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# Discrete control of resonant wave energy devices

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Aiming at amplifying the energy productive motion of wave energy converters (WECs) in response to irregular sea waves, the strategies of discrete control presented here feature some major advantages over continuous control, which is known to require, for optimal operation, a bidirectional power take-off able to re-inject energy into the WEC system during parts of the oscillation cycles. Three different discrete control strategies are described: latching control, declutching control and the combination of both, which we term latched–operating–declutched control. It is shown that any of these methods can be applied with great benefit, not only to mono-resonant WEC oscillators, but also to bi-resonant and multi-resonant systems. For some of these applications, it is shown how these three discrete control strategies can be optimally defined, either by analytical solution for regular waves, or numerically, by applying the optimal command theory in irregular waves. Applied to a model of a seven degree-of-freedom system (the SEAREV WEC) to estimate its annual production on several production sites, the most efficient of these discrete control strategies was shown to double the energy production, regardless of the resource level of the site, which may be considered as a real breakthrough, rather than a marginal improvement.

**Keywords:** wave energy; wave energy converter; wave energy device; discrete control

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## 1. Introduction

Many different principles have been conceived to harvest the energy of water waves [1,2]. Among them, wave energy converters (WECs), which exploit the motion of a body in response to the waves, generally exploit one or several mechanical resonances of the system to enhance their performances. Basically, they are mechanical oscillators, thus performing better as the wave frequency approaches their natural frequency. However, because random seas have a distributed frequency spectrum, the efficiency of these devices may fall rapidly in narrow-banded sea states. One of the means to counteract this loss is to equip

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the device with an active controller, able to adapt the response of the mechanical system to the continuously varying wave properties.

In the early 1980s, Budal & Falnes [3] showed that, for point absorbers in long waves, one condition for maximizing the energy production is to keep the velocity in phase with the excitation force. As a result, they introduced a special kind of phase control, called *latching control*, in order to achieve this phase condition on a heaving buoy WEC. Latching consists of locking the motion of the body at the instant when its velocity vanishes at the end of one oscillation, and waiting for the most favourable instant to release the body. The determination of the duration during which the system remains locked is the problem to be solved. The optimal solution is non-causal relative to the wave excitation, but can be found in the frequency domain where one knows the future, which, by definition, is the same as the past. In the real world, which is in the time domain, however, sub-optimal solutions must be sought to overcome this difficulty. The main advantage of latching control is that it is passive, which means that it does not need reactive energy to be fed into the system. Latching has been the subject of many studies in the last few years, showing the capability of this discrete control to substantially increase the amount of absorbed energy in regular and irregular waves. Most of these studies dealt with a one degree of freedom (1 d.f.) heaving buoy WEC [4–6], but some of them also considered several d.f. WECs [7–9]. Following a review of some general results obtained with latching, we focus here on the possibility to extend its applicability throughout the whole frequency range, beyond the first natural frequency of the system.

Declutching control (also called *unlatching* in Salter *et al.* [10]) consists of uncoupling the power take-off (PTO), which means that the PTO force is set equal to 0 in the equations, during some portion of the power cycle. As with latching control, it allows the generation of parametric resonance in the response of the mechanical system, and hence an improvement in energy production. It is even simpler than latching control to implement practically, since it requires only valves in the case of a hydraulic PTO [11] or a switch in the case of an electric PTO, instead of mechanical brakes or locks as required for latching control. Declutching has already been considered by Justino & Falcao [12], using relief valves, for the Portuguese oscillating water column in Pico. However, declutching control was used in the Pico study in order to prevent damage in the system rather than to improve the energy production. In Babarit *et al.* [13], declutching was applied to the SEAREV WEC, and it was shown that such control can lead to a large increase in energy production.

In the present study, we additionally consider the combination of latching control *and* declutching control in a new discrete control mode called latched–operating–declutched (LOD). We will apply it successively to (i) a mono-oscillator WEC and (ii) a bi-oscillator WEC, both in regular and irregular wave conditions. Even in regular waves, solutions and simulations are performed in the time domain, which allows us to find non-harmonic solutions. Practically, the switching sequences are derived by the optimal command method, which was first applied to the wave energy area by Hoskin & Nichols [14]. Simulations in regular and irregular waves show that this new three-state mode of control enhances dramatically the performance of both two-state control philosophies (latching or declutching).

## 2. A mono-oscillator system

### (a) Mathematical model

A linear approach will be adopted here for modelling the hydrodynamic forces, under the usual assumptions of a perfect fluid, small wave steepness and small body motions. We will start by considering the simplest generic wave energy device, a heaving cylinder of mass  $m$ , as described in figure 1. The body is able to heave in response to the excitation of the waves, all the other motions being perfectly restrained. The vertical motion of the cylinder is measured by the variable  $x$ . All the restoring forces (hydrostatic and mooring) are gathered and represented by a simple linear spring of stiffness  $k$ . The PTO unit is represented as a simple linear damper of coefficient  $b$ . Under the above assumptions, the behaviour of the body in waves is governed by the following integro-differential equation:

$$(m + \mu_\infty)\ddot{x} + \int_0^t \dot{x}(\tau)K(t - \tau) d\tau + b\dot{x} + kx = F_{\text{ex}}(t), \quad (2.1)$$

in which we recognize the so-called Cummins decomposition [15] of the radiation forces into an instantaneous added mass term  $\mu_\infty\ddot{x}$ , and a memory term expressed by a convolution product  $\int_0^t \dot{x}(\tau)K(t - \tau) d\tau$ .

The kernel  $K$  of this integral, generally named the impulse response function or sometimes the retardation function, depends fundamentally on the shape of the body. It has been computed directly in the time domain by the seakeeping boundary-element method computational codes ACHIL3D [16]. In the present time-domain linear formulation, the free motion  $x(t)$  of the device may be calculated by integrating (2.1) for a given history of the wave excitation forcing term  $F_{\text{ex}}(t)$ , and any given initial conditions  $x(0)$  and  $\dot{x}(0)$ . But the convolution integral prevents the use of compact methods developed in systems theory to work with the state equation representation of linear systems. In order to get such a form for the system in (2.1), the impulse response function  $K$  is first approximated by a sum of complex exponential functions using Prony's method [17] as detailed by Duclos *et al.* [18],

$$K(t) \simeq \sum_{j=0}^N \alpha_j e^{\beta_j t}, \quad (2.2)$$

where  $(\alpha_j, \beta_j)_{j=0, N}$  are a set of complex coefficients. Since the radiative impulse response function for such a simple floating body has a regular behaviour, five to six pairs of complex conjugate coefficients generally give an excellent approximation. Given this approximation, the convolution product  $\int_0^t K(t - \tau)\dot{x}(\tau) d\tau$  appearing in (2.1) is itself approximated by a sum of additional complex radiative states  $I_j$ ,

$$I(t) = \sum_{j=0}^N I_j = \int_0^t K(t - \tau)\dot{x}(\tau) d\tau, \quad (2.3)$$

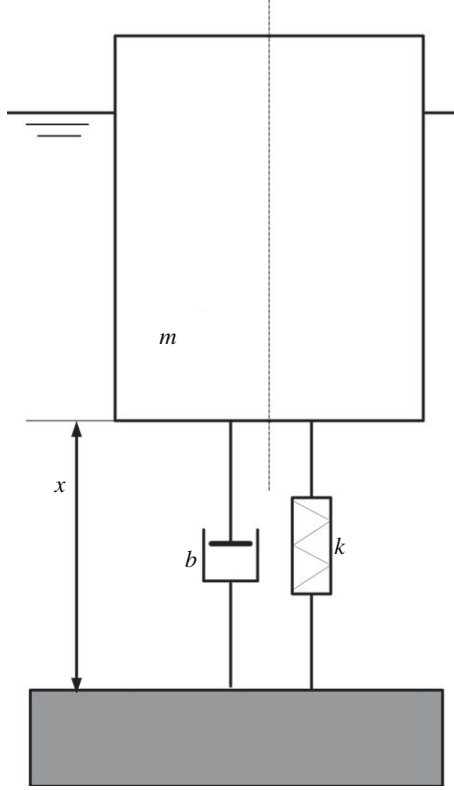


Figure 1. The heaving cylinder: a generic 1 d.f. wave energy converter.

where each  $I_j(t)$ ,

$$I_j(t) = \int_0^t \dot{x}(\tau) \alpha_j e^{\beta_j(t-\tau)} d\tau, \quad (2.4)$$

is the solution of a simple ordinary differential equation

$$\dot{I}_j = \beta_j I_j + \alpha_j \dot{x}. \quad (2.5)$$

Denoting the real and imaginary parts of the complex state  $I_j$  as  $I_j^{\text{R}}$  and  $I_j^{\text{I}}$ , respectively, equation (2.1) can then be replaced by a system of ordinary differential equations,

$$\left. \begin{aligned} (m + \mu_\infty) \ddot{x}(t) + \sum_{i=0}^N I_i(t) + b \dot{x}(t) + kx(t) &= F_{\text{ex}}(t), \\ \dot{I}_i^{\text{R}}(t) &= \beta_i^{\text{R}} I_i^{\text{R}}(t) - \beta_i^{\text{I}} I_i^{\text{I}}(t) + \alpha_i^{\text{R}} \dot{x}(t) \\ \dot{I}_i^{\text{I}}(t) &= \beta_i^{\text{I}} I_i^{\text{I}}(t) + \beta_i^{\text{R}} I_i^{\text{R}}(t) + \alpha_i^{\text{I}} \dot{x}(t). \end{aligned} \right\} \quad (2.6)$$

and

Defining an extended state vector as  $\mathbf{X} = [x \ \dot{x} \ I_1^{\text{R}} \ I_1^{\text{I}} \ \dots \ I_N^{\text{R}} \ I_N^{\text{I}}]^{\text{T}}$  incorporating the additional wave radiation states  $I_i$ , the system of equations in (2.6) can now be

written under the classical state equation matrix form

$$\dot{\mathbf{X}}(t) = \mathbf{A}\mathbf{X}(t) + \mathbf{B}(t). \quad (2.7)$$

The fully developed form of the matrices  $\mathbf{A}$  and  $\mathbf{B}$  are found in Babarit & Clément ([9], eqns (20,21)). Given the known excitation force in a time interval  $[0, T]$ , and the initial conditions  $\mathbf{X}(0)$ , the linear system (2.7) is then solved numerically by any standard integration algorithm, e.g. Runge–Kutta as used here. The mean power  $\hat{P}$  extracted by the PTO over  $[0, T]$  is then given by

$$\hat{P} = \frac{1}{T} \int_0^T b \dot{x}^2(\tau) d\tau. \quad (2.8)$$

(b) Which control strategy optimizes power extraction?

