A fuzzy handling of trend objective declaration and trend performance expression

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Abstract: On the basis of previous works, we choose to focus here on a fuzzy processing of both the declaration of the objective and the performance expression. The new idea developed in this study consists of handling a "trend" objective declaration, based on linguistic declarations of trends and Zadeh's precisiation concept, and then to analyse the impact of this on the performance expression. Indeed, knowing that an objective is declared by a target value and a temporal horizon, such a value can be more or less precise, declared in the form of a single final value or a trend value, throughout the temporal horizon. Moreover, the temporal horizon can be explicitly described by a numerical interval or only by its boundary. Given that the performance expression is defined as the achievement degree of the objective, such an expression can be enriched, in the case where the objective is declared by a trend value, by an expression that gives information about the trend of the objective achievement. Instantaneous and trend performances are thus jointed, leading in this sense to the handling of the temporal aspect that is inherent to the achievement of the objective and that suggests the trend performance expression. Throughout the study, the propositions are illustrated *via* a *ratio rate* objective.

Keywords: industrial objectives, trend declaration, temporal trajectory, trend performance expression, fuzzy symbolic description.

1 INTRODUCTION – PROBLEM STATEMENT

The objective notion is the key point of any action plan or improvement process, particularly in the industrial context. Generally, industrial objectives are directly declared by decision-makers with regards to the physical considered systems. They give thus, on the one hand, the target values to reach and, on the other hand, the temporal horizons that are associated with these values. The target value is given with regards to a selected criterion, such as *quality*, *cost*, *service rate*... The temporal horizon is often the result of an estimation of the required time, by the physical considered system, for reaching the expected value. It can also be deduced from the deadlines from which decision-makers need to see the objectives achieved. Even if the objective is not directly declared, it can be obtained by the break down of more overall objectives. Structural trees are the major tools used in this case.

We propose in this work to consider the case where the objective is declared by a decision-maker. According to the industrial practice, many parameters intervene in this declaration, such as the associated criterion (or variable) nature, which is cumulative (e.g. *product quantity*) or subjective (e.g. *skill*). The hierarchical decision level in which it is declared can also have an impact on the declaration. Indeed, on the strategic and tactical decision levels, the decision-makers can use more or less imprecise formulations, either with regards to the value or to the temporal horizon. They can also give the only target value that they expect at the end of the temporal horizon, as well as specifying a set of values. One can then talk about temporal objective trajectory. Looking for overall appreciations, in higher decisional levels, the decision-makers can declare trend evolutions rather than absolute values. These considerations lead to easily imagining many ways of declaring the objectives, by combining various situations, going from the case where everything is precisely specified to the case where only some indications are imprecisely given.

Previous works have already dealt with a fuzzy handling of the declared target value, leading thus to taking into account imprecise and subjects aspects [1], [2]. The developed idea here is the enrichment of our framework, by introducing the handling of what we call trend objective, especially when it is declared in an imprecise manner; this case being more general than the precise case.

Moreover, since we subscribe to continuous improvement processes, let us recall that the performance expressions are given in order to check the achievement of the objectives and to make the decision-making easier and more reactive, with regards to the improvement actions to be launched. The other suggested idea here is then to think about, always in the continuity of formal works developed before, the potential impact of a trend declaration of the objective on the way the performance expression is obtained. More particularly, the temporal trajectory notion will be discussed and enhanced in this sense. Classical instantaneous performance expression will be recalled and associated with the trend performance expression. That is to say that when the temporal dimension is taken into account, the instantaneous performance expression is no longer sufficient to handle the achievement of the considered objective. Moreover, in our opinion, taking

into account the temporal characteristic of the objective under the form of trend declarations allows us a better handling of what is implicitly done in the industrial practice.

This paper is thus organised as follows. We explain in Section 2 the main points concerning the objective declaration focusing on the relevance of a trend declaration and the major encountered cases. We recall then in Section 3 the essential background elements concerning the performance expression. Finally we explain in Section 4 our approach and develop the fundamentals of our fuzzy proposed formalism. Some illustrations of the objective trend representation and the performance expression are given, by considering a *ratio rate* variable.

