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Local controllability of the two-link magneto-elastic swimmer

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Abstract : A recent promising technique for moving a robotic micro-swimmers is to apply an external magnetic field. This paper focuses on a simple micro-swimmer model with two magnetized segments connected by an elastic joint, which is able to move in a plane by using a magnetic field. By considering the latter as control functions, we prove that the swimmer is locally controllable around the straight position.

1 Introduction

The propulsion at micro scale through a fluid faces a number of challenges [10]. At this scale, the inertia is negligible compared to the viscous effects. In this regime, one of the typical obstructions, well-known as the scallop theorem [16], imposes to micro-swimmer to use time reciprocal strategies of deformation for achieving their self-propulsion. Micro-swimming has led to a growing interest because of the potential of micro-scale robotic swimmer. Such devices could be used in the new therapeutic and diagnostic procedures such as targeted drug delivery, minimally invasive microsurgical operations [13]. A challenging task consists in designing a remote controlled micro-robot able to swim through a narrow channel.

A recent promising technique resides in using an external magnetic fields to act on a magnetized micro-object by creating an external force or torque

[15]. The robot experiences a deformation and then moves without requirement for chemical fuel and reaction. There are various approaches for creating a magnetically driven propulsion [14]. A first method, inspired by the motility of the bacterial cell called *Escherichia coli* [19], consists in rotating rigid helical tail fixed into a head ([7]). An alternative one is to use a surface in addition to the external field for breaking the spatial symmetry to overcome the feature of the scallop theorem ([18]). The last approach relies on the flexible filaments for propulsion ([5, 14]). In this paper, we focus on a simplified model of magnetic micro-swimmer introduced in [9] which derives from the latter type of propulsion mechanism, using a flexible magnetized filament for swimming such as the prototype introduced in [5].

In this context, the study of the controllability of such devices is a significant theoretical task. Starting from the pioneering work of A. Shapere and F. Wilczek [17], the connexion between control theory and the micro-swimming [12] has been addressed in numerous of works up to now (see for instance [2, 6, 11]). This paper focus on a 2-link elastic-magneto swimmer. A generalized version of the latter, called the *N-link magneto-elastic swimmer*, which is made of a head fixed to N magnetized segments linked each other by a spring, was studied in [1]. More precisely, it has been proved in [1] that the dynamics of the swimmer is governed by a nonlinear ordinary differential equation (ODE) which is linear with respect tho the external magnetic fields. A surprising behavior already mentioned in [9] is that the elasticity in this context allows to overcome the feature of the scallop theorem. The authors also prove that a sinusoidal magnetic field leads to steer the swimmer along one direction. In this paper, by considering the magnetic field as a control function, we establish the local controllability of the 2-link magneto-elastic swimmer around the straight configuration. An important difficulty faces by this paper derives from the fact that the linearized dynamical system of the swimmer at the straight configuration is not controllable. In the rest, we overcome this feature by using a pioneering approach, developed by J.-M. Coron in [3] and called the "return method". Applying this latter for a micro-swimming problem consists in an original task which allows promising perspectives.

The paper is organized as follows: Section 2 describes the swimmer and formulates its equations of motion with the 2-dimensionnalmagnetic field as controls and Section 3 states and proves the main result of this paper.

2 Modeling

This paper focuses on the swimmer model introduced in [9]. We consider a magneto-elastic 2-link swimmer moving in a plane. This two-dimensional setting is suitable for the study of slender, essentially one-dimensional swimmers exploring planar trajectories as explained in [1, 9].

The swimmer consists of 2 rigid segments, of length $\ell_1 > 0$ and $\ell_2 > 0$ respectively, with one articulated joint. It can move in the horizontal 2d-plane of the lab-frame, defined by the vectors $(\mathbf{e}_x, \mathbf{e}_y)$. We set $\mathbf{e}_z := \mathbf{e}_x \times \mathbf{e}_y$. In the rest, we call $\mathbf{x} = (x, y)$ the coordinates of the central point of the second segment, θ the angle that it forms with the x-axis, α the relative angle between the first and second segments (see Figure 1). The position and orientation of the swimmer are characterized by the triplet (x, y, θ), and its shape by α. We denote by

$$\mathbf{e}_{1,\parallel} = \begin{pmatrix} \cos(\theta+\alpha) \\ \sin(\theta+\alpha) \end{pmatrix}, \quad \mathbf{e}_{2,\parallel} = \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix}$$

the unit vectors aligned with segments $[A1, A2]$ and $[A2, A3]$, their orthogonal vectors by

$$\mathbf{e}_{1,\perp} = \begin{pmatrix} -\sin(\theta+\alpha) \\ \cos(\theta+\alpha) \end{pmatrix}, \quad \mathbf{e}_{2,\perp} = \begin{pmatrix} -\sin(\theta) \\ \cos(\theta) \end{pmatrix} .$$

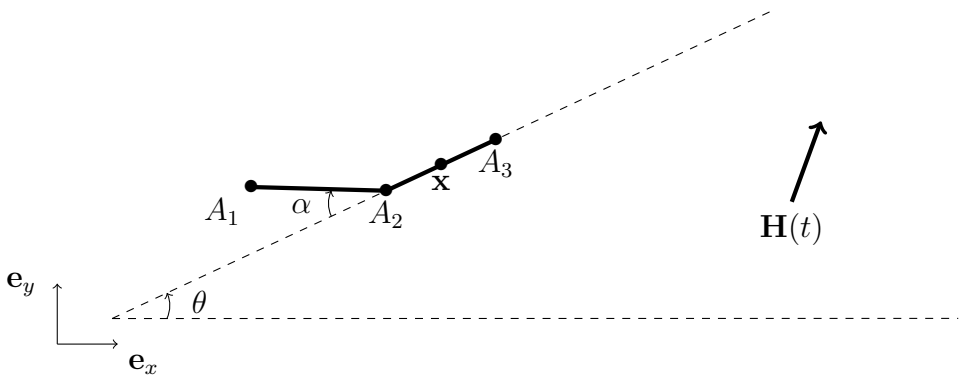

Figure 1: Magneto-elastic 2-link swimmer in plane subject to an external magnetic field $\mathbf{H}(t)$.

