322 the first 75 paths supply 145.05 dB of energy (99.9% of the overall energy at the target). At this point, it  
 323 should be remarked that for practical applications in vibroacoustic problems differences of less than 1 dB  
 324 become almost negligible (this constitutes in fact an eligible criteria to make a decision on the number of  
 325 paths to be computed for each graph). Therefore, it can be observed that with the sole contribution of the  
 326 five most dominant transmission paths it becomes possible to justify the whole transmission of energy from  
 327 C1 to C2.

328 More insight can be gained by having a look at the results in Table 2. Its first column contains the  
 329 number of transmission paths taken into account in the analysis, their cumulative energy contribution to  
 330 the target subsystems in dB and percentage respectively being presented in columns two and three. The  
 331 next three columns include the number of resonant modes of each subsystem (C1, panel and C2) that are  
 332 involved in the corresponding paths of the first column, followed by the total summation of resonant modes  
 333 in column number seven. Finally, the last column of the table indicates the difference between the total  
 334 number of modes in the system and the number of modes appearing in the paths of the first column.

335 With regard to the row information in the table, the first row is a reference containing all data involved  
 336 in the exact transmission process from source to target. The second row shows the results when considering  
 337 75 dominant paths, which justify, as said, 99.9% of the energy at the receiver cavity. 38 modes from the  
 338 45 modes of the complete problem still play a role in the energy transmission process. However, by simply  
 339 considering 25 paths, the accumulated contribution is 99.16%, which leads to a negligible difference in  
 340 terms of decibels compared to the reference (less than 0.04 dB) and only 27 modes become involved in the  
 341 transmission. Focusing on the 8 most dominant paths, it can be observed that the difference in dB is still  
 342 insignificant (less than 1 dB) and that only 14 modes are of importance. There is also a very important  
 343 point to note in this case. As opposed to the preceding ones, it can be appreciated that no plate resonant  
 344 mode intervenes. In other words, the energy transmission between the two cavities at the 630 one-third  
 345 octave band is mainly non-resonant, as already noticed from the analysis of the results in Table 1. The  
 346 proposed graph approach not only allows one to corroborate this well-known point, but to determine which  
 347 are the most important non-resonant modes dominating the transmission. In Table 3, the ranking of the 10  
 348 stronger dominant paths has been listed. Non-resonant paths can be easily identified as they do not contain  
 349 any  $q_i$  element. As seen in the table, the first eight transmission paths are completely non-resonant, being  
 350 the ninth path the first one to include a  $q_i$  mode (in particular  $q_{21}$ ).

351 It should be remarked that obtaining the type of information in Tables 1, 2 and 3 is a straightforward  
 352 task from the output of the MPS algorithm. As opposed to this, trying to get this kind of information  
 353 directly from the analysis of the whole SmEdA system is a rather intricate and lengthy task that involves  
 354 several simplifying hypotheses [22]. This can be realized when trying to determine the first two or three  
 355 dominant paths in the SmEdA model following the procedure in [22]. To do so, the SmEdA model has  
 356 to be first simplified assuming weak coupling and taking advantage of the fact that the three subsystems  
 357 are connected in series. Then, it may be expected that the modal energies of the excited subsystem C1  
 358 will be much higher than those of the homogeneous plate, which in turn will be higher than those in the  
 359 receiving cavity C2. The simplified SmEdA model relates the modal energies of C2 with the modal injected  
 360 powers through non-resonant and resonant paths, by means of equations (51) and (52) in [22], respectively.  
 361 Inspecting the values of the analytical factors relating these quantities, it becomes possible to determine

Number of paths	Accumulated contribution [dB]	Accumulated contribution [%]	$\hat{P}$	$\hat{Q}^R$	$\hat{R}$	Total involved modes	Missing modes
$\infty$	145.06	100.00	12	21	12	45	0
75	145.05	99.91	12	14	12	38	7
25	145.02	99.16	12	5	10	27	18
10	144.86	95.59	7	1	7	15	30
8	144.73	92.82	7	0	7	14	31

Table 2: Cavity - bare plate - cavity example. Transmission path analysis and involved modes for the 630 Hz 1/3 octave band.

Path	Cavity – Homogenous plate – cavity
1	$S \rightarrow p_6 \rightarrow r_7 \rightarrow T$
2	$S \rightarrow p_{10} \rightarrow r_{10} \rightarrow T$
3	$S \rightarrow p_{12} \rightarrow r_{11} \rightarrow T$
4	$S \rightarrow p_1 \rightarrow r_3 \rightarrow T$
5	$S \rightarrow p_4 \rightarrow r_5 \rightarrow T$
6	$S \rightarrow p_3 \rightarrow r_6 \rightarrow T$
7	$S \rightarrow p_6 \rightarrow r_{10} \rightarrow T$
8	$S \rightarrow p_7 \rightarrow r_9 \rightarrow T$
9	$S \rightarrow p_{12} \rightarrow q_{21} \rightarrow r_{11} \rightarrow T$
10	$S \rightarrow p_{10} \rightarrow r_7 \rightarrow T$

Table 3: Cavity - bare plate - cavity example. Ranking of the 10 most dominant modal energy paths from a total of 75 computed paths for the 630 Hz 1/3 octave band

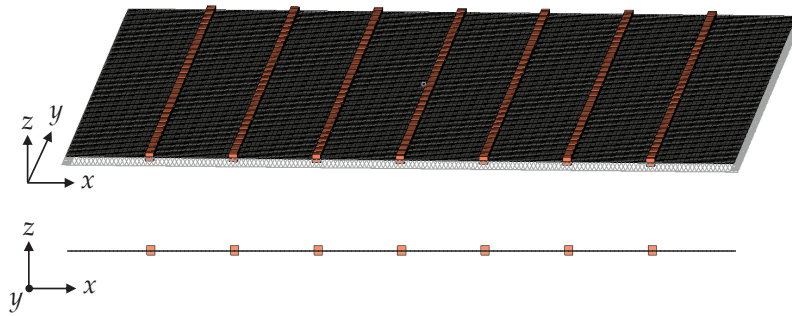


Figure 5: Ribbed plate finite element model

362 the two or three most dominant paths, which coincide with those in the ranking of Table 3. Therefore, one  
363 can resort to the procedure in [22] to validate the first paths resulting from the application of the MPS  
364 algorithm, but it should be emphasized that by no means is it possible to apply that procedure to generate  
365 an automatic list of dominant paths. Moreover, the approach in [22] only works under some restrictive  
366 assumptions which may not be valid in the case of complex SmEdA models. As seen, such difficulties can  
367 be overcome by resorting to the MPS algorithm.

