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OPTIMIZATION AND VALIDATION OF A CRUCIFORM GEOMETRY FOR THE IDENTIFICATION OF ELASTOPLASTIC MODELS

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A method based upon the sensitivity fields is proposed to optimize biaxial experimental procedures. The objective is to improve the identifiability of elastic parameters related to an a priori chosen constitutive law. This is achieved by minimizing the covariance matrix of the material parameters. This approach unifies full field measurements via image correlation, numerical simulations and identification procedure. The latter is performed on a biaxial experiment and aims at identifying elasto-plastic models.

Introduction

One of the main issues of mechanical engineering is to understand, describe and predict how materials behave and break. The related objectives are to model observed phenomena with the best accuracy. The study focuses on in-plane biaxial tests of cruciform specimens (*1*). The first in-plane biaxial loading test system has been proposed by Feron and Makinde (*2*). Chen and Matthews (*3*), and Alan and Peter (*4*) have discussed various aspects associated with biaxial experimental systems on sheet rolled steel and composite materials. Some of the protocols are able to apply complex loading conditions (*5*). Two main issues have been raised by the authors cited above: $i)$ how to reveal specific material behaviors? and $ii)$ how to quantify and identify material parameters? These two questions are related to one goal, namely, improving the quality of the identified parameters. The identification quality of constitutive parameters is both related to the experimental procedures and the identification methods. In the sequel, the identification quality is estimated by the covariance matrix of the identified material parameters $[\boldsymbol{C}_p]$. In the case of a displacement field, it is expressed with the sensitivity field gathered in a matrix $[\boldsymbol{S}]$. The covariance matrix of the identified parameter $[\boldsymbol{C}_p]$ accounts for different aspects of the problem, namely, the geometry of the studied structure, the chosen constitutive law, the set of parameters, the boundary conditions, the measurement uncertainties, the identification method and to a lesser extent the numerical simulation method.

Theoretical background

Integrated digital image correlation (I-DIC) is a global DIC technique that relies on a mechanical choice for the measured displacement field. It allows to perform the measurement and identification of the sought parameters in only one step. It is based on the global DIC technique (*6*) that relies on the registration of an image f in the reference configuration and a series of pictures g in the deformed configurations. The problem consists of minimizing the sum of squared differences between the deformed image corrected by the measured displacement $\boldsymbol{u}(\boldsymbol{x})$ and the reference image (written for each time t independently)

$$\chi_f^2(t) = \frac{1}{2\gamma_f^2 N_\Omega} \sum_\Omega ((g(\boldsymbol{x} + \boldsymbol{u}(\boldsymbol{x}, t), t) - f(\boldsymbol{x}))^2 \tag{1}$$

with respect to the sought displacement fields $\boldsymbol{u}(\boldsymbol{x}, t)$, where $\boldsymbol{x}$ is any considered pixel. In this expression Ω denotes the Region of Interest (ROI), N_Ω its area in terms of the number of pixels it contains, and γ_f the standard deviation (expressed in gray levels) of the white noise assumed to affect each image independently (including the reference one, what is responsible for the factor of $1/2$ coming as a multiplicative term in this functional). The load is also in importance for the identification because

it increases the number of the measured quantities and as a consequence diminishes the relative uncertainty by increasing the material parameters sensitivities (*7*). Thus, the same approach is followed with the reaction forces for which $\chi^2_{\boldsymbol{F}}$ is minimized

$$\chi^2_{\boldsymbol{F}} = \frac{1}{N_F N_t \gamma_F^2} \{\boldsymbol{F}_m - \boldsymbol{F}_c\}^t \{\boldsymbol{F}_m - \boldsymbol{F}_c\} \tag{2}$$

where $\{\boldsymbol{F}_m\}$ are the measured reaction forces and $\{\boldsymbol{F}_c\}$ are the computed levels with respect to the chosen material parameter set, N_F the number of load cells for each actuator ($N_F = 1$), N_t the number of steps and $\gamma^2_{\boldsymbol{F}}$ the load variance. The identification based upon both observables, i.e., displacement field and reaction force, is achieved by minimizing the global functional $\chi^2_{\boldsymbol{I}}$