The ultimate goal of the control strategy discussed here is to maximize the power extracted through the PTO of the system in a realistic situation, namely in irregular waves. As described above, the device is a linear mechanical oscillator, set into motion by the external forcing term  $F_{\text{ex}}(t)$ . It has a single natural resonant frequency  $\omega_0$ .

The optimization of such a device in regular waves (i.e. the frequency domain) has been performed previously [19–21]. It consists of determining the PTO characteristics ( $k(\omega)$ ,  $b(\omega)$ ) that maximize the mean extracted power when the incident waves are plane monochromatic waves at a frequency  $\omega$ . In the present case of a 1 d.f. heaving device, with a damping coefficient  $CA_{33}(\omega)$  and added mass coefficient  $CM_{33}(\omega)$ , it is easy to establish that

— if only the PTO coefficient  $b$  can be tuned, then its optimal value is

$$b_{\text{opt},k} = \sqrt{CA_{33}^2 + \left[ \omega(m + CM_{33}) - \frac{k}{\omega} \right]^2}, \quad (2.9)$$

resulting in a mean power capture

$$\hat{P}_{\text{opt},k} = \frac{1}{4} \frac{|F_{\text{ex}}|^2}{CA_{33} + \sqrt{CA_{33}^2 + \left[ \omega(m + CM_{33}) - \frac{k}{\omega} \right]^2}} \quad (2.10)$$

and

— if both of the coefficients  $k$  and  $b$  are adjustable, then the square brackets in (2.10) can be cancelled by setting  $k = \omega^2(m + CM_{33})$ . The mechanical system is therefore in a resonance state, and we get the simple result

$$\left. \begin{aligned} b_{\text{opt}} &= CA_{33}(\omega), \\ \hat{P}_{\text{opt}} &= \frac{|F_{\text{ex}}|^2}{8b_{\text{opt}}}, \end{aligned} \right\} \quad (2.11)$$

with, as a consequence,

$$\dot{x}_{\text{opt}} = \frac{F_{\text{ex}}}{2b_{\text{opt}}} = \frac{F_{\text{ex}}}{2CA_{33}(\omega_0)}. \quad (2.12)$$

Since, the coefficients  $b$  and  $CA$  are essentially real, this last relation (2.12) indicates that, in the optimal device configuration, the velocity of the buoy and the excitation force are exactly in phase. This is the basis of the so-called *phase-control* proposed by Budal & Falnes [22] in the late 1970s. The above relations have been derived in the frequency domain, and are valid when and only when the waves are regular and monochromatic, and the system is linear.

From these results, we can derive a basic principle for the control we seek to develop and ultimately use in the real world (i.e. in the time domain), by trying to continuously vary the PTO parameters  $k$  and  $b$  as the wave frequency  $\omega$  varies continuously in the time domain, in order to maintain this state of resonance (phase condition) and to satisfy the condition on amplitude (2.12). In fact, this continuous optimal control cannot be transposed straightforwardly in the current form, in the time domain, for several reasons.

- In the time domain and irregular waves,  $F_{\text{ex}}$  will be a random signal and the ‘phase’ concept is meaningless with such a signal. One can possibly require the two signals to have their peaks at the same instant, but not to be ‘in phase’, which should mean that they are proportional to each other.
- An ‘instantaneous frequency’ needs to be defined if one wants to directly apply the above rules in the time domain; this has been attempted with limited success by some authors [23], using Kalman filtering or other signal processing tools. Otherwise, one must revert to the time domain using the inverse Fourier transform of the frequency domain results, which is the only mathematically justifiable way to proceed.
- When proceeding via the inverse Fourier transform route, we are led, at last, to the fundamental property that the optimal controller derived in the time domain is not causal [24], which means that one must know the future wave excitation force to adapt the PTO in real time.

Another drawback of a continuous optimal control is the fact, established rigorously in the frequency domain, that the total power flowing out of the system through the PTO is not always positive during a cycle, meaning that one must be able to feed reactive power into the device in order to achieve optimal control. This is not really a great problem as long as we consider the problem in the mathematical space, but it could become a serious drawback when we have to devise the technology of the PTO to transfer energy in both directions, inward and outward. The required level of reactive power can become very large, sometimes 10 times larger than the mean output power. If the efficiency of the converter is not sufficient, then all the energy production could potentially be dissipated via energy losses.

In contrast to this continuous control approach, the discrete control modes that we are presenting in the following sections circumvent this implementation inconvenience.

### (c) *Latching control*

Basically, we consider that we have added a technological component (e.g. a brake) to the original system, which is able to lock the moving body in its current position  $x_0$  at the exact instant  $t_0$  when its velocity vanishes ( $\dot{x}(t_0) = 0$ ), and is

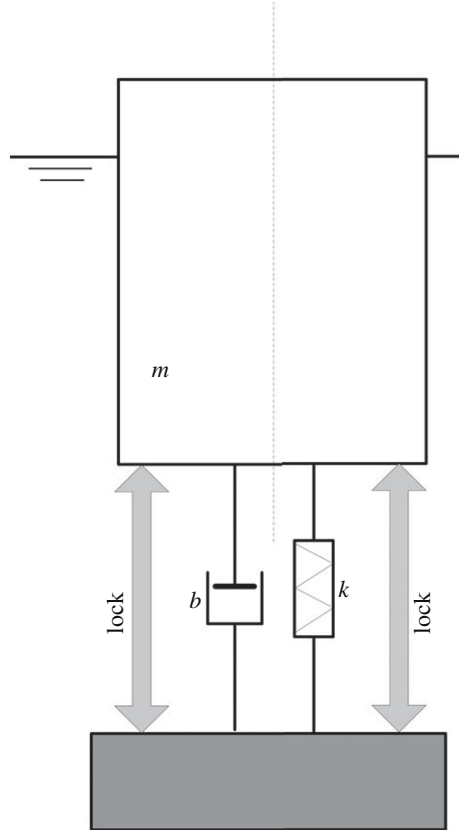


Figure 2. The generic 1 d.f. wave energy converter in latching mode.

able to release it instantaneously on demand (see the lock on figure 2). Proposed in 1980 by Budal & Falnes [3], this control strategy was introduced primarily to satisfy the phase condition between excitation force and body velocity implied by (2.12), in the cases where the wave frequency does not match the natural frequency of the system. It was called *phase-control* for this reason. More precisely the principle was to ‘slow down’ the natural response of the heaving buoy when the wave frequency was lower than the natural frequency, in order to have the velocity and the force reaching their maxima at the same time.

This principle was validated by experiments in the Norwegian University of Science and Technology in the early 1980s. A significant amplification of the WEC motion was obtained (by parametric resonance) compared with the uncontrolled system, without feeding any energy into the system, and expending only a tiny fraction of the available energy in the locking system.

(i) *Optimized latching control of a 1 degree-of-freedom system in regular waves*

There are two important instants in this control mode, one when the system is switched from operating mode to latched mode, the other when the body is released (‘unlatched’) from the latched mode to return to the operating mode. This is why this control mode is termed *discrete*. The latching instant



is determined by the system itself when the velocity vanishes, the remaining practical problem one has to solve to implement latching control in a real system is to determine the release time (or equivalently the duration of the latched state).

If we assume a regular monochromatic plane wave, then the problem can be solved quasi-analytically by a time-domain approach, as shown in Babarit *et al.* [25]. Under this assumption, with the wave excitation force given by  $F_{\text{ex}}(t) = |F_{\text{ex}}| \cos(\omega t + \varphi_0 + \varphi_{\text{ex}})$ , the system is simply switched between two state equations, namely,

$$\left. \begin{aligned} (m + \mu_{\infty})\ddot{x}(t) + \sum_{i=0}^N \lambda_i I_i(t) + b\dot{x}(t) + kx(t) &= |F_{\text{ex}}| \cos(\omega t + \varphi_0 + \varphi_{\text{ex}}), \\ \dot{I}_i^{\text{R}}(t) &= \beta_i^{\text{R}} I_i^{\text{R}}(t) - \beta_i^{\text{I}} I_i^{\text{I}}(t) + \alpha_i^{\text{R}} \dot{x}(t) \\ \dot{I}_i^{\text{I}}(t) &= \beta_i^{\text{R}} I_i^{\text{I}}(t) + \beta_i^{\text{I}} I_i^{\text{R}}(t) + \alpha_i^{\text{I}} \dot{x}(t), \end{aligned} \right\} \quad (2.13)$$

and

or in matrix form

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \Re(\mathbf{B} e^{i(\omega t + \varphi_0)}) \quad (2.14)$$

in operating mode, and the second system

$$\left. \begin{aligned} \ddot{x}(t) &= 0, \\ \dot{x}(t) &= 0, \\ \dot{I}_i^{\text{R}}(t) &= \beta_i^{\text{R}} I_i^{\text{R}}(t) - \beta_i^{\text{I}} I_i^{\text{I}}(t) \\ \dot{I}_i^{\text{I}}(t) &= \beta_i^{\text{R}} I_i^{\text{I}}(t) + \beta_i^{\text{I}} I_i^{\text{R}}(t), \end{aligned} \right\} \quad (2.15)$$

and

or

$$\dot{\mathbf{X}} = \mathbf{A}'\mathbf{X} + \Re(\mathbf{B}' e^{i(\omega t + \varphi_0)})$$

when the system is latched.

