Analysis Software for Interplanetary Trajectories using Three Body Problem

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ANALYSIS SOFTWARE FOR INTERPLANETARY TRAJECTORIES USING THREE BODY PROBLEM

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ABSTRACT

The aim of this paper is to present the PRIAM Toolbox (Poincare Resolution of Interplanetary and Astrodynamic Missions), designed to study interplanetary missions and particularly suited to the dynamics analysis near collinear libration points defined as isolated zeros of a smooth gravitational vector field.
Orbits computation and optimal transfers constitute the core of the toolbox.

1. INTRODUCTION

The study of the dynamics around the libration points has brought out the existence of low-energy interplanetary transfers, exploiting geometric invariant objects under Hamiltonian phase flow.

In order to design future missions near and between libration points, we have developed the PRIAM toolbox which offers advanced methods providing a full analysis of the three-body dynamics. These methods are derived from the theory of dynamical systems using techniques originally developed by Poincaré [13] and taken up by several authors.

Different semi-analytical methods has been carried out in order to compute accurately all the orbits around the libration points. The study of the central manifold enables to provide the entire family of orbits for each level of energy. Our tool is able to give a suitable 2D representation of all these trajectories from which users can choose the desired one.

Then, stable and unstable manifolds associated to these orbits have been computed to provide the low-energy trajectories approaching or leaving libration points.

The tool offers many applications from the computation of these manifolds, such as the search of heteroclinic connection and optimal transfer between a planet and its libration points.

Once we have reached this point, the user can run a station keeping procedure or can carry out a transfer to another orbit around the same libration point.

Finally we have developed a perturbing model which integrates JPL ephemerides and takes into account the perturbations of the Moon and other planets of the solar system.

2. ORBITS COMPUTATION

Orbits in the circular restricted three body problem (CR3BP) are view as integral curve of the Hamiltonian vector field (expressed in Eq. 2).

As well known there exists five fixed points of this Hamiltonian flow called libration points.
According to Lyapunov definition [16], two of them are stable (L4, L5) and the others are unstable (L1, L2 and L3).

Around the unstable collinear libration points, at a given level of energy, there is a family of orbits which are contained in invariant 2D tori related to Lissajous solutions. The proof of their existence can be associated to the KAM theorem [14],[15].

The PRIAM toolbox is able to compute the entire family of theses orbits, including periodic ones like

Halo and Lyapunov orbits (vertical and horizontal ones).
These orbits have been computed using numerical integration of the 3 body problem, coupled with a parallel shooting algorithm which uses the transition state matrix to refine nominal orbits .

In order to provide accurate initial conditions for the integration, we have developed two analytic methods suited to the Hamiltonian systems resolution: the first one is the Lindstedt-Poincare procedure and the second one is the reduction of the centre manifold.

2.1. Lindstedt-Poincare procedure

This L-P method is based on finding solutions in the form of trigonometric expansions which depends on orbits excursions Ax and Az, and satisfy the CR3BP's equations of motion.

The trigonometric series are expanded up to a fixed order N and appears as the product of two oscillatory components of frequencies Ω_1 and Ω_2 :

$$x(t)=\sum_{i,j=1}^{N}\left(\sum_{k<i,m\le j} x_{i,j,k,m}\cos\left(k\,\Omega_1 t+m\,\Omega_2 t\right)\right)A_x^i A_z^j$$
$$y(t)=\sum_{i,j=1}^{N}\left(\sum_{k<i,m\le j} y_{i,j,k,m}\sin\left(k\,\Omega_1 t+m\,\Omega_2 t\right)\right)A_x^i A_z^j \quad (1)$$
$$z(t)=\sum_{i,j=1}^{N}\left(\sum_{k<i,m\le j} z_{i,j,k,m}\cos\left(k\,\Omega_1 t+m\,\Omega_2 t\right)\right)A_x^i A_z^j$$

A recursive algorithm has been implemented to compute the coefficients $x_{i,j,k,m}$, $y_{i,j,k,m}$ and $z_{i,j,k,m}$.
It performs the factorisation of the potential in Legendre polynomials and put them on the form of Eq. 1. Starting from the linear solutions of the CR3BP and identifying the terms of each order, we are able to compute the desired coefficients.

