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Decision support with ill-known criteria in the collaborative supply chain context

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A B S T R A C T

In the field of Supply Chain Risk Management, the attitude of managers toward risk affect the tactical decision-making process in collaborative supply chains under an uncertain environment, concerning especially capacity levels, lot-sizing rules, purchasing strategies, production scheduling,…, etc. The issue can be formulated as a sequential decision problem under uncertainty where the customer decisions affect the decisions made by the supplier. In this paper we deal with two kinds of uncertainties. The first one is the uncertainty on the indicators of performance (which are not comparable) used by the decision maker to choose a solution (for example: service quality or inventory cost). Hence, we propose an approach based on subjective probability to evaluate the probability that a decision is optimal for the first actor and the probability that it is optimal for both. From these two evaluations, we propose a ranking function to help the first actor to take into account the second one when selecting a decision. The second kind of uncertainty pertains to the demand. A classical criterion under total uncertainty is Hurwicz criterion where a weight expresses a degree of pessimism. Nevertheless, the degree of pessimism is itself ill-known. Thus, it becomes difficult to take into account the behavior of the actors. Hence, we propose an approach based on possibility theory and the so-called pignistic transform, which computes a subjective probability distribution over the criteria. Then, we apply the method used for uncertain criterion. This approach is illustrated through an example and an industrial case study.

1. Introduction

In an increasingly competitive business world, where the sources of disturbance are drastically changing and increasing, supply chains actors are faced with the necessity to constantly improve their decision-making practices. The companies identifying supply chain risk as "an unavoidable and necessary task that continues to pose certain problems" (Lavastre et al., 2012). Faced with a "networked environment", "companies deepen their relationship with partners and thus become more dependent on each other" (Hallikas et al., 2005). Risk, in the context of an enterprise, is defined by Zsidisin (2003) as: "the danger that events or decisions will obstruct the company's achievement of its objectives". In this context, "The process of supply chain actors main can either amplify or absorb the effect of risks in the supply chain" (Juttner, 2005) when the principal risk comes from supply and demand. Mastering the decision making processes of actors is therefore a key to minimizing the risks.

In this paper, we focus on the problem of designing the collaborative purchasing processes in the supply chain context under uncertainty. Moreover, we specifically consider a supplier-customer relationship in a dyadic supply chain where actors are independent. This situation may be described as a 2-actor sequential decision problem. For an industrial Decision-Maker (DM) in a supply chain, the anticipation of the decisional behavior of his/her partners is common practice (capacity level, lot sizing rules, purchasing strategies, production scheduling…). He/she knows that his/her decision will be followed by a series of partner's decisions, which will impact the performance of his decision. The decisional behaviors of independent partners are extremely difficult to anticipate. These potential different behaviors can be interpreted as different sources of uncertainty for a particular actor of the chain.

In this paper, we deal with two sources of uncertain behavior. First, we consider the uncertainty of an actor about the performance criteria of the other actor (for instance one considers the inventory level whereas the other the service quality), criteria that are not commensurate. Moreover, we deal with behaviors under uncertainty

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(the pessimism or optimism of decision maker) which comes down of the occurrence of uncertain events (scraps, breakdowns, delays, demand fluctuations…) for which a probability distribution may not be accessible. Hence, in the context of uncertainty, the criterion should take into account the level of pessimism of decision makers.

This paper is organized in 7 sections. First, a literature review on supply chains is made (Section 2), then we provide some background on decision trees, possibility theory, pignistic probability, criteria under uncertainty and multi-actors decision that will be used in our proposal (Section 3). In the fourth section, we present our approach to model and support the decision-making process with one DM and then we propose a model for sequential multi-DM problem (Sections 5 and 6). Finally, Section 7 illustrates this proposal through an industrial case study.

2. Literature review

In our review, we distinguish the literature that focuses on minimizing risk in supply chains under uncertainty and the literature on coordination mechanisms inside the supply chain.

On the first topic, mostly optimization approaches have been proposed with a global supply chain optimization model and a single decision maker (Liang and Cheng, 2009; Peidro et al., 2009; Wang and Shu, 2005; Petrovic et al., 1999). Besides there exist robust optimization methods where suppliers share information with the customer (Guillaume et al., 2012, 2013). In these studies, the sources of uncertainty pertain to the demand, supply and process. Nevertheless, predefined criteria are used and the problem of distributed decision is not investigated, whereas it creates risks since the decisions made by suppliers impact decisions made by the customer.

Besides, the studies on coordination mechanisms focus on the minimization of sub-optimality of the distributed decision in the supply chain. Hence, an important part of the literature proposes and studies coordination mechanisms to obtain the optimal decision for the supply chain as a whole. For example, game theory is used for designing coordination mechanisms that may optimize the distributed system and studying coordination mechanisms as a form of cooperative advertising; see Aust and Buscher (2014) for a recent review. On their side, Chen, 2007, 2012 and Li et al., 2005 focus on procurement policy (price-only policies, quantity discount policy, etc.). Recently, Xiao et al. (2014) added the lead time as decision variable, on top of the price in context of make to order supply chains. Furthermore, the supply chains studied in this literature are mostly manufacturer-retailer supply chains, and these studies do not consider the planning process. Another part of this literature proposes coordination mechanism when the actors use linear mathematic models for production planning (Dudek and Stadtler, 2005) under perfect demand. In this part of the literature, the criterion of decision makers is predefined and the uncertainty is not taken into account. Moreover, in addition to this important academic research works, empirical analysis based on industrial case studies and decision-maker interviews have emphasized the fact that conceptual research has focused on the supply disruption risk with a little attention to the questions:

(i) "How views of supply disruption risk are developed and how these views affect the decision-making process" (Ellis et al., 2010),
(ii) What are supply chain managers' attitudes toward risk?
(iii) What are the ways in which decisions are made? (Lavastre et al., 2012)?

Moreover, Singh and Benyoucef (2013) emphasize the role of decision-making processes inside collaborative supply chains. It shows the difficulty to establish decisions when confronted with conflicting individual interests and where "every company is responsible for its own risks and identifies the risks from its own viewpoint" (Hallikas et al., 2005).

3. Background

In this section, we recall formal tools we shall use to build the proposed approach.

3.1. Tools for decision under imprecision

In this section, we recall a model to represent the imprecision on the information (possibility distributions), how to derive a subjective probability from it (pignistic probability), a well-known criterion under total uncertainty.

3.1.1. Possibility distributions

Imprecise information is modeled by expressions of the form $v \in A$ where A is a subset of S that contains more than one element. Imprecision is always expressed by a disjunction of values (Dubois and Prade, 2009) that form a possibility distribution on S. The assertion $v \in A$ implies that all values from v outside A are supposed to be impossible.

A possibility distribution π_v attached to an ill-known quantity v quantifies the plausibility of values taken by v (Dubois and Prade, 1988). It is a function from S to a plausibility scale L ([0,1] for numerical possibility). A numerical possibility distribution taking a finite number of values $\lambda_i \in [0, 1]$, for $i=1,\ldots,M$, may express imprecise probabilistic knowledge of the form $P(E_i) \geq 1-\lambda_i$, $i=1, \ldots,M$, where E_i is a confidence set provided by the DM (Dubois and Prade, 2009). It can also be viewed as a random set $(m,F)_\pi$, with focal sets E_i and masses $m(E_i)$, such that:

$$\begin{cases} E_i = \{x \in S | \pi(x) \geq \lambda_i\} \\ m(E_i) = \lambda_i - \lambda_{i-1} \end{cases} \quad (1)$$

The possibility distribution is then such that: $\pi(x) = \sum_{x \in E_i} m(E_i)$ (Dubois and Prade, 1982).

