Evidential Risk Graph Model for Determining Safety Integrity Level

Boutheina Ben Yaghlane, Christophe Simon, Narjes Ben Hariz

Evidential Risk Graph Model for Determining Safety Integrity Level

Boutheina Ben Yaghlane[1], Christophe Simon[2], and Narjes Ben Hariz[3]

[1] LARODEC Laboratory - Higher Institute of Management of Tunis
[2] LARODEC Laboratory - Institute of High Commercial Studies of Carthage
[3] CRAN - Nancy University - CNRS UMR 7039, 2 Rue Jean Lamour, 54506 Vandoeure, France

Abstract. Risk analysis becomes very important especially with the increase of risk accidents in the industrial fields. In this context, we present in this paper a new approach based on belief functions theory for determining the safety integrity level of a safety instrumented system. This approach consists on collecting data from expert opinions by eliciting judgements using a qualitative method, dividing them in groups using the k-means algorithm and aggregating them by applying a hierarchical method. The output of the data collecting process will be integrated into a risk evaluation model in order to get the safety integrity level. As an evaluation method we proposed a new generalized risk graph named Evidential Risk Graph which is able to deal with imperfect data modeled with the belief functions theory.

1 Introduction

Currently, industrial facilities present different risks for persons, equipment and environment. Serious accidents are still caused by these risks. One of the solutions for dealing with these problems is having good safety systems. To design, implement and maintain these systems various standards can be used. For instance the IEC61508 standard [7] presents the Safety Instrumented Systems (SIS) whose main objective is reducing the occurrence probability of the risk.
The risk reduction process is based on the evaluation of the necessary risk reduction level according to the Safety Integrity Level (SIL) of the SIS. Several methods can be used for risk evaluation such as the risk graph and the risk matrix [7].
Risk evaluation methods are based on various parameters. Getting these data becomes more and more difficult spatially with the fast changes in the current society. Experts can be a good source of information to deal with these parameters. Collecting data from experts requires two basic steps: elicitation of expert opinions and aggregation of expert opinions.
Experts when giving their opinions can not be always sure and precise. Thus, data originating from experts are usually imperfect. Many mathematical theories are able to deal with this type of data such as probability theory, possibility theory [16] and evidence theory [11].

In this paper, we propose an approach for SIL allocation based on the belief functions theory. After eliciting and aggregating expert opinions in order to get different parameters of our risk evaluation method, we propose a generalized risk graph based on evidence theory.
The rest of this paper is organized as follows: section 2 presents the risk reduction process, it defines many notions related to this process. In section 3, we describe the approach of collecting expert opinions in the evidence framework. We describe the proposed evidential risk graph and the schema of the adopted approach in section 4, a case study will be presented at the end of this section in order to illustrate the proposed evidential risk graph.

2 Risk Reduction Process

The risk reduction process aims to reduce the occurrence probability of the risk. To achieve this objective, it is important to implement a safety system and evaluate the risk using a risk evaluation methods.

2.1 Risk and Safety Systems

Risk. According to the IEC 61508 standard [7], the risk is "A combination of the probability of a damage and its gravity". Farmer [5] has developed this relation between risk, probability and gravity in his curve shown in figure1. Risks classified under the curve are considered as acceptable, they are considered as unacceptable if they are placed over the curve. There are two main strategies to

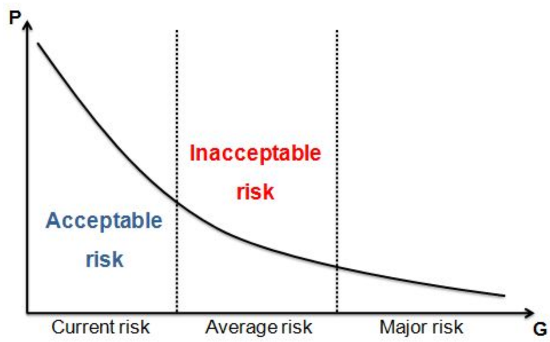

Fig. 1. Farmer's curve [5]

reduce the risk and solve these problems: the prevention strategy which consists on minimizing the occurrence probability and the protection strategy by limiting the consequence of a dysfunction [12]. To reduce the occurrence probability of the risk, industrial facilities have to design a good safety system.

Safety Systems. Safety is often defined by its opposite. it can be seen as the absence of danger, risk, accident or disaster [17].
A safety system is a system aimed to achieve a safe state and maintain it for an equipment, a machine or any other device. Safety systems are based on different types of technologies. The IEC 61508 standard [7] presents the Electric/Electronic/Programmable Electronic Systems (E/E/PES) which are systems of command, protection or surveillance based on one or more Programmable Electronic devices.

Safety Instrumented Systems(SIS). Safety Instrumented Systems are one of the most used E/E/PES. The main objective of these systems is to take a process into a safe state when it is in a real risk situation.
A SIS is composed of three parts. The sensor part is used to supervise the drift of a parameter towards a dangerous state. The logic unit is dedicated to collect the signal coming from the sensor, treat it and compute the actuator's input. The main objective of the third part called the actuator part is to put the process into a safe state and maintain it [12].
A SIS is used to implement Safety Instrumented Functions (SIF) that is intended to control parameters and implement actions in order to achieve or maintain a safe state for the supervised process with respect to the specific hazardous event. Each SIF affords a measure of risk reduction indicated by its Safety Integrity Level (SIL).

