Black Holes in Einstein's General Relativity versus Modern Theoretical Astrophysics, and Observations

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March 25, 2015

Abstract

Everybody knows that Einstein's General Relativity predicted Mercury's anomalous motion, Black Holes, and other relativistic phenomena. But what specifically those predictions say to physicists? In this work, conflicts between hypothetical properties of the Black Hole concept in Einstein's General Relativity and in Modern Astrophysics are outlined as a platform for further studies of possible conceptual inconsistencies and paradoxes inside the theory. The arising issues are discussed in view of current Astrophysical observations of still mysterious galactic objects, Black Holes "swallowing" the surrounding matter, – forever, but really?

"The growth of knowledge depends entirely upon disagreement."
— Karl Popper, *The Myth of the Framework*

On the Black Hole concept from Einstein General Relativity

The year 2015 marks the 100th anniversary of Einstein's General Relativity (GR) completed by the equation of point particle, the planet Mercury, motion in the Schwarzschild field [1]. We write this equation in dimensionless form in polar coordinates (r, θ), the energy of the proper rest mass included. There are two independent parameters: the initial angular

*Essay written for the Gravity Research Foundation 2015 Awards for Essays on Gravitation. The topic is a part of the Author's independent research program on Planetary and Galactic Dynamics of massive objects. This includes Schwarzschild geodesics in the whole range of field strength, specifically, in weak fields for the Solar planetary motion and a ballistic motion above the Earth, and under moderate and strong field conditions, such as in Black Hole environment. An essential part is devoted to alternative relativistic theories dealing with the singularity from the potential $1/r$ analogous to the Coulomb potential. The author is greatly encouraged by communications with individual researchers interested in Modern Physics problems and liberated from customization of common believes and group thinking.

velocity β_0, and the field strength parameter $\rho_0 = r_g/r_0$ with scaling $r_0 = 1$, where $\xi = r_0/r$ and the Schwarzschild radius $r_{sch} = 2\,r_g$:

$$\left(\frac{\partial\xi}{\partial\theta}\right)^2 = \left(1 - \frac{2\rho_0}{\beta_0^2} - 2\rho_0\right) + \frac{2\rho_0}{\beta_0^2}\,\xi - \xi^2 + 2\rho_0\,\xi^3\,. \tag{1}$$

The conserved momentum $l_0 = r^2(\mathrm{d}\theta/\mathrm{d}\tau)$ and the conserved total energy

$$\epsilon_0^2 = 1 - 2\rho_0\,\xi + \beta_0^2\,\xi^2 - 2\rho_0\,\beta_0^2\,\xi^3 + \left(\frac{\mathrm{d}r}{\mathrm{d}\tau}\right)^2\,.$$

The equation (1) is derivable only in the coordinate system treated as the "comoving" observer's frame. The vacuum solution $r(\theta(\tau))$ presents the observer's continuous view of BH exterior and interior regions, but there is no connection of proper time τ with the coordinate time t of "far-away" observer in this equation.

Nowadays, the equation (1) is used in the whole range of field strength including the Black Hole (BH) environment. In combination with the equation of radial fall onto BH, i.e. the case of zero angular momentum [2, 3], one has the complete GR framework for BH studies.

In [4], some simplifying assumptions in formulation of (1) are used for numerical computations, that makes the solution not exact, still realistically reflecting the picture of particle spiral fall onto BH center as a function of angle $r(\theta)$. Unfortunately, under made assumptions, the velocities of the particle motion and time dependent solutions with their functionals, the crucially important characteristics in BH Physics, could not be calculated there.

We developed methods of *exact* solutions to (1) in both numerical and analytical (in terms of Weierstrass elliptic function $\wp$) forms and obtained new results, such as spatial and time dependent trajectories and their functionals, in particular, velocities, as well as GR "shifts" of angular and time periods [5].

Einstein's General Relativity Black Hole concept is an inseparable part of the more general Astrophysical Black Hole concept, what raises the question on their consistence. *To reveal the possible theoretical conflicts is the main goal of this work.*

Based on previously known and our new results, the following main issues concerning Einstein's classical GR framework for defining the BH concept are outlined and discussed.

Issue 1: Is the central singularity follows from the GR framework?