The analytical solution method for the determination of the optimal latching delay  $\delta$ , for various wave frequencies [25], is based on the general solution of (2.14) in the time domain under a sinusoidal forcing, which can be expressed as

$$\begin{aligned} \mathbf{X}(t) &= \exp(\mathbf{A}(t - t_i))\mathbf{X}_i + \Re((\mathbf{I}e^{i\omega(t-t_i)} - \exp(\mathbf{A}(t - t_i))) \\ &\quad \times (i\omega\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} e^{i(\omega t_i + \varphi_0)}), \end{aligned} \quad (2.16)$$

with  $\exp(\mathbf{A}t)$  the notation for the matrix exponential, and  $\mathbf{X}_i = \mathbf{X}(t_i)$  the initial condition. Requiring the solution to be periodic with a period as a multiple of the wave period  $2\pi/\omega$  when a steady-state regime is established, we found two sets of optimal solutions. The first one corresponds to these responses where the latching occurs at alternating positions  $x_{\text{max}}, -x_{\text{max}}, x_{\text{max}}, \dots$ ; we call it *alternating maxima* modes. In these modes, the period of the response, given by  $T_{\text{out}} = (2k + 1)2\pi/\omega, k \in [0, 1, 2, \dots]$ , is successively  $T, 3T, 5T, \dots$  (figure 3). Therefore, using the time-domain formulation and representation of the solution, we have found latching modes with periods greater than the excitation period. These *sub-harmonic* modes are optimal solutions when the waves have a frequency higher than the natural frequency  $\omega_0$  of the device. More precisely, the device period  $T_{\text{out}}$  is three times the wave period  $T$  if  $\omega_0 < \omega < 3\omega_0$ , five times when  $3\omega_0 < \omega < 5\omega_0$ , and so on.

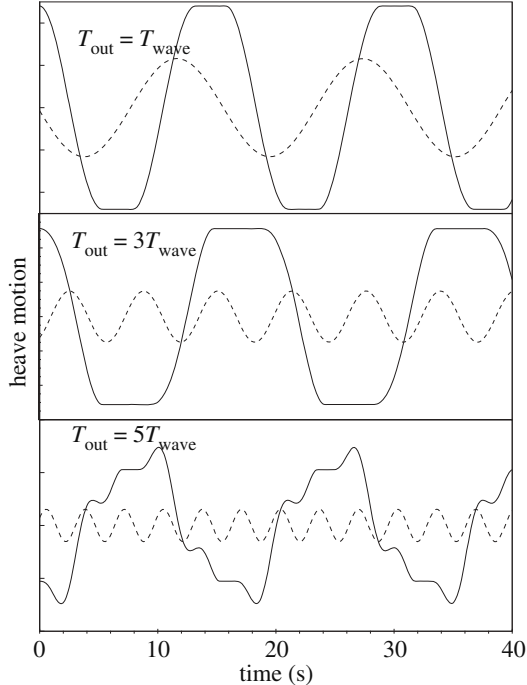


Figure 3. Optimal solutions for latching with alternated maxima in regular waves; harmonic ( $k=0$ ) and sub-harmonic ( $k=1,2$ ) solutions (dashed lines, no control; solid lines, latching control).

Other sets of optimal solutions can be found when we require the body to be always latched at the same position  $x_{\max}$ , in a mode we have called *equal ending ramps* (figure 4). The period of the optimal response is now  $T_{\text{out}} = 2l\pi/\omega$ ,  $l \in [0, 1, 2, \dots]$  in a series  $T, 2T, 3T, \dots$

These new sub-harmonic modes allow us to extend the application of latching control beyond the natural frequency of the system, with a real gain in the heaving amplitude of the body. This can be observed in figure 5 showing the absorbed power curve versus wave frequency for a 1 d.f. heaving cylinder of 5 m radius, 10 m draft,  $m = 360 \text{ T}$ ,  $k = 240 \text{ kN m}^{-1}$  and  $b = 27 \text{ kN s}^{-1} \text{ m}^{-1}$ .

In figure 5, we have also plotted the maximum power absorbed by an axisymmetric body,  $P = \rho g^3/4\omega^3$ , as established by Evans [20] and Falnes [21]. One can clearly see the benefit of latching control even beyond the natural frequency ( $0.62 \text{ rad s}^{-1}$ ), provided one switch to the first sub-harmonic mode with  $T_{\text{out}} = 3T$ . One must also realize that when the system runs in such a sub-harmonic mode, the situation is drastically different from the optimal harmonic response described in §2b, based on a frequency-domain approach and, in particular, that the optimal condition consisting of phasing the body velocity with the excitation force is no longer valid.

(ii) *Optimized latching control of a 1 degree-of-freedom system in irregular waves*

Since the ultimate goal is to apply such discrete control strategies in real sea conditions, let us now turn to the case of irregular waves for this simple 1 d.f. system. As usual in such a linear approach, we shall consider irregular wave

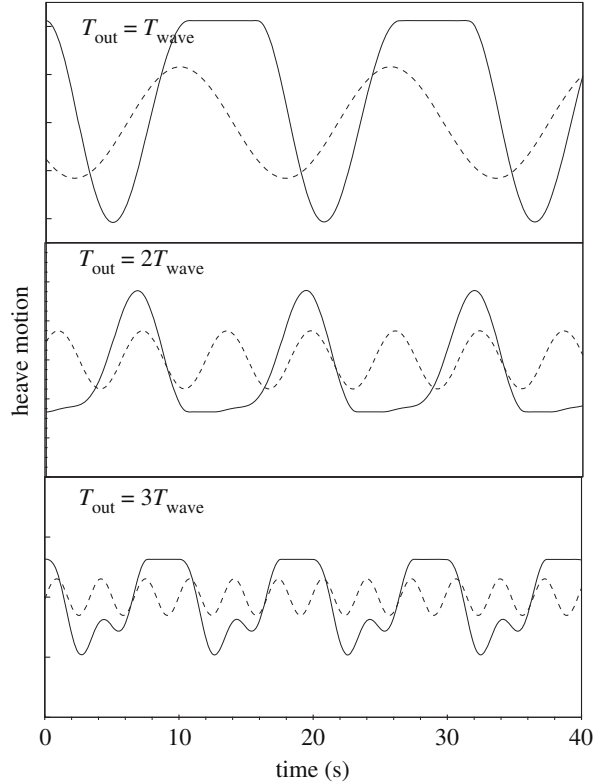


Figure 4. Optimal solutions for latching with equal ending ramps in regular waves; harmonic ( $l = 0$ ) and sub-harmonic ( $l = 1, 2$ ) solutions (dashed lines, no control; solid lines, latching control).

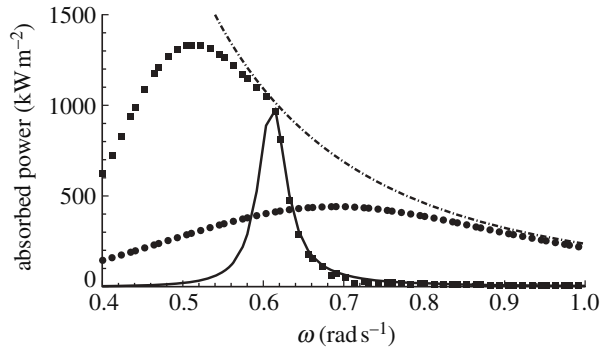


Figure 5. One degree of freedom heaving device; power absorbed in harmonic and first sub-harmonic latching modes, compared with uncontrolled, and ideal modes (solid line, without control; squares, latching:  $T_{\text{out}} = T_{\text{in}}$ ; circles, latching:  $T_{\text{out}} = 3 \times T_{\text{in}}$ ; dashed dotted line, maximum power).

trains as the sum of regular waves. The free-surface elevation,  $\eta(t)$  is a sum of  $N_c$  elementary sinus functions whose amplitudes  $(a_j)_{j=1, N_c}$  are derived from the standard Pierson–Moskowitz energy spectrum [26] and whose phases  $(\varphi_j)_{j=1, N_c}$  are chosen randomly. The problem to be solved remains the same as above, namely

finding the optimal latching delay  $\delta$  that determines the instant when the body is released from the latched mode (2.15), and the system is switched to the operating mode (2.13). As stated above, it is now impossible, in this context of irregular waves, to rigorously define a phase for the signals involved, and therefore to base the release criterion on a phase condition.

The solution of the problem can be found using optimal command theory and Pontryagin's principle, as first suggested in this context by Hoskin & Nichols [14].

