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Incorporating the FAVAD Leakage Equation into Water Distribution System Analysis

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Abstract

The standard formulation of the hydraulic network equations incorporates a power function for modeling pressure-dependent consumption such as leakage. However, recent research has shown that the FAVAD leakage model more accurately accounts for the behavior of leaks in practice. The objective of this paper is to propose a method for solving the hydraulic network equations incorporating the FAVAD model. An energy minimization problem is used to prove the existence and the uniqueness of the solution to this problem, and to provide the basis for a robust solver. A damped Newton algorithm is proposed for solving the system equations.

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Keywords: Extended period simulation; Energy minimisation; Leakage; FAVAD model; Water distribution system analysis

1. Introduction

Most hydraulic software packages simulate leakage in water distribution systems using an emitter function, which models leakage flow rate as a power function of the nodal pressure. Water losses often make up a significant component of the distributed water and thus it is important to model its behavior as accurately as possible. Recent studies have showed that the Fixed and Varied Area Discharges (FAVAD) equation provides a more realistic description of the behavior of leaks in elastic materials than the conventional power equation. The purpose of this paper is to investigate the incorporation of the FAVAD equation into the standard hydraulic network model.

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1.1. Leakage rate and pressure

It is well known that the flow rate from pipe leaks is a function of the available pressure. This behavior allows water utilities to find leaks based on pressure measurements and to use active pressure management to reduce leakage rates and increase pipe service life.

The Torricelli orifice equation forms the basis for the pressure-leakage relationship, and can be used to describe the flow rate from an orifice as:

$$Q_L = C_d A\sqrt{2gh} \tag{1}$$

Where Q_l is the leakage flow rate through the orifice; C_d a discharge coefficient; A the leak area; g the acceleration due to gravity; and h the pressure head at the orifice.

In water loss practice, the flow rate is written in a more general power equation (i.e. the same form as used for emitters in hydraulic network software) in the form:

$$Q_L = ch^{N1} \tag{2}$$

Where c is the leakage coefficient and N1 the leakage exponent. In field studies the leakage coefficient and exponent are calculated from the system leakage before and after pressure reduction. While the orifice equation Eq. (1) predicts the leakage exponent to be 0.5, values as high as 2.9 have been reported in field studies. The higher exponents are mainly due to leak areas changing with changes in system pressure.

1.2. The FAVAD leakage model

Recently, Cassa and van Zyl [1] showed that the power equation does not provide a good characterization of the pressure response of a leak, and different leakage exponents result for the same leak when measured at different pressures. Van Zyl and Cassa [2] found that the FAVAD model is particularly suited to model individual leaks in elastic materials. Replacing a linear equation for the leak area as a function of pressure into the orifice equation (Eq. 1), results in:

$$Q_L = C_d\sqrt{2g}\left(A_0 + mh\right)h^{0.5} \tag{3}$$

where A_0 is the leak area intercept; and m is the head-area slope.

The head-area slope is a function of the properties of the leak, as well as the pipe material and section properties. For round holes, m is very small (i.e. expands very little with increasing pressure); and for longitudinal, spiral and circumferential cracks, formulae for m have been proposed based on CFD studies [3]. The assumption of elastic deformation inherent in the *FAVAD* model (Eq. 3) is considered a reasonable assumption, although leaks in plastic pipe are also affected by hysteresis and plastic deformation [4].

For the combined response of many leaks, Schwaller and van Zyl [5] showed that the FAVAD model can also be used to describe the behavior of pressure management zones with many leaks, and that the parameters of the FAVAD model are strongly related to the sum of all the individual leak areas and head-area slopes in the system.

1.3. Diffuse leakage and pipe momentum equation

In current hydraulic models, leakage and water use along a link are lumped at the start and end nodes of these links. Several authors have proposed energy conservation corrections for leaky links in a WDS model (a model link typically consists of a number of consecutive elements in the real system, including pipes, joints, off-takes and minor losses). Jaumouille *et al.* [6] assumed a uniform leakage rate along the link (*i.e.:* a gradually varied flow) for deriving a rigid column equation that considers time and convective inertia terms and integration of the head loss function along the

pipes. Under a different framework, Ferrante *et al.* [7] proposed an additional term with an experimentally determined C_β coefficient to account for the loss in axial momentum.

1.4. The energy minimization problem formulation

Hydraulic software solutions rely on solving steady state equations for conservation of mass and energy of an incompressible fluid. An extended period simulation consists of a sequence of consecutive steady state simulations. The steady state equations are solved to obtain the unknown flow rates in links, and energy heads at nodes. The steady state equations are:

$$\begin{cases} \mathbf{AQ} + \mathbf{c}(\mathbf{H}) = \mathbf{0}_{nu} \\ \Delta\mathbf{h} - \mathbf{A}^T\mathbf{H} - \mathbf{A}_f{}^T\mathbf{H}_f = \mathbf{0}_{np} \\ \Delta\mathbf{h} = \mathbf{h}(\mathbf{Q}) \end{cases} \tag{4}$$

Where **Q** is the vector of link flow rates with size np (number of links); **c(H)** the vector of head-dependent consumptions, *i.e.*: leakage and water demand lumped at nodes, with size nu (number of unknown-head nodes), **A** an nu x np incidence matrix representing of unknown-head node connectivity, $\mathbf{A}_f$ an nf (number of fixed-head nodes) x np incidence matrix of fixed-head nodes, **H** the vector of piezometric heads for the unknown-head nodes, $\mathbf{H}_f$ the vector of hydraulic heads for the fixed-head nodes, Δ**h** a vector of link head losses. The first two equations describe the conservation of mass and energy respectively, and are linear. The last equation is a nonlinear equation that describes the relationship between the link flow rates and head losses, typically based on the Darcy-Weisbach formula.