#### 368 4.2. Cavity – ribbed plate – cavity

369 In this second example the homogeneous bare plate becomes stiffened with some ribs. The ribs are  
370 regularly placed 100 mm apart parallel to the plate  $y$ -axis and have a cross section of 10 mm  $\times$  10 mm, see  
371 Fig. 5. It is well-known that the acoustic transmission of a ribbed plate differs from that of a bare plate  
372 because its stiffness becomes increased in the rib direction. As a first approximation for low frequencies,  
373 the ribbed plate may be considered equivalent to an orthotropic plate. Whereas the bare plate has a single  
374 critical frequency ( $\sim 11$  kHz in the previous example), an orthotropic plate is characterized by having two  
375 critical frequencies. The lowest one is that corresponding to the coincidence of waves traveling in the plate  
376 stiffest direction (rib direction) with acoustic waves. In our case, this first critical frequency occurs at  
377  $\sim 1.7$  kHz (the second being that of the bare plate at  $\sim 11$  kHz). Therefore and contrary to the bare plate,  
378 for which resonant plate modes are only in spatial coincidence with the acoustic modes beyond 11 kHz, the  
379 stiffened plate may have some resonant modes in spatial coincidence with the acoustic ones for frequencies  
380 well below 11 kHz. One would then expect the resonant paths to play a more important role in the acoustic  
381 transmission than the one they played for the bare plate. It is however difficult to make an a priori estimation  
382 of the importance of the resonant paths compared to the non-resonant ones. As it will be shown below the  
383 graph theory approach can substantially ease this task.

384 In order to build the SmEdA matrix for this case, the subsystem modes of the ribbed plate have been  
385 calculated with FEM. The plate has been modelled with 19200 quadrilateral shell elements while 900 one-  
386 dimensional beam elements have been used for the ribs (see Fig. 5). Uncoupled extensional (in-plane)

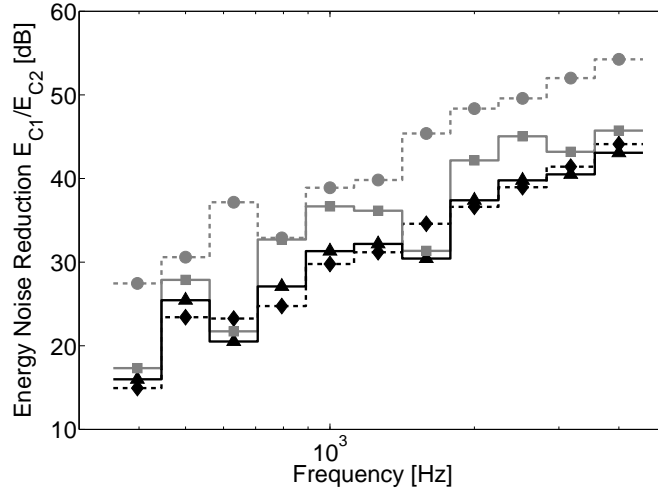


Figure 6: Energy ratio between the two cavities. -●- (grey) bare plate with only resonant transmission, -◆- (black) bare plate with resonant and non-resonant transmission, —■— (grey) ribbed plate with only resonant transmission —▲— (black) ribbed plate with resonant and non-resonant transmission.

387 and bending (out-plane) motions have been considered for the thin plate whereas bending, extensional and  
388 torsional motions have been taken into account for the beam-like stiffeners. Based on the classical FEM  
389 sampling criterion of using six elements per wavelength in the generated mesh, the results are expected to  
390 be valid up to 10 kHz. The normal modes have been computed using the SDTools code [43]. Besides, the  
391 cavity modes have been calculated analytically as in the cavity-bare plate-cavity example of the preceding  
392 subsection.

393 As expected, the energy transmission no longer presents two distinct ranges of behavior like for the bare  
394 plate. This can be first observed in Table 4, built again from the outputs of the MPS algorithm, which  
395 is the analogous to Table 1 but for the ribbed case. As seen, it is no longer true that the contribution to  
396 the energy level at C2 becomes dominated by non-resonant paths, the resonant ones playing a determinant  
397 role for several 1/3 octave bands. More graphically, this can be appreciated when analyzing the energy  
398 noise reduction (ENR) between cavities [22]. The ENR is defined as  $ENR = 10 \log_{10}(E_{C1}/E_{C2})$ , with  $E_{C1}$   
399 and  $E_{C2}$  respectively standing for the time averaged total energies at cavities C1 and C2 respectively. In  
400 Fig. 6 the ENR has been plotted in one-third octave bands for both, the bare plate of the previous example

Band [Hz]	Target Energy	Number of Paths	Accum. Contr. [%]	Accum. Contr. [dB]	Type of paths [%]			Accum. Contr. [%]			Accum. Contr. [dB]		
					PNR	PR	M	PNR	PR	M	PNR	PR	M
400	146.84	100	99.84	146.83	7.00	68.00	25.00	25.83	73.28	0.73	140.96	145.49	125.45
500	133.89	100	99.93	133.89	6.00	75.00	19.00	42.60	56.89	0.44	130.19	131.44	110.37
630	147.46	200	99.65	147.44	11.50	83.50	5.00	24.36	75.24	0.05	141.33	146.22	114.63
800	141.37	200	99.49	141.36	23.00	75.50	1.50	72.43	27.04	0.03	139.97	135.69	105.93
1000	140.53	300	99.48	140.51	27.00	72.00	1.00	70.77	28.69	0.02	139.03	135.11	104.14
1250	140.88	1000	98.79	140.83	17.70	82.10	0.20	59.76	39.03	0.01	138.64	136.79	98.38
1600	144.98	2000	98.55	144.91	16.00	83.55	0.45	18.92	79.62	0.02	137.74	143.99	107.25
2000	140.07	2000	94.89	139.84	33.35	66.65	0.00	66.07	28.82	0.00	138.27	134.67	−∞
2500	139.50	5000	92.79	139.17	35.08	64.92	0.00	69.51	23.29	0.00	137.92	133.17	−∞
3150	141.12	10000	82.84	140.30	27.56	72.44	0.00	44.81	38.03	0.00	137.63	136.92	−∞
4000	140.32	10000	78.70	139.28	42.98	57.02	0.00	42.66	36.05	0.00	136.62	135.89	−∞

Table 4: Cavity - ribbed plate - cavity example. Contributions of resonant, non-resonant and mixed transmission paths for the considered 1/3 octave frequency bands.

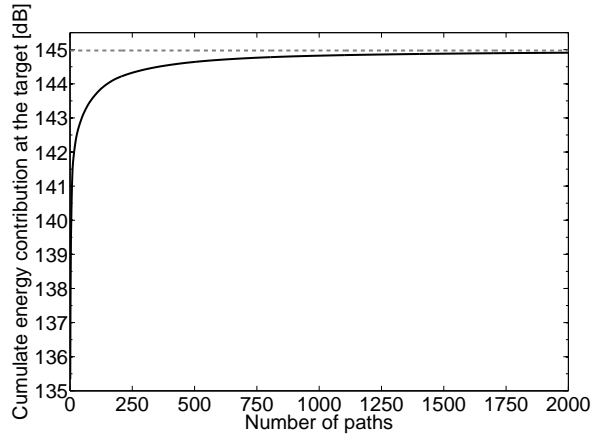


Figure 7: Cumulative path energy contribution at the receiver cavity C2 versus number of considered dominant paths, for the cavity - ribbed plate - cavity example for the 1600 Hz 1/3 octave band. Continuous black line: cumulative energy contribution, Continuous straight grey line: total energy at the receiver C2.

