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HANDLING LOCALIZATION IN DAMAGE MODELS WITH THE THICK LEVEL SET APPROACH (TLS)

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ABSTRACT

In this paper, we discuss a new way to model damage growth in solids. A level set is used to separate the undamaged zone from the damaged zone. In the damaged zone, the damage variable is an explicit function of the level set. This function is a parameter of the model. Beyond a critical length, it is assumed that the material is totally damaged, thus allowing a straightforward transition to fracture. The damage growth is expressed as a level set propagation. The configurational force driving the damage front is non local in the sense that it averages information over the thickness in the wake of the front. Three important theoretical advantages of the proposed approach are as follows : (a) The zone for which the materials is fully damaged is located inside a clearly identified domain (given by an iso- level set). (b) The non-locality steps in gradually in the model. At initiation the model is fully local. At initiation, micro-cracks being absent no length scale should prevail. (c) It is straightforward to prove that dissipation is positive. A numerical experiment of the cracking of a multiply perforated plate is discussed.

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INTRODUCTION

It is well known that fracture mechanics alone may not model the full scenario of the degradation of solids under mechanical loading. Crack initiation for instance requires damage mechanics to model the gradual loss of stiffness in a small area [1, 2]. When it comes to modeling damage, special care needs to be taken to avoid so called spurious localizations. Several damage models were proposed in the literature to avoid spurious localization as detailed in [3, 4]:

* non local integral damage model: the damage evolution is governed by a driving force which is non-local i.e. it is the average of the local driving force over some region [5], [6];
* higher order, kinematically based, gradient models through the inclusion of higher-order deformation gradient [7–9] or additional rotational degrees of freedom [10] ;
* higher order, damage based, gradient models: the gradient of the damage is a variable as well as the damage itself. This leads to a second order operator acting on the damage: [11–13].

In this paper, we wish to model damage as a propagating level set front which is yet another way to avoid spurious localization.

A level set based damage model

We follow here the development of the TLS model described in [14]. The free energy per unit volume φ depends on the strain ε and a scalar damage variable denoted d.

$$\varphi = \varphi(\varepsilon, d) \tag{1}$$

The state laws are obtained by differentiating the free energy

$$\sigma = \frac{\partial \varphi}{\partial \varepsilon}, \qquad Y = -\frac{\partial \varphi}{\partial d} \tag{2}$$

where σ is the stress and Y the local energy release rate. For instance, if we consider a symmetric behavior in tension and compression, we may use the potential

$$\varphi(\varepsilon, d) = \frac{1}{2}(1-d)\, \varepsilon : E : \varepsilon \tag{3}$$

leading to

$$\sigma = (1-d)\, E : \varepsilon, \quad Y = \frac{1}{2}\varepsilon : E : \varepsilon \tag{4}$$

The damage description is depicted in Fig. 1. The level set $\phi = 0$ separates the domain Ω into an undamaged zone and damaged one. In the damaged zone, the damage variable d is an explicit function of the level set. Note that a single level set is needed even if damaged zones are disconnected as shown in Fig. 1.

The damage increases progressively as the level set value rises. Mathematically this is expressed by the following inequalities:

$$d(\phi) = 0, \quad \phi \leq 0 \tag{5}$$
$$d'(\phi) \geq 0, \quad 0 \leq \phi \leq l_c \tag{6}$$
$$d(\phi) = 1, \quad \phi \geq l_c \tag{7}$$

Consider a body Ω. Along the external boundary $\partial\Omega$, loading $T^d(t)$ is applied on $\partial\Omega_T$ and displacements $u^d(t)$ are prescribed on the complementary part $\partial\Omega_u$; $\partial\Omega = \partial\Omega_T \cup \partial\Omega_u, \quad \partial\Omega_T \cap \partial\Omega_u = \emptyset$. Some parts of Ω may be totally damaged i.e. $d = 1$. We denote by Ω_c, the part of Ω not fully damaged:

$$\Omega_c = \{x \in \Omega : \phi(x) \leq l_c\} \tag{8}$$

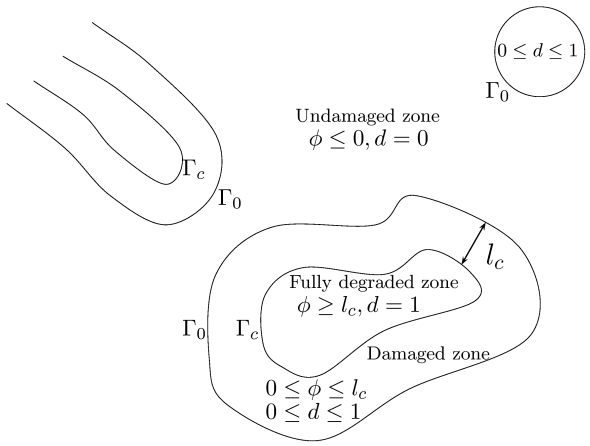

FIGURE 1. Multiple damaged, undamaged and transition zones are described using only one thick level set.