Methodologically, the central singularity in GR replicates similar singularities in classical and relativistic field theories, all inevitably dealing with sources of point masses and charges in the Lagrangian framework.

In the radial fall problem, $l_0 = 0$, the connection between the proper time τ and the coordinate time t is determined from the expression for the conserved total energy [2]:

$$\epsilon_{rad} = \left(1 - \frac{2\,r_g}{r}\right)\frac{\mathrm{d}t}{\mathrm{d}\tau}\,.$$

Similarly to (1) with the time variable τ, the central divergence is present. However, the solution with t is apparently diverges at the Schwarzschild radius r_{sch}, where the test particle stops but it takes an infinite time to observe the event.

The central divergence is not a GR generic property. This fact is seen, for example, from the original metric form published by Schwarzschild [6]. There, the singularity is eliminated by introduction of a small "cut-off" material parameter R_m, consistently with the spherical symmetry requirement in the metric derivation from Einstein's field equations. The similar view on the singularity elimination was expressed in [7] in connection with the problem of gravitational collapse of super-massive fading stars. As pointed out in [8], the central divergence could be also eliminated by the introduction of a non-divergent potential.

One should bear in mind that the geodesic equation (1) in 3-space and time is derived from the spherically symmetric metric with constraints imposed from the conservation laws and the corresponding symmetries. The metric itself is not a geodesic equation.

In our calculations, the conventional (divergent) metric is used to make the results comparable with those in the current GR studies. Next, the following physical properties of (1) are outlined.

Issue 2: The periodicity and reversibility of the solution

Given the initial conditions, the solution is unique, periodic, and continuous in the whole region in space and time. It is governed by the conservation laws for the total energy and the angular momentum.

A particle trajectory is reversible in space and time. The reversibility of time and the time translation symmetry take place. This is what ensures the total energy conservation, in accordance with Noether's theorem on the conservation law origin for conservative fields.

In fact, our above statements are not new; they refrain old results of famous theoretical astronomer Y. Hagihara. In [9], he presented results of his substantial study of elliptic functions applied to Schwarzschild geodesics. The statements are in contradiction with the Astrophysical BH concept.

Issue 3: The Event Horizon

"The Event Horizon" concept is defined as the Schwarzschild "no return" boundary between interior and exterior regions. The speed of particle crossing the sphere equals the speed of light. However, the spiral fall solution to (1) shows that a particle, at the point $r = r_{sch}$, can reach a speed equal to, or less, or greater than the speed of light, depending on the initial conditions, see Figures 1 and 2. This is the illustration of the statement that, judging by the GR laws, the gravity phenomenon is incompatible with Special Relativity [2].

One can think that the Schwarzschild metric signature changes with the particle speed $\beta(r) \geq 1$, as occurred either in the radial equation of motion in the case $l_0 = 0$ or in the equation (1). As we argue before, the metric is not the geodesic equation. The particle speed is governed by the GR effective potential, which goes to infinity as $r \to 0$ due to the central singularity in those equations. At the same time, both the total energy and the angular momentum remain intact, consistently with the metric.

Astrophysical Black Hole concept

The Astrophysical BH concept was developed as multi-conceptional collection of hypotheses from beyond GR, – as the extension of the academic GR framework by assignment of additional properties from other physical disciplines, including modern theoretical Physics. Briefly, these are:

- Massive matter and light, once trapped by BH, would never return back to outer space.
- Falling onto BH bodies could reach the speed of light at the distinct BH boundary (the Schwarzschild radius), and might move in the interior region with even greater speed until vanishing at the center.
- From "the Event Horizon" concept and "No Hair Theorem" it follows that information carried by swallowed matter is lost forever, consequently, paradoxes arise. At the same time, BH could evaporate through "Hawking's quantum radiation" and eventually disappear.
- The "firewall" hypothesis was suggested to mitigate "the information paradox problems". All these concepts could be reconciled with the GR radial equation for a particular case $l_0 = 0$, which is actually not a realistic case, but it is in plain contradictions with the GR basic geodesic equation (1).