401 (dashed lines) and the ribbed one (solid lines). For each case two ENRs have been considered, the first one  
 402 only considers resonant modes in the transmission (grey lines) whereas the second one takes into account  
 403 resonant and non-resonant transmission as well (black lines).

404 Given that the maximum frequency limit in the figure is 4KHz, which is well below the critical frequency  
 405 of the bare plate, the resonant transmission ENR (dashed grey line) is clearer higher than the resonant plus  
 406 non-resonant ENR (dashed black line) because, as explained in Section 4.1 (see Table 1), transmission is  
 407 mainly dominated by non-resonant modes at this frequency range. However, when comparing the ENRs for  
 408 the ribbed plate, there is no clear dominance from one type of transmission or another. Besides, note from  
 409 Fig. 6 that the ENR of the ribbed plate considering all type of paths is generally higher than that of the  
 410 bare plate, except for the 630 Hz and 1600 Hz bands. At these frequencies, the transmission seems to be  
 411 mainly resonant for the ribbed plate, since the grey and black solid lines have really close values and there  
 412 are two dips in the ENR curves. This is confirmed by the results in Table 4.

413 If the transmission is mostly resonant, one could suspect that there is a group of modes of the cavities  
 414 and the plate which exhibit spatial and frequency coincidences, i.e. a group involving a path of the type  
 415  $p_i \rightarrow q_i \rightarrow r_i$ . One could then be tempted to check from all possible combinations of three modes which  
 416 of them present a high intermodal work  $W_{ij}$  (to account for the spatial matching) or which are closer in  
 417 frequency (to account for the frequency matching), to try to determine which modes control the energy  
 418 coupling. However, this is not a very good option because the effects of spatial and frequency coincidence  
 419 are studied separately. To determine modal energy transmission it is better to focus the analysis on the  
 420 modal coupling factors, see (5), which include the spatial and frequency coincidence effects at the same  
 421 time. Inspecting the values of these factors between the excited cavity and the ribbed panel on the one side,  
 422 and between the ribbed panel and the receiving cavity on the other side, it is possible to identify the most  
 423 dominant paths between each pair of coupled subsystems. However, it still becomes very difficult to identify  
 424 the most dominant paths when considering the coupling of all three subsystems together. For example,  
 425 one plate mode could be strongly coupled with a mode of the excited cavity whereas poorly coupled with  
 426 the modes of the receiving cavity. Such a mode could exchange much less energy than another one being  
 427 moderately coupled with the modes of both cavities. The analysis of the modal coupling may be a lengthy  
 428 and time consuming manual procedure which cannot fully guarantee that the inspected modes will be the  
 429 ones dominating energy transmission. Alternatively, resorting to graph theory allows one to gather this type  
 430 of information.

431 It is interesting to focus on the 1600 Hz frequency band for which resonant modes play a significant role.  
 432 In Fig. 7 the cumulative energy contribution of the first 2000 paths in this band is depicted. The total energy  
 433 at the receiver cavity is 144.97 dB and the contribution e.g., of the first 250 paths already provides 144.32 dB,

Path	Cavity – Ribbed plate – cavity
1	$S \rightarrow p_{49} \rightarrow q_{20} \rightarrow r_{49} \rightarrow T$
2	$S \rightarrow p_7 \rightarrow q_1 \rightarrow r_{13} \rightarrow T$
3	$S \rightarrow p_{14} \rightarrow q_5 \rightarrow r_{12} \rightarrow T$
4	$S \rightarrow p_{82} \rightarrow q_{31} \rightarrow r_{72} \rightarrow T$
5	$S \rightarrow p_{82} \rightarrow q_{31} \rightarrow r_{55} \rightarrow T$
6	$S \rightarrow p_{78} \rightarrow q_{20} \rightarrow r_{49} \rightarrow T$
7	$S \rightarrow p_{49} \rightarrow q_{20} \rightarrow r_{30} \rightarrow T$
8	$S \rightarrow p_{141} \rightarrow q_{57} \rightarrow r_{129} \rightarrow T$
9	$S \rightarrow p_{82} \rightarrow q_{31} \rightarrow r_{81} \rightarrow T$
10	$S \rightarrow p_{33} \rightarrow q_{20} \rightarrow r_{49} \rightarrow T$

Table 5: Cavity - ribbed plate - cavity example. Ranking of the 10 most dominant modal energy paths from a total of 2000 computed paths for the 1600 Hz 1/3 octave band.

434 which is fairly close to the total value (less than 1 dB). Therefore, it becomes feasible to circumscribe the  
435 analysis to the ranking of the first 250 paths. For every mode, the number of appearances in the list of the  
436 250 paths and the number of paths that contain it have been counted. The results are presented in Fig. 8.  
437 It can be observed that for every subsystem, there is a relatively small group of outstanding modes, whose  
438 maxima can respectively be identified as  $p_{49}$ ,  $q_{20}$  and  $r_{49}$ . One may presume that these modes will play a  
439 predominant role in energy transmission. Actually, if one inspects the ranking of the 250 paths (see Table 5  
440 for the first 10 paths), it precisely follows that the preminent path is  $p_{49} \rightarrow q_{20} \rightarrow r_{49}$  which contributes