2.2 Risk Evaluation Methods

Several methods are used to evaluate the risk level. These methods are divided on three categories: quantitative methods, semi-quantitative methods and qualitative methods. In our approach we are interested in semi-quantitative and qualitative methods such as risk matrix and risk graph. Risk graph is one of the most popular methods used in industry problems. Because it is a simple and clear way to model the relation between the risk and its components. This method is used to measure the risk level by determining the SIL of a SIF in a security instrumented system which gives the necessary risk reduction level. The parameters used in the risk graph are generally imperfect and the main sources of these parameters are experts.
Figure 3 shows an example of a risk graph for SIL allocation according to the IEC standard [7]. This model is based on four parameters, C, F, P and W. The meaning of each parameter is given in table 1. This graph is explained as follow: The use of parameters C, F, and P gives as a result several exits (X_1, X_2, X_3..., X_n). Each exit is recorded in one of three scales (W_1, W_2, W_3). Each scale gives the SIL allocation level for the SIS. There are four levels of risk reduction ($SIL \in 1, 2, 3, 4$). Level a means that a SIF is not necessary and level b indicates that only one safety system is not sufficient.

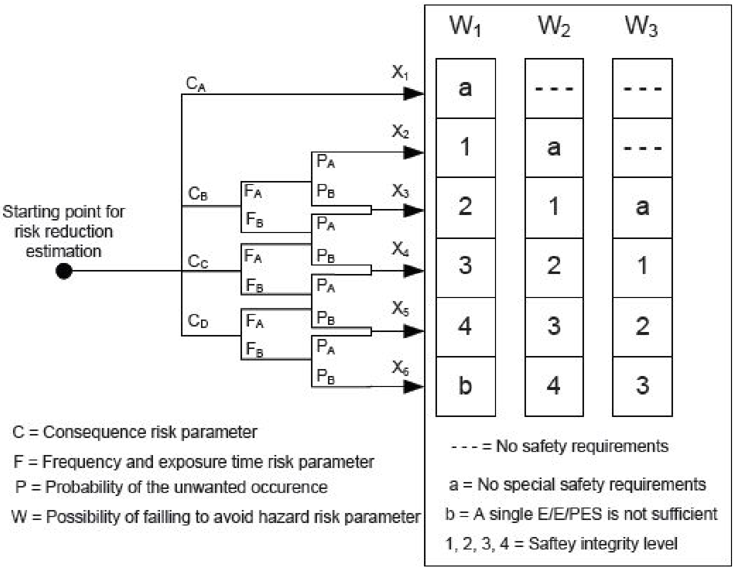

Fig. 2. Risk Graph in IEC 61508 for SIL allocation

Table 1. Risk graph parameters

Parameter	Meaning	Possible values
C	Consequence of the dangerous event	C_A;C_B;C_C;C_D
F	Frequency and exposure time	F_A;F_B
P	Possibility of avoiding the dangerous event	P_A;P_B
W	Probability of the unwanted occurrence	W_1;W_2;W_3

2.3 Risk Evaluation Under Uncertainty

Risk evaluation methods are based on several parameters. Usually, these parameters are imperfect. Indeed, they can be incoherent, imprecise or/and uncertain. Many theories can be used for dealing with imperfect data, such as probability theory, possibility theory and evidence theory. This theory becomes more and more popular. It is a simple and flexible framework for dealing with imperfect information. It generalizes the probabilistic framework by its capacity to model the total and partial ignorance. Also, it is a powerful tool for combining data. Thus we are interested, in this paper, on the treatment of uncertainty using the evidence theory.

Evidence Theory. The evidence theory also known as belief functions theory or Dempster-Shafer theory was introduced by Dempster in 1968 and Shafer in 1976.

Several models have been proposed from this theory. One of the most used is the Transferable Belief Model developed by Smets to represent quantified beliefs. In the following, we remind some basic concepts of the TBM. More details can be found in [11, 13, 14].

Basic Concepts. Let Ω a finite set of exclusive and exhaustive elements called the frame of discernment and 2^{Ω} its power set defined by:

$$2^{\Omega} = \{A : A \subseteq \Omega\} \tag{1}$$

A basic belief assignment (*bba*) is a function $m : 2^{\Omega} \rightarrow [0, 1]$, such that:

$$\sum_{A \subseteq \Omega} m(A) = 1 \tag{2}$$

$m(A)$ is the portion of belief supporting exactly A. Any subset A of Ω such that $m(A) > 0$ is called a focal element.
The belief function corresponding to m is a function $bel : 2^{\Omega} \rightarrow [0, 1]$, defined as:

$$bel(A) = \sum_{\emptyset \neq B \subseteq A} m(B) \tag{3}$$

$bel(A)$ gives the amount of support given to A.
The plausibility function associated with m is a function $pl : 2^{\Omega} \rightarrow [0, 1]$, defined by:

$$pl(A) = \sum_{\emptyset \neq B \cap A} m(B) \tag{4}$$

$pl(A)$ represents the maximum amount of potential specific support that could be given to A, it contains parts of belief that do not contradict A.
The commonality function associated with a bba m is a function $q : 2^{\Omega} \rightarrow [0, 1]$, defined by:

$$q(A) = \sum_{A, B \subseteq \Omega, B \supseteq A} m(B) \tag{5}$$

Combination of belief functions. Let m_1 and m_2 two $bba's$ representing two sources of information and having the same frame of discernment Ω. Different rules can be used to combine these pieces of information. The conjunctive rule of combination (CRC) denoted by $\bigcirc\!\!\!\!\cap$ is defined as follow:

$$(m_1 \bigcirc\!\!\!\!\cap\, m_2)(A) = \sum_{B, C \subseteq \Omega:\ B \cap C = A} m_1(B).m_2(C) \tag{6}$$