Potential energy is written as a function of the displacement field and the level set function ϕ:

$$E(u, \phi) = \int_{\Omega_c} \varphi(\varepsilon(u), d(\phi))\, \mathrm{d}\Omega - \int_{\partial\Omega_T} T \cdot u \ \mathrm{d}S \tag{9}$$

where $\varepsilon(u)$ denotes the strain, i.e. symmetric part of the gradient of u. The displacement field must satisfy the kinematic boundary conditions whereas the gradient of the level set field must be of norm 1 since it is a signed distance function.

Taking the derivative of the potential energy with respect the the front location, we get the definition of the configurational force g along the front (whose curvilinear abscissa is denoted s) :

$$g(s) = \int_0^l Y(\phi, s) d'(\phi)(1 - \frac{\phi}{\rho(s)})\, \mathrm{d}\phi \tag{10}$$

where d' is the derivative of d with respect to ϕ and $\rho(s)$ is the radius of curvature of the front Γ_0. The integral starts on Γ_0 and runs over a segment orthognal to Γ_0. The distance l is at most l_c. We consider a very brittle damage model.

$$Y \leq Y_c, \quad \dot{d} \geq 0, \quad (Y - Y_c)\dot{d} = 0 \, . \tag{11}$$

It may be shown [15] that the corresponding relationship between the front speed $\dot{a}$ and the configurational force g reads

$$g \leq g_c, \quad \dot{a} \geq 0, \quad (g - g_c)\dot{d} = 0 \, . \tag{12}$$

where g_c is computed by (10) replacing $Y(\phi, s)$ by Y_c. It may shown that the condition $g = g_c$ tends to $Y = Y_c$ as the damage zone tends to zero meaning the initial condition for damage to grow is local.

DISCRETIZATION

The space discrtization is carried out by finite elements. The displacement and level set fields are approximated over each element. In the numerical experiment described next section, second order element are used for the displacement and first order (nodal values) for the level set field. For a given loading and level set location, the problem of finding the displacement field is a linear problem (or non linear if dissymetric tension compression behavior is considered). Once the displacement field is known, one may compute the configurational force g (see Eqn. (10)). Eqn. (10) is in fact not very convienent to be used with finite element. Instead, we compute g by solving a variational principle in the damage band (it is detailed in [15]). Since we are dealing with a brittle behavior the loading is not a proper parameter because there might be no solution above some loading (limit point). A bit like with arc length technique, we thus consider the loading as an unknown and we force the maximum front advance, $a_{\max}$ to be a given value at each time step. At time t_n, the front advance a at each point of the front is determined by an explicit formula

$$a = k \left\langle (1 + |\Delta\mu|/\mu_n)^2 \left(\frac{g}{g_c}\right)_n - 1 \right\rangle_+ \tag{13}$$

where μ_n is the known load fractoir at time t_n and

$$\Delta\mu = \mu_n - \mu_{n-1} \tag{14}$$

$$k = \frac{a\max}{\left(1 + \frac{|\Delta\mu^n_{n-1}|}{\mu_n}\right)^2 - 1} \tag{15}$$

Motivations for such a formula are detailed in [15]. Using the front advance, the level set is updated and the loading at step $n+1$ is given by the maximum load μ_{n+1} such that $g \leq g_c$ at every point on the front. In case, some point in the domain has reached the condition $Y \geq Y_c$ with the new loads, a level set (little circle) is placed around this point.

Cracking of a multiply perforated plate

As a numerical experiment, we consider the problem of a plate presenting three holes. Loading and boundary conditions are represented in Fig. 2, taking into account the symmetry of the geometry. The upper part of the plate presents a Young modulus ten times larger, the imposed load is therefore very close to an imposed displacement during a large part of the computation. This computation illustrates the potential of the method, especially for the initiation of damage and cracks. The computational mesh presents a characteristic element size of about $l_c/10$. The damage profile is an arc-tangent profile

$$d(\phi) = c_2 atan\left(c_1\left(\frac{\phi}{l_c} - c_3\right)\right) + c_4 \tag{16}$$

where $c_1 = 10, c_3 = 0.5$ and c_2 and c_4 are such that $d(0) = 0$ and $d(l_c) = 1$. The Young modulus and Poisson ratio used are respectively $E = 36.5GPa$ and $\nu = 0.2$, while we choose $l_c = 4\ 10^{-2}m$ and $Y_c = 10^2 Pa$.

The damage evolution and the computational mesh are shown in Fig. 3. The corresponding load-displacement curve is depicted in Fig. 2. During the first computation steps, we observe that six small defects have been initiated on the holes, on the horizontal symmetry axis. Then, one after the others, those defects first grow until reaching a complete damage, and then propagate the crack (fully degraded zone) horizontally. We also observe that the global stiffness of the structure is not much affected as a defect grows from $d = 0$ to $d = 1$. On the contrary, the crack propagation corresponds to an unstable (strong snapback) process with large loss of stiffness.

REFERENCES

[1] Kachanov, L., 1958. "Time of rupture process under creep conditions". *Izvestia akademii nauk SSROtd. Tekh. Nauk.,* ***8***, pp. 26–31.