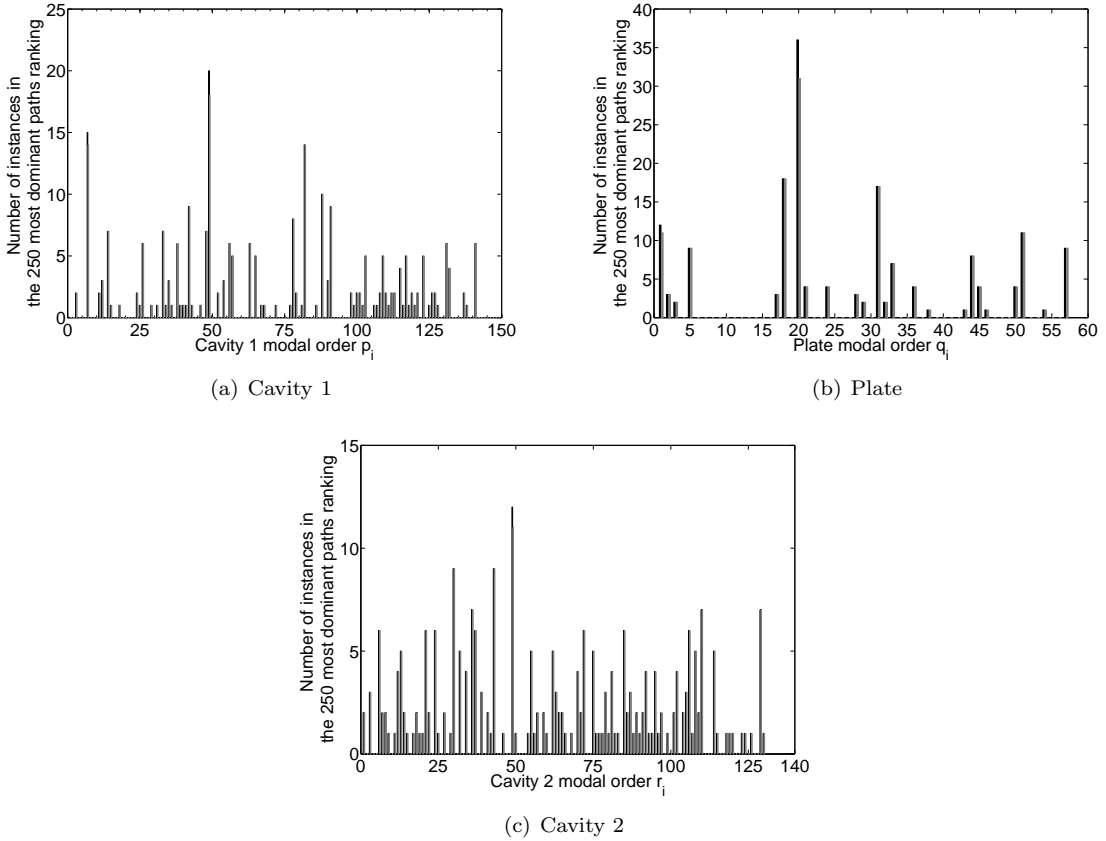


Figure 8: Number of instances of every mode in the 250 most dominant paths ranking. Black columns: total number of instances in the path ranking. Grey columns: number of paths containing a particular mode.



441 135.36 dB to the energy at the receiver cavity. The natural frequencies of these modes are 1547 Hz for  $p_{49}$ ,  
 442 1549 Hz for  $q_{20}$ , and 1565 Hz for  $r_{49}$ . These modes are relatively close in frequency but for the receiving  
 443 cavity, other modes closer to  $q_{20}$  than  $r_{49}$  can be found. For example,  $r_{42}$  has a frequency of 1547 Hz. Thus,  
 444 the matching in frequency cannot explain in itself why  $p_{49} \rightarrow q_{20} \rightarrow r_{49}$  dominates.

445 It is worth examining this point in more detail. In Fig. 9 the spatial distribution (pressure for cavity  
 446 modes and displacement for the ribbed plate) on the plate surface is plotted, for the three involved modes.  
 447 Although the natural flexural wavelength of the plate is  $\sim 0.08$  m at 1.5 kHz, a wavelength for mode  $q_{20}$   
 448 along the  $y$ -direction can be appreciated well above this value, which is comparable to the wavelengths of  
 449 acoustic modes. This is due to the stiffness effect of the ribs and it leads to a spatial matching between this  
 450 plate mode and the acoustic modes  $p_{49}$  and  $r_{49}$  in the the  $y$ -direction. The intermodal works between  $p_{49}$   
 451 and  $q_{20}$ , and  $q_{20}$  and  $r_{49}$  are, respectively, 66.7 J and 71.4 J and their modal coupling factors turn out to be  
 452 22.5 Hz and 12.8 Hz. The latter is lower than the former whereas the opposite happens for the intermodal  
 453 works. This can be simply explained by the fact that  $p_{49}$  and  $q_{20}$  are closer in frequency than  $q_{20}$  and  $r_{49}$ .  
 454 The spatial matching is here the key phenomenon which leads the path  $p_{49} \rightarrow q_{20} \rightarrow r_{49}$  to be the most  
 455 dominant one, as identified by the MPS algorithm.

456 In order to give the reader a point for comparison, an analysis is made of the interactions in the path  
 457  $p_{34} \rightarrow q_{14} \rightarrow r_{33}$  which does not appear in the MPS ranking of the first 250 paths, though its modes have very  
 458 close natural frequencies (1514.2 Hz, 1512.0 Hz, 1512.6 Hz, respectively). The pressure and displacement  
 459 spatial patterns of those modes on the plate surface have been plotted in Fig. 10 to be compared with  
 460 those in Fig. 9. As observed, the plate mode  $q_{14}$  presents short wavelengths along the  $x$  and  $y$  directions

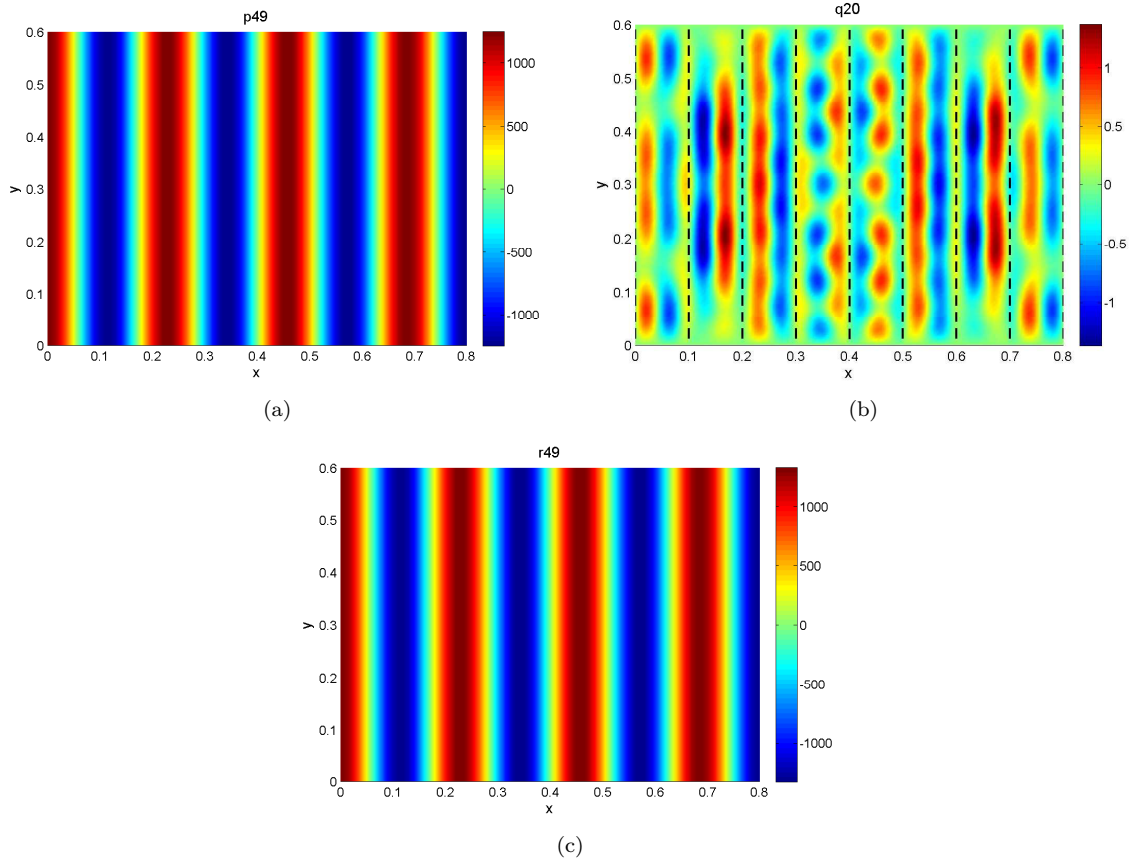


Figure 9: Magnitudes of the mode spatial shapes on the coupling surface: (a)  $p_{49}$ ; (b)  $q_{20}$  (Vertical dashed line: rib positions); (c)  $r_{49}$ .