The disjunctive rule of combination (DRC) denoted by $\bigcirc\!\!\!\!\cup$ is defined as:

$$(m_1 \bigcirc\!\!\!\!\cup\, m_2)(A) = \sum_{B, C \subseteq \Omega:\ B \cup C = A} m_1(B).m_2(C) \tag{7}$$

As an extension of the conjunctive rule we have the cautious conjunctive rule [3] denoted by $\owedge$. This rule is obtained by the following formula:

$$m_1 \owedge m_2 = \cap_{A \subsetneq \Omega} A^{\omega_1(A) \wedge \omega_2(A)} \tag{8}$$

where $\wedge$ denotes the minimum operator and $\omega(A)$ is the weight of every $A \in 2^\Omega \setminus \{\Omega\}$ obtained by:

$$\omega(A) = \prod_{B \supseteq A} q(B)^{(-1)^{|B|-|A|+1}} \tag{9}$$

where $|A|$ is the cardinality of A, and A^ω denotes the Generalized Simple BBA (GSBBA). It is a function $\mu : 2^\Omega \longrightarrow \Re$ verifying:

$$\begin{array}{l} \mu(A) = 1 - \omega \\ \mu(\Omega) = \omega \\ \mu(B) = 0, \;\; \forall B \in 2^\Omega \;\; A, \Omega \end{array} \tag{10}$$

for $A \neq \Omega$ and $\omega \in [0, +\infty)$.
The use of all these rules of combination depends on the dependency and reliability of the data sources.

Decision Making. The problem of making decisions from beliefs is resolved in the TBM by the pignistic transformation which gives a probability measure denoted by $BetP$ in order to make a decision. The pignistic transformation is defined as follow:

$$BetP(A) = \sum_{B \subseteq \Omega} \frac{|A \cap B|}{|B|} \frac{m(B)}{1 - m(\emptyset)} \;\; \forall A \subseteq \Omega \tag{11}$$

where $|A|$ denotes the cardinality of A.

3 Collecting Expert Opinions

As we previously mentioned, risk evaluation methods are based on different parameters. The process of collecting these parameters is very important to get correct and relevant results. However, getting needed information is currently very difficult. Experiences are not always possible and they can not give at all times the expected and useful results. So, experts' opinions can be a good solution for these problems.
Two steps are very important in the process of collecting expert opinions: eliciting these opinions and aggregating them.

3.1 Elicitation of Expert Opinions in the Evidence Theory

Getting efficient information from expert opinions needs to model them in a proper way. Two main approaches are generally adopted for elicitation of expert opinions [2].The quantitative approaches and the qualitative approaches.

Quantitative Approach. In the quantitative approaches the expert is asked to give his judgement using numbers. Depending on the problem, these numbers can be modeled according to the probability, possibility or evidence theory.
It is very difficult to experts to express their opinions especially when they are not familiar with the theory used in the elicitation problem. Then, the qualitative approach can be more suitable to elicit experts' opinions.

Qualitative Approach. In this approach experts can easily express their opinions using natural language. Several methods have been proposed for eliciting qualitatively expert opinions.
Ben Yaghlane et al. [1, 2] proposed a method for constructing belief functions from qualitative expert opinions. The main idea of this method is expressing opinions using preferences. These preferences are based on two binary relations: the preference relation denoted by $\succ$ and the indifference relation denoted by $\sim$. These relations will be transformed into belief functions as follows:

$$A \succ B \Leftrightarrow bel(A) > bel(B) \tag{12}$$

$$A \sim B \Leftrightarrow bel(A) = bel(B) \tag{13}$$

where A and B two propositions in 2^{Ω}.
These relations constitute the constraints of an optimization problem in order to get the evidence distribution suitable to expert opinions.

3.2 Aggregation of Expert Opinions

Once the elicitation step is achieved, an aggregation process will be very important in order to get a unique and reliable information that represents all experts' opinions. Here, we are interested in aggregating data using the belief functions framework.

Combination of Expert Opinions in the Belief Functions Theory. As mentioned in section 2.3, many rules in evidence theory can be used for the fusion of expert judgements. The efficiency of these rules depends on the reliability and dependence of the sources of information. For instance, the conjunctive rule is usually used for combining two $bba's$ produced by distinct and reliable sources of information. For the fusion of evidences provided by sources which are distinct but not considered all reliable, the disjunctive rule is generally used. The cautious conjunctive rule is suitable when sources are correlated [3, 15]. Then, it will be interesting to have a combination method based on more than one rule of combination which can be able to combine different types of information sources.

Hierarchical Method for Aggregation of Expert Opinions. Ha-Duong [6] proposed a hierarchical method for aggregating expert opinions based on two rules of combination of expert opinions. The main idea of this method is to combine conjunctively coherent sources of information and then combining disjunctively partially aggregated opinions. It is based on three essential steps:

1. Dividing expert opinions into schools of thought, i. e. experts which have similar opinions will be in the same group.
2. Combining information within each group using the cautious conjunctive rule assuming that sources in each group are reliable but not independent.
3. Combining the different results of the second step using the disjunctive rule supposing that the groups of experts are independent but not all reliable.