It seems that every new hypothesis intended to resolve the original GR interpretation problems, actually, requires a further extension of the theoretical framework, – that makes a good intellectual room for origination of a chain of paradoxes, which are likely caused by conceptual inconsistence and the corresponding conflicts inside the collection. Indeed, one needs, in the spirit of work [10], to take an inquisitive look at the bottom line of the conflicts between the basics of Einstein's GR framework and additional hypotheses from the Theoretical Astrophysics in view of observational opportunities dealing with the Astrophysical conceptual ingredients.

Black Hole Theoretical models and Observational Astrophysics

Let us sum up the BH model consistence arguments.

- The gravitational collapse into a central (singularity) point is admitted by but is not a necessary attribute of the GR framework.
- Periodicity of the solution to (1) means that a particle, crossing the Event Horizon enters the interior region and returns back into the exterior. Yet, the Horizon boundary occur to be dependent on the initial conditions. This gives to "Hawking's radiation" and "Loss of Information" problems completely different view. Hawking's recent work [11], in fact, presents new arguments, actually doubting the existence of such a radiation in relationship with "the event horizon". Still, "Hawking's radiation" as a physical hypothesis is interesting since its speculative study seems to be compelling in a continuing search for Quantum Gravity.
- Astrophysical BH model is very much based on the existence of the central singularity. However, the singularity could be eliminated in several ways within the GR framework.
- Astrophysical BH model suggests a definition of the Event Horizon at the Schwarzschild radius, where a particle must reach the speed of light. This does not follow from (1).

Currently, the observational precision still is not sufficient for accurate determination of actual size and mass values in terms of r_{sch} and R_m to verify or falsify the theory. At this stage, the Astrophysical model does not seem to provide Astrophysical observational community with a solid guidance on a strategy and practice of observations of the BH phenomenon.

We conclude that theorists currently working on the BH physical phenomenon, have a good cause to focus efforts on providing the BH observational community with a creative self-consistent BH model formulated in Physical and Mathematical rigorous terms to identify BH Nature basics in the labyrinth of multi-aspect physical outlets, and avoiding, if possible, "New Physics". We argue that the BH concept can be developed in a more consistent way preserving the GR methodology, or, if needed, addressing the alternatives.

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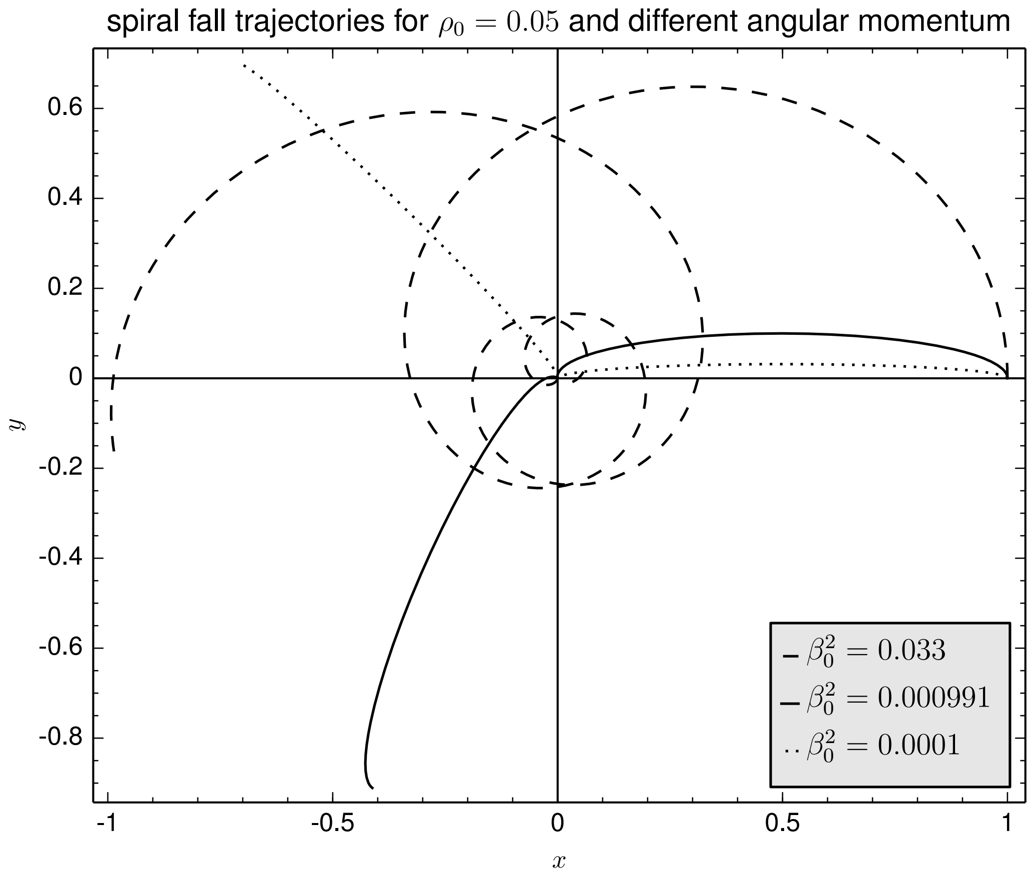