3.1.2. Pignistic probability distribution

The so-called pignistic probability extends Laplace principle of insufficient reason to possibility theory and to belief functions. It presupposes the idea that, while the knowledge or an actor can be too imprecise to be represented by a single probability distribution, the latter is needed when evaluating decisions in order to comply with the classical (Savage) decision theory (Smets, 2005). This probability distribution reflects betting odds used by the actor possessing a certain body of information. When the actor has no information, all alternatives are viewed as equally possible and the actor will bet on them at equal odds. Deriving the pignistic probability from a belief function consists in equally sharing the masses (m,F) over each element of focal set E for a random set (m,F)

$$Pg^s(x) = \sum_{E \subseteq S} \frac{m(E)}{|E|} \forall x \in S \quad (2)$$

It can be viewed as the subjective probability distribution the decision-maker would provide via betting rates, had his knowledge been faithfully represented by the possibility distribution π_v. This probability distribution has been proposed by Dubois and Prade (1982) and axiomatized by Smets (2005), who coined it "pignistic". It coincides with the (older) Shapley value (Shapley, 1953) in the game theory. The pignistic probability distribution can be applied to possibility distributions and is also used in the simulation of "fuzzy variables" (Chanas and Nowakowski, 1988).

Example 1. For example, let $\Pi(c^1)=1$ and $\Pi(c^2)=0.8$ be a possibility distribution over two possible criteria. It expresses the incomplete information that the probability that the actor uses criterion c^1 is at least 0.2, while the other one is unknown. Let us compute the pignistic probability of each criterion. First, we compute the masses $m(E_i)$. In this case, the values of λ_i are discrete values: $\lambda_0=0$; $\lambda_1=0.8$; $\lambda_2=1$. Then:

- $E_1=\{c^1;c^2\}$ with $m(E_1)=0.8-0=0.8$
- $E_2=\{c^1\}$ with $m(E_2)=1-0.8=0.2$

From Eq. (6) we have: $Pg(c^1)=(m(E_1)/2)+(m(E_2)/1)=(0.8/2)+0.2=0.6$ and $(c^2)=(m(E_1)/2)=(0.8/2)=0.4$.

While in the finite case, letting the DM directly provide such subjective probability degrees may be possible, it is too difficult for a DM to provide precise continuous subjective probability. In that case, it is more user-friendly to ask for weak information (like support and mode, or confidence intervals), to represent it faithfully in possibility theory, and then to extract the pignistic probability from it.

3.1.3. Hurwicz criterion

Hurwicz (1951) criterion for decision under total uncertainty is a parametric criterion which takes into account the optimism of decision maker. Let $\mathbb{D}=\{d_1,..,d_n\}$ be the set of possible decisions, S the set of possible states of the world, X the set of the potential consequences of the decisions, $f_{d_i}(s)\in X$ the function defined for each decision $d_i\in\mathbb{D}$ which associates to each state $s\in S$ a precise consequence $x\in X$, $u(f_d(s))\in\mathbb{R}$ the utility function that attaches a value to each consequence $x\in X$ and α the degree of pessimism of decision maker. The Hurwicz criterion achives a trade-off between the most cautious and the most risk evaluations:

$$H_u(f_d)=\alpha\times\min_{s\in S}(u(f_d(s)))+(1-\alpha)\times\max_{s\in S}(u(f_d(s))) \tag{3}$$

$$d^*=\underset{d\in\mathbb{D}}{\operatorname{argmax}}(H_u(f_d)) \tag{4}$$

3.2. Tools for multi-actors decision making

In this section, we recall the model of sequential decision more precisely, namely decision trees and an egalitarist approach to multifactorial ranking of decisions: leximin.

3.2.1. Decision tree

In a real dynamic situation, the DM does not make a single decision, but a sequence thereof, according to the successive arrivals of relevant pieces of information. This type of problem is called *dynamic decision under uncertainty*. The decision made at time t depends on the information available at t. By hypothesis, the information known at time t is still known at time $t+\Delta t$. The incoming information is commonly viewed as "events". They are the results of an external independent entity, for example nature. In such conditions, we can call $\beta_t=\{e_t^1,\ldots,e_t^m\}$ and $\beta_{t+1}=\{e_{t+1}^1,\ldots,e_{t+1}^n\}$ the sets of mutually exclusive and exhaustive events at time t and $t+1$. The set β_{t+1} refines a partition of states induced by the set β_t. We call $\mathbb{D}=\{D_1,\ldots,D_T\}$ the set of decisions that have been made at various times, where decision D_t is made at time t.

This kind of problem has motivated many research works especially in the Artificial Intelligence literature (Nielsen and Jaffray, 2006; Jeantet et al., 2008; Jaffray and Jeleva, 2008). They are relevant in situations where a DM has a sequence of decisions (at prescribed times) to make. In this context, a strategy, called Δ, is defined as a particular choice of decisions (one decision per possible situation). The set of all strategies is denoted by $\overline{\Delta}$. The target is therefore to support the DM who must choose the best strategy, $\Delta^*=\underset{\Delta\in\overline{\Delta}}{\operatorname{argmax}}(u(\Delta))$. All decisions are known when the strategy is applied.

A Decision Tree (DT) is often used to represent this kind of decisions. A DT may be defined as a directed acyclic graph $T=(\mathcal{N},\mathscr{E})$ where $\mathcal{N}$ is the set of nodes and ε the set of arcs and there exists a unique node (root node), from which there is a single path leading to any other node. The set of nodes is made of (Nielsen and Jaffray, 2006):

- $\mathcal{N}_D$: the set of decision nodes (represented by squares). They characterize states where the DM has to decide and to choose one alternative among several ones. Each output arc of a decision node represents an alternative (some $d\in\mathbb{D}$);
- $\mathcal{N}_c$: the set of chance (or event) nodes (represented by circles). Event nodes represent the sources of uncertainty in the problem, i.e. states of nature. Each output arc of an event node shows a possible state of the world after the event has occurred (some $e\in\mathbb{E}$);
- C: the set of terminal nodes (leaves). A leaf is defined as a node without children ($child(N)=\varnothing,\forall N\in C$) and represents a terminal state of the sequential decision problem (a final consequence). A utility value is associated to each terminal node ($u(N),\forall N\in C$).

In a decision tree, a strategy Δ is therefore defined as a set of arcs: $\Delta=\{(N,N'):N\in\mathcal{N}_{D^\Delta},N'\in\mathcal{N}^\Delta\}\subseteq\mathscr{E}$ where $\mathcal{N}_{D^\Delta}=\mathcal{N}_D\cap\mathcal{N}^\Delta$ and $\mathcal{N}^\Delta\subseteq\mathcal{N}$ is the set of nodes involved in the strategy Δ, i.e. the set of nodes made of:

- The root node: N_r (a decision by hypothesis);
- A unique child for each decision made according to the strategy, i.e. $N\in\mathcal{N}_{D^\Delta}$;
- All the children of an event node met in the strategy, i.e. $N\in\mathcal{N}_{c^\Delta}=\mathcal{N}_c\cap\mathcal{N}^\Delta$.

We call $\overline{\Delta}^T$, the set of strategies in a given DT, T. An example of DT is given on Fig. 1.

It represents a decision situation where a DM has to decide at node D_1, then the event E_1 will occur, after what a second decision at D_2 will be made followed by a last event E_2. Formally:

- $\mathcal{N}_D=\{D_1,D_2\}$, with $\mathbb{D}_1=\{d_1^1,d_1^2\}$ and $\mathbb{D}_2=\{d_2^1,d_2^2\}$;
- $\mathcal{N}_c=\{E_1,E_2\}$, with $S_1=\{e_1^1,e_1^2\}$ and $S_2=\{e_2^1,e_2^2\}$;
- $\beta_1=\{e_1^1,e_1^2\}$ and $\beta_2=\{e_1^1\cap e_2^1,e_1^1\cap e_2^2,e_1^2\cap e_2^1,e_1^2\cap e_2^2\}$.