These steps are presented by the following formula:

$$m_{Hierarchical} = \bigcirc\!\!\!\!\cup_{k=1..N} \ \bigcirc\!\!\!\!\wedge_{i \in G_k} discount(m_i, 0.999) \tag{14}$$

where N denotes the number of groups and $G_1...G_N$ present the different groups of experts.
This approach is extremely dependent on the step of dividing experts. This step becomes more difficult when the number of experts is very large. So, we propose to divide them automatically using a clustering algorithm.

Clustering Method for Dividing Expert Opinions. The clustering is the process of organizing objects into groups (clusters) by maximizing the similarity of objects within the same group and maximizing the dissimilarity of objects belonging to different clusters [10].
One of the well spread techniques of clustering is the k-means [8] algorithm proposed by MacQueen in 1967. The k-means algorithm is based on the following steps [15]:

1. Select arbitrarily an initial partition with K clusters;
2. Compute cluster centers;
3. **Repeat**:
   (a) Generate a new partition by assigning each object to its nearest cluster center.
   (b) Compute new cluster centers.
   Until cluster membership stabilizes (clusters do not change from an iteration to another);

Two parameters are essential in this algorithm: the number of clusters K and the metric used to measure the similarity (distance) between objects. In our work, we assume that K is given by a manager. For the metric used in this algorithm we need one able to measure the distance between two bodies of evidence. The distance of Jousselme et al. [9] is widely used in the belief functions framework. This distance takes into account the specificity of the belief function by inducing the cardinalities of the focal elements in the distance's calculation. It is defined as follow:

$$d_{BPA}(m_1, m_2) = \sqrt{\frac{1}{2}(\overrightarrow{m_1} - \overrightarrow{m_2})^T D(\overrightarrow{m_1} - \overrightarrow{m_2})} \tag{15}$$

where m_1 and m_2 are two bodies of evidence defined on the same frame of discernment Ω. $\overrightarrow{m_1}$ and $\overrightarrow{m_1}$ are vectors containing the $bba's$ of m_1 and m_2. D is a $2^{|\Omega|} \times 2^{|\Omega|}$ matrix whose elements are defined as:

$$D(A,B) = \begin{cases} 1 & \text{if A=B=}\emptyset \\ \frac{|A\cap B|}{|A\cup B|} & \forall A,B \in 2^{\Omega} \end{cases} \tag{16}$$

where $|A|$ denotes the cardinality of A.

4 Evidential Risk Graph

The main objective of the process of collecting expert opinions is to prepare the different parameters that will be the input of the adopted risk evaluation method called the Evidential Risk Graph.

4.1 Evidential Risk Graph

The standard risk graph as described in section 2.2 needs four essential parameters: C, F, P and W. To get the SIL, the risk graph works as a decision tree: if C=.. *and* F=.. *and* P=.. *and* W=.. *then* SIL=.. .
We propose in this paper an evidential risk graph which has the same parameters as a standard risk graph. These parameters are uncertain and presented using the belief functions theory. Our main objective is to propagate these parameters in the evidential risk graph in order to get the SIL of a SIS. To achieve this objective, we simulate the same inference engine (classification procedure) of the Belief Decision Trees (BDT) proposed by Elouedi et al.[4]. Several steps are needed in this process:

1. Generate a global frame of discernment Ω_G relative to all the parameters using the cross-product of the different frames of discernment:
$$\Omega_G = \Omega_C \times \Omega_F \times \Omega_P \times \Omega_W \tag{17}$$
2. Extend the $bba's$ (m_C; m_F; m_P; m_W) of the different parameters to the global frame of discernment Ω_G. the extended $bba's$ are denoted by: $m_{C\uparrow G}$; $m_{F\uparrow G}$; $m_{P\uparrow G}$; $m_{W\uparrow G}$
3. Calculate the body of evidence corresponding to the global frame of discernment m_G by aggregating the different extended $bba's$ using the conjunctive rule of combination:
$$m_G = m_{C\uparrow G} \textcircled{\cap} m_{F\uparrow G} \textcircled{\cap} m_{P\uparrow G} \textcircled{\cap} m_{W\uparrow G} \tag{18}$$
4. Calculate the belief function $bel^{\Omega_{SIL}}[x]$ of each focal element x of the *bba* m_G generated by the third step. As in the belief decision tree, this calculation depends on the cardinality of the treated focal element x:
   - If the focal element is a singleton ($|x| = 1$), then $bel^{\Omega_{SIL}}[x]$ is equal to the belief function of the leaf attached to the treated focal element.
   - If the focal element is not a singleton ($|x| > 1$), then $bel^{\Omega_{SIL}}[x]$ depends on the different paths corresponding to the values of the parameters:

- If all paths bring to the same leaf, then $bel^{\Omega_{SIL}}[x]$ is given by the belief function of the leaf related to these paths.
- If paths lead to distinct leaves, then $bel^{\Omega_{SIL}}[x]$ is computed by combining the belief functions corresponding to each leaf using the disjunctive rule of combination.

