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Experimental observation of two-dimensional Anderson localization with the atomic kicked rotor

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Dimension 2 is expected to be the lower critical dimension for Anderson localization in a time-reversal-invariant disordered quantum system: the dynamics is generically localized in dimension lower than 2, while it presents a transition from a diffusive regime at weak disorder to a localized regime at strong disorder in dimension larger than 2. We use an atomic quasiperiodically kicked rotor – equivalent to a two-dimensional Anderson-like model – to experimentally study Anderson localization in dimension 2. We observe a localized dynamics with a localization length depending exponentially on the disorder strength and show that a self-consistent theory of localization quantitatively predicts the localization length.

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The metal-insulator Anderson transition plays a central role in the study of quantum disordered systems. Using a “tight-binding” description of an electron in a lattice, Anderson [1] postulated in 1958 that the dominant effect of impurities in a crystal is to randomize the diagonal term of the Hamiltonian, and showed that this generally leads to a localization of the wavefunction, in sharp contrast with the Bloch-wave solution for a perfect crystal. In a weakly disordered crystal, the eigenstates are delocalized leading to a diffusive (metallic) transport, while strong disorder produces an insulator with localized states. From its original solid-state physics scope [1–4] this approach has been applied to a large class of systems in which waves propagate in disorder. This includes quantum-chaotic systems [5, 6] and electromagnetic radiation [7–9]. Important theoretical progress was obtained in [10], which postulated that Anderson localization can be described by a one-parameter scaling law, leading to the prediction that, for $d \leq 2$, the dynamics is generically localized, even if the disorder is very weak. For $d > 2$, it predicted the existence of the so-called Anderson transition between a diffusive dynamics at weak disorder and a localized dynamics at strong disorder. The transition is expected to be a continuous second-order phase transition, and the critical point is approximately given by the Ioffe-Regel criterion (for $d = 3$):

$$k\ell \approx 1 \quad (1)$$

where k is the wavevector and ℓ the mean-free path for propagation in the disordered medium.

It is difficult to be more quantitative: For example, the position of the critical point in the Anderson model with nearest-neighbor uniform coupling and diagonal disorder [11] depends on microscopic details such as the dispersion relation or the statistical properties of the disorder.

However, the Anderson transition itself is universal: The critical exponents [11] do not depend on the microscopic details, but only on the dimension and on symmetry properties.

The exact value of the critical exponent is unknown. Supersymmetry techniques [12] make it possible to derive expansions in powers of $d - 2$ of the various quantities of interest, but reaching even $d = 3$ is difficult. One reason is probably the existence of very large fluctuations of the wavefunction near the critical point [13] making the definition of an order parameter and the calculations of quantities averaged over disorder realizations rather delicate.

An alternative, simplified, theoretical approach is to use the self-consistent theory of localization. In few words, it can be thought as a mean field theory where large fluctuations are neglected, but where weak localization corrections to transport, due to interference between time reversed multiply scattered paths, are included self-consistently. For spinless time-invariant systems, belonging to the orthogonal symmetry class [12], this approach correctly predicts the existence of the metal-insulator Anderson transition for $d > 2$ and provides us with a microscopic justification of the Ioffe-Regel criterion, Eq. (1), although it fails to predict the correct critical exponent. For $d = 1$, it quantitatively predicts the localization length in a weak disorder. Other approaches lead to approximate values for the critical exponent not far from the numerical prediction [14].

Dimension $d = 2$ – the lower critical dimension – is very special, the localization properties depending on the symmetry class. In the orthogonal symmetry class [12], the dynamics is always localized, but the localization length is predicted to scale *exponentially* with the (inverse) disorder strength, i.e. $\xi \propto \ell \exp(\pi k\ell/2)$ [15]. The 2D case

has been previously studied experimentally in optical and ultracold atom systems [8, 16], although no direct observation of the localization was possible in the last case. In the present Letter, we use the well-known correspondence between the d -dimension Anderson model and the d -frequency quasiperiodic kicked rotor [6, 17] to test experimentally these predictions.