2 THE OBJECTIVE DECLARATION

2.1 Hypothesis

In previous works we have considered that an objective is necessarily identified by an expected value associated with a temporal horizon [2]. Indeed, according to what is found in the literature [11], [16], [5], we enhance the idea that an objective is defined for indicating the value to obtain, this being the reason for which this value initialises the associated action plan to launch. Besides, Ducq et al. and other authors [14], [9] propose another vision, in which the authors consider that an absolute value has no more relevance than a trend value. Such a trend value is thus handled by a verb, with regards to a variable. Hence, the objective "*must be expressed with a verb explaining the expected trend (i.e. to increase, to decrease, to maintain) associated to a considered performance domain (i.e. cost, quality, lead time, flexibility)*" [4].

Formally representing an objective by its final instantaneous value has been the purpose of previous works [1], leading to the consideration, by using a fuzzy formalism, of precise and imprecise, numerical or linguistic declarations. Performance expressions, which yield the achievement degree of the objectives, have also been defined in the same logic. For instance, let us recall several definitions which will be used in the following [2].

Definition 1: Let V be the set of variables related to objectives. Any attribute of an objective is obtained through a function defined on V. Thus, $o(v)$ represents the target value of the objective. In the same manner, $T_i(v)$ and $T_f(v)$ are respectively the initial and final dates for the objective action plan.

Because trends, with regards to values, are related to time, this notion implicitly induces a form of continuity in time leading to the association of the considered values with temporal trajectories.

Definition 2: Let V be the set of variables of the system under consideration and $v \in V$. The objective temporal trajectory is defined by the function q, called the quantification function, and $q(v,t)$ is the value of the

objective associated with v at the time $T_i(v) \le t \le T_f(v)$. Obviously we have: $o(v) = q(v, T_f(v))$.

By considering the objective being declared under a trend manner, of course, one can say that one way to proceed consists of translating the trend declaration into a final value and to apply to this value the classical handling. Nevertheless, it would be easy to see that a trend declaration handles more indications than a single final value declaration. It indicates not only the target value but also, in a sense, the way the decision-maker imagines the achievement of his declared objective. More particularly, we propose to distinguish the case where a precise trajectory is given from the case where only the final result is asked about. Trend performance expressions will thus be introduced. For example, let us consider a *ratio rate* over a *6 month* horizon. The decision-maker may express the objective as "*Slightly increase the ratio rate during 6 months*". Beyond the trend of the achievement of the final absolute value, the decision-maker gives some indications about the temporal associated trajectory.

2.2 From value to trends

Using and handling trends or more generally qualitative information is an idea which is close to the intuitive thinking. More particularly, it can be found in qualitative economics from the 60's in the work of Lancaster [13] where signs of parameters, *i.e.* {+, 0, -}, were used to represent the qualitative properties of a system. The field of qualitative physics has also been very active in the 80's [10], [6], [12]. An extension to fuzzy order of magnitude using fuzzy intervals was proposed by Dubois and Prade [3].

Concerning the handling of the trend objective declaration, we propose to consider such a declaration in a qualitative manner, which corresponds, on the one hand, to the decision-makers way of expressing themselves and is, on the other hand, more general than a quantitative manner. Two consecutive steps are thus carried out. The first step leads to the translation of the decision-maker declaration into a symbolic description on a set of terms. The second step consists of translating once again the obtained description into numerical values that can be easy to handle.

Moreover, in *Computing with Words*, Zadeh introduced the "*precisiation*" notion as a means for translating a human declaration into a formal homogeneous declaration, which is based on the association of a target value to a variable or criterion [19], [20], [21]. In Zadeh's *precisiation*, the meanings of terms are addressed, when needed, by means of fuzzy sets which are functions of variables. Sentences expressed in natural language are thus translated into mathematical expressions from which computation is possible. Precisiating our example "*Slightly increase the ratio rate during 6 months*" can be done by qualifying the *ratio rate trend* on a qualitative scale using orders of magnitude expressed by terms such as *Negative Big, Negative Medium, Negative Small, Zero, Positive Small, Positive Medium* or *Positive Big*. For example, the objective could be rewritten as

"*The ratio rate trend has to be Positive Medium after 6 months*".

Even if trends have been illustrated on the objective declaration, the concept remains general and can be applied to other quantities, such as the measurement, the performance expression... Before considering objectives defined by trends, let us explain how the general concept of trend can be handled in the fuzzy context.

2.3 Fuzzy handling of the trends

The concept of trend is linked to the variation of a quantity over a given time interval for our applications.

Definition 3: Let u(v) be a quantity associated to a variable $v \in V$ and ΔT a time interval. Let us denote u(v, t) the value of the quantity at time t. The variation of the quantity u(v) at time t is $\Delta(u(v),t) = u(v,t) - u(v,t - \Delta T)$.