Elastic effects As in [9], there is a torsional spring at the joint that tends to align the segment one with the other. It apples a torque

$$\mathbf{T}^{el} = \kappa\, \alpha\, \mathbf{e}_z ,$$

to segment $[A_1, A_2]$ and $-\mathbf{T}^{el}$ to segment $[A_2, A_3]$, here κ is a spring constant (more detail on its physical unit can be found in [1]).

Magnetic effects Additionally, we assume that each segment are constantly magnetized. the two segments submit torques deriving from the effect of an external magnetic field

$$\mathbf{T}_i^m = M_i \left(\mathbf{e}_{i,\|} \times \mathbf{H}\right) , \quad i = 1, 2 ,$$

where M_i is a constant of magnetization and $\mathbf{H}$ is the external magnetic field. The reader could find more details on the physical units used for M_i and $\mathbf{H}$ in [1]. We assume that the external magnetic field $\mathbf{H}$ is horizontal in such a way that the motion holds in the plane generate by $\mathbf{e}_x$ and $\mathbf{e}_y$ and we call $H_\|$ and $H_\perp$ its coordinates into the into the moving frame $\mathbf{H} = H_\| \, \mathbf{e}_{2,\|} + H_\perp \, \mathbf{e}_{2,\perp}$.

Hydrodynamic effects Moreover, the swimmer is assumed to be sufficiently small for modeling the hydrodynamic interaction between the swimmer and the fluid by the local drag approximation of Resistive Force Theory [8]. The latter assumes a linear dependance between the hydrodynamic drag force per unit length acting and the velocity at that point. It means that if the point of abscissa s on segment number i has velocity $\mathbf{u}_i(s) = u_{i,\|}(s) \, \mathbf{e}_{i,\|} + u_{i,\perp}(s) \, \mathbf{e}_{i,\perp}$ (namely $u_{i,\|}(s)$ and $u_{i,\perp}(s)$ are the tangent and normal components of the velocity), then the drag force applied to the a part of length ds of this segment around this point is $\mathbf{f}_i(s)\mathrm{d}s$ with

$$\mathbf{f}_i(s) = -\, \xi_i \, u_{i,\|}(s) \, \mathbf{e}_{i,\|} \; - \; \eta_i \, u_{i,\perp}(s) \, \mathbf{e}_{i,\perp} , \quad i = 1, 2 , \tag{1}$$

with ξ_i and η_i for $i = 1, 2$ the positive drag coefficient such as $\xi_i \leq \eta_i$.

Denote by $\mathbf{R}_\varphi$ the matrix of the rotation of angle φ, $\mathbf{R}_\varphi = \left(\begin{smallmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{smallmatrix}\right)$ for any angle φ. By noticing that the matrix $\mathbf{R}_{\theta+\alpha}$ sends the basis $(\mathbf{e}_x, \mathbf{e}_y)$ onto the basis $(\mathbf{e}_{1,\|}, \mathbf{e}_{1,\perp})$ and the matrix $\mathbf{R}_\theta$ sends the basis $(\mathbf{e}_x, \mathbf{e}_y)$ onto the basis $(\mathbf{e}_{2,\|}, \mathbf{e}_{2,\perp})$, relation (1) translates into

$$\mathbf{f}_1(s) = -\mathbf{R}_{(\theta+\alpha)} \, \mathbf{D}_1 \, \mathbf{R}_{-(\theta+\alpha)} \, \mathbf{u}_1(s) \quad \text{and} \quad \mathbf{f}_2(s) = -\mathbf{R}_\theta \, \mathbf{D}_2 \, \mathbf{R}_{-\theta} \, \mathbf{u}_2(s) , \tag{2}$$

with $\mathbf{D}_i$ the matrix $\left(\begin{smallmatrix} \xi_i & 0 \\ 0 & \eta_i \end{smallmatrix}\right)$ for $i = 1, 2$ and where the vectors are coordinates in the bases $(\mathbf{e}_x, \mathbf{e}_y)$.

If the origin of the abscissa s is the point A_2 on segment 1 and the point $\mathbf{x}$ on segment 2, one has

$$\mathbf{x}_1(s) = \mathbf{x} - \frac{\ell_2}{2} \, \mathbf{e}_{2,\|} \; - s \, \mathbf{e}_{1,\|} , \qquad 0 \leq s \leq \ell_1 , \tag{3}$$

$$\mathbf{x}_2(s) = \mathbf{x} + s \, \mathbf{e}_{2,\|} , \qquad -\frac{\ell_2}{2} \leq s \leq \frac{\ell_2}{2} , \tag{4}$$

hence

$$\mathbf{u}_1(s) = \dot{\mathbf{x}} - \frac{\ell_2}{2}\dot{\theta}\mathbf{e}_{2,\perp} - s\left(\dot{\alpha} + \dot{\theta}\right)\mathbf{e}_{1,\perp}\,, \qquad 0 \le s \le \ell_1\,, \tag{5}$$

$$\mathbf{u}_2(s) = \dot{\mathbf{x}} + s\dot{\theta}\mathbf{e}_{2,\perp}\,, \qquad -\frac{\ell_2}{2} \le s \le \frac{\ell_2}{2}\,. \tag{6}$$

Noting $\dot{\mathbf{x}}_\theta$ the rotated vector $\dot{\mathbf{x}}$ through the angle θ, i.e., $\dot{\mathbf{x}}_\theta = \mathbf{R}_{-\theta}\mathbf{x}$, we deduce from (2) the total hydrodynamic force acting on the first segment as

$$\mathbf{F}_1^h = \int_0^{\ell_1} \mathbf{f}_1(s')\mathrm{d}s' \;=\; -\mathbf{R}_\theta\mathbf{R}_\alpha\mathbf{D}_1\mathbf{R}_{-\alpha} \left(\ell_1\dot{\mathbf{x}}_\theta - \ell_1\frac{\ell_2}{2}\dot{\theta}\mathbf{e}_y - \frac{{\ell_1}^2}{2}\left(\dot{\theta} + \dot{\alpha}\right)\mathbf{R}_\alpha\mathbf{e}_y\right)\,. \tag{7}$$