In order to reach suitable order, we have created specific tools for algebraic operators, in particular for the factorisation of trigonometric series. We have also taken the advantages of the symmetries of CR3BP.

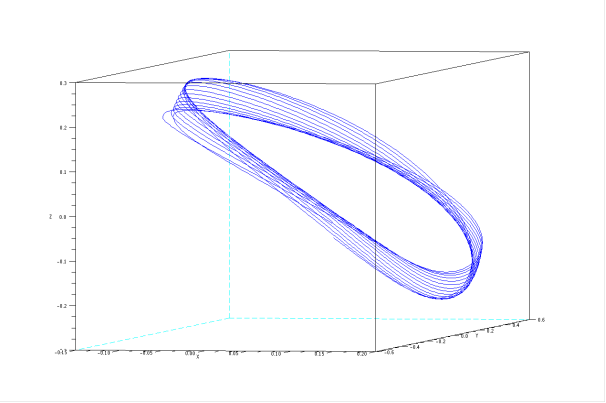

Figure 1 : Quasi-Halo of the CR3BP in the Sun-Earth system around L1 (Energy = 0.5, Az = 175 000 km) by order 10

The convenience of this method is that solutions are known at any time by direct valuation of the series. By covering all the excursions parameters, one can obtain the entire family of orbits.
However, the procedure is very sensitive to resonance and does not provide good results for low number of rotation (orbits near halo).

The coefficients of series expansions have been compared with [1] and [6].

2.2. Reduction of the centre manifold

The second method is based on expanding the initial Hamiltonian of the system around the collinear libration point and carrying out a partial normal form of this Hamiltonian, uncoupling the hyperbolic modes from the elliptic ones.

To provide the Hamiltonian normal form we used the method of Lie transforms developed by Deprit [12]. According to this method we consider a continuous map of symplectic change of coordinates depending on a small parameter. This map is the general solution of a new Hamiltonian system called G [17].

Beforehand, the Hamiltonian is expanded in a series of Legendre polynomials up to a fixed
order as expressed in Eq. 2 :

$$H=\frac{1}{2}(p_x^2+p_y^2+p_z^2)+yp_x-xp_y-\sum_{n\ge 2} c_n\rho^n P_n\left(\frac{x}{\rho}\right) \quad (2)$$

The canonical transformation has been performed by using Lie algebraic operators, dealing with homogeneous polynomials.

$$\tilde{H} = H + \left[H,G\right] + \frac{1}{2}\left[\left[H,G\right],G\right] + \frac{1}{3!}\left[\left[\left[H,G\right],G\right],G\right] + \ldots \quad (3)$$

The main algorithm, based on a recursive scheme, compute the monomials of H in the new coordinates and kill those which are related to the hyperbolic directions [6].

Once the reduction have been performed, selecting a level of energy and a 2D Poincaré section, we choose a point on this section and use it as initial conditions for a numerical integration of the restricted Hamiltonian.

At each time the trajectories crosses the Poincare section z=0, an initial condition is taken. After a certain time of computation, we obtained a 2D representation of all the dynamics in the centre manifold. This representation is known as a 'Poincare map'. Each point on this map stand for an intersection of an orbit when it crosses the section.

The boundary of this map is the Lyapunov horizontal periodic orbit. If the energy is sufficient, one can see two fixed point by the side of the map. They are related to the Halo orbit.

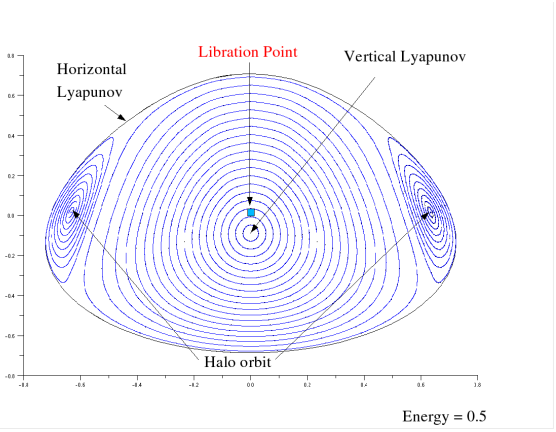

Figure 2 : Poincare map : representation of the whole family of orbits near L1

The PRIAM toolbox open the possibility of generating orbits starting from a point in the Poincare map.