Now let us consider a fuzzy partition of the variation of a quantity by means of fuzzy intervals respectively associated with the qualitative terms *Negative Big, Negative Medium, Negative Small, Zero, Positive Small, Positive Medium* or *Positive Big*, introduced before. In other words, the fuzzy intervals are the fuzzy meaning of these labels [18]. In short, the labels are abbreviated, leading to consider the set of terms $L = \{NB, NM, NS, Z, PS, PM, PB\}$. An example of such fuzzy partition is represented in Fig. 1 with regards to the quantity example, where the variation is expressed in %.

Let us note that membership functions generalise the characteristic functions of crisp intervals and therefore, the crisp case is included in the fuzzy case.
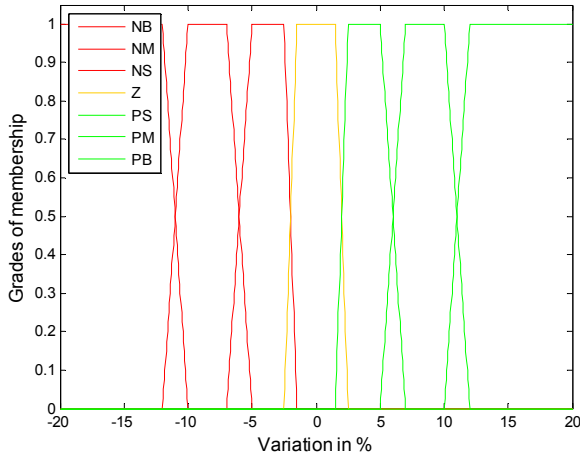

Figure 1: Example of fuzzy partition

From a general point of view, the fuzzy meaning of a term l in a set of terms L, denoted $M(l)$, is defined by its membership function $\mu_{M(l)}$ on a set of number X. Thus $\mu_{M(l)}(x)$ is the grade of membership of $x \in X$ to the meaning of l. As explained by Zadeh [18], the relation between terms and numbers can also be characterised by the fuzzy description (descriptor set) of a number x, denoted

$D(x)$, is defined by its membership function $\mu_{D(x)}$. Thus $\mu_{D(x)}(l)$ represents to which grade the value x can be represented by the term l. Since the fuzzy meaning and the fuzzy description are two ways of characterising the relation, we have the following equality:

$\forall l \in L, \forall x \in X, \mu_{D(x)}(l) = \mu_{M(l)}(x)$.

Based on the previous concepts, the trend can now be formally defined.

Definition 4: Let u(v) be a quantity associated to a variable $v \in V$. The trend of u(v) at time t is a fuzzy subset of L defined as the fuzzy description of the variation of u at time t, i.e. $Tr(u(v),t) = D(\Delta(u(v),t))$.

For example, using Zadeh's representation of fuzzy set [17], a trend could possibly be represented by the fuzzy set $0.8/NS + 0.2/Z$ meaning that the trend can be described as *Negative Small* to a grade of 0.8 but also considered as *Zero* to a grade 0.2.

2.4 Objectives declared by trends

Let us now go back to our example where trends are used in the objective declaration. As already seen, "*Slightly increase the ratio rate during 6 months*" could be rewritten as "*The ratio rate trend has to be PM after 6 months*". For the sake of the simplicity, let us denote the *ratio rate* by the variable *ratioRate*.

The available pieces of information are respectively the production at the initial time $T_i(ratioRate) = 0$, *i.e.* $q(ratioRate, 0)$ and the trend at $T_f(ratioRate) = 6$ months, *i.e.* $Tr(o(ratioRate), 6) = PM$ but the value of the objective at any time, *i.e.* the quantification function $q(ratioRate, t)$ and, consequently, the temporal trajectory are not known. Nevertheless, let us denote $\tilde{o}(ratioRate)$ the possible target value for the objective, *i.e.* at $t = T_f(ratioRate) = 6$. Let us denote $\Delta(\tilde{o}(ratioRate), 6) = \tilde{o}(ratioRate) - q(ratioRate, 0)$ the variation of the ratio rate over the 6 months. To be coherent with the objective declaration, the possible target value must be such that:

$Tr(\tilde{o}(ratioRate), 6) = D(\Delta(\tilde{o}(ratioRate), 6)) = PM$.

In this case, the trend is a crisp singleton taken in the set of terms L. As we have assumed that the meanings of the terms were fuzzy interval defined on a set X, crisp singletons can be obtained only when x belongs to the kernel[1] of fuzzy intervals. Indeed, we have:

$\forall l \in L, \mu_{D(x)}(l) = 1 \Leftrightarrow \mu_{M(l)}(x) = 1$.