5. Compute the belief functions of the different classes (SIL levels) by averaging the belief functions computed in the previous step using the following formula:

$$bel^{\Omega_{SIL}}[\Omega_G](\omega) = \sum_{x \subseteq \Omega_G} m_G(x).bel^{\Omega_{SIL}}[x](\omega) \;\; for \;\; \omega \in \Omega_{SIL} \tag{19}$$

6. Transform the beliefs resulting from the fifth step to probabilities using the pignistic transformation in order to make a decision. The adopted SIL will be the SIL having the highest probability value.

Thus, the result output of the evidential risk graph will be the SIL based on the given experts' opinions.

4.2 General Scheme of the Proposed Process

We now introduce the general process summarized in figure 3. The first step in this process is the elicitation of expert opinions by the qualitative method of Ben Yaghlane et al [2]. The second step is dividing expert opinions using the k-means algorithm. The output of this step will be the input of the third step which is aggregating expert judgements by means of the hierarchical method of fusion of expert opinions proposed by Ha-Duong [6]. These steps will be performed for each parameter (C, F, P, W) in order to get $bba's$ corresponding to these parameters. The resulting $bba's$ will be integrated in the evidential risk graph which will generate the safety integrity level of the SIS.

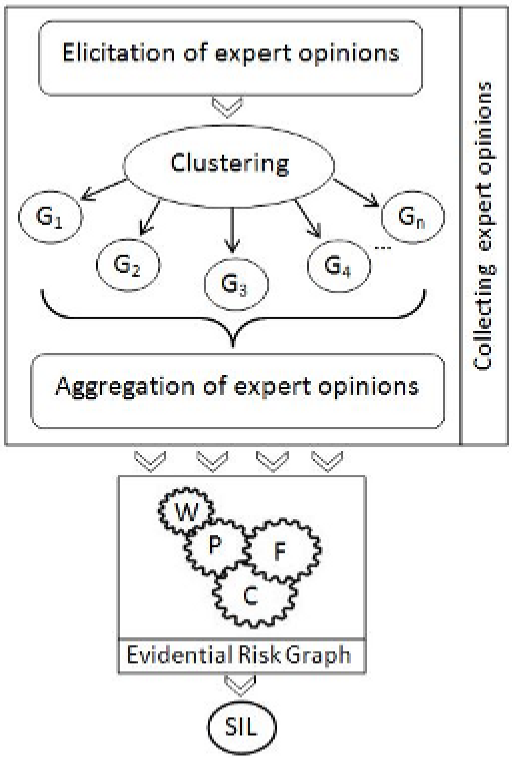

Fig. 3. SIL allocation by elicitation and aggregation of expert opinions using evidential risk graph

4.3 Advantages of the evidential risk graph

The evidential risk graph has many advantages:

- It is a clear and simple way to determine the safety integrity level as it maintain the same graphical structure of a standard risk graph.
- It can be considered as a qualitative or a semi quantitative method for risk evaluation.
- It is based on the belief functions theory. Thus, it can be used with perfect data as well as imperfect data.
- It can be applied with different types of systems in different fields.

5 Case Study

In order to illustrate the proposed approach for SIL allocation we present in the following a case study detailing its different steps.

5.1 Problem's description

Let us consider an example from the IEC standard. A process composed of a pressurized vessel containing volatile flammable liquid (see figure 4) can reject material in the environment. The acceptable risk is defined, it has an average level of gas rejection less than 10 years. A hazard analysis has shown that the current protection systems (alarm and protection layers) are insufficient to warrant the risk level. Our goal is to determine the SIL level of a safety integrated function that allows to reach the acceptable level risk. This determination is based on the known risk about the vessel [7, 18]. Below are the different values

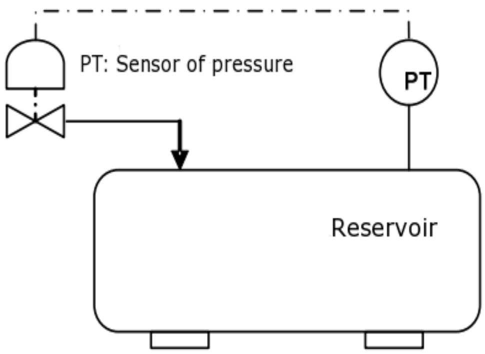

Fig. 4. Vessel under pressure

of the risk parameters used:

- Significance of parameter C:
  - Low: minor harm
  - Medium: serious harm affecting one or more persons
  - High: Death of several people
  - Very High: Several killed people
- Significance of parameter F:
  - Medium: exposure from rare to frequent in a dangerous area
  - High: exposure from frequent to permanent in a dangerous zone
- Significance of parameter P:
  - Medium: possible under some conditions
  - High: almost impossible
- Significance of parameter W:
  - Low: a very weak probability that undesired events occur or only some undesired occurrences is probable
  - Medium: a weak probability that undesired events occur or only some undesired occurrences is probable

- High: a high probability that undesired events occur or it is probable that undesired events frequently occur

The values and the frames of discernment of these parameters are presented in table 2.