Identically, according to (2), we have the one acting on the second segment as

$$\mathbf{F}_2^h = \int_{-\frac{\ell_2}{2}}^{\frac{\ell_2}{2}} \mathbf{f}_2(s')\mathrm{d}s' \;=\; -\ell_2\mathbf{R}_\theta\mathbf{D}_2\dot{\mathbf{x}}_\theta\,. \tag{8}$$

Moreover, the total hydrodynamic torque generated by the each segment is given by

$$\mathbf{e}_z\cdot\mathbf{T}_i^h := \mathbf{e}_z\cdot\int_{\text{i-th segment}} (\mathbf{x}_i(s) - \mathbf{x})\times\mathbf{f}_i(\mathbf{s})\,\mathrm{d}\mathbf{s}\,, \qquad \text{with } i = 1, 2\,,$$

here $\times$ stands for the cross product. Let us express

$$\begin{aligned}
\mathbf{e}_z\cdot\mathbf{T}_2^h &= \int_{-\frac{\ell_2}{2}}^{\frac{\ell_2}{2}} (\mathbf{R}_\theta s\mathbf{e}_x)\times(\mathbf{R}_\theta\mathbf{D}_2\mathbf{R}_{-\theta}\mathbf{u}_2(s))\,\mathrm{d}s \\
&= \int_{-\frac{\ell_2}{2}}^{\frac{\ell_2}{2}} (s\mathbf{e}_x)\times(\mathbf{D}_2\mathbf{R}_{-\theta}\mathbf{u}_2(s))\,\mathrm{d}s \\
&= -\xi_2\frac{{\ell_2}^3}{12}\dot{\theta}\,.
\end{aligned} \tag{9}$$

Similarly, we get

$$\begin{aligned}
\mathbf{e}_z\cdot\mathbf{T}_1^h \;=\; & -\xi_1\left(\frac{{\ell_1}^2}{2}\left(-\sin(\alpha)(\dot{x}_\theta - \frac{\ell_2}{2}\dot{\theta}) + \cos(\alpha)(\dot{y}_\theta - \frac{\ell_2}{2}\dot{\theta})\right)\right) \\
& -\xi_1\frac{{\ell_1}^3}{3}\left(\dot{\alpha} + \dot{\theta}\right) + \frac{\ell_2\ell_1}{2}\sin(\alpha)\cos(\alpha)\left(\eta_1 - \xi_1\right)\dot{x}_\theta \\
& \frac{\ell_2\ell_1}{2}\left(\sin^2(\alpha)\eta_1 + \cos^2(\alpha)\xi_1\right)\left(\dot{y}_\theta - \frac{\ell_2}{2}\dot{\theta}\right) \\
& -\xi_1\frac{\ell_2}{2}\frac{{\ell_1}^2}{2}\cos(\alpha)(\dot{\alpha} + \dot{\theta})\,.
\end{aligned} \tag{10}$$

Equations It remains to assemble the equations governing the motion of the magneto-elastic 2-link swimmer. We then write the total balance of forces and of the total torque for the whole system and to express the torque component on the subsystem consisting of one segment (see [1]). Since inertia is assumed to be negligible, these equations take the form

$$\left\{\begin{array}{rcl} \mathbf{F}_1^h + \mathbf{F}_2^h & = & 0 \\ \mathbf{T}_1^h + \mathbf{T}_2^h & = & -\left(\mathbf{T}_1^m + \mathbf{T}_2^m\right) \\ \mathbf{T}_2^h & = & -\mathbf{T}_2^m + \mathbf{T}^{el} \end{array}\right. \tag{11}$$

In the previous system of equations (11) only 4 equations are non-trivial, the latter reads

$$\mathbf{M}^h(\theta,\alpha)\begin{pmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\\ \dot{\alpha}\end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -M_1\left(\cos(\alpha)H_\perp - \sin(\alpha)H_\parallel\right) - M_2 H_\perp \\ +\kappa\alpha - M_2 H_\perp \end{pmatrix} \tag{12}$$

where

$$\mathbf{M}^h(\theta,\alpha) = \left(\begin{array}{c|c}\mathbf{R}_{\theta+\alpha} & 0 \\ \hline 0 & \mathbf{I}_2\end{array}\right) \underbrace{\left(\begin{array}{c|c}E_{11}(\alpha) & E_{12}(\alpha) \\ \hline E_{21}(\alpha) & E_{22}(\alpha)\end{array}\right)}_{=\mathbf{E}(\alpha)} \left(\begin{array}{c|c}\mathbf{R}_{-\theta} & 0 \\ \hline 0 & \mathbf{I}_2\end{array}\right) \tag{13}$$

where

$$E_{11} = \begin{pmatrix} -\left(\xi_1\ell_1 + \xi_2\ell_2\right)\cos\alpha & -\left(\xi_1\ell_1 + \eta_2\ell_2\right)\sin\alpha \\ \left(\eta_1\ell_1 + \xi_2\ell_2\right)\sin\alpha & -\left(\eta_1\ell_1 + \eta_2\ell_2\right)\cos\alpha \end{pmatrix}, \tag{14}$$

$$E_{12} = \begin{pmatrix} \frac{1}{2}\xi_1\ell_1\ell_2\sin\alpha & 0 \\ \frac{1}{2}\eta_1\ell_1\left(\ell_1 + \ell_2\cos\alpha\right) & \frac{1}{2}\eta_1{\ell_1}^2 \end{pmatrix}, \tag{15}$$

$$E_{21} = \frac{1}{2}\begin{pmatrix}\eta_1\ell_1^2 & \eta_2\ell_2^2 \\ 0 & \eta_2\ell_2^2\end{pmatrix}\begin{pmatrix}-\sin\alpha & \cos\alpha \\ 0 & -1\end{pmatrix}, \tag{16}$$

$$E_{22} = -\begin{pmatrix}\eta_1\ell_1^2 & \eta_2\ell_2^2 \\ 0 & \eta_2\ell_2^2\end{pmatrix}\begin{pmatrix}\frac{1}{4}\,\ell_2\cos\alpha + \frac{1}{3}\,\ell_1 & \frac{1}{3}\,\ell_1 \\ \frac{1}{12}\,\ell_2 & 0\end{pmatrix}. \tag{17}$$