Figure 1: Three spiral fall trajectories for one full period for $\rho_0 = 0.05$ starting from the same initial point. A particle moving along the initially top trajectory ($\beta_0^2 = 0.033$, dashed line) achieves superluminal speed at r_{sch}, see Figure 2, and makes more than three and a half full revolutions before radius becomes 1 again. When $\beta_0^2 \approx 0.000991$ (solid line), a particle achieves the speed of light at r_{sch}, the full period does not make a full revolution. For a smaller angular momentum (shown for $\beta_0^2 = 0.0001$, dotted line), a particle moves slower then speed of light at r_{sch}.

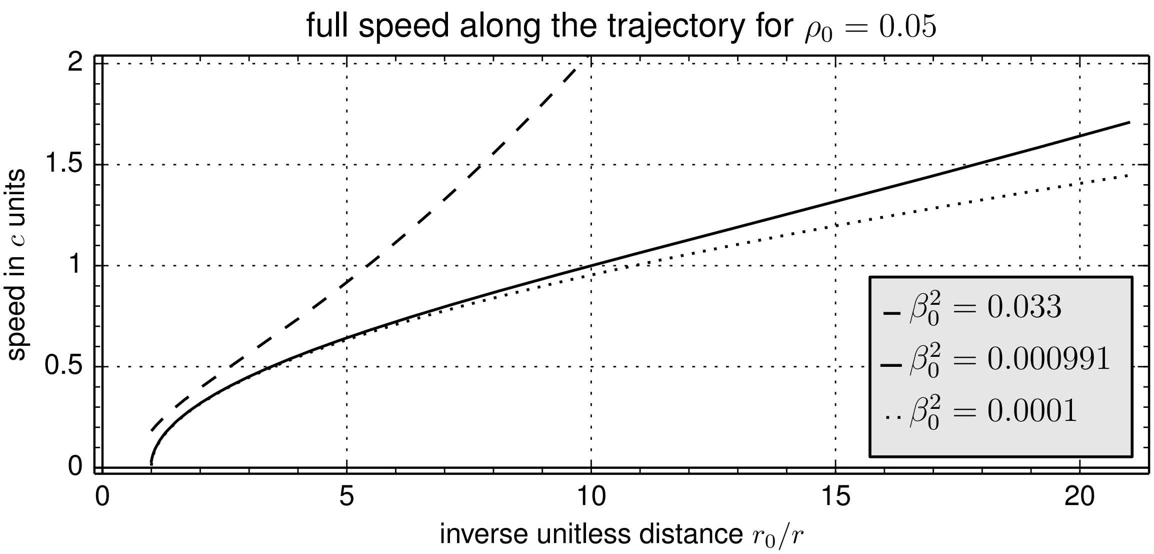

Figure 2: The total speed graph for three spiral fall trajectories. Here, as in Figure 1, we consider a strong field $\rho_0 = 0.05$. A particle achieves speed of light at Schwarzschild radius $r_{sch} = 0.1$ for angular momentum value ≈ 0.000991 (solid line). A particle moves faster than speed of light for a larger angular momentum, shown $\beta_0^2 = 0.033$ (dashed line), at r_{sch} the speed is more than double that c. A particle moves slower than speed of light for a smaller angular momentum, shown $\beta_0^2 = 0.0001$ (dotted line), at r_{sch} the speed is approximately $0.95\,c$.