The quasiperiodic kicked rotor (QPKR) [5, 6, 17, 18] is a spatially one-dimensional system, with an engineered time-dependence such that its dynamics is similar to the dynamics of a time-independent multidimensional system. The QPKR can be simply realized experimentally by exposing cold cesium atoms to a delta-pulsed (kicked) laser standing wave with a time period T_1 . The *amplitude* of the kicks is quasiperiodically time-modulated with a frequency ω_2 . The dynamics is effectively one-dimensional along the axis of the laser beam, as transverse directions are uncoupled. The corresponding Hamiltonian is thus:

$$H = \frac{p^2}{2} + K \cos x \left[1 + \varepsilon \cos \omega_2 t \right] \sum_{n=0}^{N-1} \delta(t - n), \quad (2)$$

where x is the particle position, p its momentum, K the kick intensity and ε the amplitude of the modulation. We have chosen conveniently scaled variables such that $x/2\pi$ is measured in units of the spatial period of the potential, the particle's mass is unity and time is measured in units of pulse period T_1 . In the quantum case, a crucial parameter is $\tilde{k} \equiv 4\hbar k_L^2 T_1 / M$, the reduced Planck constant ($2\pi/T_1$, ω_2 and \tilde{k} must be incommensurable numbers [17, 19]).

For $\varepsilon = 0$, one obtains the periodic kicked rotor, which can be mapped onto a one-dimensional Anderson-like model [6], and displays “dynamical” localization [5, 20], that is Anderson localization in momentum space instead of configuration space. For non-zero ε , it has been shown in [17, 19] that the temporal dynamics of the QPKR is exactly that of a two-dimensional periodically kicked system, which itself can be mapped onto a two-dimensional Anderson-like anisotropic model, whose anisotropy is controlled by ε and where the ratio of hopping to diagonal disorder is controlled by K/\tilde{k} .

The existence of a mapping of the kicked rotor onto an Anderson-like model has been used to experimentally observe Anderson 1D localization with atomic matter waves as early as 1995 [18]. The two-frequency modulation of the QPKR – which can be mapped on a 3D Anderson model [17, 19] – was used to experimentally observe 3D Anderson localization and the metal-insulator Anderson transition [21], measure accurately the critical exponent and demonstrate its universality [22].

The experimental study of the 2D case is more difficult than the 3D one, as the observation of the exponential behavior of the localization length p_{loc} requires a range of p_{loc} of about one order of magnitude. The localization

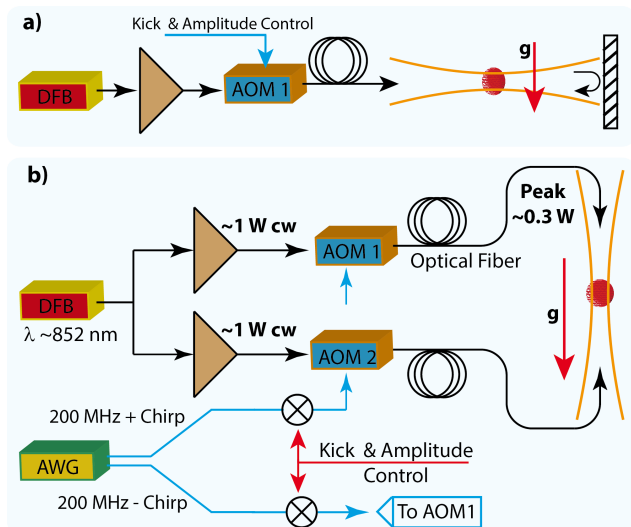


FIG. 1. (Color online) Evolution of the experimental setup a) Horizontal SW setup used in former experiments. The kicks are controlled by a single acousto-optic modulator (AOM), and the SW is built by retro-reflecting the incoming laser beam. b) Schematic view of the new vertical SW setup used in the present work. A Distributed Feedback Laser Diode (DFB) seeds two optical amplifiers. In addition to the pulsed sequence, a linear chirp is added to the driving RFs so that the SW “falls” simultaneously with the atomic cloud.

time increasing with p_{loc} , this also requires the ability to preserve coherence over several hundreds of kicks. This needed important evolutions of our experimental setup.