Noticing that $\det\mathbf{M}^h(\theta,\alpha) = \det\mathbf{E}(\alpha)$, a straightforward computation yields to get $\det\mathbf{M}^h(\theta,\alpha)$ as

$$\begin{aligned}\det\mathbf{M}^h(\theta,\alpha) = -\tfrac{1}{9}\eta_1\eta_2\,\ell_1^3\ell_2^3\,\big(&\tfrac{1}{4}\left(\xi_1\ell_1 + \xi_2\ell_2\right)\left(\eta_1\ell_1 + \eta_2\ell_2\right)\cos^2\alpha \\ &+\big(\xi_1\ell_1 + \tfrac{1}{4}\,\eta_2\ell_2\big)\big(\tfrac{1}{4}\,\eta_1\ell_1 + \xi_2\ell_2\big)\sin^2\alpha\big)\,,\end{aligned} \tag{18}$$

hence the matrix $\mathbf{M}_h$ is invertible. Summing this up, the dynamics of the swimmer (12) is given by the following control system, affine with respect to

the control functions $H_\perp$ and $H_\parallel$

$$\dot{\mathbf{z}} = \mathbf{F}_0(\mathbf{z}) + H_\parallel \, \mathbf{F}_1(\mathbf{z}) + H_\perp \, \mathbf{F}_2(\mathbf{z}) \quad \text{with } \mathbf{z} = \begin{pmatrix} x \\ y \\ \theta \\ \alpha \end{pmatrix} , \tag{19}$$

calling $\mathbf{X}_3$ and $\mathbf{X}_4$ the vector fields whose vector of coordinates in the basis $(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial \theta}, \frac{\partial}{\partial \alpha})$ are respectively the third and fourth columns of

$$\left(\begin{array}{c|c} \mathbf{R}_{-\theta} & 0 \\ \hline 0 & \mathbf{I}_2 \end{array} \right) (\mathbf{M}^h(\theta, \alpha))^{-1} , \tag{20}$$

the vector fields $\mathbf{F}_0$, $\mathbf{F}_1$, $\mathbf{F}_2$ are defined by the following expressions

$$\begin{aligned} \mathbf{F}_0(\mathbf{z}) = \kappa \alpha \mathbf{X}_4 , \quad \mathbf{F}_1(\mathbf{z}) = M_1 \sin(\alpha) \mathbf{X}_3 , \\ \mathbf{F}_2(\mathbf{z}) = -\left(M_1 \cos(\alpha) + M_2\right) \mathbf{X}_3 - M_2 \mathbf{X}_4 . \end{aligned} \tag{21}$$

3 Controllability result

This section is devoted to the main result of this paper. We are interested in the equilibrium points $\big((\mathbf{x}_e, \theta_e, \alpha_e), (H_\parallel, H_\perp)\big)$ of the system (19) given by

$$\mathcal{M}_e = \big\{ \left((\mathbf{x}, \theta, 0) , (0, 0) \right) , \mathbf{x} \in \mathbf{R}^2 , \theta \in [0, 2\pi] \big\} .$$

The latter corresponds to a straight configuration for the swimmer. Since Jacobian of the drift term $\mathbf{F}_0$ is not invertible, it could be checked that the Kalman rank condition (see Theorem 1.16 on page 9 of the book [4]) does not hold around this particular configuration. However, this paper establishes a local controllability result which confirm that a straight swimmer can be locally controlled by the exterior magnetic field. As the following section will show, getting this controllability result requires to use an advanced method of control theory well-known as the return method, which has been introduced by J. -M. Coron in [3].

Before stating the main result of this paper, let us first point two cases where this controllability will certainly not hold. First, a straightforward computation leads to get the first point of the following proposition.

Proposition 3.1. • *If $\eta_1 - \xi_1 = \eta_2 - \xi_2 = 0$, then the two first equations in* (19) *imply*

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(x - \frac{\eta_1\ell_1\left(\ell_1\cos(\alpha+\theta)+\ell_2\cos\alpha\right)}{2(\eta_1\ell_1+\eta_2\ell_2)}\right) = 0\,,$$
$$\frac{\mathrm{d}}{\mathrm{d}t}\left(y - \frac{\eta_1\ell_1\left(\ell_1\sin(\alpha+\theta)+\ell_2\sin\alpha\right)}{2(\eta_1\ell_1+\eta_2\ell_2)}\right) = 0\,.$$

• *If*

$$\left(3+4\frac{\ell_2}{\ell_1}+\frac{\eta_2{\ell_2}^2}{\eta_1{\ell_1}^2}\right)M_1 - \left(3+4\frac{\ell_1}{\ell_2}+\frac{\eta_1{\ell_1}^2}{\eta_2{\ell_2}^2}\right)M_2 = 0\,,$$

then the set $\{\alpha = 0\}$ is invariant for equations (19).