In order to ensure global convergence for any initial solution, it is useful to adopt an optimization approach. Similar primal-dual formulations for demand-driven steady state problems (with **c(H)** = **d**, a vector of known demands) were proposed by several authors [8, 9 , 10]. An optimization approach allows step-size correction to be made to the solution, thus allowing numerical instabilities to be avoided. In addition, the existence and uniqueness of a solution to the equations can be proven, and thus convergence on a unique solution is guaranteed.

The FAVAD leakage model has not been incorporated in a hydraulic modeling package yet. For that purpose, first an energy minimization problem is introduced that is equivalent to solve. Next, the latter is used to prove the existence and the uniqueness of the solution of head-dependent consumption problem Eq. (4). Finally, a robust Newton-based solver algorithm is proposed.

2. Energy problem formulation for steady state solution with FAVAD leakage model

Piller *et al.* [11] introduced a primal-dual framework suitable for solving Eq. (4) for pressure-driven model (PDM). One important difference is that the water consumption PDM function is bounded above by a fixed demand; also, the primal formulation is not directly applicable for FAVAD leakage model that is not bounded above (one constraint should be released). On the other hand, their PDM dual formulation may be applicable for the solution of Eq. (4) with **c(H)** including the FAVAD Eq. (3) as a component. Thus, we consider the following minimization problem:

$$\min_{\mathbf{H}} CC(\mathbf{H}) := \sum_{i=1}^{np} \int_{\Delta h_i(0)}^{\left(\mathbf{A}^T\mathbf{H}+\mathbf{A}_f{}^T\mathbf{H}_f\right)_i} g_i(u)du + \sum_{j=1}^{nu} \int_0^{H_j} c_j(v)dv \tag{5}$$

Were CC is the co-content function; and g_i is the head loss inverse function for the ith link. CC has the dimension of power and is expressed with unknown nodal head as basic unknowns. For making the connection with system Eq. (4) one has to differentiate the CC criterion. The gradient vector is given by:

$$\nabla CC(\mathbf{H}) = \mathbf{AQ}(\mathbf{H}) + \mathbf{c}(\mathbf{H}), \text{ with } \mathbf{Q}(\mathbf{H}) = \mathbf{g}(\mathbf{A}^T\mathbf{H} + \mathbf{A}_f{}^T\mathbf{H}_f) \tag{6}$$

For all **H**, the solution (**Q**(**H**), **H**) verifies the energy balance equation by definition of **Q** function of **H**. Let us observe that minimization problem Eq. (5) is not constrained. We can deduce from it that the gradient taken at a minimum solution $\mathbf{H}^*$ equals zero. It follows **Q**(**H***) satisfied the mass conservation as well, and (**Q**(**H***), **H***) is solution of system Eq. (4). By convexity of CC, one **H*** solution of Eq. (4) is also a minimum of minimization problem (Eq. 5). Solving Eq. (4) is then equivalent to solving the co-content minimization problem.

Now the Darcy-Weisbach head loss function is differentiable with **Q** and its inverse g is also differentiable. There are both strongly monotone which leads to conclude that CC is elliptic (strongly convex). Its Hessian matrix is a symmetric positive-definite matrix and may be calculated as:

$$\mathbf{Hess}(CC)(\mathbf{H}) = \mathbf{AD}(\mathbf{Q}(\mathbf{H}))^{-1}\mathbf{A}^T + \Delta(\mathbf{H}), \text{ with } \Delta(\mathbf{H}) = \partial_{\mathbf{H}}\mathbf{c}(\mathbf{H}) \text{ and } \mathbf{D}(\mathbf{Q}) = \partial_{\mathbf{Q}}\Delta\mathbf{h}(\mathbf{q}) \tag{7}$$

An elliptic criterion possesses level sets that are closed and bounded. This is enough for demonstrating the uniqueness and existence of the solution of Eq. (5) and then, by equivalence, of Eq. (4).

3. The damped Newton method

Newton method converges given a sufficiently close initial guess, but may converge slowly or fail to converge even for elliptic functions due to numerical instabilities (caused by the inexact representation of numbers in computer memory), particularly for high accuracies. However, we can ensure that the method converges by incorporating a damping factor. This leads to the proposal of a global damped Newton method that, for the minimization of Eq. (5), consists of the iteration:

$$\mathbf{H}^{k+1} = \mathbf{H}^k - \alpha_k \left[\mathbf{AD}(\mathbf{Q}(\mathbf{H}^k))^{-1}\mathbf{A}^T + \Delta(\mathbf{H}^k)\right]^{-1}\left(\mathbf{AQ}(\mathbf{H}^k) + \mathbf{c}(\mathbf{H}^k)\right) \tag{8}$$

Were $0 < \alpha_k \leq 1$ is a damping factor that is such that the following Goldstein condition holds:

$$\lambda \leq \frac{CC(\mathbf{H}^k) - CC(\mathbf{H}^{k+1})}{\alpha_k \nabla CC(\mathbf{H}^k)^T \mathbf{Hess}(CC)(\mathbf{H}^k)^{-1}\nabla CC(\mathbf{H}^k)} \leq \mu \tag{9}$$

With $0 < \lambda < 0.5 < \mu < 1$ two fixed numbers. The left-hand side of the inequality ensures a sufficient reduction in CC whereas the right-hand side guarantees sufficient distance between iterations. There exists a number N such that if $k > N$, then there is no need for damping and α_k may be chosen to be 1.

4. Conclusion

In this paper we propose an energy formulation for water distribution network hydraulics incorporating the FAVAD leakage equation. It is formulated as a minimization problem with nodal heads as basic unknowns. This formulation is useful both to demonstrate the uniqueness and existence of the solution and to derive a global descent algorithm. A damped Newton method is proposed for solving the equations.

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