Experimentally, an atomic sample of a few million atoms is prepared in a thermal state ($3.2 \mu\text{K}$) whose momentum distribution is much narrower than the expected localization length. The atomic cloud is then “kicked” by a far-detuned ($\Delta \approx 13 \text{ GHz}$) pulsed standing wave (SW). Pulse duration is typically $\tau = 300 \text{ ns}$, while the typical pulse period $T_1 = 27.778 \mu\text{s}$ corresponds to an effective Planck constant $\tilde{k} = 2.89$. According to Eq. (2), an adjustable amplitude modulation with $\omega_2/2\pi = \sqrt{5}$ is superimposed to the kick sequence. In all of our previous experiments, to minimize coupling with gravity, the SW was horizontal, see Fig. 1a. This geometry has however several drawbacks. For 1000 kicks, the atoms would fall of 3.8 mm, to be compared to the 1.5 mm of the SW waist! Moreover, the SW was built by retro-reflecting the incoming beam, which leads to a $\sim 1 \text{ m}$ optical path difference between the two beams overlapping on the atomic cloud region, a source of phase noise detrimental to dynamical localization. These restrictions limited the kick number to less than 200 in our previous experiments. In order to overcome this limit, we built a new SW system with several improvements, as shown in Fig. 1b. The SW is now vertical, and, between kicks, the atoms fall freely. The intensity inhomogeneity is negligible, even for 1000 kicks. The SW is now formed by two independent beams,

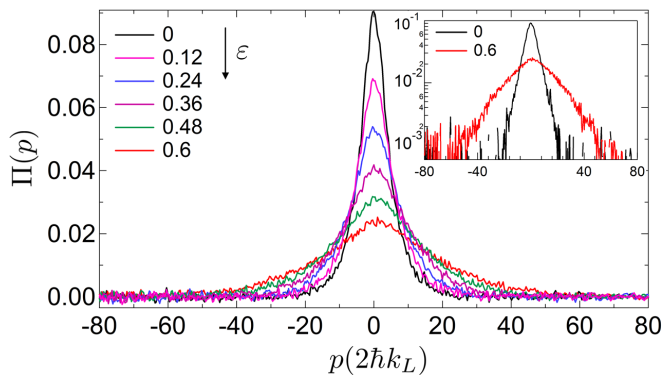


FIG. 2. (Color online) Experimentally measured velocity distributions after 1000 kicks for the quasi-periodically kicked rotor, showing two-dimensional Anderson localization (linear scale on the main plot, logarithmic scale in the inset). All curves correspond to the same kicking strength $K = 5.34$, and same reduced Planck constant $\hbar k = 2.89$. The modulation amplitude ϵ increases from the top to the bottom curve. The distributions have an exponential shape characteristic of Anderson localization, with a localization length increasing with ϵ .

which has many advantages, as each arm can be independently controlled, both in amplitude and phase through the radio-frequency wave (RF) that drives the acousto-optic modulators. This allows us to accurately cancel gravity effects, by imposing a linear frequency chirp of one of the arms with respect to the other, so that the SW itself “falls” with an acceleration g . A kicked rotor is thus realized in the free-falling reference frame. Finally, the SW phase noise induced by the laser linewidth is minimized by accurately balancing the optical paths of the arms to better than 1 cm. This is performed directly by minimizing the kinetic energy dispersion at 1000 kicks with a 1D kicked rotor. At the end of the process, the velocity distribution is measured by a standard time of flight technique.

Residual decoherence processes are likely to be due to spontaneous emission, atomic collisions or technical noise on the lasers producing the kicks. By using $\epsilon = 0$ where the localization time is short (of the order of few or few tens of kicks), we have been able to monitor the velocity distribution over a much longer time – up to 1000 kicks – and observe that it does not significantly broaden, an *in situ* demonstration that decoherence processes breaking the Anderson localization are almost negligible on this time scale. In order to prevent classical localization effects, we use $K > 4$.