A time series of the irregular wave excitation force  $F_{\text{ex}}(t)$  is supposed to be known over an interval of time  $[0, T]$ . The latching force that is necessary to cancel the acceleration is represented here by an infinite damper added in parallel to the PTO damper at the instant when one wants to lock the system. A control variable  $u \in [0, 1]$  is used to command this force, cancelling it when  $u = 0$ . Therefore,  $F_{\text{latch}} = -uG\dot{x}$ . When  $G$  is infinite and  $u \neq 0$ , we recover an absolute latching control with this additional term [27]. In simulation, we have rather implemented a weaker version of latching with  $G$  large compared with the PTO damper, but not infinite. When  $G$  is large enough, the correct behaviour of the latching sequences, as described above, is obtained numerically. With this implementation of latching, systems (2.6) and (2.15) can be merged into a single system,

$$\left. \begin{aligned} (m + \mu_\infty)\ddot{x}(t) + \sum_{i=0}^N I_i(t) + (b + Gu(t))\dot{x}(t) + kx(t) &= F_{\text{ex}}(t), \\ \dot{I}_i^{\text{R}}(t) &= \beta_i^{\text{R}} I_i^{\text{R}}(t) - \beta_i^{\text{I}} I_i^{\text{I}}(t) + \alpha_i^{\text{R}} \dot{x}(t) \\ \dot{I}_i^{\text{I}}(t) &= \beta_i^{\text{R}} I_i^{\text{I}}(t) + \beta_i^{\text{I}} I_i^{\text{R}}(t) + \alpha_i^{\text{I}} \dot{x}(t). \end{aligned} \right\} \quad (2.17)$$

and

The problem is now to find  $u(t)$ , which maximizes the energy recovered through the PTO damper during the complete time range  $[0, T]$ ,

$$\max_u E(u) = \int_0^T b\dot{x}^2 dt. \quad (2.18)$$

Let  $\mathbf{X}(t)$  be the generalized state vector of the system in (2.17) when written in the compact form  $\dot{\mathbf{X}}(t) = \mathbf{f}(t, \mathbf{X}, u)$ . A Hamiltonian  $\mathbb{H}$  can be defined by

$$\mathbb{H} = b\dot{x}^2 + \boldsymbol{\lambda}^t \mathbf{f}, \quad (2.19)$$

where  $\boldsymbol{\lambda}^t$  is an adjoint vector solution of the adjoint system

$$\dot{\boldsymbol{\lambda}}_i = -\frac{\partial \mathbb{H}}{\partial X_i}(t, \mathbf{X}, u), \quad (2.20)$$

with final conditions  $\lambda_i(T) = 0$ .

From (2.17) and (2.19), we have that

$$\mathbb{H} = b\dot{x}^2 + \lambda_1 \dot{x} + \lambda_2 \left[ F_{\text{ex}} - \frac{1}{m + \mu_\infty} \left( (b + Gu)\dot{x} + kx + \sum_{i=1}^N I_i^{\text{R}} \right) \right] \quad (2.21)$$

$$+ \sum_{i=0}^N \lambda_i^{\text{R}} (\beta_i^{\text{R}} I_i^{\text{R}} - \beta_i^{\text{I}} I_i^{\text{I}} + \alpha_i^{\text{R}} \dot{x}) + \lambda_i^{\text{I}} (\beta_i^{\text{R}} I_i^{\text{I}} + \beta_i^{\text{I}} I_i^{\text{R}} + \alpha_i^{\text{I}} \dot{x}), \quad (2.22)$$

and hence the adjoint system is given as

$$\left. \begin{aligned} \dot{\lambda}_1 &= \frac{k}{m + \mu_\infty} \lambda, \\ \dot{\lambda}_2 &= -2b\dot{x} - \lambda_1 + \lambda_2(b + Gu) - \sum_{i=1}^N (\lambda_i^R \alpha_i^R + \lambda_i^I \alpha_i^I), \\ \dot{\lambda}_i^R &= -\lambda_i^R \beta_i^R - \lambda_i^I \beta_i^I + \frac{\lambda_2}{m + \mu_\infty} \\ \dot{\lambda}_i^I &= \lambda_i^R \beta_i^I - \lambda_i^I \beta_i^R. \end{aligned} \right\} \quad (2.23)$$

and

The solution principle is based on a double time-stepping integration of the two dynamical systems (2.17) from 0 forward to  $T$ , and (2.23) backwards, from  $T$  (with initial conditions  $\lambda_i(T) = 0$ ) to 0. During this backward integration, we determine  $u(t)$  by applying Pontryagin's principle, requiring that the optimal command should maximize the Hamiltonian at each time  $t \in [0, T]$ . Since the Hamiltonian is a linear function of the command variable  $u$  (see equation (2.21)), its maximization will require  $u$  to be either 0 or 1 depending on the sign of its coefficient, so

$$u = \begin{cases} 1, & \text{if } (-\lambda_2 G \dot{x}) < 0, \\ 0, & \text{otherwise.} \end{cases} \quad (2.24)$$

This procedure of forward and backward integration is initialized by a given sequence for  $u(t)$ , and then iterated up to final convergence of the absorbed energy over  $[0, T]$  [9].

Two noteworthy properties follow from (2.24).

- The optimal control is a ‘bang-bang’ control,  $u$  being either 0 or 1, but never an intermediate value in  $[0, 1]$ . Thus, in that case, discrete control is superior to a continuous control.
- The system switches every time  $\dot{x}$  vanishes and changes its sign. This (re)establishes the latching control principle mathematically, by applying the Hamilton–Pontryagin principle, as proposed heuristically by Budal & Falnes [22] in the late 1970s.

A simulation of latching control in irregular waves is illustrated by figure 6. One can easily see the magnification of the response, especially for  $t > 260$  s. It seems that after this time value, a regime of parametric resonance is reached by the system. Since the absorbed energy is a quadratic function of the motion amplitude, one can understand the interest in getting such an amplified response.

#### (d) *Declutching control*

Latching control is not the only way of getting such parametric resonance by switching, at discrete times, between sub-models of the system. Latching was implemented here as switching between a finite and an ‘infinite’ PTO damping coefficient; namely between  $b$  and  $b + G$ . We will now instead consider that we have the possibility of uncoupling the PTO at chosen discrete instants. In a real

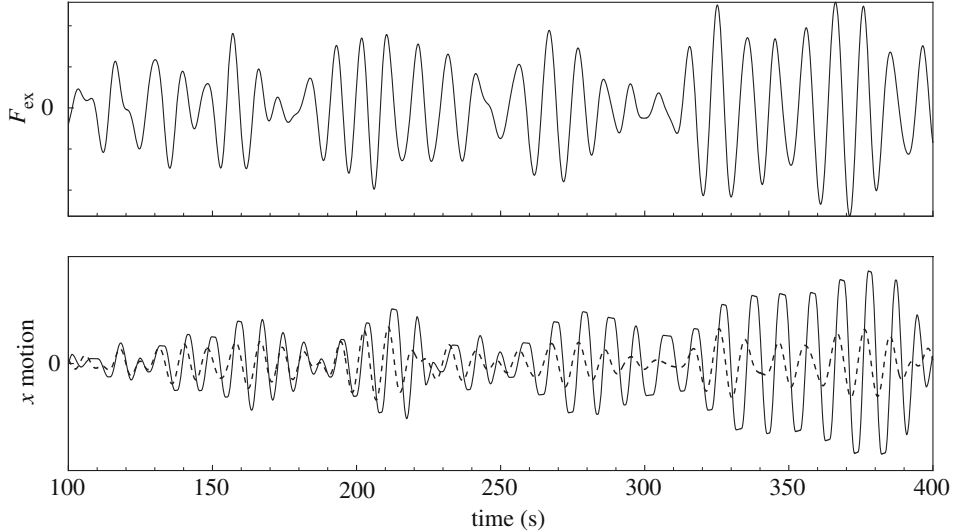


Figure 6. 1 d.f. heaving device in irregular waves. Natural- and latching-controlled heave response versus excitation force (solid lines, latching control; dashed lines, no control).

device, this could be done by a by-pass valve, if the PTO is hydraulic [28], or using a power-electronic switch in the case of a direct electrical generator, or via a mechanical clutch. We named this action *declutching*, as opposed to unlatching, which was used in Salter *et al.* [10], but which could also be mistakenly interpreted as the release action during latching control. In declutching control, the system will now switch between a finite and a null PTO damping coefficient, namely between  $b$  and 0. Since the switching occurs at discrete instants (to be determined), this control mode also belongs to the class of what we call discrete control. Retaining the heaving cylinder as an example, the system of differential equations will differ from the previous one (2.6) only with respect to the damping term,

$$\left. \begin{aligned} (m + \mu_\infty)\ddot{x}(t) + \sum_{i=0}^N I_i(t) + vb\dot{x}(t) + kx(t) &= F_{\text{ex}}(t), \\ \dot{I}_i^{\text{R}}(t) &= \beta_i^{\text{R}} I_i^{\text{R}}(t) - \beta_i^{\text{I}} I_i^{\text{I}}(t) + \alpha_i^{\text{R}} \dot{x}(t) \\ \text{and} \quad \dot{I}_i^{\text{I}}(t) &= \beta_i^{\text{I}} I_i^{\text{I}}(t) + \beta_i^{\text{R}} I_i^{\text{R}}(t) + \alpha_i^{\text{I}} \dot{x}(t), \end{aligned} \right\} \quad (2.25)$$

where the command variable  $v \in [0, 1]$  must be set to 0 to model declutching.

The complete study for latching control was repeated with this new discrete mode of control, following the same steps.

We found that declutching can also be profitable, based on the same principle of modifying (online) the natural frequency of the system to best fit the incoming wave properties. In regular waves, figure 7 shows a simulation of the heaving response of a cylinder computed by integrating (2.25) by a standard (Runge–Kutta) iterative method.