Table 2. Possible values of parameters

Parameter	**Possible values**	**Frame of discernment**
C	Low (L_C); Medium (M_C); High (H_C); Very High (VH_C)	$\Omega_C = \{L_C; M_C; H_C; VH_C\}$
F	Medium (M_F); High (H_F)	$\Omega_F = \{M_F; H_F\}$
P	Medium (M_P); High (H_P)	$\Omega_P = \{M_P; H_P\}$
W	Low (L_W); Medium (M_W); High (H_W)	$\Omega_W = \{L_W; M_W; H_W\}$

5.2 Collecting of expert opinions

For data collecting process we consider the opinions of fives experts. The judgement of each expert concerning each parameter is summarized in table 3. These opinions are expressed as preferences that will be transformed into an optimization problem. The result of the elicitation step will be the *bba's* of the different

Table 3. Expert opinions

Expert	**C**	**F**	**P**	**W**
Expert1	$\{VH_C\}$	$\{H_F\}$	$\{M_P\}$	$\{L_W\}$
Expert2	$\{VH_C\}$	$\{H_F\}$	$\{M_P \cup H_P\}$	$\{L_W\} \sim \{L_W \cup M_W \cup H_W\}$
Expert3	$\{VH_C\}$	$\{H_F\}$	$\{H_P\} \succ \{M_P\}$	$\{L_W\}$
Expert4	$\{VH_C\}$	$\{H_F\}$	$\{M_P \cup H_P\} \succ \{M_P\}$	$\{M_W\} \succ \{L_W\}$
Expert5	$\{VH_C\}$	$\{H_F\}$	$\{M_P\}$	$\{M_W \cup H_W\} \succ \{L_W\}$

parameters. These results are summarized in table 4.

5.3 Clustering

The next step consists on dividing the different opinions given for each parameter using the clustering algorithm.
The graphical representation of the opinion of each expert (figure 5) can be helpful for choosing the number of clusters k.

Table 4. The result of the elicitation step

Parameter C	Expert1	$m(\{VH_C\}) = 1$
	Expert2	$m(\{VH_C\}) = 1$
	Expert3	$m(\{VH_C\}) = 1$
	Expert4	$m(\{VH_C\}) = 1$
	Expert5	$m(\{VH_C\}) = 1$
Parameter F	Expert1	$m(\{H_F\}) = 1$
	Expert2	$m(\{H_F\}) = 1$
	Expert3	$m(\{H_F\}) = 1$
	Expert4	$m(\{H_F\}) = 1$
	Expert5	$m(\{H_F\}) = 1$
Parameter P	Expert1	$m(\{M_P\}) = 1$
	Expert2	$m(\{M_P \cup H_P\}) = 1$
	Expert3	$m(\{H_P\}) = 0.9267$ $m(\{M_P\}) = 0.0733$
	Expert4	$m(\{H_P\}) = 0.242$ $m(\{H_P \cup M_P\}) = 0.758$
	Expert5	$m(\{M_P\}) = 1$
Parameter W	Expert1	$m(\{L_W\}) = 1$
	Expert2	$m(\{L_W\}) = 1$
	Expert3	$m(\{L_W\}) = 1$
	Expert4	$m(\{M_W\}) = 0.9267$ $m(\{L_W\}) = 0.0733$
	Expert4	$m(\{M_W\}) = 0.242$ $m(\{H_W \cup M_W\}) = 0.758$

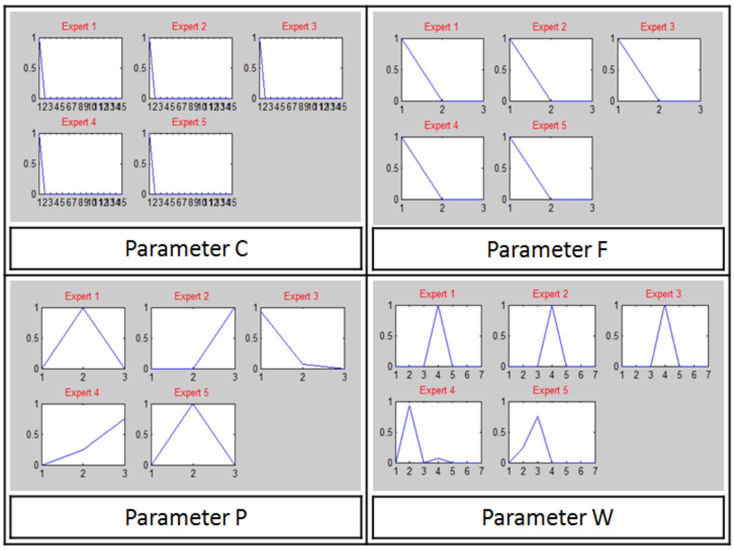

Fig. 5. Graphical representation of expert opinions

It is clear for both parameters C and F that we have one cluster that contains all experts. Assuming that we have three groups of experts for P and two groups for parameter W, the result of the clustering step is as follows:

Parameter P
$Expert1, Expert5 \subset Cluster1$
$Expert2, Expert4 \subset Cluster2$
$Expert3 \in Cluster3$.
Parameter W
$Expert1, Expert2, Expert3 \subset Cluster1$
$Expert4, Expert5 \subset Cluster2$

5.4 Aggregation of expert opinions

Once the groups are generated, then we can aggregate the different opinions using the hierarchical method explained in section 3.2.

Conjunctive combination: The information provided by experts belonging to the same group are combined using the cautious conjunctive rule [3]. In this use case, all experts assert that parameters C and P are certain. So, the bba0s of these parameters do not change from a step to another. The result of the conjunctive combination within each group of experts for parameters F and W is summarized in table 5.

Disjunctive combination: After aggregating data within each cluster, it is necessary now to combine the different results of the previous step in order to have a unique and useful information for each parameter.The result of this step will be the result of the collecting data process. It is summarized in table 6.