Proof. Substituting $\alpha = 0$ in (19) and solving for $\dot\alpha$ yields

$$\alpha = 0 \;\Rightarrow\; \dot\alpha = 3\frac{\left(3+4\frac{\ell_2}{\ell_1}+\frac{\eta_2{\ell_2}^2}{\eta_1{\ell_1}^2}\right)M_1 - \left(3+4\frac{\ell_1}{\ell_2}+\frac{\eta_1{\ell_1}^2}{\eta_2{\ell_2}^2}\right)M_2}{\ell_1\,\ell_2\,(\eta_1\ell_1+\eta_2\ell_2)}\,H_\perp\,. \tag{22}$$

and the second point; the first one results from a straightforward but long computation. □

We have to rule out these two cases, where no controlability may hold:

Assumption 3.2. *The constants ℓ_1, ℓ_2, ξ_1, ξ_2, η_1, η_2, M_1, M_2, κ characterizing the system are such that ℓ_1, ℓ_2, ξ_1, ξ_2, η_1, η_2 and κ are positive, M_1 and M_2 are nonzero, and*

$$(\,\eta_1-\xi_1\,,\,\eta_2-\xi_2\,) \neq (0,0)\,, \tag{23}$$
$$\eta_1 \geq \xi_1\,,\quad \eta_2 \geq \xi_2\,, \tag{24}$$
$$\left(3+4\frac{\ell_2}{\ell_1}+\frac{\eta_2{\ell_2}^2}{\eta_1{\ell_1}^2}\right)M_1 - \left(3+4\frac{\ell_1}{\ell_2}+\frac{\eta_1{\ell_1}^2}{\eta_2{\ell_2}^2}\right)M_2 \neq 0\,. \tag{25}$$

Theorem 3.3 (Main local controllability result)**.** *Let assumption 3.2 hold. For any equilibrium $\mathbf{z}_e = (\mathbf{x}_e, \theta_e, 0) \in \mathcal{M}_e$, any $T > 0$, any $\varepsilon > 0$ and any neighborhood $\mathcal{W}$ of $(\mathbf{x}_e, \theta_e, 0)$, there exists a neighborhood $\mathcal{V} \subset \mathcal{W}$ of $(\mathbf{x}_e, \theta_e, 0)$ such that, for any $(\mathbf{x}^i, \theta^i, \alpha^i) \in \mathcal{V}$ and $(\mathbf{x}^f, \theta^f, \alpha^f) \in \mathcal{V}$, there exist bounded measurable functions $H_\parallel$ and $H_\perp$ in $L^\infty([0,T]), I\!R)$ such that*

$$\|H_\perp\|_\infty < \varepsilon\,,\quad \|H_\parallel\|_\infty < 2\,\kappa\left|\frac{M_2+M_1}{M_2\,M_1}\right| + \varepsilon \tag{26}$$

and, if $t \mapsto (\mathbf{x}(t), \theta(t), \alpha(t))$ is the solution of (19) *starting at $(\mathbf{x}^i, \theta^i, \alpha^i)$, then $(\mathbf{x}(t), \theta(t), \alpha(t)) \in \mathcal{W}$ for all time $t \in [0,T]$ and $(\mathbf{x}(T), \theta(T), \alpha(T)) = (\mathbf{x}^f, \theta^f, \alpha^f)$.*

Remark 3.4 (Small time local controllability)**.** *Theorem 3.3 establishes a property weaker than small time local controllability (STLC) [4, Definition 3.2]. Indeed STLC around an equilibrium with zero control requires that the control can be taken arbitrarily small to reach any point sufficiently close to the equilibrium, whereas in Theorem 3.3, the bounds on the control functions are greater than* $2\,\kappa\,\frac{|M_1+M_2|}{|M_2\,M_1|}$*. We do not know whether STLC holds when* $M_1 + M_2 \neq 0$*. Note that, in our context, we expect to have this kind of constraint on the magnetic field since it has to be sufficiently strong to deform the swimmer.*

Let us make some remarks concerning the proof. It could be checked that the linearized of system (19) at an equilibrium $(\mathbf{x}_e, \theta_e, 0)$ is not controllable. In what follows, we use the return method to overcome this feature. Let us recall the idea of the return method (see chapter 6 of the book [4] for more details). If one can find a trajectory of the nonlinear control system (here system (19)) such that it starts and ends at the equilibrium and the linearized control system around this trajectory is controllable, then we can conclude by using the implicit function theorem that one can go from any state close to the equilibrium to any other final state close to the equilibrium.

Let us proceed with the proof. First, we identify in Lemma 3.5 a family of bounded magnetic fields that lead the swimmer to move while starting and ending at an equilibrium point $(\mathbf{x}_e, \theta_e, 0)$. Secondly, Lemma 3.6 shows the controllability of the linearized system (19) around some of these trajectories. These lemmas are then used ot prove the theorem.

Lemma 3.5 (return trajectory)**.** *Let assumption 3.2 hold. There exists positive numbers* k *and* $\overline{T}$ *with the following property: For any* T*,* $0 < T < \overline{T}$*, and any bounded measurable control* $t \mapsto \overline{H}(t) = (\overline{H}_\perp(t), \overline{H}_\parallel(t))$ *defined on* $[0, T/2]$*, there is a bounded measurable control* $t \mapsto H^*(t) = (H^*_\perp(t), H^*_\parallel(t))$ $t \mapsto \overline{H}(t) = (\overline{H}_\perp(t), \overline{H}_\parallel(t))$ *defined on* $[0, T]$ *such that*

$$H^*_\perp(t) = \overline{H}_\perp(t) \text{ and } H^*_\parallel(t) = \overline{H}_\parallel(t)\,, \qquad 0 \leq t \leq \frac{T}{2}\,, \tag{27}$$

the sup norm of control functions $H^*_\perp(.)$ *and* $H^*_\parallel(.)$ *satisfy*

$$\|H^*_\perp(.)\|_\infty \leq (1 + k\,T)\,\|\overline{H}\|_\infty\,, \tag{28}$$

$$\|H^*_\parallel(.)\|_\infty \leq 2\kappa \left|\frac{M_1 + M_2}{M_1\,M_2}\right| + (1 + k\,T)\,\|\overline{H}\|_\infty\,, \tag{29}$$

and, if $t \mapsto \mathbf{z}^*(t)$ *is the solution of system* (19) *with control* H^* *and initial condition* $\mathbf{z}^*(0) = \mathbf{z}_e$*, then*

$$\mathbf{z}^*(t) = \mathbf{z}^*(T - t)\,, \quad 0 \leq t \leq T\,. \tag{30}$$

Let us recall that the linearized control system around a trajectory $(\mathbf{z}^*(.), H_\perp^*(.), H_\parallel^*(.))$ (defined on $[0,T]$) is the time-varying control system

$$\dot{\mathbf{y}} = \mathbf{A}(t)\mathbf{y} + \mathbf{B}(t)\mathbf{v}\,, \tag{31}$$

where $\mathbf{A}(t)$ is the Jacobian of $\mathbf{z} \mapsto \mathbf{F}_0(\mathbf{z}) + H_\perp^*(t)\mathbf{F}_2(\mathbf{z}) + H_\parallel^*(t)\mathbf{F}_1(\mathbf{z})$ with respect to $\mathbf{z}$ at $\mathbf{z} = \mathbf{z}^*(t)$ and the two columns of $\mathbf{B}(t)$ are $\mathbf{F}_1(\mathbf{z}^*(t))$ and $\mathbf{F}_2(\mathbf{z}^*(t))$.