One example of strategy is pictured in Fig. 1 (in bold). Enumerating the strategies may become a very hard computational problem because of the complexity of the decision situation (the number of strategies increases exponentially). Different methods have been proposed to find the best strategy (Hammond, 1988; Machina, 1989; McClennen, 1990; Nielsen and Jaffray, 2006).

Considering a multiple actor sequential game, algorithms based on the *Dynamic Programming* principle, i.e., backward induction, have to be preferred to search and find (if they exist) equilibria in this kind of game (Cachon and Netessine, 2006).

3.2.2. Leximin criterion

In the next section we shall use the leximin criterion to build a ranking function taking into account the collective satisfaction of several decision-makers without favoring any of them.

Definition 1. (Barbera and Jackson, 1988) Let $u_j(\overline{d})$, $j\in\{1,\ldots,m\}$ be the levels of satisfaction of the DMj for decision $\overline{d}$, and $a\in[0,1]$,

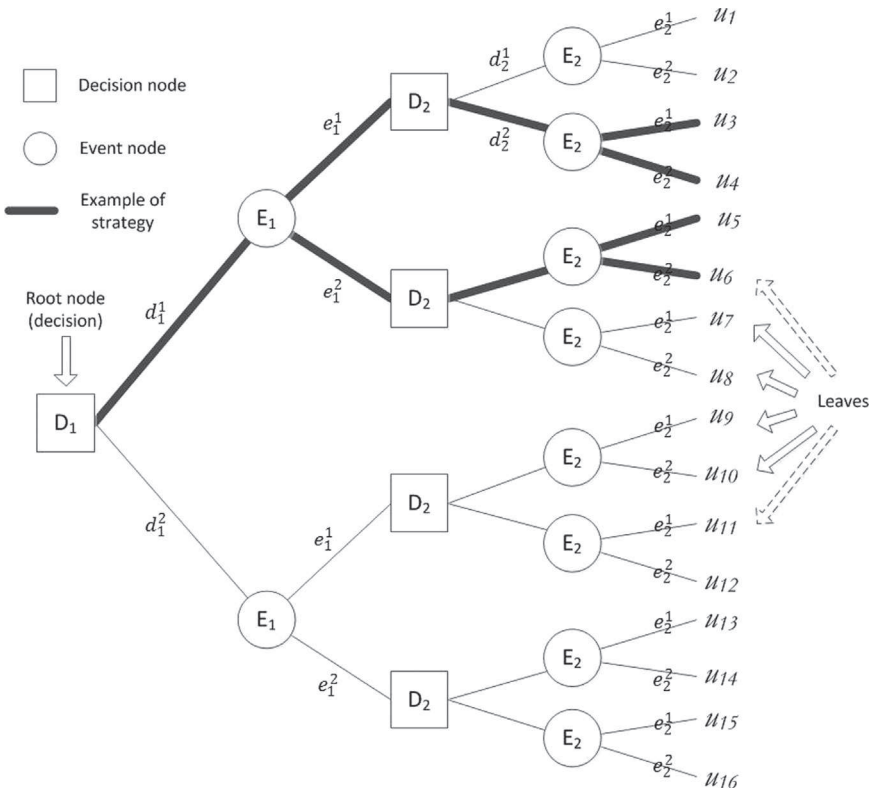


Fig. 1. Example of Decision Tree.

we define $J(a,\overline{d}) = \{j | u_j(\overline{d}) \leq a\}$ and $|J(a,\overline{d})|$ the cardinality of $J(a,\overline{d})$. We write $\overline{u} \succ_{Lm} \overline{v}$ if decision $\overline{u}$ is preferred to the decision $\overline{v}$ using the *leximin* criterion. The *leximin* is defined as follows (Eq. (5)):

$$\overline{u} \succ_{Lm} \overline{v} \leftrightarrow \exists \text{a such that } |J(a,\overline{u})| < |J(a,\overline{v})| \& \forall b < a | J(b,\overline{u})| = |J(b,\overline{v})| \quad (5)$$

This criterion can also be defined by first reordering components of vectors $\overline{u}$ and as $\overline{v}$ in increasing order, and then ranking the two rearranged vectors in lexicographic order (see also Dubois et al., 1996). The leximin optimal decision can be interpreted in the context of multi-actor decision-making as an egalitarist solution.

4. Ranking decisions taking into account uncertainty on criteria

In this section, we present how a DM can make decision under uncertainty about his/her own criterion. Here we focus on the case where the criteria are not commensurable so an aggregation of ratings is not allowed. In this context, we suppose that the DM is satisfied if the decision $d \in D$ is optimal and not satisfied otherwise. More formally, the utility function of DM is:

$$u(d) = \begin{cases} 1 & \text{if decision d is optimal} \\ 0 & \text{otherewise} \end{cases}$$

Moreover, we distinguish the case where the criteria are discrete (set of distinct criteria) and the case where the criterion has an imprecise parameter especially we focus on the case where the optimism degree of Hurwicz criterion is imprecise.

4.1. Discrete case

In the discrete case, we consider that the DM is able to give a subjective probability on each possible criterion $c \in C$ noted $Pg(c)$. The DM will take the decision that maximizes the expected value: $d^* = argmax_{d \in D}(\sum_{c \in C} Pg(c)u(d))$.

Example 2. $C = \{c^1, c^2, c^3\}$ with the subjective probability $Pg(c^1) = 0.4$, $Pg(c^2) = 0.3$ and $Pg(c^2) = 0.3$ and the set of decisions $D = \{d^1, d^2, d^3\}$. Decisions d^1 and d^3 are both optimal for the criterion c^1, d^2 is optimal for criteria $c^2$and c^3. So, the evaluation for each decision is: $0.4 \times 1 + 0.3 \times 0 + 0.3 \times 0 = 0.4$ for d^1, $0.4 \times 0 + 0.3 \times 1 + 0.3 \times 1 = 0.6$ for d^2 and 0.4 for d^3. The optimal decision is d^2.

4.2. Model of imprecise degree of optimism

The model is based on the hypothesis that DM is able to give possibility distributions on the value of α: a possibility distribution $\tilde{\alpha}$ on his/her degree of optimism. Those possibility distributions can model vague adjectives like: *pessimistic*, *optimistic*, *neutral* (see Fig. 2(a)) or more precise as *very pessimistic*, *little pessimistic*, *strongly neutral*, (see Fig. 2(b)). Another way is to ask for the most plausible value of α and the maximal interval that contains α. We can then build a triangular possibility distribution with these three values (see Fig. 2(c)).

Based on these possibility distributions and the study of change decision point (see Fig. 3), we can evaluate the stability of decisions (which in our approach comes down to computing the pignistic probability of being optimal). More precisely, we first build the pignistic probability distribution from the possibility distribution (cf Section 3.1.2), then compute the pignistic probability for each decision to be optimal. Namely, $Pg(d = optimal) = Pg(\alpha \epsilon [\underline{\alpha_d}; \overline{\alpha_d}])$ with $[\underline{\alpha_d}; \overline{\alpha_d}]$ the interval of values of α where d is optimal. As before, DM will choose the decision that maximizes the expected value: $d^* = argmax_{d \in D}(Pg(d = optimal)u(d))$.