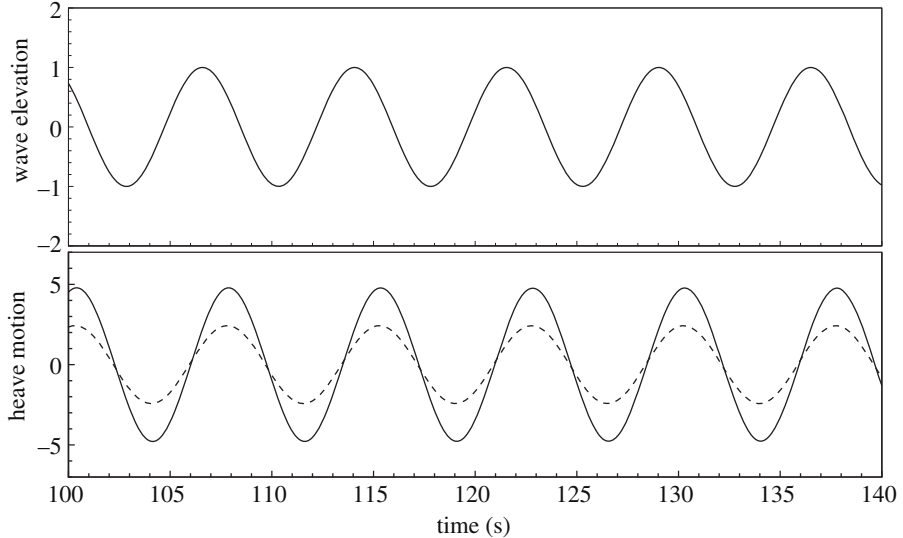


Figure 7. Response of a heaving cylinder in regular waves with (solid lines) and without (dashed line) declutching control.

In irregular waves, the same methodology was applied, using the Hamiltonian formulation as exposed above, but now applied to (2.25). Again, owing to the linearity of the Hamiltonian with respect to the control variable  $v$ , a bang-bang control is shown to be optimal.

Figure 8 shows the response of the cylinder to a sequence of irregular waves. The amplification of the response with declutching observed here is smaller than with latching control. Nevertheless, declutching control alone is still beneficial. As with latching control, declutching has the property that it does not require any energy to be fed into the system. Furthermore, we will see in §2e that the combination of latching and declutching can further enhance the performance of such oscillating wave energy devices.

(e) *Latching and declutching applied to a bi-oscillator system*

We consider now a more complex wave energy device featuring 2 d.f. as described in figure 9. It is composed of the following.

- A positively buoyant free-surface piercing cylindrical hull of mass  $m_1$ , moored to the sea bottom and restrained to move only in heave motion (measured by  $x_1$ ). We will assume all its other motion to be perfectly restricted.
- An internal moving mass  $m_2$ , which can slide without friction along the vertical axis  $x$  inside the floating cylinder. Let  $x_2$  be the relative displacement of  $m_2$  from its equilibrium position.

When the system is excited by the wave forces, both the cylinder and the internal mass are set into motion, in such a way that the relative motion between the two parts can be converted into energy by means of an appropriate PTO.

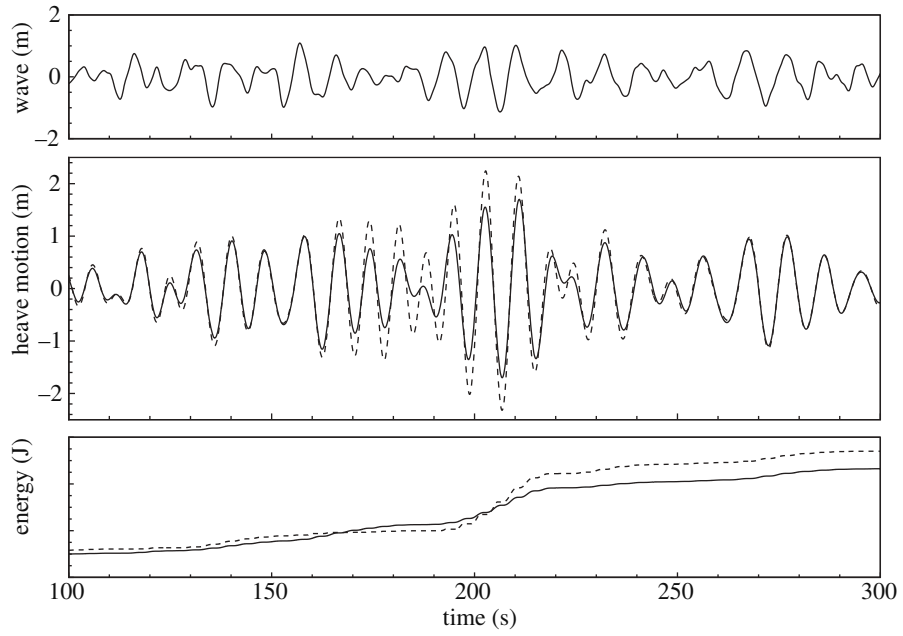


Figure 8. Response of a heaving cylinder in irregular waves with (dashed lines) and without (solid lines) declutching control.

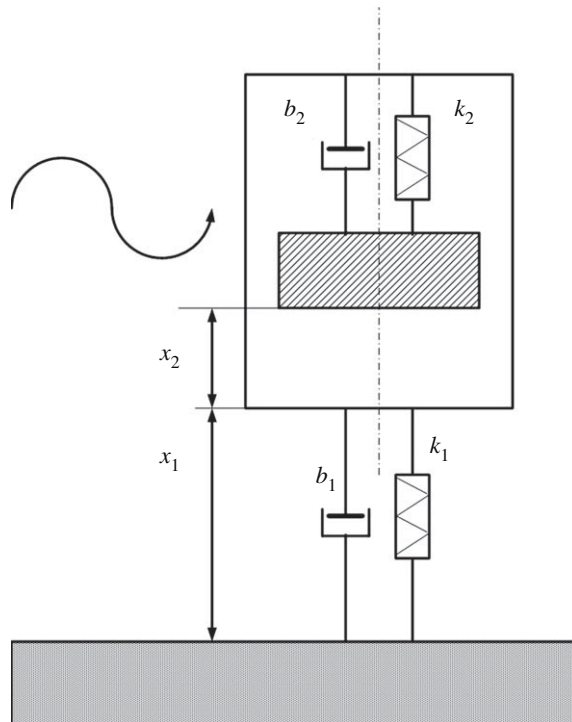


Figure 9. 2 d.f. heaving wave energy converter: definition sketch.



Table 1. Mechanical parameters of the wave energy converter.

$m_1$	705 032	kg
$m_2$	100 000	kg
$k_1$	789 736	N m <sup>-1</sup>
$k_2$	700 000	N m <sup>-1</sup>

We assume, as before, that the PTO can be modelled by a linear spring and damper with coefficients,  $k_2$  and  $b_2$ , respectively. The equations of motion for this 2 d.f. system in the time domain are

$$\left. \begin{aligned} m_1 \ddot{x}_1 + m_2(\ddot{x}_1 + \ddot{x}_2) &= F_{\text{ex}} - \mu_{\infty} \ddot{x}_1 - \int_0^t K(t - \tau) \dot{x}_1(\tau) d\tau - b_1 \dot{x}_1 - k_1 x_1 \\ \text{and } m_2(\ddot{x}_1 + \ddot{x}_2) &= -b_2 \dot{x}_2 - k_2 x_2, \end{aligned} \right\} \quad (2.26)$$

where

- $b_1$  is a damping force coefficient associated with an external linearized viscous force; it can be set to zero ( $b_1 = 0$ ) here without essentially changing the conclusions of the paper;
- $k_1$  is the model for the restoring force applied to the buoy (mooring + hydrostatics), we can further neglect the contribution from the moorings, and  $k_1$  is simply the hydrostatic force given by  $k_1 = \rho g S_{\text{WP}}$ ; and
- $F_{\text{ex}}$ ,  $\mu_{\infty}$  and  $K$  have been defined in preceding sections.

Using King’s approach [29], let  $K_{\text{ex}}(t)$  be the force response associated with an impulsive elevation on the free surface at a given reference point propagating along the  $x$ -axis. Using the superposition principle, the excitation force can be expressed as

$$F_{\text{ex}}(t) = \int_0^t K_{\text{ex}}(t - \tau) \eta(\tau) d\tau, \quad (2.27)$$

where  $\eta(t)$  is the free-surface elevation. For simulation, we have considered a 5 m radius and 10 m depth vertical cylinder as the hull of the WEC, figure 9, the mechanical characteristics of which are summarized in table 1.

The equations of motion now become

$$(m_1 + \mu_{\infty}) \ddot{x}_1 + m_2(\ddot{x}_1 + \ddot{x}_2) = \int_0^t K_{\text{ex}}(t - \tau) \eta(\tau) d\tau - \sum_{j=0}^N I_j - \rho g S_{\text{WP}} x_1, \quad (2.28)$$

$$\left. \begin{aligned} m_2(\ddot{x}_1 + \ddot{x}_2) &= -b_2 \dot{x}_2 - k_2 x_2 \\ \text{and } \dot{I}_j &= \beta_j I_j + \alpha_j \dot{x}_1. \end{aligned} \right\} \quad (2.29)$$

Fundamentally, this system comprises two oscillators with two distinct natural frequencies: one for the heaving buoy and the other for the moving mass. These oscillators are coupled through equation (2.28). A Runge–Kutta second-order

scheme was used to numerically integrate the equations of motion in order to perform time-domain simulations of this 2 d.f. WEC, with a time step of 0.01 s.