Lemma 3.6 (linear controllability along trajectories)**.** *Let $\mathbf{z}_e$ be an equilibrium belonging to $\mathcal{M}_e$. For any number β, denote by $t \mapsto \mathbf{z}^\beta(t)$ the solution of* (19) *with initial condition $\mathbf{z}^\beta(0) = \mathbf{z}_e$ and (constant) controls*

$$H_\perp^\beta(t) = \beta\,, \quad H_\parallel^\beta(t) = 0\,. \tag{32}$$

It is defined on $[0,+\infty)$. Under Assumption 3.2, there exists, for any $\beta > 0$, some β^* positive no larger than β such that the linearized system (31) *of* (19) *around $(\mathbf{z}^{\beta^*}(.),\ H_\perp^{\beta^*}(.), H_\parallel^{\beta^*}(.))$ is controllable on $[0,\tau]$ for any positive τ.*

These two lemmas are proved later. Let us first use them.

Proof of Theorem 3.3. Let $\varepsilon > 0$, $T > 0$ the equilibrium $\mathbf{z}_e$ and its neighborhood $\mathcal{W}$ be given. In the sequel we may make T smaller without loss of generality because the property for some $(\varepsilon, T, \mathcal{W})$ implies the same property for $(\varepsilon, T', \mathcal{W})$, $T' \geq T$.

Set $\beta = \varepsilon/4$ and apply Lemma 3.6; then set

$$\overline{H}_\perp(t) = H_\perp^{\beta^*}(t) = \beta^*\,,\ \overline{H}_\parallel(t)) = H_\parallel^{\beta^*}(t) = 0\,, \tag{33}$$

with β^* given by Lemma 3.6, and apply Lemma 3.5 (this requires $T < \overline{T}$): this yields, if $T < 1/k$, a control $t \mapsto (H_\perp^*(t), H_\parallel^*(t))$ associated with a solution $t \mapsto \mathbf{z}^*(t)$ of (19) with this control, both defined on $[0,T]$, such that

$$\begin{aligned} &\|H_\perp^*\|_\infty < \varepsilon/2\,,\ \|H_\parallel^*\|_\infty < \frac{2\kappa\,|M_+M_2|}{M_1\,M_2} + \varepsilon/2\,, \\ &\mathbf{z}^*(0) = \mathbf{z}^*(T) = \mathbf{z}_e\,, \quad \mathbf{z}^*(t) \in \mathcal{W}\,,\ t \in [0,T]\,, \end{aligned} \tag{34}$$

and the linear approximation of (19) along this solution is controllable. Note that $\mathbf{z}^*([0,T]) \subset \mathcal{W}$ is obtained by taking T small enough.

Let the end-point mapping $\mathcal{E} : L^\infty([0,T],\mathbb{R}^2) \to \mathbb{R}^4$ be the one that maps a control on $[0,T]$ to the point $\mathbf{z}(T)$ with $\mathbf{z}(.)$ the solution of the system (19) with that control and initial value $\mathbf{z}(0) = \mathbf{z}_e$, we have $\mathcal{E}(\,H^*(.)\,) = z_e$ and linear controllability amounts to $\mathcal{E}$ being a submersion at this point; hence

$\mathcal{E}$ sends any neighborhood of H^* in $L^\infty([0,T],\mathbb{R}^2)$ to a neighborhood of $\mathbf{z}_e$; this yields all the properties of the theorem with $(\mathbf{x}^f,\theta^f,\alpha^f)=\mathbf{z}_e$; this trick may be done twice to obtain arbitrary $(\mathbf{x}^f,\theta^f,\alpha^f)$. This concludes the proof of Theorem 3.3. $\square$

Proof of Lemma 3.5. Take for instance $\overline{T}=1$ (it can be chosen arbitrary) and let $\mathbf{z}_e\in\mathcal{M}_e$ be an equilibrium point. Consider T positive no larger than $\overline{T}$ and an arbitrary $t\mapsto(\overline{H}_\perp(t),\overline{H}_\parallel(t))$ defined on $[0,T/2]$. Let $\overline{u}=\|\overline{H}\|_\infty=\max\{\|\overline{H}_\perp\|_\infty,\|\overline{H}_\parallel\|_\infty\}$ and let $t\mapsto\mathbf{z}^*(t)=(\mathbf{x}^*(t),\theta^*(t),\alpha^*(t))$ be the solution of system (19) on $[0,\frac{T}{2}]$ associated with the control $(\overline{H}_\perp(.),\overline{H}_\parallel(.))$ and starting at $\mathbf{z}_e$. We now construct particular control functions $H^*_\perp(.)$ and $H^*_\parallel(.)$ on $[0,T]$ which extend the previous ones:

$$H^*_\perp(t)=\overline{H}_\perp(t)\,,\quad H^*_\parallel(t)=\overline{H}_\parallel(t)\,,\quad t\in\left[0,\frac{T}{2}\right]\,, \tag{35}$$

and allow the swimmer to come back to the starting point $\mathbf{z}_e$ at time T by using the same path:

$$\mathbf{z}^*(t)=\mathbf{z}^*(T-t)\,,\quad t\in\left[\frac{T}{2},T\right]\,. \tag{36}$$