If the dynamics were classical, the momentum distribution would keep its Gaussian shape and the average kinetic energy increase linearly with time, $E_{\text{kin}} = Dt$, where D is the classical diffusion coefficient. In contrast, in the quantum localized regime, the evolution of the momentum distribution is “frozen” after the localization time into an exponential curve $\exp(-|p|/p_{\text{loc}})/2p_{\text{loc}}$. As

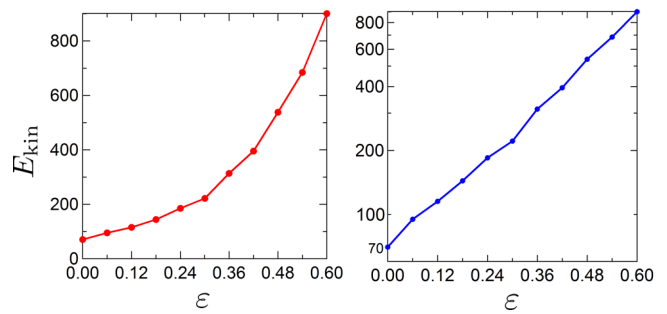


FIG. 3. (Color online) Kinetic energy E_{kin} of the quasi-periodically kicked rotor vs. the modulation amplitude ϵ (same data as in Fig. 2). The left plot uses a linear scale, the right plot in logarithmic scale clearly showing the exponential dependence of the localization length with ϵ .

we can use long kick sequences, the freezing of the distribution combined with the exponential shape are clear-cut proofs of 2D Anderson localization. Fig. 2 shows experimental momentum distributions recorded after 1000 kicks for fixed values of K and $\hbar k$, from the 1D regime at $\epsilon = 0$ to the 2D regime for increasing values of ϵ . One clearly sees the characteristic localized exponential shape, with the obvious observation that the localization length increases with ϵ .

Instead of measuring the full momentum distribution, it is sufficient to measure the population $\Pi_0(t)$ of the zero velocity class as

$$E_{\text{kin}} \propto \frac{1}{2\Pi_0^2(t)} \quad (3)$$

is proportional to $\langle p^2 \rangle(t)$ (as the total number of atoms is constant) [32]. In Fig. 3, we show E_{kin} after 1000 kicks vs. ϵ . A clear exponential increase over one order of magnitude is observed, resulting in a straight line in the logarithmic plot.

Fig. 4 displays E_{kin} (at 1000 kicks) vs. ϵ for a fixed value of $K = 4.6$, for three values of the reduced Planck constant $\hbar k$, showing that the exponential dependence in ϵ is a general feature, with a rate decreasing with $\hbar k$. Note the overall dynamics in E_{kin} of a factor of 60 (corresponding to a 8 fold increase in the localization length), a key feature of this experiment. We also have checked that the exponential rate increases with K .

The self-consistent theory of localization [24], successfully used to predict properties of the Anderson transition [25, 26], has been transposed to the case of the periodically kicked rotor in [20, 27] and to the quasiperiodic kicked rotor with two additional driving frequencies (equivalent to a 3D Anderson model) in [28]. We use a similar approach for a single driving frequency. It consists in computing perturbatively the weak localization correction to the (anisotropic) diffusion constant and to extrapolate to the strong localization regime. This is approximate only, but has been shown to semi-

quantitatively describe the Anderson transition in the 3D case. The detailed calculation of the localization length is too long to be reproduced here [29]. The result is:

$$p_{\text{loc}} = \frac{K^2}{4\bar{k}^2} \exp\left(\frac{\pi\varepsilon K^2}{\sqrt{32}\bar{k}^2}\right) \quad (4)$$

The first term $K^2/4\bar{k}^2$ is nothing but the usual expression of the localization length for the 1D periodically kicked rotor (corresponding to $\varepsilon = 0$) [19, 20], the second exponential term is the non-trivial 2D prediction. The term inside the exponential is essentially the geometric average of the classical diffusion constants along the two directions of the anisotropic Anderson problem. For the kicked rotor it plays the role of the $k\ell$ term in a usual 2D disordered system where the localization length scales like $\exp(\pi k\ell/2)$ [15].