(i) *Latching control*

In this 2 d.f. device, latching control consists of locking the relative motion  $x_2$  when the velocity  $\dot{x}_2$  vanishes, and releasing it after an optimal time, to be determined. As above, latching control is implemented in the model by adding in the equation of motion a very large damping force on the relative motion  $x_2$  into the equations of motion, as

$$f_{\text{latching}} = -u_1 G(m_2 + \mu_\infty) \dot{x}_2, \quad (2.30)$$

where  $u_1 \in [0, 1]$  is the control variable. This weak modelling of latching control may have a lower efficiency than an absolute formulation, but it allows the more convenient implementation of optimal command theory in order to compute the control law in a time-step numerical procedure, as described in previous sections. It is also more realistic when one wants to model real mechanical components, e.g. brakes, which do not necessarily have instantaneous action.

Practically, a delay or lag will exist between the time when the controller decides to apply the control and the time when brakes will hold the body. In order to take this delay into account, we refine the PTO model and we now consider the coefficient  $G$  to be time-varying, instead of a constant, solution to the differential equation

$$G(t) + \tau_{\text{latching}} \dot{G}(t) = u_1 G_0. \quad (2.31)$$

When the controller switches to the latching mode, the control variable  $u_1$  is set to 1, and  $G(t)$  grows exponentially to its final value  $G_0$  with a time constant  $\tau_{\text{latching}}$ . When the controller switches the system back to the normal operating mode,  $u_1$  is set to 0 and  $G(t)$  decays exponentially to 0.

(ii) *Declutching control*

For the 2 d.f. device, we also consider declutching control, which effectively disconnects the PTO between the buoy and the moving mass at some instants of the motion, which have also to be determined. Again, to take into account possible delays or lags in the response of the real actuators, with a time constant  $\tau_{\text{PTO}}$ , we consider the  $b_2$  coefficient as a function of time, as

$$b_2(t) + \tau_{\text{PTO}} \dot{b}_2(t) = u_2 B_0. \quad (2.32)$$

For declutching, the control variable is  $u_2$ .

(iii) *Latched–operating–declutched: a three-state discrete control mode*

As for the single degree of freedom device, and using the same methodology, we have shown that latching control or declutching control are also efficient with the 2 d.f. device. Now, we want to go further, to assess the result that may be obtained by combining these two modes of control. We shall therefore implement a three-state discrete control, where the PTO can be alternatively latched, operating or declutched (we use *LOD control* as an abbreviation).

Hence, we have defined a three-stage controller, depending on the combinations of the value of the control variables  $u_1$  and  $u_2$ :

- when  $u_1 = 1$ , irrespective of the value of  $u_2$ , the system is latched;
- when  $u_1 = 0$  and  $u_2 = 0$ , the system is operating; and
- when  $u_1 = 0$  and  $u_2 = 1$ , the system is declutched.

The optimal command method is again used to solve for the determination of the switching times in the time-domain simulation.

Let  $\mathbf{X} = (x_1 \ x_2 \ x_3 \ x_4 \ G \ b_2 \ I_1 \ \dots \ I_N)^T$  be the new state vector, with  $x_3 = \dot{x}_1$  and  $x_4 = \dot{x}_2$ . One can rewrite the equation of motion as a standard state equation,

$$\dot{\mathbf{X}} = \mathbf{f}(t, \mathbf{X}, \mathbf{u}), \quad (2.33)$$

with

$$\mathbf{f}(t, \mathbf{X}, \mathbf{u}) = \begin{pmatrix} x_3 \\ x_4 \\ \mathbf{M}^{-1} \begin{pmatrix} F_e x - k_1 x_1 - \sum_{j=0}^N I_j \\ -(b_2 + G(m_2 + \mu_\infty))x_4 - k_2 x_2 \end{pmatrix} \\ \frac{1}{\tau_{\text{latching}}} (u_1 G_0 - G), \\ \frac{1}{\tau_{\text{PTO}}} (u_2 B_0 - b_2) \\ \beta_1 I_1 + \alpha_1 x_3 \\ \vdots \\ \beta_N I_N + \alpha_N x_3 \end{pmatrix}, \quad (2.34)$$

$$\mathbf{M}^{-1} = \frac{1}{(m_1 + \mu_{\text{inf}})m_2} \begin{pmatrix} m_2 & -m_2 \\ -m_2 & m_1 + m_2 + \mu_{\text{inf}} \end{pmatrix} \quad (2.35)$$

and the initial conditions

$$\mathbf{X}(0) = \mathbf{X}_0. \quad (2.36)$$

The optimization problem is to maximize the energy extracted,

$$\max_{\mathbf{u}} E = \int_0^T b_2 \dot{x}_4^2 dt. \quad (2.37)$$

Now define the Hamiltonian

$$\mathbb{H} = b_2 \dot{x}_4^2 + \boldsymbol{\lambda}^t \mathbf{f}, \quad (2.38)$$

where  $\boldsymbol{\lambda} \in \mathbb{R}^{N+6}$  is the adjoint state vector.  $\boldsymbol{\lambda}$  is the solution of the set of adjoint differential equations

$$\dot{\lambda}_i = -\frac{\partial \mathbb{H}}{\partial X_i}(\mathbf{X}, u), \quad (2.39)$$

with the final condition  $\boldsymbol{\lambda}(T) = 0$ .

Using equations (2.34) and (2.40), one can show that the Hamiltonian is linear in  $\mathbf{u}$ ,

$$\mathbb{H}(t) = h(\mathbf{X}, \boldsymbol{\lambda}) + u_1 \lambda_5 + u_2 \lambda_6. \quad (2.40)$$

Hence, the switching instants will be determined by the signs of the adjoint states  $\lambda_5$  and  $\lambda_6$ , and we again have a bang-bang control defined by the protocol

$$u_1 = \begin{cases} 1, & \text{if } \lambda_5 \geq 0, \\ 0, & \text{if } \lambda_5 < 0, \end{cases} \quad (2.41)$$

and

$$u_2 = \begin{cases} 1, & \text{if } \lambda_6 \geq 0, \\ 0, & \text{if } \lambda_6 < 0. \end{cases} \quad (2.42)$$

Since the iterative process includes forward and backward integration of (2.34) and (2.39), which are used to determine the optimal latching control law, knowledge of the excitation force during the complete duration of the simulation, up to the final instant  $T$ , is required. The decision to latch or declutch the body at a current time  $t$  then depends on the future of the system beyond  $t$ , which means that this process is anti-causal. Its practical implementation in the real world requires some wave prediction, or necessitates the adoption of sub-optimal causal algorithms for calculating the control law online. These issues are not addressed here, since the aim of this study is simply to assess the ability of the discrete LOD control to increase energy production.

(f) *Latched–operating–declutched control in regular waves*

A comparison of the results of time-domain simulations of the motion without control, with latching control, with declutching control and finally, with LOD control is plotted in figure 10. The period of the incident wave is 10 s and its height is 1 m, with the damping coefficient  $b_2$  set to  $10000 \text{ N m}^{-1} \text{ s}^{-1}$ . The latching coefficient is  $G_0 = 80$  and the time constants for the PTO and the latching are  $\tau_{\text{PTO}} = \tau_{\text{latching}} = 0.1 \text{ s}$ . The control is activated at  $t = 40 \text{ s}$ .

In this example, both latching and LOD control increase the energy production in comparison with the case without control (2.5 kW), whereas the declutching control does not bring any improvement. When the steady state is reached, the mean power produced with LOD control (61.7 kW) is considerably higher than with latching control (23.5 kW). The power is measured here as the slope of the bottom curve: energy versus time. The increase in the amplitude of the relative motion  $x_2$  (the ‘working’ d.f.) with LOD control is also spectacular in comparison with the other control strategies. This is one of the major findings of this study: the combination of latching and declutching control together can give far better results than each one used separately. The price to pay is that the duration of the transient period between the activation of the control (40 s) and the establishment of the steady state (approx. 220 s) is longer with LOD control than with latching control.

It was shown in Babarit & Clément [9] that the weak modelling of latching control we use here (i.e. a large but not infinite damping force) has more influence on the computed amplitude of the system response, than on the mean absorbed power. Figure 11 shows that the closer we are to absolute latching (the