Example 3. consider two decisions d^1 and d^2 with respectively the minimal value equal 10 and 8 and maximal equal 25 and 30. So the

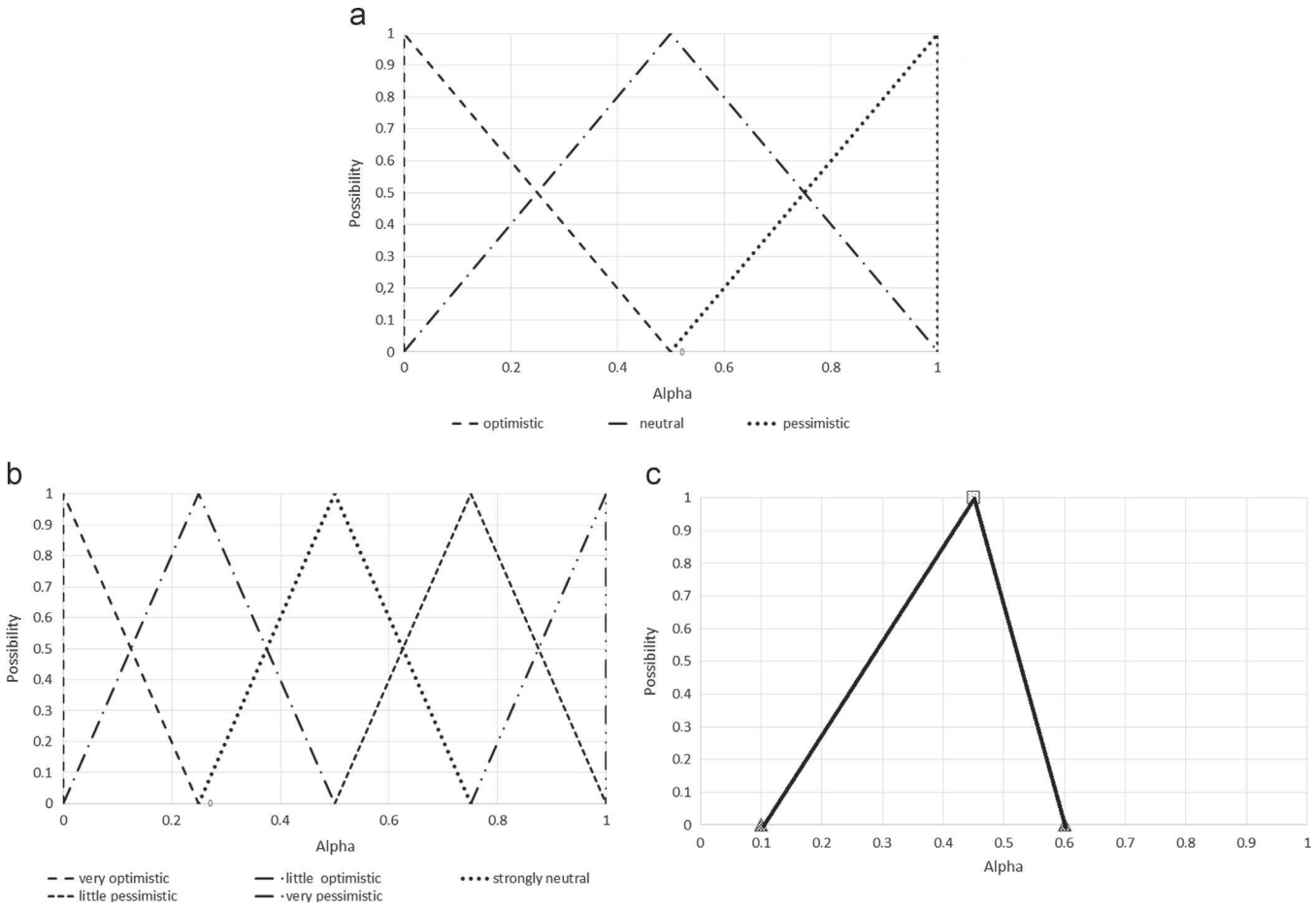


Fig. 2. Choice of the possibility distribution.

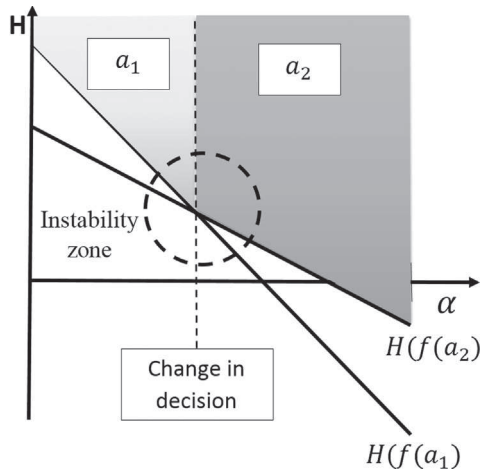


Fig. 3. Decision using Hurwicz criterion.

Hurwicz criteria of both decisions yield $H_u(d^1) = \alpha \times 10 + (1-\alpha) \times 25$ and $H_u(d^2) = \alpha \times 8 + (1-\alpha) \times 30$ (see Fig. 4).

If $\alpha \in [(5/7); 1]$ the decision d^1 is optimal so $[\underline{\alpha_{d^1}}; \overline{\alpha_{d^1}}] = [(5/7); 1]$ and $Pg(\alpha \epsilon [(5/7); 1]) \cong 0.9$.

If $\alpha \in [0; (5/7)]$ the decision d^2 is optimal so $[\underline{\alpha_{d^2}}; \overline{\alpha_{d^2}}] = [0; (5/7)]$ and $Pg(\alpha \epsilon [0; (5/7)]) \cong 0.1$.

Hence, the utility of DM for decision d^1 is 0.9 and 0.1 for d^2. Finally, DM chooses d^1.

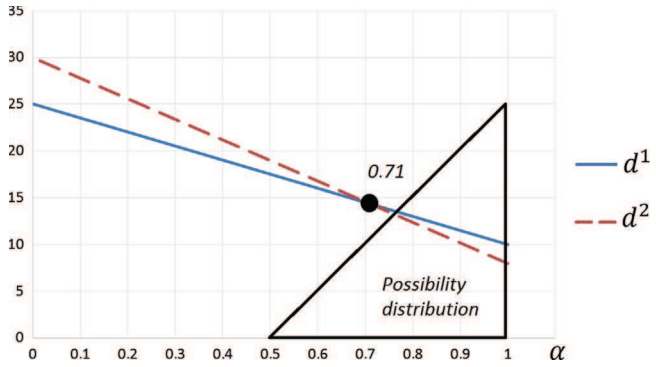


Fig. 4. Decision under imprecise optimism degree.

5. Sequential decision problem taking into account uncertainty on criteria

In this section, we first introduce the main principles of our method of ranking decisions in a game with two players when (i) DM_1 makes his decision before DM_2 and (ii) DM_1 only partially knows his/her own criterion and likewise for DM_2 criterion. Moreover, we consider that under uncertainty on the criteria, the DM focuses on optimal decisions for each criterion. In other words, a decision may satisfy the DM if it is optimal for at least one criterion. Otherwise the decision is not considered.

We adopt the following notations:

- C^i: set of criteria c^i of DM_i with $i = 1, 2$
- D^1: set of decisions d^1 of DM_1

- D_j^2: set of decisions d_j^2 of DM_2 such that DM_1 chooses decision j, j being the index of decision node of DM_1 with $j = 1, \ldots, J$ and $J = |D^1|$
- $D = D_1^2 \times \ldots \times D_J^2$: set of decision vectors $\overrightarrow{d^2} = (d_1^2, \ldots, d_J^2)$ of DM_2
- $C_2(\overrightarrow{d^2})$: set of criteria $c^2 \in C^2$ for which decision vector $\overrightarrow{d^2}$ is optimal.
- $C_{1,2}(d^1, \overrightarrow{d^2})$: set of pairs of criteria (c^1, c^2) for which d^1 is optimal for $c^1 \in C^1$ and $\overrightarrow{d^2}$ is optimal for $c^2 \in C^2$.
- $C_1(d^1)$: set of pairs of criteria (c^1, c^2) for which decision d^1 is optimal
- $C_2^1(d^1)$: set of criteria $c^2 \in C^2$ for which decision d^1 is the best decision of DM_1 for the DM_2 using c^2
- $Pg(c^i)$: Pignistic probability of criterion c^i
- $Pg(d^1|DM_1)$: is the pignistic probability that d^1 is an optimal decision from the point of view of DM_1. The optimality of the decision depends on the criterion $c^1$of DM_1 and the criterion $c^2$of DM_2. It depends on the probability of using $c^1$and c^2. So $Pg(d^1|DM_1)$ is the sum of the probabilities of pairs (c^1, c^2) for which d^1 is optimal.
- $Pg(d^1|DM_2)$: is the pignistic probability that d^1 is an optimal decision from the point of view of DM_2. In other terms, the pignistic probability induced when the choice of d^1 is optimal for the DM_2.
- $u_i(d) = \begin{cases} 1 & \text{if decision d is optimal} \\ 0 & \text{otherwise} \end{cases}$ the utility function for DM_i with $d \in D^i$