[2] Lemaitre, J., 1992. *A course on damage mechanics*. Springer.

[3] Peerlings, R., 1994. *Gradient damage for quasi-brittle materials*. Master Thesis. University of Eindhoven.

[4] Peerlings, R., de Borst, R., Brekelmans, W., and Vree, J. D., 1996. "Gradient enhanced damage for quasi-brittle materials". *International Journal for Numerical Methods in Engineering,* ***39***, pp. 3391–3403.

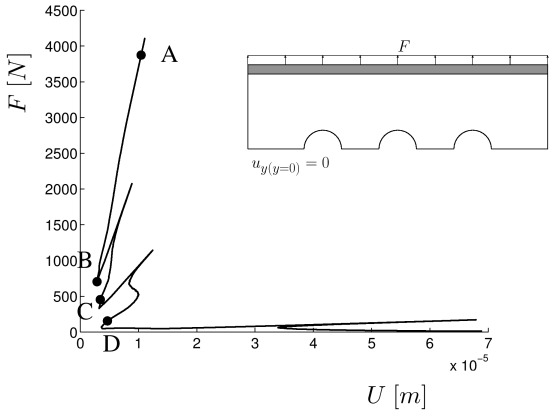

FIGURE 2. Load-displacement curve and domain definition and boundary conditions for the three holes plate.

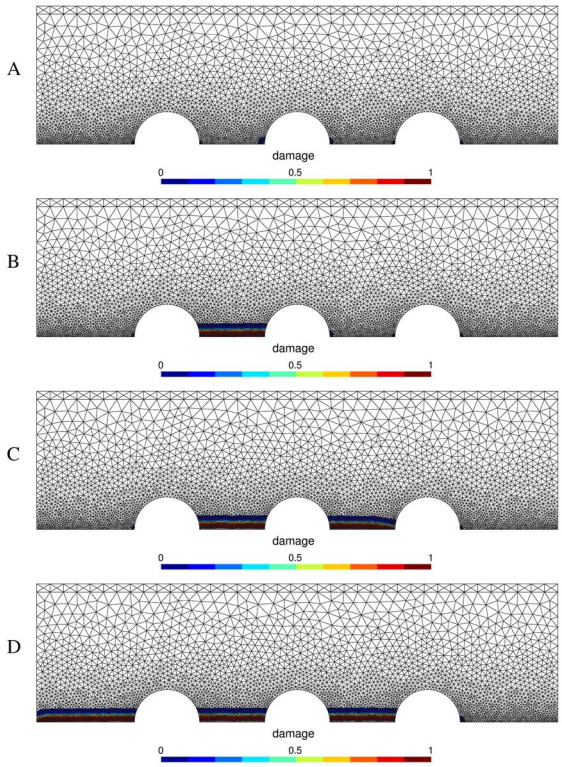

FIGURE 3. Evolution of the damage field with the underlying computational mesh.

[5] Bazant, Z., Belytschko, T., and Chang, T., 1984. "Continuum theory fo strain-softening". *Journal of Engineering Mechanics,* ***110***, pp. 1666–1692.

[6] Pijaudier-Cabot, G., and Bazant, Z., 1987. "Non local damage theory". *Journal of Engineering Mechanics,* ***113***, pp. 1512–1533.

[7] Aifantis, E., 1984. "On the structural origin of certain inelastic models". *Journal of Engineering Materials and Technology,* ***106***, pp. 326–330.

[8] Triantafyllidis, N., and Aifantis, E., 1986. "A gradient approach to localization of deformation. I. Hyperelastic materials". *Journal of Elasticity*.

[9] Schreyer, H., and Chen, Z., 1986. "One dimensional softening with localization". *Journal of Applied Mechanics,* ***53***, pp. 791–979.

[10] Mühlhaus, H. B., and Vardoulakis, L., 1987. "The thickness of shear bands in granular materials". *Geotechnique*.

[11] Frémond, M., and Nedjar, B., 1996. "Damage, gradient of damage and principle of virtual power". *International Journal of Solids and Structures,* ***33***(8).

[12] Pijaudier-Cabot, G., and Burlion, N., 1996. "Damage and localisation in elastic materials with voids". *Cohesive-frictional Materials,* ***144***, pp. 129–144.

[13] Nguyen, Q.-s., and Andrieux, S., 2005. "The non-local generalized standard approach: a consistent gradient theory". *Comptes rendus de l'Académie des sciences: Mécanique, physique, chimie, astronomie,* ***333***, pp. 139–145.

[14] Moës, N., Stolz, C., Bernard, P.-E., and Chevaugeon, N., 2011. "A level set based model for damage growth : the thick level set approach". *International Journal For Numerical Methods in Engineering,* ***86***, pp. 358–380.

[15] P.E. Bernard, N. M., and Chevaugeon, N., 2012. "Damage growth modeling using the thick level set (tls) approach: efficient discretization for quasi-static loadings". *Computer Methods in Applied Mechanics and Engineering*. submitted.