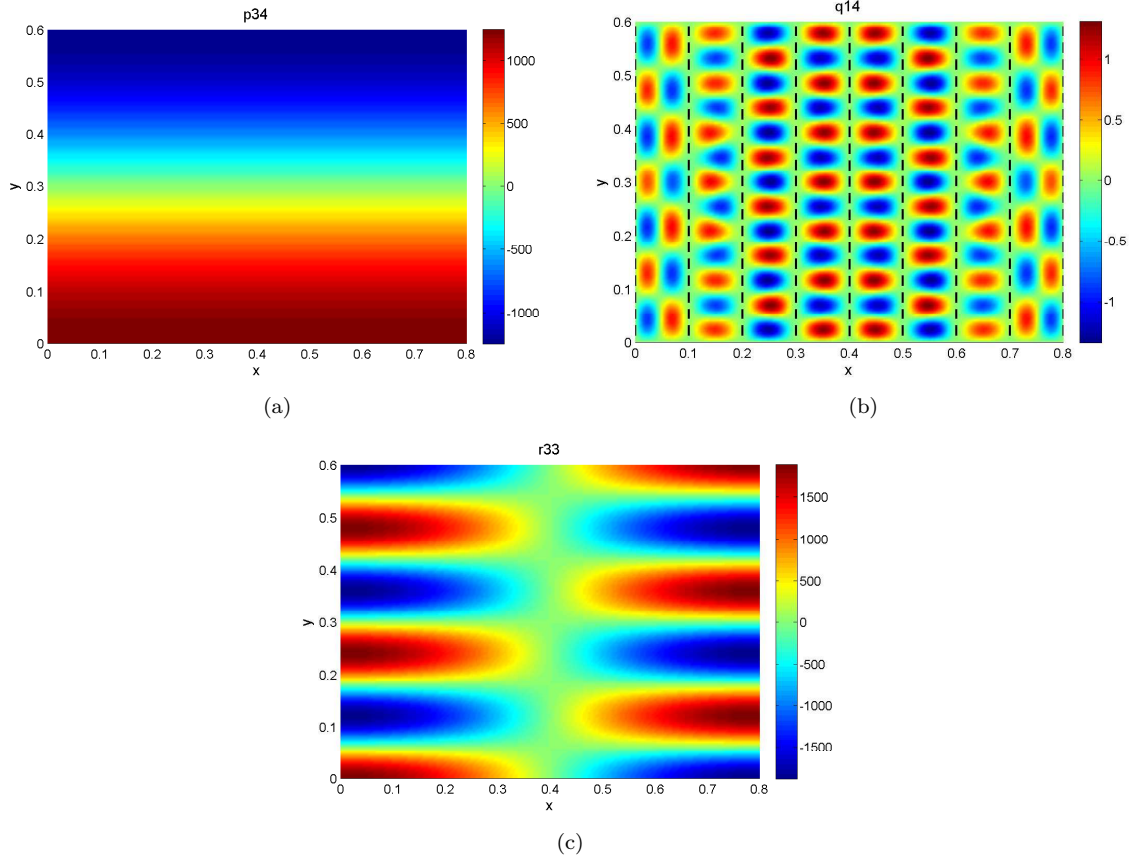


Figure 10: Magnitudes of the mode spatial shapes on the coupling surface: (a)  $p_{34}$ ; (b)  $q_{14}$  (Vertical dashed lines: rib positions); (c)  $r_{33}$ .

461 compared to the wavelengths of the acoustic modes. This results in low values for the intermodal works  
 462 ( $1.9 \times 10^{-7}$  J between  $p_{34}$  and  $q_{14}$  and  $8.7 \times 10^{-7}$  J between  $q_{14}$  and  $r_{33}$ ) and in very low values for the modal  
 463 coupling factors ( $1.9 \times 10^{-16}$  Hz between  $p_{34}$  and  $q_{14}$  and  $3.9 \times 10^{-17}$  Hz between  $q_{14}$  and  $r_{33}$ ), despite the  
 464 frequency matching. Hence, it becomes clear that the main difference between the paths  $p_{49} \rightarrow q_{20} \rightarrow r_{49}$   
 465 and  $p_{34} \rightarrow q_{14} \rightarrow r_{33}$  is the effect of spatial matching. The former path is the most dominant one and this  
 466 is a consequence of the rib effects which lead to a low energy noise reduction in comparison to that of the  
 467 bare plate for the 1600 Hz third octave band (see Fig. 6). Therefore, it becomes clear again how from an  
 468 automatic and quick inspection of the outputs of the MPS algorithm it is possible to identify those modes  
 469 responsible of a sudden decrease in the ENR curves, and act upon them if necessary. Otherwise, one could  
 470 waste lots of time analysing paths with good frequency matching but no relevant energy contribution such  
 471 as  $p_{34} \rightarrow q_{14} \rightarrow r_{33}$ .

## 472 5. Application to a shipbuilding structure

### 473 5.1. Path analysis in *SmEdA*

474 The above developments will be applied in this section to a more complex case consisting of a shipbuilding  
 475 structure made of 6 rooms distributed in two decks (see Fig. 11). The floor between decks and the vertical  
 476 walls between rooms are made of steel ( $\rho = 7800$  kg/m<sup>3</sup>, Young modulus  $E = 2 \times 10^{11}$  Pa and internal  
 477 damping  $\eta = 0.01$ ). The thicknesses of the panels are 6 mm for the floor and 2 mm for the walls. The floor  
 478 and the walls are stiffened with 6 T-shaped ribs. These ribs are regularly spaced along the floor's longest

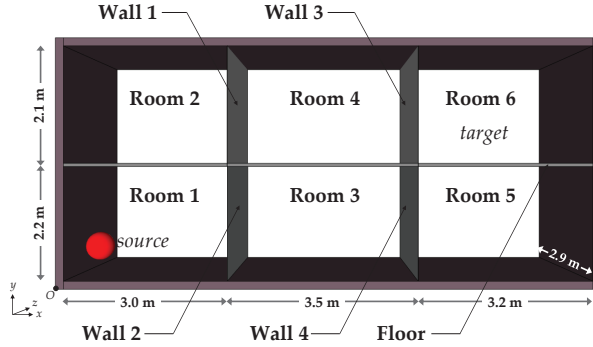


Figure 11: Sketch of the shipbuilding built-up structure model

479 edge and along the vertical wall edges. The rib spacing is 0.4 m and the first and the last ribs are respectively  
 480 placed at 0.46 m and 0.44 m from the edges. The T-cross section dimensions are  $[80 \times 8] \text{ mm}^2 / [80 \times 8] \text{ mm}^2$   
 481 for the floor ribs and  $[60 \times 5] \text{ mm}^2 / [60 \times 5] \text{ mm}^2$  for the wall ones. Besides, to deal with a more amenable  
 482 model the external walls and floors of the structure have been assumed rigid. The rooms are filled with  
 483 air (mass density  $\rho_0 = 1.29 \text{ kg/m}^3$ , wavespeed  $c_0 = 340 \text{ m/s}$ , damping loss factors  $\eta = 0.01$ ). Overall,  
 484 this results in a model with eleven subsystems: six rooms  $R1 - R6$ , the floor  $F$  and four separating walls  
 485  $W1 - W4$  (see Fig. 11).