5.1. Overview of the approach

To evaluate the probability that the decision d is optimal in front of the possible criteria, we use the concept of pignistic probabilities (i.e. Section 3.1.2). Indeed the decision that has the maximal expected value to be optimal (using pignistic probability) is the one that is most likely to be optimal, taking into account the uncertainty on the criterion. Note that the expected value of DM_i, $E_i[d]$, with utility function $u_i(d)$, is equivalent to the pignistic probability that d is an optimal decision from the point of view of DM_i: $Pg(d|DM_i)$. So in the next part of the paper we will use the notation $Pg(d|DM_i)$ to refer to this expected value.

Since the decision of DM_1 affects the possible decision of DM_2, we propose to evaluate the decision of DM_1 using the points of view of DM_1 and DM_2. Formally we assign a pair of values(a_{d^1}, b_{d^1}) to each decision $d^1 \in D^1$ where $a_{d^1} = Pg(d^1|DM_1)$ and $b_{d^1} = Pg(d^1|DM_2)$.

In order to compute the pignistic probability that decision is optimal, we have to know for which criteria this decision is optimal and then to sum the pignistic probabilities of these criteria. Moreover, we propose to rank the decisions using the importance given by DM_1 to DM_2. This importance is linked to the kind of relationship among DM_1 and DM_2. For instance DM_1 will grant a high level of importance to her/his strategic or/and constraining partners whereas his/her non strategic and/or dependant partners will have a low level of importance (Marcotte et al., 2009).

5.2. Evaluation of decisions under uncertain criteria

The problem of computing the chance of optimality for DM_1 and DM_2 can be computed in 6 steps:

- **Step 1.** Computation of $C_2(\overrightarrow{d^2})$ for each vector $\overrightarrow{d^2} \in D$, as follows:

$$C_2(\overrightarrow{d^2}) = \{c^2 | \overrightarrow{d^2} \text{ is optimal}\} \quad (6)$$

- **Step 2.** Computation of $C_{1,2}(d^1, \overrightarrow{d^2})$ for each vector $\overrightarrow{d^2} \in \{\overrightarrow{d^2} | C_2(\overrightarrow{d^2}) \neq \varnothing\}$ and each decision $d^1 \in D^1$, as follows:

$$C_{1,2}(d^1, \overrightarrow{d^2}) = \{(c^1, c^2) | d^1 \text{ and } \overrightarrow{d^2} \text{ are optimal}\} \quad (6)$$

- **Step 3.** Computation of $C_1(d^1) = \cup_{\overrightarrow{d^2} \in D} C_{1,2}(d^1, \overrightarrow{d^2})$ for each $d^1 \in D^1$
- **Step 4.** Computation of $Pg(d^1|DM_1)$ as follows:

$$Pg(d^1|DM_1) = \sum_{(c^1,c^2) \in C_1(d^1)} Pg(c^1) \times Pg(c^2) \quad (8)$$

- **Step 5.** Computation of $C_2^1(d^1)$ for each decision $d^1 \in D^1$as follows:

$$C_2^1(d^1) = \{c^2 | d^1 = argmin_{d^1 \in D^1}(min_{d^2 \in D_{d^1}^2} c^2(d^1, d^2))\} \quad (9)$$

- **Step 6.** Computation of $Pg(d^1|DM_2)$ as follows:

$$Pg(d^1|DM_2) = \sum_{c^2 \in C_2^1(d^1)} Pg(c^2) \quad (10)$$

5.3. Taking into account the importance given to the point of view of DM_2

To help ssthe DM_1 choose a decision, we propose to rank the possible optimal solutions according to the importance given to the point of view of DM_2. If the importance given by DM_1 to DM_2 is very low, then she/he chooses the decision which maximizes her/his probability to be optimal for her/him. On the contrary, if the importance given by DM_1 to DM_2 is very high, then she/he chooses the decision which maximizes the probability for DM_2. Between these extremes, different levels of importance will be considered. More formally let f_2^1 be the utility of a decision of DM_1 from the point of view of DM_2 (the latter being measured by $Pg(d^1|DM_2)$).

The function f_2^1 is a mapping $Pg(d^1|DM_2) \in [0;1] \mapsto [-1;1]$ that has to respect two requirements:

- If DM_1 grants very low importance to DM_2:

$$Pg(d^1|DM_1) > f_2^1(Pg(d^1|DM_2)),\ \forall Pg(d^1|DM_1) \in [0;1]$$

- If DM_1 grants the same importance to DM_2 as to himself:

$$Pg(d^1|DM_2) = f_2^1(Pg(d^1|DM_2)),\ \forall Pg(d^1|DM_2),\ \in [0;1]$$

To respect these characteristics, we propose that $f_2^1(Pg(d^1|DM_2)) = Pg(d^1|DM_2) + s_2^1$ with $s_2^1 \in [-1;1]$ such that if $s_2^1 = 1$ the importance given by DM_1 to DM_2 is very low otherwise if $s_2^1 = -1$ then the importance given by DM_1 to DM_2 is very high.

In order to find the decision that ensures equity between the two decision makers DM_1 and DM_2 taking into account the importance given by DM_1 to DM_2, we propose to use the leximin criterion on the vector $\overline{v} = \langle Pg(d^1|DM_1); f_2^1(Pg(d^1|DM_2))\rangle$. There are sevsseral cases:

- if $s_2^1 = 1$ (very low importance given by DM_1 to DM_2): we maximize first $Pg(d^1|DM_1)$ and then $f_2^1(Pg(d^1|DM_2))$ such that $Pg(d^1|DM_1) \leq Pg(d^1|DM_2) + 1,\ \forall\ Pg(d^1|DM_1),\ Pg(d^1|DM_2) \in [0,1]$
- if $s_2^1 = 0$ (no difference between $DM_1$and DM_2): we maximize indifferently the minimum between $Pg(d^1|DM_1)$ and $Pg(d^1|DM_2)$ and the other

- if $s_2^1 = -1$ (very high importance given by DM_1 to DM_2): we maximize first $f_2^1(Pg(d^1|DM_2))$ and then $Pg(d^1|DM_1)$ such that $Pg(d^1|DM_2) - 1 \le Pg(d^1|DM_1)$, $\forall Pg(d^1|DM_1), Pg(d^1|DM_2) \in [0,1]$.

Note that we do not ask the decision-maker DM_1 for parameter s_2^1. It is just an artifact of the method. By letting parameter s_2^1 range over its domain, we can explain whether a decision made by DM_1 is likely to be accepted or not by DM_2 in all situations. For instance:

- If $\forall s_2^1 \in [-1;1]$ optimal decision is the same then this decision will be accepted by the two DMs.
- If they exist an optimal decision (d^{1*}) for $s_2^1 \in [-1;a]$ with a close to -1 and other one (d^{2*}) for $s_2^1 \in [a;1]$ the decision d^{1*} will be to be accepted by DM_2.