An example is with the membership function represented in Figure 1 when $D(8) = 1/PM$ because $\mu_{M(PM)}(8) = 1$.

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[1] The kernel of a fuzzy set E, characterised by its membership function μ_E, is the crisp set $\{x \in E \mid \mu_E(x) = 1\}$

The kernel of the fuzzy meaning of the terms used in the trend is a crisp interval whose bounds provide a lower limit and an upper limit for the variation. In other words, declaring an objective as a qualitative trend is equivalent to declaring an imprecise objective by an interval.

Let us emphasise this equivalence with our example. Indeed, all $\tilde{o}(ratioRate)$ such that $\Delta(\tilde{o}(ratioRate), 6)$ is in the kernel of the fuzzy meaning of the term PM, will be coherent with the objective trend declaration. In other words, based on the fuzzy meaning represented in Fig. 1, all $\Delta(\tilde{o}(ratioRate), 6) \in [7\%, 10\%]$ verify the following equality for the trend: $Tr(\tilde{o}(ratioRate), 6) = PM$.

This interval provides the lower bound and the upper bound for the variation. It means that target value for the objective $o(ratioRate)$ is no longer a crisp value but is an interval. It represents the imprecision induced by the decision-maker declaration in terms of trends.

Assuming for example that the *ratio rate* was initially 80%, the objective "*Slightly increase the ratio rate during 6 months*" is equivalent to declaring an imprecise final objective target, which is represented by the interval $o(ratioRate) = [85.6\%, 88\%]$.

Using the same approach as in [2], the objective temporal trajectory, *i.e.* the quantification function q, can be built by interpolation, except that, in our case, t $q(ratioRate, t)$ is an interval. The quantification function associated with the objective "*Slightly increase the ratio rate during 6 months*" is represented in Fig.2 by its lower and upper bounds.
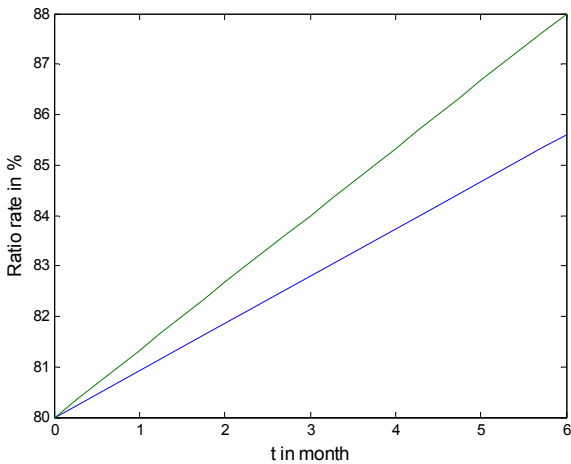
Figure 2: the resulting objective temporal trajectory

It is interesting to note that more complex trend objectives can be handled as, for example, "*Slightly increase the ratio rate during 4 months and then a little bit less during the last 2 months*". This type of declaration provides a temporal declaration with an intermediate objective also expressed by a trend. Once again, the objective must be precisiated to be processed. For example it could be rewritten as "*The ratio rate trend has to be between PM and PS after 4 months and*

then PS after the next 2 months". Assuming that the membership functions in Fig.1 are kept, first of all the membership function of "*between PM and PS*" has to be defined. A possible representation is given in Fig.3.
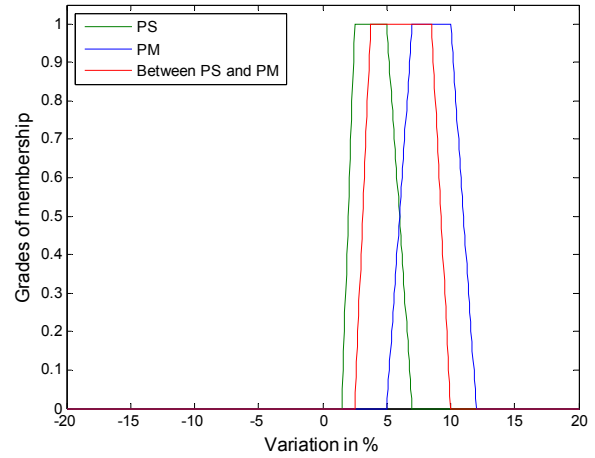
Figure 3: Fuzzy meaning of "Between *PS* and *PM*"

The first part of the objective declaration generates the interval $\Delta(\tilde{o}(ratioRate), 4) \in [3.75\%, 8.5\%]$ which verifies $Tr(\tilde{o}(ratioRate), 4) = $ *Between PS and PM* . The second part generates the interval $\Delta(\tilde{o}(ratioRate), 6) \in [2.5\%, 5\%]$ which verifies $Tr(\tilde{o}(ratioRate), 6) = PS$.