486 In this case, to build the SmEdA model only flexural vibration modes have been considered for the panels.  
 487 Different studies on ship structures [44, 45] showed that this type of vibration transmission is predominant  
 488 in the low and mid frequency range. For high frequencies (several kHz for typical naval frames), in-plane  
 489 modes may be of importance for some one third octave bands. Actually, it has been noticed that in-plane  
 490 longitudinal and shear motions may have significant contributions on the energy transmitted to a subsystem  
 491 far away from the excited one [46, 47]. However, given that the frequency range of analysis has been fixed  
 492 to  $[80 \text{ Hz} - 500 \text{ Hz}]$ , which is below the first in-plane mode of the floor (616 Hz), contemplating only bending  
 493 motions is a fair enough approximation. The modes of the floor and walls have been computed with FEM  
 494 and the SDTool code [43], whereas the cavity modes have been calculated analytically, like in the previous  
 495 examples.

496 An acoustic monopole has been located at point  $M_0 = (0.8, 0.9, 0.7) \text{ m}$  in Room 1 (source subsystem),  
 497 with unit power spectral density. Room 6 is considered as the receiver. In Fig. 13(a) the ENR between  
 498 R1 and R6 has been plotted, whereas the ENR for the two adjacent rooms R1 and R3 is also presented  
 499 in Fig. 13(b) for comparison. As observed, the energy reduction between R1 and R6 is considerably high  
 500 except for the 200 Hz band where strong energy transmission takes place. One may then wonder which  
 501 modes are responsible for the transmission at this frequency band to act upon them e.g., by increasing their  
 502 damping and thus reducing the energy level at the receiver.

503 In Table 6 a path analysis is presented for the shipbuilding built-up structure, analogous to that in

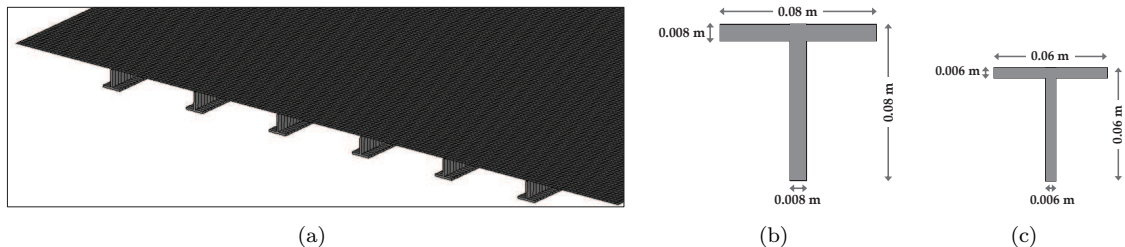


Figure 12: (a) Zoom view of the floor in the shipbuilding structure of Fig. 11. (b) Section of the floor ribs. (c) Section of the wall ribs.

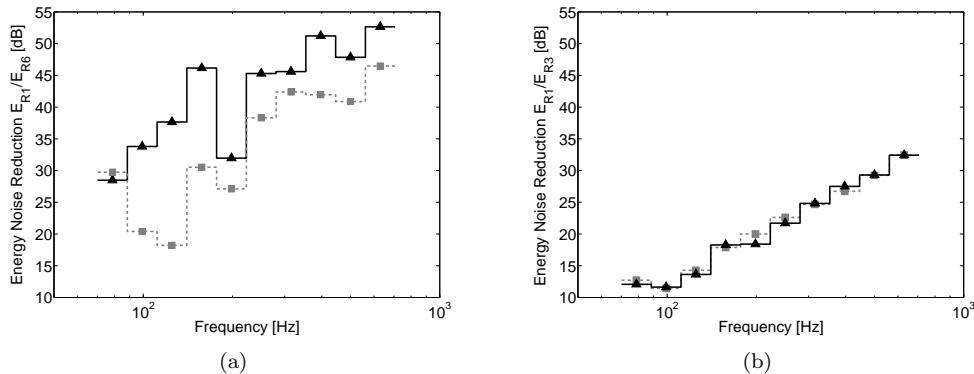


Figure 13: (a) Noise Energy Reduction  $E_{R6}/E_{R1}$  between Room 1 and Room 6. —▲— (black) SmEdA results. - -■- - (grey) SEA results. (b) Noise Energy Reduction  $E_{R3}/E_{R1}$  between Room 1 and Room 3. —▲— (black) SmEdA results. - -■- - (grey) SEA results.

504 Table 2 for the cavity-homogeneous plate-cavity system. As seen from the table, there are a total of 326  
 505 modes involved in the 200 Hz band, the overall energy at the receiver subsystem being 45.06 dB (first row).  
 506 Considering 1000 paths 93.77% of the energy at the receiver can be recovered, with a drastic reduction in the  
 507 number of involved modes (from 326 to 112). The strongest reduction of modes takes place for subsystems  
 508  $R2$ ,  $R3$  and  $W1 - W4$ , which is logical. With the sole consideration of 25 paths the reconstructed energy at  
 509 the receiver is less than one dB of its overall value, which may suffice for the analysis of the vibroacoustic  
 510 behaviour of the structure.

511 Computing the number of instances of every mode in the ranking of paths (see Fig. 14) reveals that the  
 512 dominant modes of the floor are  $f_{22}$  and  $f_{23}$ . This can also be appreciated in the ranking list of Table 7  
 513 for the first seven dominating transmission paths. These paths contribute 58.11% of the energy at the receiver  
 514 and only contain modes belonging to the source room, the receiver room and the floor. This indicates that  
 515 energy is mainly transmitted along the floor separating the two decks.

516 Similarly to what occurred for the cavity - ribbed plate - cavity example, a close inspection of modes  
 517  $f_{22}$  and  $f_{23}$  shows that they present high wavelengths in the floor's stiffest direction, which results in good  
 518 spatial matching with the cavity modes (see Figs. 15a, b). Despite the amplitudes of the mode shapes  
 519 being smaller in the junction area with room 1 than with room 6, there is still good spatial matching of  $f_{22}$   
 520 and  $f_{23}$  with the cavity modes of the former. In contrast, mode  $f_{25}$ , for example, exhibits lower vibration  
 521 values in the junction areas with rooms 1 and 6 than in the junction with room 2, and also presents shorter  
 522 wavelengths than  $f_{22}$  and  $f_{23}$  (see Fig. 15c). This explains the lack of spatial matching leading to destructive  
 523 interference between the floor mode and the cavity modes. In turn, that results in low intermodal works  
 524 within room 1 and room 6 modes.