5.4. Example

We illustrate the method in a general context, where DM_1 does not know if DM_2 will take him/her decision according to the minmax criterion (with probability 0.6) or Laplace's weighted average (with probability 0.4) on the indicator $f(d^1, d^2, n)$ and DM1 hesitates between the utility functions $g(d^1, d^2, n)$ (with probability 0.7) and $h(d^1, d^2, n)$ (with probability 0.3) within the minmax criteria:

- $C^2 = \{minmax; Laplace\}$
- $C^1 = \{g; h\}$

5.4.1. Evaluation of decision under uncertain criteria

DM_1 has 2 possible decisions {1; 2} and DM_2 has two possible decisions {one, two} and nature induces three possible realizations {a, b, c}. The evaluation of decision strategies is represented in Table 1 and Fig. 5.

To solve this problem, we apply the method presented in Section 5.2

- **Step 1.** Computation of $C_2(\overrightarrow{d^2})$ for each vector $\overrightarrow{d^2} \in D$

  We have 4 vectors $\overrightarrow{d^2}$: $\langle one, one\rangle, \langle one, two\rangle, \langle two, one\rangle, \langle two, two\rangle$. For this 4 vectors we compute $C_2(\overrightarrow{d^2})$:
  1. $C_2(\langle one, one\rangle) = \varnothing$
  2. $C_2(\langle one, two\rangle) = \{minmax\}$
  3. $C_2(\langle two, one\rangle) = \{Laplace\}$
  4. $C_2(\langle two, two\rangle) = \varnothing$
- **Step 2.** Computation of $C_{1,2}(d^1, \overrightarrow{d^2})$ for each vector $\overrightarrow{d^2} \in \{D | C_2(\overrightarrow{d^2}) \neq \varnothing\}$ and each $d^1 \in D^1$

  We have 4 combinations $(d^1, \overrightarrow{d^2})$ with $C_2(\overrightarrow{d^2}) \neq \varnothing$

  $(1, \langle one, two\rangle), (2, \langle one, two\rangle), (1, \langle two, one\rangle)$ and $(2, \langle two, one\rangle)$

  For this 4 combinations we compute $C_{1,2}(d^1, \overrightarrow{d^2})$:
  1. $C_{1,2}(1, \langle one, two\rangle) = \{(h; minmax)\}$
  2. $C_{1,2}(2, \langle one, two\rangle) = \{(g; minmax)\}$
  3. $C_{1,2}(1, \langle two, one\rangle) = \{(g; Laplace), (h; Laplace)\}$
  4. $C_{1,2}(2, \langle two, one\rangle) = \varnothing$
- **Step 3.** Computation of $C_1(d^1) = \cup_{\overrightarrow{d^2} \in D} C_{1,2}(d^1, \overrightarrow{d^2})$ for each $d^1 \in D^1$
  1. $C_1(1) = \left\{ \begin{matrix} (h; minmax), (g; Laplace), \\ (h; Laplace) \end{matrix} \right\}$
  2. $C_1(2) = \{(g; minmax)\}$
- **Step 4.** Computation of $Pg(d^1|DM_1)$
  1. $Pg(1|DM_1) = \sum_{C_1(1)} Pg(c^1) \times Pg(c^2) = 0.3 \times 0.6 + 0.7 \times 0.4 + 0.3 \times 0.4 = 0.58$
  2. $Pg(2|DM_1) = \sum_{C_1(2)} Pg(c^1) \times Pg(c^2) = 0.7 \times 0.6 = 0.42$
- **Step 5.** Computation of $C_2^1(d^1)$for each decision $d^1 \in D^1$

  First we compute $min_{d^2 \in D^2_{d^1}} c^2(d^1, d^2)$ for $d^1 \in D^1$ and $c^2 \in C^2$
  1. $min_{d^2 \in D_1^2} \max_{n \in N} f(1, d^2, n) = 10$
  2. $min_{d^2 \in D_2^2} \max_{n \in N} f(2, d^2, n) = 15$
  3. $min_{d^2 \in D_1^2} \sum_{n \in N} \frac{f(1,d^2,n)}{|N|} = 7$
  4. $min_{d^2 \in D_2^2} \sum_{n \in N} \frac{f(2,d^2,n)}{|N|} = 10$

  Then we compute:
  1. $argmin_{d^1 \in D^1}(min_{d^2 \in D^2_{d^1}} \max_{n \in N} f(1, d^2, n))$
     $= argmin(min_{d^2 \in D_1^2} \max_{n \in N} f(1, d^2, n); min_{d^2 \in D_2^2} \max_{n \in N} f(2, d^2, n))$
     $= argmin(10; 15) = 1$
  2. $argmin_{d^1 \in D^1}\left(min_{d^2 \in D^2_{d^1}} \sum_{n \in N} \frac{f(1,d^2,n)}{|N|}\right)$
     $= argmin\left(min_{d^2 \in D_1^2} \sum_{n \in N} \frac{f(1,d^2,n)}{|N|}; min_{d^2 \in D_2^2} \sum_{n \in N} \frac{f(2,d^2,n)}{|N|}\right)$
     $= argmin(7; 10) = 1$

  From the previous results we build $C_2^1(1)$ and $C_2^1(2)$:
  1. $C_2^1(1) = \{minmax, Laplace\}$
  2. $C_2^1(2) = \varnothing$

Table 1
Evaluation of the decision strategies.

DM_1	DM_2	$f(d^1, d^2, n)$		$g(d^1, d^2, n)$	$h(d^1, d^2, n)$
		max	Laplace	max	max
1	one	10	8	10	12
	two	14	7	11	10
2	one	20	10	14	11
	two	15	12	9	15

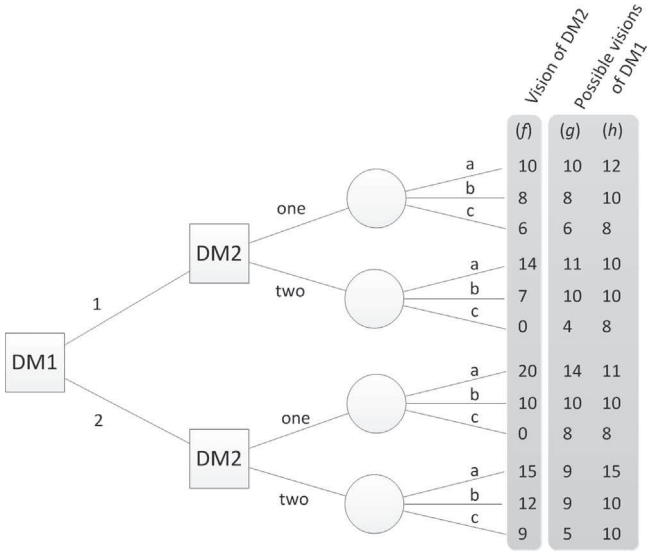


Fig. 5. DT of the example.

Table 2
Results taking into account the importance given to the DM_2.

Optimal decision	$d=1$	$d=1$	$d=1$	$d=1$	$d=1$	$d=1$	$d=1$	$d=1$	$d=1$	$d=1$	$d=1$	$d=1$	$d=1$	$d=1$	$d=1$	$d=1$	$d=1$	$d=1$	$d=1$	$d=1$	$d=1$
s_2^1	−1	−0.9	−0.8	−0.7	−0.6	−0.5	−0.4	−0.3	−0.2	−0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$d=1$	0	0.1	0.2	0.3	0.4	0.5	0.58	0.58	0.58	0.58	0.58	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
	0.58	0.58	0.58	0.58	0.58	0.58	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
$d=2$	−1	−0.9	−0.8	−0.7	−0.6	−0.5	−0.4	−0.3	−0.2	−0.1	0	0.1	0.2	0.3	0.4	0.4	0.4	0.4	0.4	0.4	0.4
	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.4	0.4	0.4	0.4	0.5	0.6	0.7	0.8	0.9	1

- **Step 6.** Computation of $Pg(d^1|DM_2)$
  1. $Pg(1|DM_2)=\sum_{c^2\in C_2^1(1)}Pg(c^2)=0.7+0.3=1$
  2. $Pg(2|DM_2)=\sum_{c^2\in C_2^1(2)}Pg(c^2)=0$

5.4.2. Ranking of decisions taking into account the importance given to DM_2

In this example, we have $Pg(1|DM_1)=0.58$; $Pg(2|DM_1)=0.42$; $Pg(1|DM_2)=1$ and $Pg(2|DM_2)=0$.