Differentiating the latter with respect to time, we get:

$$\dot{\mathbf{z}}^*(t)=\dot{\mathbf{z}}^*(T-t)\,,\quad t\in\left[\frac{T}{2},T\right]\,. \tag{37}$$

From equation (19), this yields

$$\begin{aligned}&\mathbf{F}_0(\mathbf{z}^*)+\mathbf{F}_1(\mathbf{z}^*)H^*_\parallel+\mathbf{F}_2(\mathbf{z}^*)H^*_\perp\\ =&-\mathbf{F}_0(\mathbf{z}^*)-\mathbf{F}_1(\mathbf{z}^*)\overline{H}_\parallel-\mathbf{F}_2(\mathbf{z}^*)\overline{H}_\perp\,.\end{aligned}$$

The latter reads

$$2\mathbf{F}_0(\mathbf{z}^*)+\mathbf{F}_1(\mathbf{z}^*)\left(H^*_\parallel+\overline{H}_\parallel\right)+\mathbf{F}_2(\mathbf{z}^*)\left(H^*_\perp+\overline{H}_\perp\right)=0\,.$$

By using the expression (21) for $\mathbf{F}_0$, $\mathbf{F}_1$ and $\mathbf{F}_2$ and because $\mathbf{X}_3$ and $\mathbf{X}_4$ are linearly independents, the projection onto the vector space generated by $\mathbf{X}_i$, $i=3,4$ of the previous equality has to vanish. Solving the following system of linear equation with the two unknowns, $H^*_\parallel$ and $H^*_\perp$,

$$\left\{\begin{array}{rl}M_1\sin(\alpha^*)\left(H^*_\parallel+\overline{H}_\parallel\right)&=\Big(M_2+M_1\cos(\alpha^*)\Big)\Big(H^*_\perp+\overline{H}_\perp\Big)\,,\\ M_2\left(H^*_\perp+\overline{H}_\perp\right)&=2\,\kappa\,\alpha^*\,,\end{array}\right. \tag{38}$$

we deduce that the magnetic field $H^*_\perp(.)$ and $H^*_\parallel(.)$ on $[\frac{T}{2}, T]$ has to be defined by

$$H^*_\perp(t) = \frac{2\,\kappa}{M_2}\alpha^*(t) - \overline{H}_\perp(T-t)\,, \quad t \in \left[\frac{T}{2}, T\right]\,, \tag{39}$$

and

$$\begin{aligned} H^*_\parallel(t) =& \frac{2\,\kappa\,\alpha^*(t)}{M_2\,M_1\,\sin(\alpha^*(t))}\,(M_2 + M_1\cos(\alpha^*(t))) \\ & - \overline{H}_\parallel(T-t)\,, \quad t \in \left[\frac{T}{2}, T\right]\,. \end{aligned} \tag{40}$$

To sum up, defining the magnetic field $H^*_\perp(.)$ and $H^*_\parallel(.)$ as a following piecewise functions

- on the interval $[0, \frac{T}{2}]$, $H^*_\perp(.)$ and $H^*_\parallel(.)$ are equal to the fixed functions $(\overline{H}_\perp(.)$ and $\overline{H}_\parallel(.)$ (see (35)) and
- on the interval $[\frac{T}{2}, T]$, $H^*_\perp(.)$ and $H^*_\parallel(.)$ are defined by the expression given in formulas (39) and (40),

the solution of system (19) associated with these control functions and $\mathbf{z}^*(0) = \mathbf{z}_e$, satisfies (30) with $\mathbf{z}^*(T) = \mathbf{z}^*(0) = \mathbf{z}_e$.

Moreover, by continuity, there exists k such that

$$|\frac{2\,\kappa}{M_2}\dot{\alpha}^*(t)| \leq k\overline{u}\,, \quad t \in [0,1]\,,$$

thus the norm sup of function $t \mapsto \frac{2\,\kappa}{M_2}\alpha^*(t)$ on $[0,T]$ is bounded by $k\,T\,\overline{u}$. Similarly, the norm sup of $t \mapsto \frac{2\,\kappa\,\alpha^*(t)}{M_2\,M_1\,\sin(\alpha^*(t))}\,(M_2 + M_1\cos(\alpha^*(t)))$ is bounded by $2\kappa\left|\frac{M_1+M_2}{M_1\,M_2}\right| + k\,T\,\overline{u}$. From the latter point and the definition of the control functions $H^*_\perp(.)$ and $H^*_\parallel(.)$, we deduce the upper bounds in equations (28) and (29). It concludes the lemma 3.5. □

The proof of Lemma Lemma 3.6 requires a more technical lemma:

Lemma 3.7. *Let Assumption 3.2 hold. For any $\beta > 0$, there exists β^*, $0 < \beta^* \leq \beta$, and $\overline{\alpha} > 0$, such that the distribution spanned by the vector fields $\mathbf{X}_3$, $\mathbf{X}_4$, $[\mathbf{X}_3, \mathbf{X}_4]$ and $\mathbf{X}_5^{\beta^*}$, with*

$$\mathbf{X}_5^{\beta^*} = -\beta^*\,(M_2 + M_1\cos\alpha)\,[\mathbf{X}_3, [\mathbf{X}_3, \mathbf{X}_4]] + (\kappa\alpha - \beta^* M_2)\,[\mathbf{X}_4, [\mathbf{X}_3, \mathbf{X}_4]]\,. \tag{41}$$

has rank 4 at all points in

$$\{\mathbf{z} = (x, y, \theta, \alpha) \in \mathbb{R}^4\,,\ |\alpha| < \overline{\alpha},\ \alpha \neq 0\}\,. \tag{42}$$

Proof. A simple computation shows that the determinant of $\mathbf{X}_3$, $\mathbf{X}_4$, $[\mathbf{X}_3, \mathbf{X}_4]$, $\mathbf{X}_5^\beta$ depends only on α and β and that it is a polynomial with respect to β, α, $\cos\alpha$ and $\sin\alpha$; this expression vanishes for $\alpha = 0$ but can easily be seen to be nonzero under Assumption 3.2, for instance by its leading coefficients. Hence, for almost all β, it is a nonzero real analytic function of α that vanishes at zero, hence the result. □