In Fig. 5, we gather the results of 275 measurements, corresponding to 12 values of the ratio $K/\bar{k} \in [1.3, 2.5]$, with $K \in [4.33, 7.26]$ and $\bar{k} = \{2.89, 3.2, 3.46\}$, and to ε values from 0 to 0.6 (step 0.06). Dividing $E_{\text{kin}}(\varepsilon)$ by $E_{\text{kin}}(\varepsilon = 0)$ makes it possible to probe the exponential term in Eq. (4), shown as a red dashed line. The experimental points are spread around this straight line, but the overall agreement is very good, proving that the self-consistent theory makes quantitative predictions. Beside unavoidable experimental uncertainties (see error bars in fig. 4), which explain a large part of the spreading, it must be noticed that the prediction Eq. (4), assumes that the classical diffusion constant is simply $K^2/4$, which is valid only for $K \gg 1$, whereas oscillatory corrections at moderate K are known to exist for the 1D kicked rotor [30] and to persist even for the 3D QPKR [31]. These corrections are responsible for a significant part of the spreading.

For large ε , the localization time can be only slightly shorter than the duration of the experiment (1000 kicks), meaning that the measured momentum distribution is not the asymptotic one for infinite time and underestimates the eventual saturation of E_{kin} at long time. This explains why the experimental points at large ε tend to lie below the theoretical prediction.

To summarize, we have presented the first experimental evidence of two-dimensional Anderson localization with atomic matter waves. We have studied the variation of the localization length with the system parameters and showed that it displays an exponential dependence characteristic of time-reversal spinless systems. To the best of our knowledge, such an experimental evidence has not been observed previously. We have shown that the self-consistent theory of localization quantitatively predicts the localization length.

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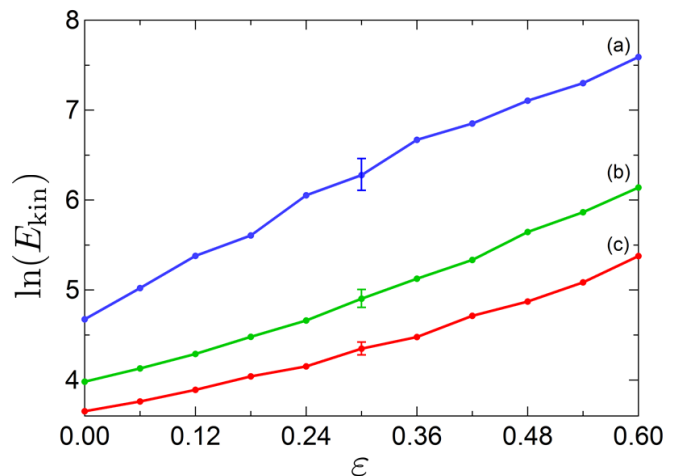


FIG. 4. (Color online) Kinetic energy E_{kin} of the quasi-periodically kicked rotor vs. the modulation amplitude ε , for the same kicking strength $K = 4.6$, but various values of the effective Planck constant $\bar{k} = 2, 2.89, 3.46$ (from upper to lower curve). The error bars indicate the typical experimental uncertainty. The three curves are straight lines in this logarithmic scale, with a slope depending on \bar{k} .

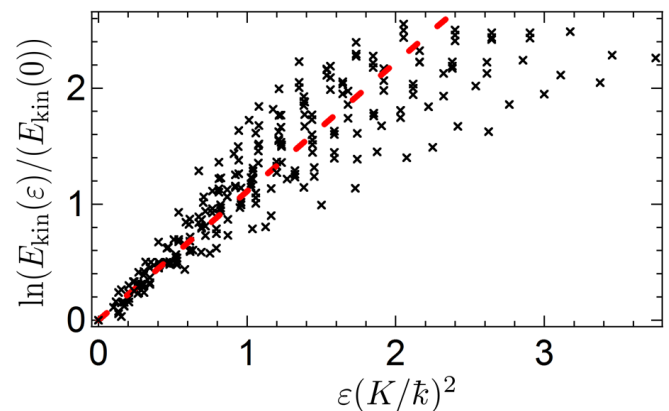


FIG. 5. (Color online) Increasing of the kinetic energy (proportional to the square of the localization length) of the quasi-periodically kicked rotor with respect to the purely one-dimensional situation $\varepsilon = 0$ vs. the scaling parameter $\varepsilon(K/\bar{k})^2$. The red dashed line is the prediction of the self-consistent theory of localization, eq. (4). The overall agreement is very good, proving that the self-consistent theory can be used for quantitative predictions.

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