To help DM_1 rank the decisions in terms of the importance given by DM_1 to DM_2, s_2^1 evolves from -1 (very low importance) to 1 (very high importance). In order to summarize the knowledge and therefore to visualize possible change in the decision-making, we collect information in a table (Table 2). On the first line, you find the optimal solution depending of the value of s_2^1 recalled in line two. Then we give for each decision $d^1\in D^1$ the vector rearranged in increasing order.

From Table 2, DM_1 chooses decision 1 whatever the importance he/she gives to DM_2 (best likelihood to be optimal for both DMs).

6. The case of imprecise optimism degrees

In this section, we consider the case of Hurwicz criterion with imprecise value of optimism degree α. We describe how to compute the sets $C_2(\overrightarrow{d^2})$, $C_{1,2}(d^1,\overrightarrow{d^2})$ and $C_2^1(d^1)$, in this context. We detail some steps of this method when (i) both DM_1 and DM_2 criteria are Hurwicz criteria with respectively $\tilde{\alpha^1}$ and $\tilde{\alpha_1^2}$ the fuzzy optimism degree of this criterion.

6.1. Determination of $C_2(\overrightarrow{d^2})$ (step 1)

In this section, we give the framework of the algorithm to compute $C_2(\overrightarrow{d^2})$:

- **Step 1.1.** Computation of the value of $\tilde{\alpha_1^2}$ for which decision d_j^2 changes, denoted by $\tilde{\alpha_{change}^2}$, for each node of decision of DM_2, (cf: Fig.3)
- **Step 1.2.** Computation of the set of $\tilde{\alpha_1^2}$ such that vector $\overrightarrow{d^2}$ is optimal for DM_2: $C_2(\tilde{d^2})$

The maximal cardinality of $C_2(\tilde{d^2})$ appears when all decisions are optimal for a given α^2 and each α_{change}^2 are different for each decision nodes of DM_2. Thus, in the worst case, we have $|D^1|\times|D^2|$ sets $C_2(\overrightarrow{d^2})$.

6.2. Determination of $C_{1,2}(d^1,\overrightarrow{d^2})$ (step 2)

After determining all $C_2(\overrightarrow{d^2})$, we compute the set $C_{1,2}(d^1,\overrightarrow{d^2})$ for each $d^1\in D^1$. The outline of the algorithm is:

- **Step 2.1.** Computation of the value of α^1 for which decision d^1 changes, denoted by α_{change}^1, for each $\overrightarrow{d^2}$ such that $C_2(\overrightarrow{d^2})\neq\varnothing$, (see Fig. 4 Section 4.2)
- **Step 2.2.** Computation of the set of α^1 such that $\overrightarrow{d^2}$ is the optimal vector of DM_2 and d^1 is the optimal decision of DM_1: $C_{1,2}(d^1,\overrightarrow{d^2})$

In the worst case, we must compute $|D_1|$ for each $C_2(\overrightarrow{d^2})\neq\varnothing$ so at most $|D^1|^2\times|D^2|$ $C_{1,2}(d^1,\overrightarrow{d^2})$.

6.3. Determination of $C_2^1(d^1)$ (step 5)

In this section we present the algorithm to compute the set of criteria giving an optimal solution the decision d^1 of DM_1. This algorithm the result of $C_2(\overrightarrow{d^2})$

- **Step 5.1.** Computation, for each vector $\overrightarrow{d^2}\in C_2(\overrightarrow{d^2})$, of the value of α^2 for which decision d^1 changes, denoted by α_{change}^1, for each $\overrightarrow{d^2}$ such that $C_2(\overrightarrow{d^2})\neq\varnothing$, (see Fig. 4 Section 4.2)
- **Step 5.2.** Computation of the set of α^1 such that $\overrightarrow{d^2}$ is optimal vector of DM_2 and d^1 is optimal decision of DM_1: $C_{1,2}(d^1,\overrightarrow{d^2})$ using Eq. (11)

$$C_2^1(d^1)=\{c^2|d^1=argmin_{d^1\in D^1}(min_{d^2\in D_{d^1}^2}u_{c^2}(d^1,d^2))\} \quad (11)$$

7. Application of the method to an industrial case-study

In this section, we apply the method on a real-life case based on a dyadic supply chain where the customer, a French worldwide dermo-cosmetic maker (Pierre Fabre Dermo-Cosmétique), has to choose a collaboration protocol (2 possibilities) with its packaging product supplier. According to the traditional collaboration protocol the customer has to release orders (a product, a quantity) and the supplier responds. A DM's decision variable is the order lead time (here 12, 8 or 6 weeks). With the advanced collaboration protocol the customer commits on purchases associated to a family of products 8 weeks in advance (product family aggregation is related to supplier's set up considerations). Then, the customer releases delivery needs about the product 1 week in advance. A DM's decision lever is the minimal volume associated to the family engagement (here 50,000, 100,000 or 150,000 products).

7.1. Problem modeling

According to the notation defined in previous parts, we denote by DM_1 the customer (PFDC) and by DM_2 one of his packaging suppliers. Two sequential decisions have to be made:

- DM_1 has to define the collaboration protocol and its parameter (6 possibilities) (Table 3),
- then, DM_2 will define his lot sizing strategy (3 possibilities).

Table 3
Decision protocols for DM1's decisions.

Notation	Protocol decision	Parameter decision
1	Advanced collaboration	Low volume (50,000)
2	Advanced collaboration	Medium volume (100,000)
3	Advanced collaboration	High volume (150,000)
4	Basic order	Little order lead time (6w)
5	Basic order	Medium order lead time (8w)
6	Basic order	Big order lead time (12w)

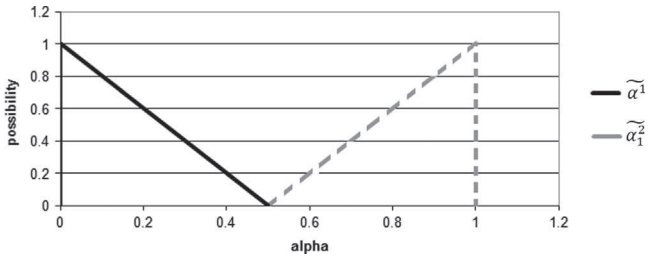


Fig. 6. Fuzzy optimism degree of DM_1 and DM_2.

In addition, the performance of the supply chain will be subject to a global uncertain event that models the uncertainty of the performance due to different risk sources (scrap, production/transport delay, breakdowns…) (7 possible situations).

According to the number of scenarios that have to be evaluated, we use a simulation tool called LogiRisk for the evaluation of each scenario (each leaf of the tree) on real data from PFDC (Marques, 2010). Developed in the Perl language, it is dedicated to tactical and mostly strategic SC planning processes. This simulator is based on a discrete event simulation modeling approach. Authors have established a generic representation of the different planning processes for each SC actor based on the MRPII (Manufacturing Resource Planning) processes. An upstream planning process is used between partners: plans are made by the customer and passed to its suppliers. The procedure is repeated all over the chain in the upstream direction. No information circulates downstream (Lamothe et al., 2007, Marques et al., 2009).

The customer's cost function is 2/3 average customer's stock-out 1/3 average customer's stock and supplier's cost function is 1/2 average supplier's stock-out 1/2 average supplier's inventory level.