To build the quantification function, a first interpolation can be performed using the quantification at $t = 4$ given by the interval $q(ratioRate, 4) = [83\%, 86.8\%]$. Then, the quantification at $t = 6$ must be performed by applying $\Delta(\tilde{o}(ratioRate), 6) \in [2.5\%, 5\%]$ for each bound of $q(ratioRate, 4)$ and keeping the largest (resp. the lowest) bound of the resulting interval. It leads to $q(ratioRate, 6) = [85.07\%, 91.14\%]$. Finally, this second interval is used to build, by interpolation, the quantification function for $t \in [4, 6]$. The resulting quantification function is represented in Fig.4.
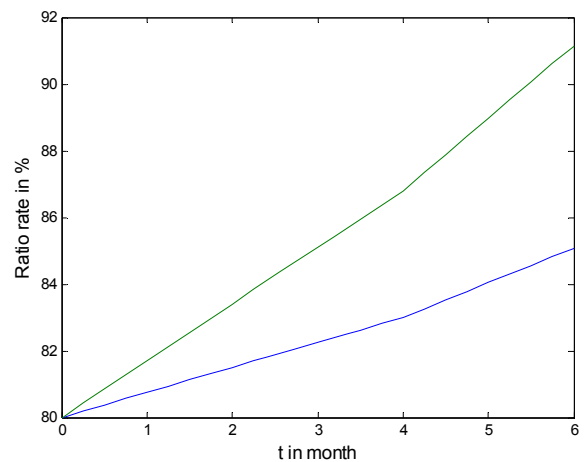
Figure 4: Quantification function for the objective "*The ratio rate trend should be between PM and PS after 4 months and then PS after the next 2 months*"

3 THE PERFORMANCE EXPRESSION

In previous works, both performance expressions and temporal trajectories were defined as follows [2]:

Definition5: Let $v \in V$ be a variable. $p(v,t)$ is the performance expression of the objective associated with v at the time t. The function p is called the performance expression trajectory.

The performance expression is obtained by directly comparing the objective to the measure. It becomes $p(v,t) = f(q(v,t), m(v,t))$ where f is the comparison function and m is the measurement function. Thus, $m(v,t)$ is the measured value for the variable v at the time t.

Let us assume that the objective is declared in terms of trends and precisiated as explained in the previous section. What can be said in terms of performance expression in such an objective declaration context is an interesting question. Two kinds of performance expression can thus be introduced, on the basis of the available pieces of information, *i.e*:

- the objective value, qualitatively expressed by the trend throughout the considered temporal horizon;
- the acquired measurement, generally expressed by a numerical value.

The first performance expression directly relies on the interval-based quantification function obtained from the trends as explained in the previous section. At any time, the crisp measurement is compared to the interval-based quantification, associated with the objective, *i.e.* $p(v,t) = f(q(v,t), m(v,t))$. The comparison function can be either based on distances or matching degrees [8]. The performance value $p(v,t)$ shows how far the measurement is from the imprecise objective which was expressed by trends.

The second performance expression can be obtained by comparing, at any time, the trend of the interval-based quantification to the trend of the measurement. This performance expression shows how far apart the two trends are. In other words, combining the two expressions, the decision-maker can know if the objective is reached and how it is reached.

In order to illustrate this idea, before handling the trend case, let us consider the precise objective declaration "*Increase the ratio rate by 15% after 6 months*". Fig.5 illustrates the quantification function associated with this objective and a possible measurement temporal trajectory. As can be observed, three specific cases can be emphasised:

- In the time interval [1.5, 2.5] the variation of the measurement is very close to the variation of the quantification function meaning that both trends are similar. Comparing the trends will lead to a "good" performance in terms of trends while the conventional performance, *i.e.*

$p(v,t) = f(q(v,t), m(v,t))$, will not be so "good" since the values are not so close.