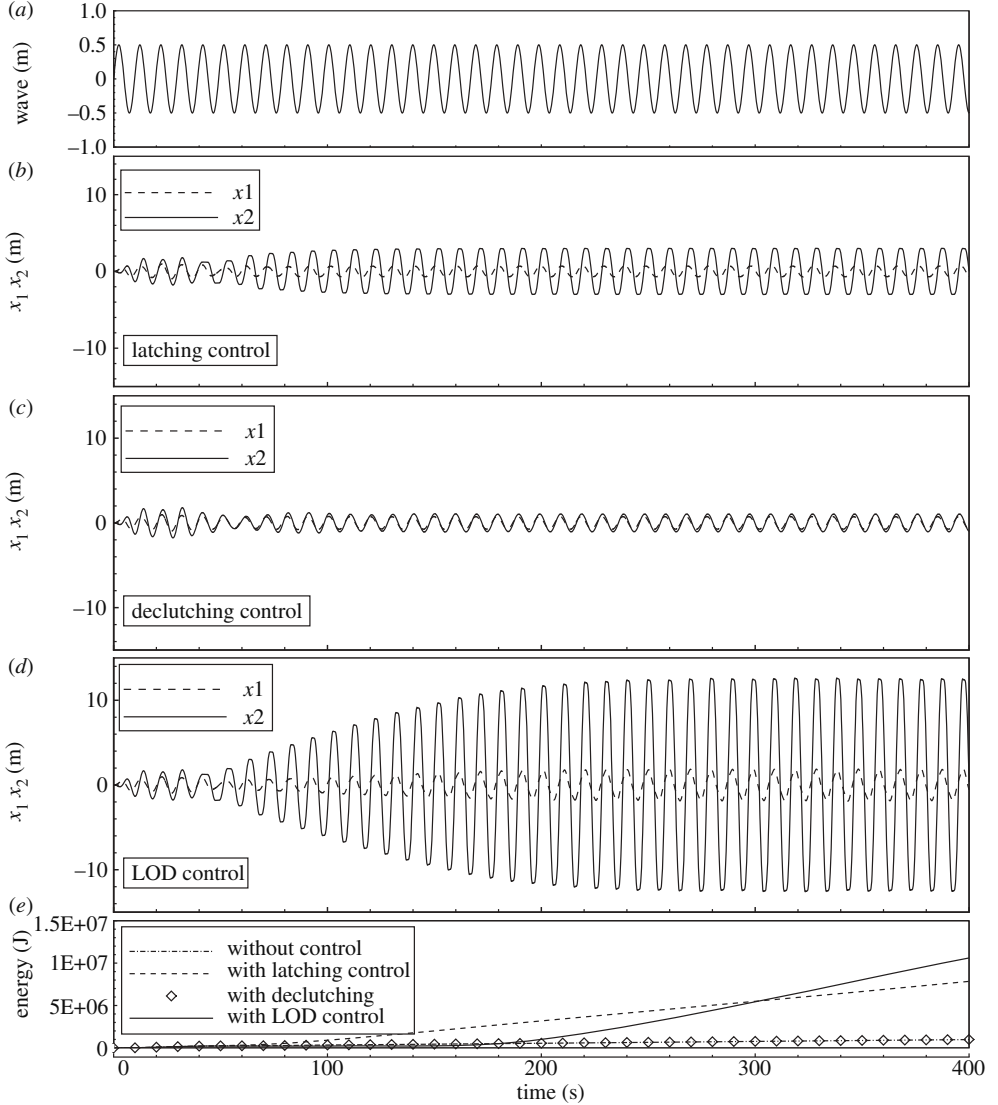


Figure 10. (a–d) Simulations of the response of the 2 d.f. wave energy device in regular waves; under latching control, declutching control or latched–operating–declutched control. Wave period  $T = 10$  s; wave height  $H = 1$  m. (e) evolution of the total extracted energy versus time.

highest value of  $G_0$ ), the larger is the computed power absorbed. However, as the dynamical system becomes stiffer and stiffer, it requires a very short time step in order to keep the numerical time integration scheme stable. Hence, a reasonable value of 80 was assigned to  $G_0$  in the following.

To take into account the delay between the time the controller decides to apply a control and the time the actuators physically apply the control, we introduce the time constants  $\tau_{\text{latching}}$  and  $\tau_{\text{PTO}}$  on the latching coefficient  $G$  and on the PTO damping coefficient  $b_2$ , respectively. Figure 12 shows the motion and the

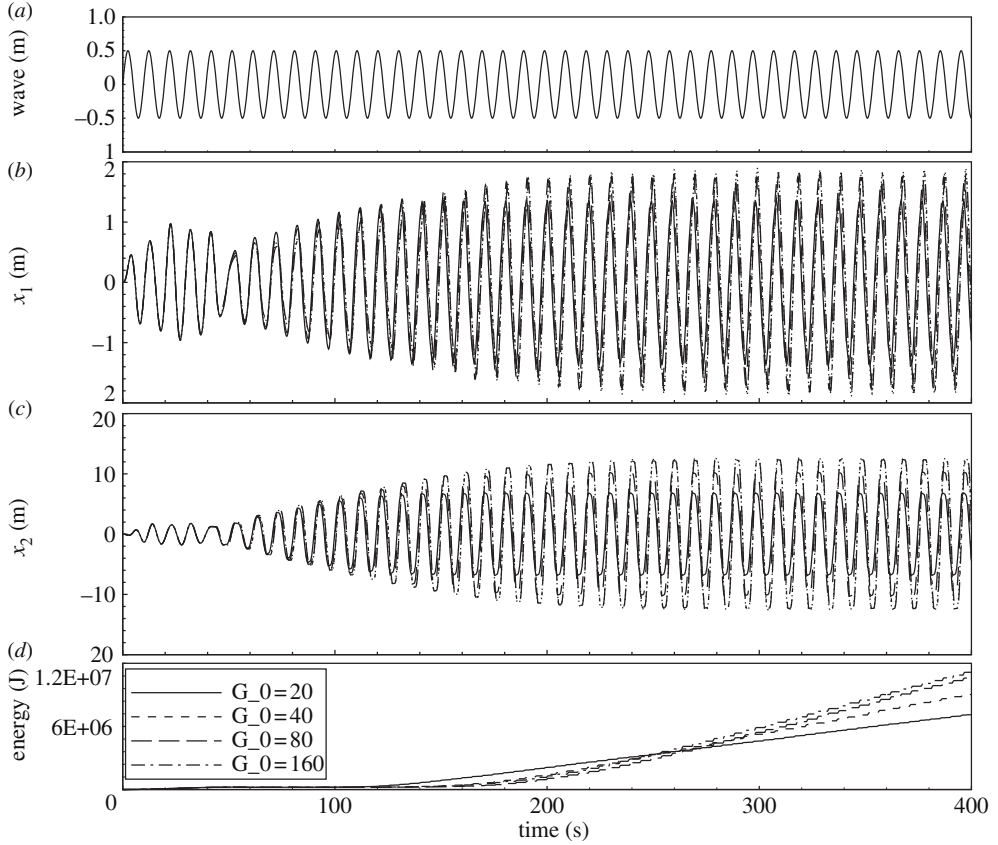


Figure 11. (a–d) Simulations of the response of the 2 d.f. wave energy device under latched–operating–declatched control, with different values of the latching damping  $G_0$ . Incident wave period  $T = 10$  s; wave height  $H = 1$  m.

energy production we get under LOD control, using different values for the time constants. Again, better results are obtained as we tend towards absolute discrete control: the shorter the time constant, the better the energy production.

Time-domain simulations were performed for a set of periods of the regular incident waves in the range (5–15 s) in order to establish the power performance curves of the 2 d.f. system under the three discrete control modes: latching; declutching; and LOD (here  $G_0$  was set to 80,  $B_0$  to 10 000, and  $\tau_{\text{latching}}$  and  $\tau_{\text{PTO}}$  to 0.05). Figure 13 shows the power produced in each mode, measured in the time-domain simulation, when the steady-state regime is established. For the sake of comparison, the theoretical maximum achievable with such an axisymmetric device is also plotted for each wave period, as a reference.

As could be expected, the response without control features two peaks associated with each resonance mode of this bi-oscillator device. One can see that all three controls considered are able to increase the energy absorption for certain wave periods. With the chosen set of coefficients, declutching control seems beneficial mostly around the two natural frequencies of the

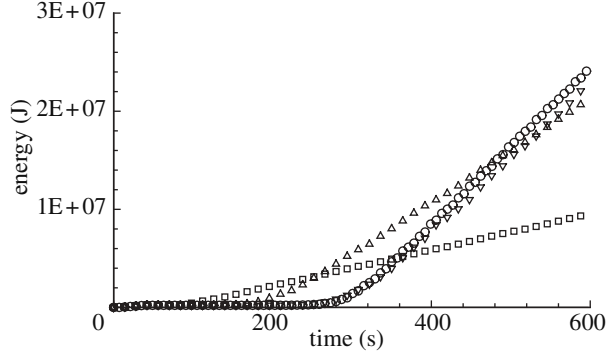


Figure 12.2 d.f. wave energy device under latched–operating–declutched control with varying time constants. Wave period  $T = 10$  s; wave height  $H = 2$  m (squares,  $\tau = 0.2$  s; triangles,  $\tau = 0.1$  s; inverted triangles,  $\tau = 0.05$  s; circles,  $\tau = 0.02$  s).

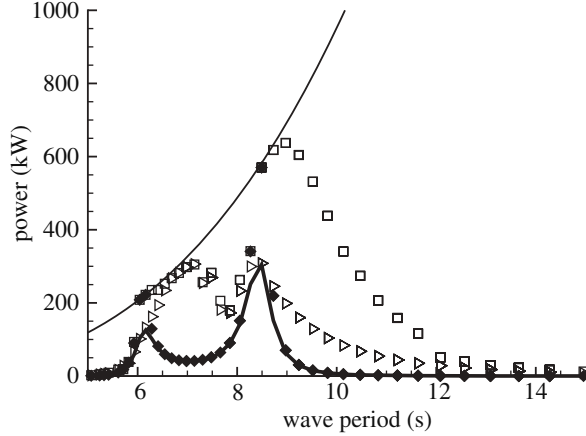


Figure 13. Power curves of the 2 d.f. wave energy device in regular waves under the three discrete control strategies (thick line, without control; right-facing triangles, latching; thin line, theoretical maximum; diamonds, declutching; squares, latched–operating–declutched).

system, amplifying the peak responses. Latching gives good results in the low-frequency range, as expected, but also between the peaks. The most interesting results on figure 13 are those associated with LOD control; it gives better energy absorption than the sum of the energy absorbed with latching control and the energy absorbed with declutching control, giving energy absorption at the theoretical maximum over a large range of wave periods. As shown by the detailed example of figure 10, combining the declutching and latching strategies seems to boost the performance of latching, while declutching alone brings no improvement, as for  $T > 9$  s in figure 13. These conclusions can be extended for other values of the system parameters ( $G_0, B_0$ ), not shown here for brevity.