Table 5. The result of the conjunctive combination

Parameter P	Cluster1	$m(M_P) = 1$
	Cluster2	$m(\{M_P\}) = 0.242$ $m(\{H_P \cup M_P\}) = 0.758$
	Cluster3	$m(\{H_P\}) = 0.9267$ $m(\{H_P \cup M_P\}) = 0.0733$
Parameter W	Cluster1	$m(\{L_W\}) = 1$
	Cluster2	$m(\{M_W\}) = 0.9267$ $m(\{H_W\}) = 0.0733$

Table 6. The result of collecting data process

Parameter	**Focal elements**
C	$m(\{VH_C\}) = 1$
F	$m(\{H_F\}) = 1$
P	$m(\{M_P\}) = 0.0177$ $m(\{H_P \cup M_P\}) = 0.9823$
W	$m(\{L_W\}) = 0.0733$ $m(\{L_W \cup M_W\}) = 0.9267$

5.5 Performing Evidential Risk Graph

These results are the input of the evidential risk graph. As mentioned previously, the inference in the evidential risk graph is similar to the classification process in the belief decision trees.

1. Generation of the global frame of discernment Ω_G:

   $$\Omega_G = \{(L_C, L_F, L_P, L_W); (L_C, L_F, L_P, M_W); (L_C, L_F, L_P, H_W); (L_C, L_F, M_P, L_W); ...\}$$

2. Extension of bba's to the global frame of discernment for each parameter:

- Parameter C:
  - $m_{C\uparrow G}(\{VH_C\} \times \Omega_F \times \Omega_P \times \Omega_W) = 1$
- Parameter F:
  - $m_{F\uparrow G}(\{H_F\} \times \Omega_C \times \Omega_P \times \Omega_W) = 1$
- Parameter P:
  - $m_{P\uparrow G}(\{M_P\} \times \Omega_C \times \Omega_F \times \Omega_W) = 0.02$
  - $m_{P\uparrow G}(\{H_P \cup M_P\} \times \Omega_C \times \Omega_F \times \Omega_W) = 0.02$
- Parameter W:
  - $m_{W\uparrow G}(\{L_W\} \times \Omega_C \times \Omega_F \times \Omega_P) = 0.08$
  - $m_{W\uparrow G}(\{L_W \cup M_M\} \times \Omega_C \times \Omega_F \times \Omega_P) = 0.92$

3. Combination of the extended bba's:
   - $m_G(\{(VH_C, H_F, M_P, L_W) \cup (VH_C, H_F, H_P, L_W)\}) = 0.072$
   - $m_G(\{(VH_C, H_F, M_P, L_W) \cup (VH_C, H_F, M_P, M_W) \cup (VH_C, H_F, H_P, L_W) \cup (VH_C, H_F, H_P, M_W)\}) = 0.9103$
   - $m_G(\{(VH_C, H_F, M_P, L_W)\}) = 0.0013$
   - $m_G(\{(VH_C, H_F, M_P, L_W) \cup (VH_C, H_F, M_P, M_W)\}) = 0.0164$

4. Computation of beliefs on levels:

   Let $\Omega_{SIL} = \{No; SIL1; SIL2; SIL3; a; b\}$ be the frame of discernment containing the safety levels generated by the evidential risk graph. Figure 6 shows the decision tree that corresponds to this problem. Leaves of this tree are numbered in order to characterize each road generated by this tree.
   This step consists on computing the beliefs on levels defined on Ω_{SIL} by taking into account roads generated by each focal element found in the previous step according to the tree in figure 6. Thus, we get:
   - $bel^{\Omega_{SIL}}[\{(VH_C, H_F, M_P, L_W) \cup (VH_C, H_F, H_P, L_W)\}] = bel_{34} \bigcirc\!\!\!\!\!\cup\ bel_{37}$
   - $bel^{\Omega_{SIL}}[\{(VH_C, H_F, M_P, L_W) \cup (VH_C, H_F, M_P, M_W) \cup (VH_C, H_F, H_P, L_W) \cup (VH_C, H_F, H_P, M_W)\}] = bel_{34} \bigcirc\!\!\!\!\!\cup\ bel_{35} \bigcirc\!\!\!\!\!\cup\ bel_{37} \bigcirc\!\!\!\!\!\cup\ bel_{38}$
   - $bel^{\Omega_{SIL}}[\{(VH_C, H_F, M_P, L_W)\}] = bel_{34}$

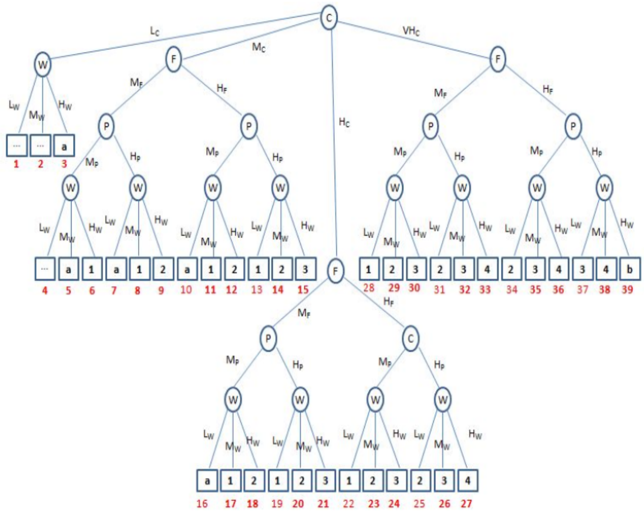

Fig. 6. The decision tree corresponding to the risk graph

- $bel^{\Omega_{SIL}}[\{(VH_C, H_F, M_P, L_W) \cup (VH_C, H_F, M_P, M_W)\}] = bel_{34} \textcircled{\scriptsize$\cup$} bel_{35}$