Proof of Lemma 3.6. Define the matrices $\mathbf{B}_j(t) = \left(\frac{\mathrm{d}}{\mathrm{d}t} - \mathbf{A}(t)\right)^j \mathbf{B}(t)$, with $\mathbf{A}(t)$ and $\mathbf{B}(t)$ given by (31). According to [4, Theorem 1.18], the linear system (31) is controllable on $[0, \tau]$ if there is at least one t, $0 < t < \tau$, such that

$$\mathcal{S}p(t) := \mathrm{Span}\left\{\mathbf{B}_j(t)\mathbf{v}; \quad \mathbf{v} \in \mathbb{R}^2; \quad j \geq 0\right\} = \mathbb{R}^4 . \tag{43}$$

By a simple computation, it turns out that, with the constant controls $H_\parallel^*(t) = 0$ and $H_\perp^*(t) = \beta$, the ith column of $\mathbf{B}_j(t)$ is the column of coordinates of the vector field $\mathbf{C}_{i,j}^\beta$ at point $\mathbf{z}^*(t)$, with

$$\mathbf{C}_{i,0}^\beta = \mathbf{F}_i\,, \;\; \mathbf{C}_{i,1}^\beta = [\mathbf{F}_0 + \beta\,\mathbf{F}_1, \mathbf{C}_{i,0}^\beta]\,, \;\; \mathbf{C}_{i,2}^\beta = [\mathbf{F}_0 + \beta\,\mathbf{F}_1, \mathbf{C}_{i,1}^\beta] \tag{44}$$

and so on (we do not need $j > 2$). We claim that

$$\left.\begin{array}{l}\text{for any } \beta > 0, \text{ there exists } \beta^*,\ 0 < \beta^* \leq \beta, \text{ and } \overline{\alpha} > 0, \text{ such that the distribution} \\ \text{spanned by } \mathbf{C}_{1,0}^\beta, \mathbf{C}_{2,0}^\beta, \mathbf{C}_{1,1}^\beta, \mathbf{C}_{2,1}^\beta, \mathbf{C}_{1,2}^\beta, \mathbf{C}_{2,2}^\beta \text{ has rank 4 at all points of (42).}\end{array}\right\} \tag{45}$$

This claim implies Lemma 3.6: take β^* in the lemma as the one given by (45), along the trajectory $t \mapsto \mathbf{z}^{\beta^*}(t) = (x^*(t), y *^(\, t), \theta^*(t), \alpha^*(t))$, one has $\alpha^*(0) = 0$, and, from (22), $\dot\alpha^*(0) \neq 0$, hence there exists $\bar t > 0$ (that depends on β^*) such that

$$0 < t < \bar t \quad \Rightarrow \quad \alpha(t) \neq 0 \text{ and } |\alpha(t)| < \overline{\alpha}\,; \tag{46}$$

hence, according to the above remark and if the claim holds, (43) holds for all t, $0 < t < \bar t$; linear controllability on $[0, \tau]$ in the lemma follows from (43) at some positive t smaller than $\min\{\bar t, \tau\}$.

Let us now prove (45). Recall that

$$\mathbf{F}_0 + \beta\,\mathbf{F}_1 = -\beta\,(M_2 + M_1\cos\alpha)\,\mathbf{X}_3 + (\kappa\alpha - \beta M_2)\,\mathbf{X}_4\,. \tag{47}$$

According to (21) and (44), and since $M_1 \neq 0$ and $M_2 \neq 0$, $\mathbf{C}_{1,0}^\beta$ and $\mathbf{C}_{2,0}^\beta$ span the same distribution as $\mathbf{X}_3$ and $\mathbf{X}_4$ at points where $\alpha \neq 0$. Hence $\mathbf{C}_{1,0}^\beta, \mathbf{C}_{2,0}^\beta, \mathbf{C}_{1,1}^\beta, \mathbf{C}_{2,1}^\beta$ span the same distribution as $\mathbf{X}_3$, $\mathbf{X}_4$, $[\mathbf{F}_0 + \beta\,\mathbf{F}_1, \mathbf{X}_3]$ and $[\mathbf{F}_0 + \beta\,\mathbf{F}_1, \mathbf{X}_4]$, that is, according to (47), $\mathbf{X}_3$, $\mathbf{X}_4$, $-\beta\,(M_2 + M_1\cos\alpha)\,[\mathbf{X}_3, \mathbf{X}_4]$ and $(\beta M_2 - \kappa\alpha)\,[\mathbf{X}_3, \mathbf{X}_4]$, i.e., if, in addition, $(M_2 + M_1\cos\alpha,\ \kappa\alpha - \beta M_2) \neq$

$(0,0)$, the same distribution as $\{\mathbf{X}_3, \mathbf{X}_4, [\mathbf{X}_3, \mathbf{X}_4]\}$, and finally $\mathbf{C}^\beta_{1,0}$, $\mathbf{C}^\beta_{2,0}$, $\mathbf{C}^\beta_{1,1}$, $\mathbf{C}^\beta_{2,1}$, $\mathbf{C}^\beta_{1,2}$, $\mathbf{C}^\beta_{2,2}$ span the same distribution as $\mathbf{X}_3$, $\mathbf{X}_4$, $[\mathbf{X}_3, \mathbf{X}_4]$ and $\mathbf{X}^\beta_5$ given by (41) at points where $\alpha \neq 0$ and $(M_2 + M_1 \cos\alpha, \kappa\alpha - \beta M_2) \neq (0,0)$. This property, together with Lemma 3.7, proves the claim (45), hence Lemma 3.6. □

4 Perspectives

This paper establishes the local controllability of a micro-swimmer model made by two magnetized links connected by an elastic joint. We show that the swimmer displacement can be locally controlled by an external magnetic field. A natural perspective would be to address the question of global controllability for this model. In addition, extending our result for more realistic swimmer would be an important issue.

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