7.2. Problem solving

The customer provides two possibility distributions on the optimism degree of himself/herself and on the supplier. The optimism degrees are represented in Fig. 6. Here, the DM_1 is optimistic (black line) and the DM_2 is known to be pessimistic (dotted line) by DM_1.

From the simulation we build the decision tree (with 6 decisions for DM_1 {1;2;3;4;5;6} and 3 decisions for DM_2 {1;2;3}) and the cost function for each DM (DM_1: customer's cost and DM_2: supplier's cost).

7.3. Determination of $C_2(\overrightarrow{d^2})$ (step 1)

Decision 1 of DM_2 is Pareto-optimal for all decisions of DM_1. In other words, decision 1 has the minimal "min" and minimal "max" for each decision of DM_1. So, whatever the optimism degree of DM_2, DM_2 chooses decision 1 for each node.

$$C_2((1,1,1,1,1,1)) = [0.5;1] \; \forall \overrightarrow{d^2} \neq (1,1,1,1,1,1) \; C_2(\overrightarrow{d^2}) \neq \varnothing$$

7.4. Determination of $C_{1,2}(d^1, \overrightarrow{d^2})$ (step 2)

Then we compute the set $C_{1,2}(d^1,(1,1,1,1,1,1)) \; \forall d^1 \in D^1$. Whatever the optimism degree of DM_1, decisions 1,4,5,6 can be chosen:

$$\forall d^1 \neq 2,3 \; C_{1,2}(d^1,(1,1,1,1,1,1)) \neq \varnothing$$

DM_1 has two possible optimal solutions: solution 2 and 3.

So we compute α^1_{change} (Fig. 7): $C_{1,2}(2,(1,1,1,1,1,1)) = [0;0.429]$ and $C_{1,2}(3,(1,1,1,1,1,1)) = [0.429;0.5]$.

7.5. Computation of $C_1(d^1)$ (step 3)

In this example: $C_1(d^1) = C_{12}(d^1,(1,1,1,1,1,1)) \; \forall d^1 \in D^1$:

$$C_1(2) = C_{12}(2,(1,1,1,1,1,1)) = [0;0.429]$$

$$C_1(3) = C_{12}(3,(1,1,1,1,1,1)) = [0.429;0.5]$$

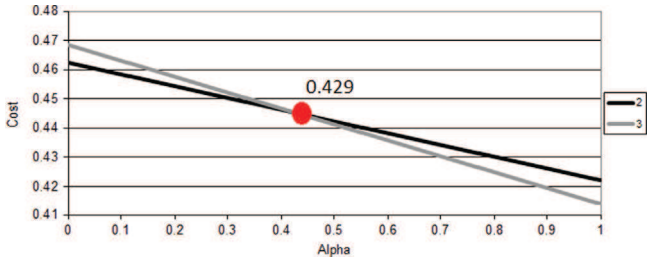


Fig. 7. Comparative analysis of decisions 2 and 3.

Table 4
Data of decision tree of the case-study.

DM_1	DM_2	Supplier's cost		Customer's cost	
		min	max	min	max
1	1	7.175	7.696	0.471	0.537
	2	14.516	17.563	0.415	0.475
	3	20.436	25.396	0.411	0.453
2	1	6.022	6.907	0.422	0.462
	2	13.078	14.34	0.380	0.425
	3	18.92	21.57	0.375	0.414
3	1	5.905	6.956	0.414	0.468
	2	12.975	14.734	0.382	0.420
	3	18.267	21.257	0.374	0.412
4	1	6.177	7.272	0.547	0.656
	2	11.862	14.444	0.505	0.605
	3	17.268	20.824	0.478	0.554
5	1	6.427	6.946	0.571	0.622
	2	12.131	13.985	0.567	0.624
	3	17.540	20.445	0.542	0.639
6	1	7.307	7.549	0.765	1.009
	2	13.010	14.628	0.763	1.009
	3	18.968	21.294	0.765	1.008

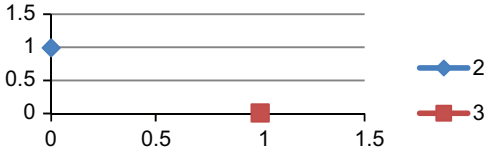


Fig. 8. Conflict of decisions.

Table 5
Results taking into account the importance given to DM_2.

Optimal decision	$d=3$	$d=3$	$d=3$	$d=3$	$d=3$	$d=3$	$d=3$	$d=3$	$d=3$	$d=3$	$d=3$	$d=2$	$d=2$	$d=2$	$d=2$	$d=2$	$d=2$	$d=2$	$d=2$	$d=2$	$d=2$
s_2^1	−1	−0.9	−0.8	−0.7	−0.6	−0.5	−0.4	−0.3	−0.2	−0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$d=2$	−1	−0,9	−0.,8	−0.7	−0.6	−0.5	−0.4	−0.3	−0.2	−0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$d=3$	0	0.02	0.02	0.02	0.02	0,02	0.02	0.02	0.02	0.02	0.02	0	0	0	0	0	0	0	0	0	0
	0.84	0.84	0.84	0.84	0.84	0,84	0.84	0.84	0.84	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2

7.6. Computation of $Pg(d^1|DM_1)$ (step 4)

To choose between decisions 2 and 3 we compute the pignistic probability that decision 2 is optimal: $Pg(2|\mathrm{DM}_1) = Pg(\alpha^1 \in [0;0.429]) \times Pg(\alpha^2 \in [0;1]) \cong 0.992 \times 1$ and the pignistic probability that decision 3 is optimal: $Pg(3|\mathrm{DM}_1) = Pg(\alpha^1 \in [0.429;0.5]) \times Pg(\alpha^2 \in [0;1]) \cong 0.008 \times 1 = 0.008$ (Table 4).

7.7. Determination of $C_2^1(d^1)$ (step 5)

We compute $C_2^1(d^1)$ for 1 to 6. Noted that the decision 1, 4, 5 and 6 are dominated by decision 2 and 3. But decision 3 is optimal for $\alpha^2 \in [0.5;1]$. So we deduce that $C_2^1(1) = C_2^1(2) = C_2^1(4) = C_2^1(5) = C_2^1(6) = \varnothing$, and $C_2^1(3) = [0.5;1]$.

7.8. Computation of $Pg(d^1|DM_2)$ (step 6)

So, $Pg(1|\mathrm{DM}_2) = Pg(2|\mathrm{DM}_2) = 0$ and $Pg(3|\mathrm{DM}_2) = 1$.

A graphical representation of the resulting conflict of decisions (step 7) is pictured (Fig. 8):

Based on the evaluation of the pignistic probability of each decision of DM_1 from the point of view of DM_1 and DM_2we compute the optimal solution taking into account the importance given by $DM_1$to DM_2 (Table 5).

In this case, the final decision depends on the balance of power between actors. Table 5 shows that the decision chosen by DM_1 strongly depends on the importance given by DM_1 to DM_2. In fact, if DM_1 gives low importance to DM_2 $(s_2^1 > 0,1)$ she/he imposes decision 2 to DM_2 otherwise DM_1 will choose decision 3 to satisfy DM_2.

8. Conclusion

In this paper we focused on a decision problem in a dyadic collaborative supply chain. More precisely we addressed the problem of decision making for a customer, taking into account the future decision of his supplier under imprecise information on the criteria of the two SC partners. We proposed a decision method ensuring optimal stability. In other words we focus on the decision that has the best chance to be optimal under an imprecise criterion.

Industrial DMs are daily faced to the issue of exploiting their empirical knowledge of their partners' decisional behavior. This knowledge is rarely precise and quantified. Being able to exploit this knowledge may be a strategic advantage in term of value creation and preservation. The model presented in this paper and the associated case study illustrate the advantage to identify the decision which has the best chance of being optimal under imprecise knowledge, even if research efforts have to be made to improve the robustness of the results and to use real life collaboration experience in order to improve the mutual knowledge of partners' decisional behaviors.

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