- In time interval [2.8, 3.2] the variation of the measurement is greater that the variation of the quantification function. Therefore the performance in terms of trends is not "good" while the performance in terms of values is better than in the previous time interval. Especially, after time $t = 3.1$, the measurements are greater than the expected *ratio rate*.
- Finally, in the time interval [5, 6] both the variations and values are close together leading to "good" performances in terms of values and trends.
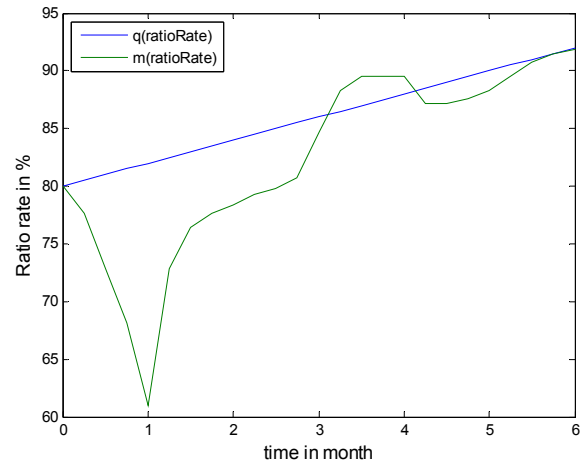

Figure 5: Quantification function and associated measurement temporal trajectory

Let us now consider the case where the objective is declared by trends. As has been shown, it leads to an interval-based quantification function. Therefore the variation in a time interval is itself an interval. In order to handle the performance in trends, this interval has to be compared with the variation of the measurement, which is still a scalar. Once again distances or matching degrees can be used. However, another interesting approach can be developed by directly comparing the trends, *i.e.* the fuzzy description of the variations, instead of comparing the variations themselves.

The comparison can be performed at the linguistic level using a so-called rule-base which, in other words, is the graph of the comparison function. An example of such a graph is given in table 1 where *VB*, *B*, *G*, *VG* respectively stand for *Very Bad*, *Bad*, *Good*, *Very Good*.

		Trend of the quantification						
		NB	NM	NS	Z	PS	PM	PB
Trend of the measurement	PB	VB	VB	VB	VB	B	G	VG
	PM	VB	VB	VB	B	G	VG	G
	PS	VB	VB	B	G	VG	G	B
	Z	VB	B	G	VG	G	B	VB
	NS	B	G	VG	G	B	VB	VB
	NM	G	VG	G	B	VB	VB	VB
	NB	VG	G	B	VB	VB	VB	VB

Table 1: Graph of the comparison function at the trend level

The comparison is represented by a function g, whose graph is provided by Table 1, for example, such that $p_{trend}(v,t) = g(Tr(q(v),t), Tr(m(v),t)) = g(A,B)$ with $p_{trend}(v,t)$ being a fuzzy subset of $L_3 = \{VB, B, G, VG\}$. Thanks to Zadeh's composition rule of inference, we have for all $l_3 \in L_3$:

$$p_{trend}(v,t) = C(l_3) = \perp_{(l_1,l_2) \in L_1 \times L_2} T(A(l_1), B(l_2), R(l_1, l_2, l_3))$$

with R the graph of g, $\perp$ a t-conorm and T a t-norm. Most often max-min operators are used as t-conorm and t-norm. It was shown in [7] that other couples of operators can be used. In particular, $\forall x, y \in [0,1] \times [0,1], \perp(x,y) = \min(x+y, 1)$ and $T(x,y) = x.y$ preserve Ruspini's fuzzy partitions [15] and lead to:

$$\forall l_3 \in L_3, C(l_3) = \sum_{(l_1,l_2) \in L_1 \times L_2} A(l_1).B(l_2).R(l_1, l_2, l_3) .$$

4 CONCLUSION

This study has focused on the way the objective can be handled when it is declared in a trend manner. Two aspects have been distinguished in this sense, the objective declaration on the one hand, and the performance expression on the other hand. The developed idea consists of a fuzzy processing of the decision-maker declarations in order to obtain symbolic or numerical values that are easy to use. This has led to the consideration of temporal trajectories that can be quantitative as well as qualitative, and consequently, more or less precise. As a corollary, the performance expression has been proposed in two forms. The first one is the result of numerical interval comparison, and the second one is more related to trend comparisons.

Fundamentally, the exercise of thinking about the way the objective is declared before looking at representing it or expressing its performance has allowed us to highlight the idea that all the performance expression requirements are imposed by the necessity of handling the temporal dimension of the objective. Works are hence in progress in order to generalise such handling in the performance expression, in particular with regards to the predictive and the aggregated performance expressions.

Another prospect of this work is to go deeper in the industrial application, leading thus to more specified analysis of the industrial practice with regards to the industrial practice.

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