525 As a consequence, one could attempt to increase the damping of modes  $f_{22}$  and  $f_{23}$  in order to diminish  
 526 the energy level in the receiver room. This has been simulated by setting their new internal damping values

Number of paths	Accum. Contr. [dB]	Accum. Contr. [%]	$\hat{R}1$	$\hat{R}2$	$\hat{R}3$	$\hat{R}4$	$\hat{R}5$	$\hat{R}6$	$\hat{F}$	$\hat{W}1$	$\hat{W}2$	$\hat{W}3$	$\hat{W}4$	Total involved modes	Missing modes
$\infty$	45.06	100.00	15	14	18	18	17	17	65	42	40	40	40	326	0
1000	44.78	93.77	12	5	6	13	8	17	37	2	5	7	0	112	214
500	44.67	91.39	11	5	3	8	4	15	30	1	3	5	0	85	241
100	44.44	81.67	8	3	3	4	3	12	14	0	0	2	0	49	277
25	44.18	70.85	3	1	0	0	1	5	6	0	0	0	0	16	310
7	42.70	58.11	1	0	0	0	0	3	3	0	0	0	0	7	319

Table 6: Shipbuilding structure. Transmission path analysis and involved modes.

Path	Shipbuilding
1	$S \rightarrow r1_6 \rightarrow f_{22} \rightarrow r6_5 \rightarrow T$
2	$S \rightarrow r1_6 \rightarrow f_{22} \rightarrow r6_6 \rightarrow T$
3	$S \rightarrow r1_6 \rightarrow f_{23} \rightarrow r6_5 \rightarrow T$
4	$S \rightarrow r1_6 \rightarrow f_{23} \rightarrow r6_6 \rightarrow T$
5	$S \rightarrow r1_6 \rightarrow f_{22} \rightarrow r6_7 \rightarrow T$
6	$S \rightarrow r1_6 \rightarrow f_{21} \rightarrow r6_5 \rightarrow T$
7	$S \rightarrow r1_6 \rightarrow f_{23} \rightarrow r1_6 \rightarrow f_{22} \rightarrow r6_5 \rightarrow T$

Table 7: Shipbuilding structure example. Ranking of the 10 most dominant modal energy paths from a total of 1000 computed paths

to  $\eta_{22} = \eta_{23} = 0.1$ , which has resulted in a 5.0 dB reduction of the transmitted energy. Of course this structural modification is only theoretical, but it confirms that the structural modes on which one should intervene for controlling the noise transmission have been well identified. In practice, viscoelastic layers can be used for increasing the modal damping. In order to optimize the increase of damping of a given mode the layers should be placed at the positions of maximum strain energy according to the mode shape [48]. The damping factors of all modes in the considered structure will be obviously influenced by these viscoelastic layers. They could be estimated using the Modal Strain Energy (MSE) method [49–51].

To summarize, the MPS algorithm has allowed for a quick identification of the two most critical modes determining the energy transmission between source and receiver in the SmEdA model of the shipbuilding built-up structure, which contains a total of 326 modes. The sole modification of the damping loss factors of two modes, from 65 modes in the floor, has resulted in a noise reduction of 5 dB.

All in all, it would be feasible to think of drawing a process for any built-up structure, which could start with the identification of the most dominant modal transmission paths from source to receiver subsystems, using the graph theory approach depicted heretofore. An analysis could then be made of the mode shapes having the most significant participation in noise transmission (like modes  $f_{22}$  and  $f_{23}$ ), and then one could try to reduce their energy contribution at the receiver by increasing their damping. A target noise reduction value could be fixed for the receiver (like the 5 dB in room 6) and appropriate values for the problematic mode damping (like 0.1 for  $f_{22}$  and  $f_{23}$ ) could be derived to reach that goal. Damping layers could judiciously be placed to reach the objective and determined according to the MSE method. A final SmEdA simulation would verify the performance of the solution in terms of noise reduction. A similar automatized procedure in the easier case of SEA was developed in the framework of graph theory in [28]. The development of an analogous process for SmEdA as described is, however, outside the scope of this paper.

## 5.2. Comparison with SEA results

A comparison with the results that can be obtained from a SEA model of the shipbuilding structure instead of the SmEdA one will be finally presented to highlight some of the advantages of the latter. A SEA

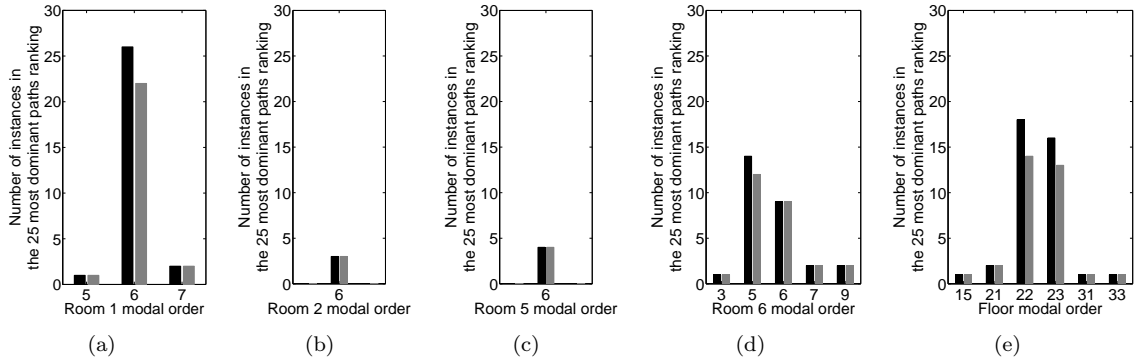


Figure 14: Number of instances of every mode in the 25 most dominant paths ranking. Black columns: total number of instances in the path ranking. Grey columns: number of paths containing a particular mode.

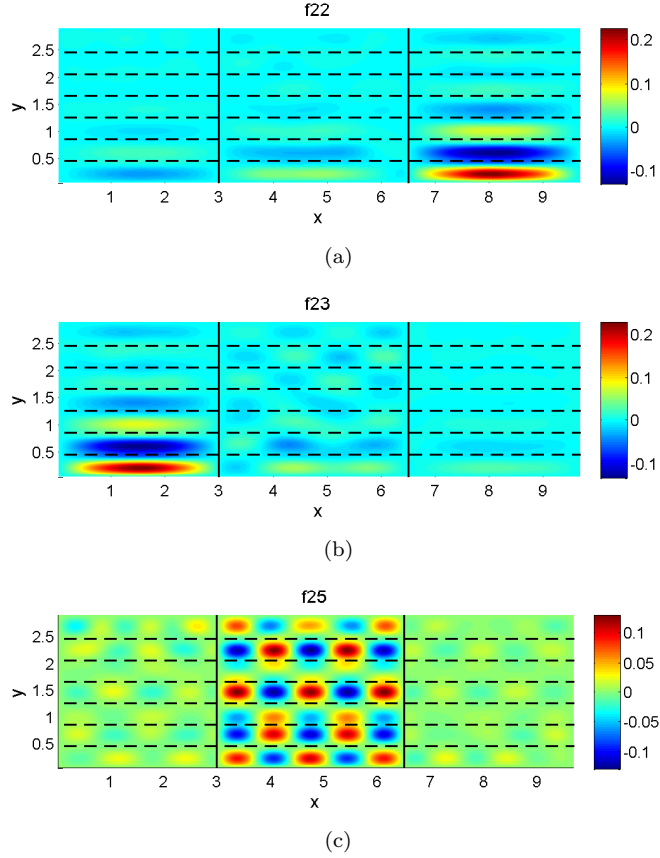


Figure 15: Magnitudes of the mode spatial shapes on the coupling surface (Horizontal dashed lines: rib positions): (a)  $f_{22}$ ; (b)  $f_{23}$ ; (c)  $f_{25}$ .