5. Aggregation of beliefs defined on Ω_{SIL}:

   This step consists on computing the belief of each level using equation 19. These beliefs are transformed into $bba's$ as follows:

   - $m^{\Omega_{SIL}}[m_G](\{SIL2\}) = 0.0013$
   - $m^{\Omega_{SIL}}[m_G](\{SIL2 \cup SIL3\}) = 0.0884$
   - $m^{\Omega_{SIL}}[m_G](\{SIL2 \cup SIL3 \cup SIL4\}) = 0.9103$

6. Decision making:

   In order to make a decision and know the risk reduction level needed for this system, it is necessary to transform the beliefs computed for each level into probabilities using the pignistic transformation. The pignistic probabilities are shown in table 7.

Thus, the risk reduction level needed in this case is SIL2.

6 Conclusion

As a method for SIL allocation, we proposed, in this paper, the evidential risk graph. It is an extension of the standard risk graph based on the belief functions

Table 7. The result of the evidential risk graph

Class	*BetP*
b	0
a	0
SIL4	0.3034
SIL3	0.3476
SIL2	0.3489
SIL1	0
No safety requirements	0

theory, which uses data originating from expert opinions in order to get the risk reduction level. The first step in our approach is collecting data from expert judgements by eliciting and aggregating them. For the fusion of expert opinions we used a hierarchical method which divides these opinions before aggregating them. To make this process faster and easier, we proposed to automate it by means of a clustering algorithm. In this work we assumed that the different risk integrity levels given by the risk graph are certain. Further work can be elaborated to deal with uncertain classes.

References

[1] Ben Yaghlane, A., Deneoux, T., Mellouli, K.: Elicitation of expert opinions for constructing belief functions. In Proceeding of IPMU'2006. Vol I, pages 403-411 (2006)

[2] Ben Yaghlane, A., Deneoux, T., Mellouli, K.: Constructing Belief Functions from Qualitative Expert Opinions. Information and communication Technologies. ICTTA'06,1, 2006, pp.1363-1368 (2006)

[3] Denoeux, T.: Conjunctive and disjunctive combination of belief functions induced by non distinct bodies of evidence. Artificial Intelligence. 172(2-3):234-264 (2008)

[4] Elouedi, Z., Mellouli, K., Smets, P.: Belief decision trees: theoretical foundations. International Journal of Approximate Reasoning, IJAR. v28. 91-124 (2001)

[5] Farmer, F. R.: Siting criteria : a new approach. Atom, 128 :152166 (1967)

[6] Ha-Duong, M.: Hierarchical fusion of expert opinions in the Transferable Belief Model, application to climate sensitivity. International Journal of Approximate Reasoning. v49. 555-574 (2008)

[7] IEC 61508.: Functional safety of Electrical/ Electronic/Programmable Electronic (E/E/PE) safety related systems. International Electrotechnical Commission (IEC) (1998)

[8] Jain, A.K.: Data Clustering: 50 Years Beyond K-Means. Pattern Recognition Letters (2009)

[9] Jousselme, A.L., Grenier, D., Bosse, E.: A New Distance between Two Bodies of Evidence. J. Information fusion 2(1), 91101 (2001)

[10] San, O.M., Huynh, V.-N. and Nakamori, Y.: An alternative extension of the k-means algorithm for clustering categorical data. International Journal of Applied Mathematics and Computer Science. v14 i2. 241-247 (2004)

[11] Shafer, G.: A mathematical theory of evidence. Princeton University Press (1976)

[12] Simon, C., Sallak, M., Aubry, J.: SIL allocation of SIS by aggregation of experts' opinions. Safety and Reliability Conference. ESREL, Stavanger: Norvege (2007)

[13] Smets, Ph., Kennes, R.: The Transferable Belief Model. Artificial Intelligence. volume 66, pages 191-234 (1994)

[14] Smets, Ph.: The transferable belief model for quantified belief representation. in: D.M.Gabbay, Ph.Smets (Eds). Handbook of Defeasible Reasoning and Uncertainty Management System, vol.1, Kluwer Academic Publishers, Doordrecht (1998)

[15] Smets, Ph.: Data Fusion in the Transferable Belief Model. Proc. 3rd Intern. Conf. Information Fusion, Paris, France PS 21-33 (2000)

[16] Zadeh, L. A.: Fuzzy sets. Information Control, 8:338353 (1965)

[17] Sallak, M.: Evaluation de paramètres de sûreté de fonctionnement en présence d'incertitudes et aide à la conception: Application aux systèmes Instrumentés de sécurité. PhD thesis, Institut National Polytechnique de Lorraine(2007)

[18] Simon, C., Sallak, M., Aubry, J.: Allocation de SIL par aggregation d'avis d'experts. 15ème Colloque National de Maîtrise des Risques et Sûreté de Fonctionnement, Lambada-mu 15(2006)