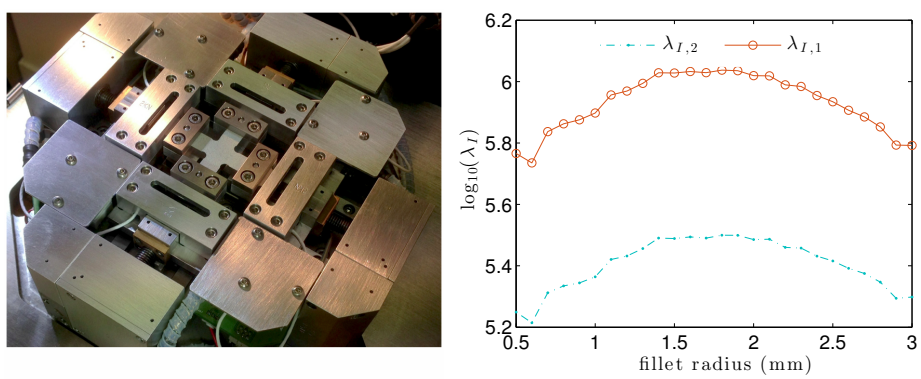
$$\chi^2_{\boldsymbol{I}} = \frac{N_\Omega}{N_\Omega + N_F} \chi^2_f + \frac{N_F}{N_\Omega + N_F} \chi^2_F \tag{3}$$

where the DIC and reaction forces functionals have been introduced in Equations. (1) and (2) respectively. The choice for the specific weight is issued from a Bayesian foundation for using a weighted quadratic difference including noise covariance (*7*).The covariance matrix of the identified parameters is written as (where $\langle \bullet \rangle$ is the mean value of $\bullet$)

$$[\boldsymbol{C}^I_p] = \langle \{\delta \boldsymbol{p}\} \otimes \{\delta \boldsymbol{p}\} \rangle \tag{4}$$

Optimization

The identification involving multiple parameters, it is somewhat subjective to choose a given criterion to express the quality of identification by a single number to be optimized. It is desirable that all parameters be determined accurately, but when increasing the quality of one determination degrades that of another one, expressing a preference is difficult. Because of this freedom, one cannot pretend to provide a universal answer. The choice made herein is to focus on the "worst" determination, namely, the one that has the largest uncertainty due to noise, and optimization is designed to reduce this uncertainty to its minimum. In another word, the maximum level of the minimum eigenvalue of $[\boldsymbol{C}^I_p]^{-1}$ is sought. Furthermore and since prior the experiment the pattern is unknown, it is useful to resort to a "mean-field" assumption. The chosen parameterization is based on $(\log(E/E_0), \log(\nu/\nu_0))$ collectively denoted $\{\boldsymbol{q}\}$. Here E_0 and ν_0 are nominal values of the chosen parameters. The choice of using a log scale for the moduli is to favor *relative* uncertainties.



(a) (b)

Figure 1: (a) Biaxial machine with the chosen sample geometry (b) Eigen value λ_I of Hessian $[\boldsymbol{C}^I_p]^{-1}$ with the ($\{\boldsymbol{p}\}$) parametrization.

The geometry optimization is applied to a linear elastic law for the sake of simplicity and relies on the four fillet radii size. The material parameters are chosen to be representative of a stainless steel (17-7 PH grade) ($E = 200$ GPa, $\nu = 0.3$ and $\sigma_0 = 300$ MPa). A non-proportional loading path is prescribed. First a displacement along direction $\boldsymbol{e}_1$ is applied up to a maximum value $d_1 = d^*$ while the transverse displacement is kept equal to 0. Then, at fixed displacement d_1, the displacement d_2 is raised to reach an equibiaxial strain state, $d_1 = d_2 = d^*$ and finally both displacement amplitudes are reduced to zero at the same rate so that in the displacement plane (d_1, d_2) the loading path is a triangle. For each filet radius, d^* is defined such as the maximum Von Mises equivalent stress is equal to the yield stress. **Figure** 1(b) shows the eigenvalues of $[\boldsymbol{C}^I_p]^{-1}$ and the optimal radius r equal to $r^{opt} = 1.7$ mm.