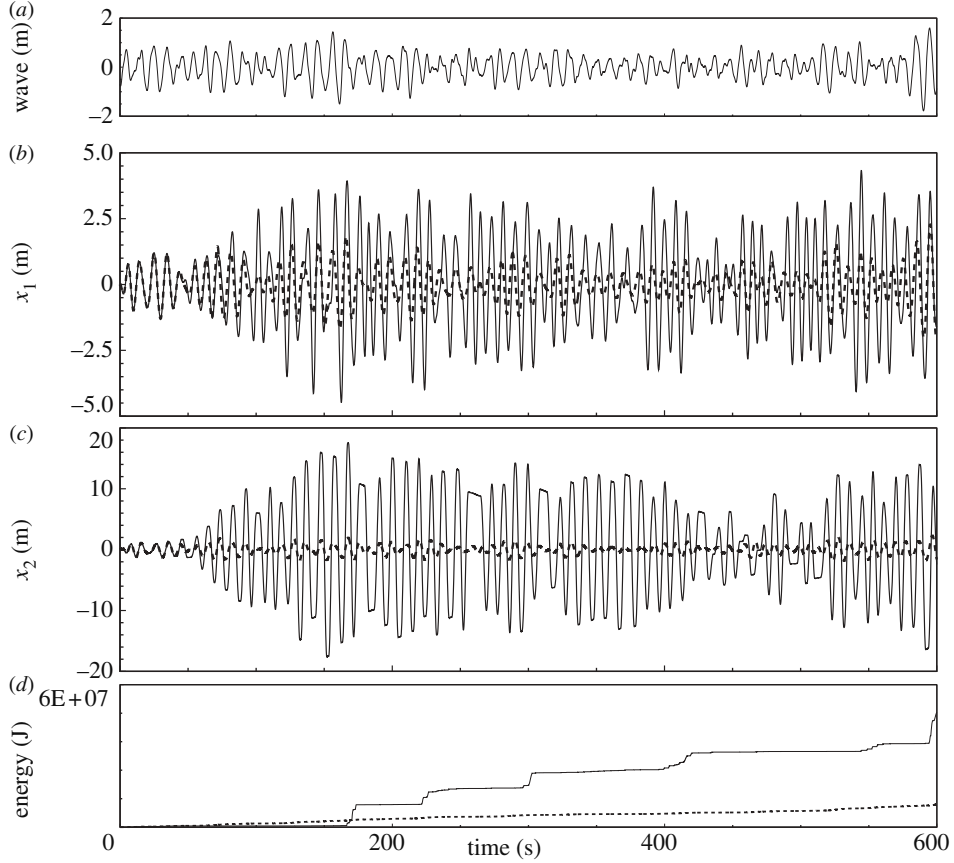


Figure 14. (a–d) Response of the 2 d.f. wave energy device with latched–operating–declutched in random waves. Peak period of the wavetrain  $T_p = 10$  s; significant wave height  $H_s = 2$  m. Power take-off damping coefficient  $B_0 = 50 \text{ kN m}^{-1} \text{ s}^{-1}$  (dashed lines, without control; solid lines, with latched–operating–declutched control).

(g) *Latched–operating–declutched control in irregular waves*

Finally, a set of time-domain simulations were performed in random wave conditions, in sequences of 600 s, each one defined by its peak period  $T_p$  and its wave height  $H_s$ , according to a spectral density following the Pierson–Moskowitz law. The three discrete control strategies were tested for each run. The control sequence was determined by the optimal command methodology exposed above, which means that the control is optimal but not causal, and needs the whole sequence (600 s) of the wave excitation to be computed at each time step.

Figure 14 shows the results of a run with peak period  $T_p = 10$  s and significant wave height  $H_s = 2$  m. The PTO damping coefficient was set to  $B_0 = 50 \text{ kN m}^{-1} \text{ s}^{-1}$ .

In this example, the absorbed power is multiplied by a factor 5.2, from 15.5 kW without control to 78 kW with LOD control. One can see that, in addition to the increase of energy absorption due to LOD control, there is also a large increase in the response of the system, with an amplification of the maximum amplitude of



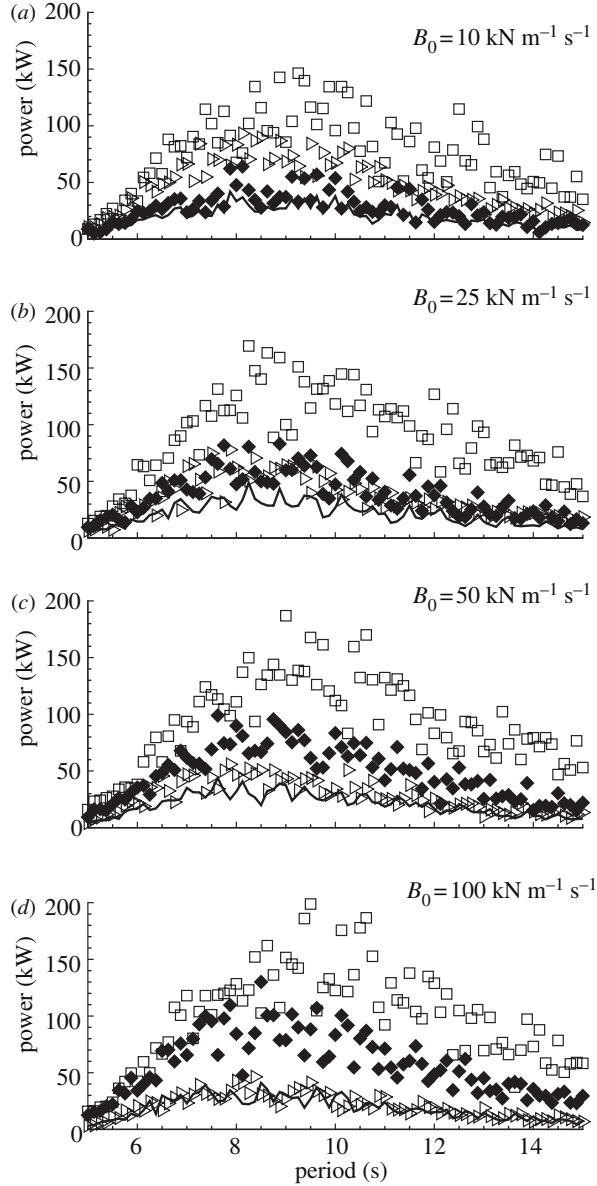


Figure 15. (a–d) Power function of the 2 d.f. wave energy converter versus peak period  $T_p$  in random waves, under latching, declutching or latched–operating–declutched control with four values of the power take-off damping  $B_0$ . Significant wave height  $H_s = 2$  m (solid lines, no control; diamonds, declutching; right-facing triangles, latching; squares, latched–operating–declutched).

the motion by a factor 2 for  $x_1$  and by a factor of 10 for  $x_2$ . The amplitude of the inner moving mass reaches almost 20 m, and would indeed be meaningless within the present context of linear theory, the total draught of the buoy considered here being only 10 m. Constraints on the amplitude of the motion should be taken into account in any serious application study.

Figure 15 shows the power absorbed by the system with and without control in random waves. Each dot on this figure corresponds to the mean power absorbed by the system during a 600 s time-domain simulation of the motion. The significant wave height was 2 m in all runs. Time constants for the controller were set equal to 0.05 s, and  $G_0$  was set equal to  $G_0 = 80$ .

Results are noisy owing to the fact that, for each sequence of irregular incident waves, the phases of the components are set randomly, and each sample is of finite duration. According to standard statistical theory implicitly assumed here, this scattering could be avoided by repeating each run a large number of times and averaging, or by extending the simulation time to infinity. However, the results given in this figure are globally similar and confirm the behaviour observed in the case of regular waves:

- whatever the peak period of the spectrum, the three discrete control strategies are able to increase the energy production in comparison with the reference case without control;
- declutching control works better with large values of the PTO damping coefficient  $B_0$ , whereas latching control works better with small values; and
- LOD control is the control strategy that leads to the largest increase in energy absorption, with gains up to 500 per cent. Moreover, LOD control seems to be relatively insensitive to the value of the PTO damping coefficient, even if large values of it seem to marginally improve the amount of absorbed energy.

### 3. Conclusion

In this study, three strategies of discrete control, applied at the PTO level to oscillating wave energy devices, were presented. The most famous of them, latching control introduced by Budal & Falnes [22] in the early 1980s, was shown to be beneficial not only for long wave periods beyond the natural period of the floating body, but also for shorter waves in sub-harmonic modes, and in irregular waves.

Declutching control, which was proposed more recently, mainly for hydraulic PTOs, is also efficient, especially with large values of the PTO damping coefficient, which is the opposite trend compared with latching.

The three-state control strategy (LOD), consisting of combining these two individual modes, appeared to give far better results than each one individually. It seems that declutching control enhances the power of latching in this combination, especially for long waves, and its performance is less sensitive to the PTO damping coefficient.

The results have been shown to be always better when the discrete control is absolute, switching instantaneously from one model to the other. However, modelling of the transients by suitable time constants have shown that the performances are only slightly affected, with reasonable values of them.

These conclusions were established not only for a single oscillator system, but also for a bi-oscillator harnessing its relative motion; this was not so evident when we began the study. We have shown, in recent unpublished extensions of this work, that LOD control is also superior when applied to the SEAREV device which is a 7 d.f. wave energy device, with four of them being resonant.

Nevertheless, one should keep in mind that, owing to the methods applied here, we have demonstrated only the potential of these discrete control strategies, but not the way to implement them in the real world. The method of Hamilton–Pontryagin, as used here, requires knowledge of the future of the excitation, and is therefore not causal. The development of causal control algorithms, necessarily sub-optimal, is still an open challenge in which we are engaged.

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