552 system can be directly deduced from a SmEdA one by assuming modal energy equipartition, see [18, 20].  
 553 The SEA coupling loss factors (CLF) and injected power can be related, respectively, to the SmEdA modal  
 554 coupling factors and modal injected powers. For the basic case of two coupled subsystems considered in  
 555 Section 2.1, the CLF between subsystems 1 and 2,  $\eta_{12}$ , can be computed as

$$\eta_{12} = \frac{\sum_{p \in \hat{P}} \sum_{q \in \hat{Q}} \beta_{pq}}{\omega_c N_p}, \quad (18)$$

556 and the injected power in subsystem 1 as

$$\Pi^1 = \sum_{p \in \hat{P}} \Pi_{inj}^p. \quad (19)$$

557 These expressions can be easily generalized in the case of having more subsystems.

558 When the subsystem modal overlap is high (typically greater than one), energy equipartition takes place  
 559 to a good extent so that SEA and SmEdA provide similar results. However, for low modal overlap, modal  
 560 energy equipartition is no longer satisfied for subsystems located far away from the excited one, which leads  
 561 SEA to overestimate energy transmission [20]. This can be observed in Figs. 13(a) and (b) for the ENR  
 562 values between rooms 1 and 6 and rooms 1 and 3. In the case of adjacent rooms SEA and SmEdA yield  
 563 fairly close results. As opposed to this, in the case of rooms 1 and 6 which are at considerable distance apart  
 564 (see Fig. 11) a strong discrepancy can be observed between the predictions of SmEdA and SEA. The latter  
 565 gives a lower ENR value and thus a higher energy level at the receiver room 6.

Path	Shipbuilding SEA
1	$S \rightarrow r1 \rightarrow f \rightarrow r6 \rightarrow T$
2	$S \rightarrow r1 \rightarrow f \rightarrow r4 \rightarrow r6 \rightarrow T$
3	$S \rightarrow r1 \rightarrow w2 \rightarrow f \rightarrow r6 \rightarrow T$
4	$S \rightarrow r1 \rightarrow r3 \rightarrow f \rightarrow r6 \rightarrow T$
5	$S \rightarrow r1 \rightarrow f \rightarrow w3 \rightarrow r6 \rightarrow T$
6	$S \rightarrow r1 \rightarrow f \rightarrow r4 \rightarrow f \rightarrow r6 \rightarrow T$
7	$S \rightarrow r1 \rightarrow f \rightarrow r3 \rightarrow f \rightarrow r6 \rightarrow T$
8	$S \rightarrow r1 \rightarrow r2 \rightarrow f \rightarrow r6 \rightarrow T$
9	$S \rightarrow r1 \rightarrow f \rightarrow r5 \rightarrow r6 \rightarrow T$
10	$S \rightarrow r1 \rightarrow f \rightarrow r6 \rightarrow f \rightarrow r6 \rightarrow T$

Table 8: Shipbuilding structure example. Ranking of the 10 most dominant energy SEA paths from a total of 100 computed paths for the 200 Hz 1/3 octave band.

566 In Table 8, the ranking for the 10 dominant energy transmission paths between rooms 1 and 6 in the  
567 SEA model is presented. As observed the first path clearly involves energy transmission through the floor.  
568 However, and contrary to the results in Table 7, in this case no information at all is available on which  
569 modes are responsible for this transmission.

## 570 6. Conclusions

571 Energy interchange between modes in subsystems rather than between subsystems (groups of modes)  
572 themselves, it is at the very core of SmEdA. As seen, avoiding modal energy equipartition allows one to  
573 extend classical SEA to the mid frequency range and dealing with low modal overlap and/or locally excited  
574 subsystems, as well as with complex geometry subsystems. However, given that many modes are considered  
575 within subsystems, the dimensions of SmEdA matrices become considerably large even for medium sized  
576 systems, like those representing sound transmission between two adjacent cavities. Energy transmission in  
577 such type of systems is more often caused not so much by mode frequency matching as by mode spatial  
578 matching. If one aims at vibroacoustic remedial actions, identifying those resonant and non-resonant modes  
579 that play an essential role in energy transmission becomes a must. However, direct mode by mode inspection  
580 may become a never-ending task even in the occurrence of frequency matching, as shown for some of the  
581 analyzed examples.

582 It has been proved that graph theory offers a way out to this problem. Analogously to what was  
583 previously done in SEA, and for some energy distribution models as well, it becomes possible to define a  
584 SmEdA graph associated to any SmEdA model and apply a ranking path algorithm to it. The latter results  
585 in a sorted list of modal energy transmission paths from source to receiver subsystems, which allows for the  
586 straightforward identification of the modes that dominate the transmission process. Then one could focus  
587 on analyzing the spatial matching of the modes in that dominating paths and see whether an increase of  
588 damping, a structure reinforcement, or whatever action deemed appropriate, may result in a decrease of  
589 the noise transmitted to the receiver. The validity of the graph approach to ease the analysis of SmEdA  
590 systems has been demonstrated for different examples, consisting of the transmission of sound between  
591 cavities separated by both bare and ribbed plates, as well as for the more complex case of a shipbuilding  
592 structure.

Number of paths	Accum. Contr. [dB]	Accum. Contr. [%]	Presence of the subsystem											
			R1	R2	R3	R4	R5	R6	F	W1	W2	W3	W4	
$\infty$	49.73	100.00	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
100	49.73	99.93	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
20	49.72	99.73	✓	✓	✓	✓	✓	✓	✓	✓	×	✓	✓	×
10	49.68	98.72	✓	✓	✓	✓	✓	✓	✓	×	✓	✓	✓	×
1	49.47	94.12	✓	×	×	×	×	×	✓	✓	×	×	×	×

Table 9: Shipbuilding structure. Transmission path analysis in the SEA model.

593 **7. Acknowledgments**

594 This work has been partially supported by the Labex CeLyA of Université de Lyon, operated by the  
595 French National Research Agency (ANR-10-LABX-0060/ANR-11-IDEX-0007). In particular, the short stay  
596 of the first author at INSA-Lyon has been funded by this support. The first author would also like to  
597 acknowledge the Generalitat de Catalunya (SUR/ECO) for the pre-doctoral FI grant No. 2014FI-B2 00141.

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