Biaxial experiment and identification

The previous procedure has been applied to a biaxial experiment with a precipitate hardened stainless steel (17-7 PH grade). The experimental system with the tested sample is shown in **Figure 1**(a). The identification rests on three constitutive laws,

namely, (A) a linear isotropic elasticity law, (B) an elasto-plastic law with linear kinematic hardening and (C) an elasto-plastic law with exponential kinematic hardening. The latter includes 360 time steps (loads and images) captured at 1 Hz and includes loading and unloading cycles. A 16-bit camera (PCO edge 2.0) with telecentric lens ($\times 0.5$) is used. The correlation residual is the natural tool to evaluate the registration quality of DIC approaches (*8*). **Figure** 2 shows the correlation residuals for the DIC solution (black line) and for the three constitutive laws (color lines). The maximum permanent strain reached is equal to $\epsilon = 18.9$.

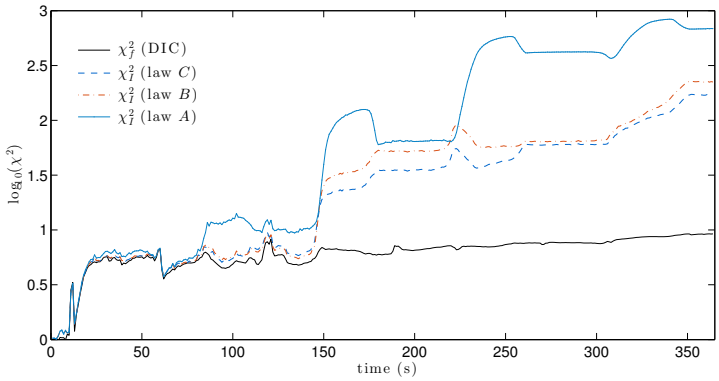


Figure 2: Correlation (DIC) and Integrated Digital Image Correlation (I-DIC) residuals for the three constitutive laws.

Table 1 gathers the results. The choice of an elasto-plastic law diminishes the model error with the lowest level's residual. However, the latter is still high with respect to the DIC one equal to $\chi_f^2 = 7.77$. This implies that the model doesn't describe efficiently all the mechanical behavior over the loading range.

Table 1: Identified parameters and residuals for the three laws ($\delta_p(\%)$).

law	χ_I^2	E(GPa)	δ_E	ν	δ_ν	σ_0(MPa)	δ_{σ_0}	C(GPa)	δ_C	γ	δ_γ
Ref	$\times$	200	$\times$	0.3	$\times$	300	$\times$	10	$\times$	10	$\times$
A	225	9	0.05	0.499	0.002	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
B	50.7	157	0.20	0.31	0.04	480	0.19	6.7	0.2	$\times$	$\times$
C	40.2	148	0.25	0.30	0.04	423	0.25	8.6	0.25	10.8	0.04

Figure 3 shows the evolution of the parameters along the iterations. The convergence is reached in few steps. However, the Poisson's ratio tends to $\nu = 0.50$ since the kinematic behavior is mostly plastic meaning incompressible. Indeed, since the experimental strain is far to be elastic, the identification of law A is mainly driven by plastic strain.

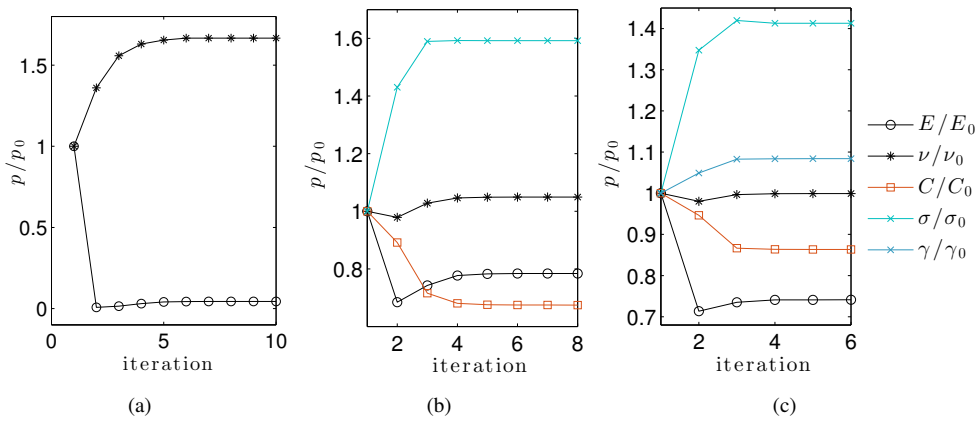


Figure 3: Evolution of the material parameters respectively for the laws A(a), B(b) and C(c).

Finally, **Figure** 3 shows the displacement field residual for component U_1 expressed in pixel (1 pixel = 13.5 µm) at the end of the experiment ($F_1 = F_2 = 0$ N), a displacement field residual being the difference between the DIC and I-DIC solution.

The choice of an elasto-plastic law diminishes the model error. In the other hand and for law A, the plastic strain is directly exhibited since no plastic strains are predicted by the model.

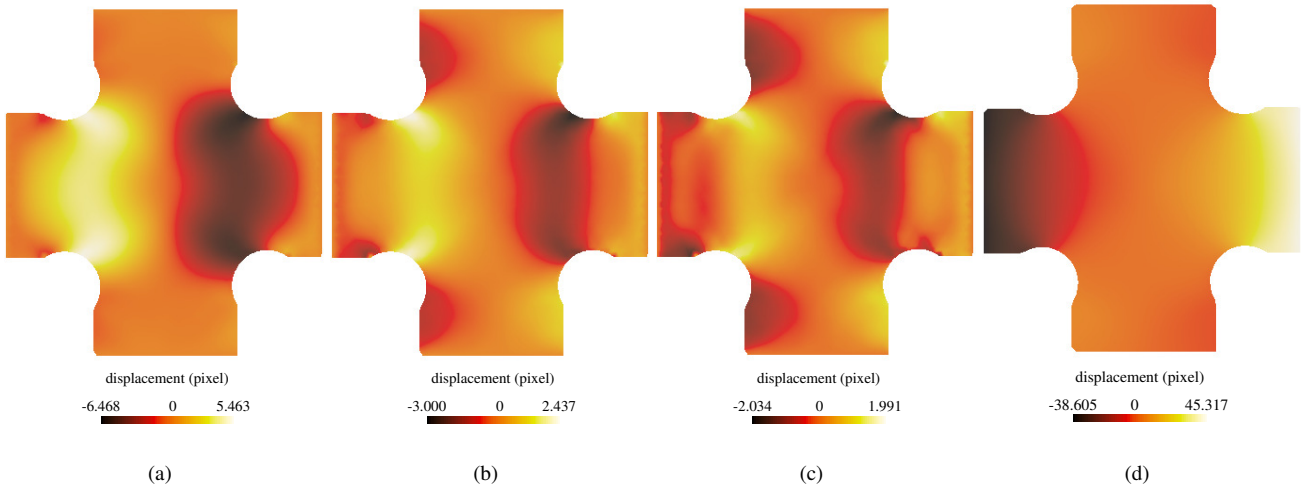


Figure 4: (a), (b) and (c) difference between DIC and I-DIC of the displacement field component U_1 respectively for laws A, B and C. (d) displacement field component U_1 from DIC.

In conclusion, a complete data procedure has been firstly described, which starts from the acquisition of experimental data (i.e., load values and images) to the extraction of material parameters whereby uncertainties could be tracked all the way down to the identification step. Secondly, the identification of constitute parameters has been performed and a procedure has been set up in order to discriminate constitutive laws with respect to experimental data and to evaluate the best one.

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