The energy parameter and the coordinates inside this map can be connected with excursions and phases parameters. Therefore we are able to make a correlation between this method and the Lindstedt-Poincare procedure, which have different input parameters.

One of the advantage of the reduction of central manifold is that it provides a global and intuitive view of the dynamics at fixed energy.

3. PERTURBATIONS ANALYSIS AND REAL EPHEMERIDES MODEL

The PRIAM Toolbox was designed initially in order to study the dynamics of the CR3BP. Nevertheless, due to the high sensitivity to the initial conditions and exterior environment, the development of perturbing model turned out to be essential.

This model takes account :

- the gravitational attraction of the Moon and other planets of the solar system (Jupiter, Venus, Saturn...)
- The contribution of the radiation pressure of the Sun**.** The force depends on the rs vector (position with respect to the Sun) and alpha, which is a constant related to the mass and the section of the satellite.

$$\gamma_{ps} = -\tilde{\alpha}\frac{\vec{r}_s}{r_s^3} \quad (4)$$

- The effect of the non-circularity of the second body orbit (like Earth). It takes into account the variation of the distance between the two primaries.

If the perturbations remain small compared to the potentials of the two primaries, the libration points are still defined.
However the non-uniform movement of the secondary

body around the primary induces a displacement of the libration point with respect to the secondary body.
The computation of the perturbing forces, including the inertial acceleration, has been carried out by taking into account the real ephemeris of the different bodies.

These ephemeris are interpolated from a JPL DE405 model available on the web site interface HORIZONS.

The basic ephemeris are expressed in the ecliptic system of reference, centred on Sun. But the equations of motions have been developed in the synodical frame of reference with origin at the libration point. Therefore, the tool attend to transfer the initial data to this frame of reference.

Simulations highlight that perturbations disturb periodic orbits and those with small excursions. The most important effect is clearly due to the real movement of the secondary body like Earth which have an eccentricity of 0.016.

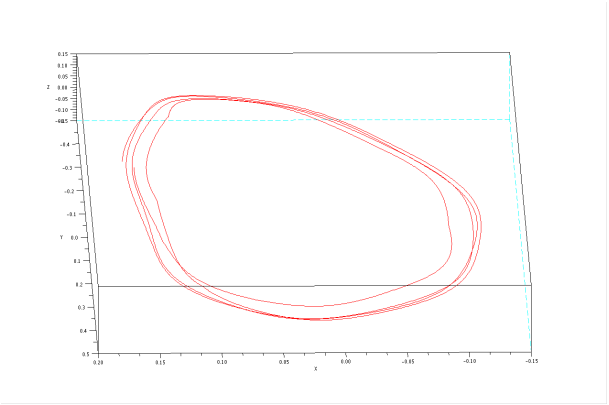

Figure 3 : Quasi around L1 computed and refine in the perturbed model with parallel shooting method.

4. STATION KEEPING

Station keeping strategies allows spacecraft to keep close to nominal orbits, defined by the mission analysis. Due to the initial injection error or to the use of a rough model of forces, the spacecraft can be derived from its nominal path.

To estimate the cost of a mission near libration points, two different method of on/off control have been used and tested during integration.
The first one is the Floquet mode approach, developed by Simó [10]. It consists in eradicating the unstable component of the error between the nominal orbit and the current path. It is based on the computation of Floquet vectors and projection factors of the orbit.

Actually the unstable component increases in an exponential way by a factor $\lambda 1$ in one period ($\lambda 1$ is the dominant eigenvalue of the monodromy matrix).
Generally a manoeuvre is done when this component has increased by a factor e, which correspond to a one-month control.

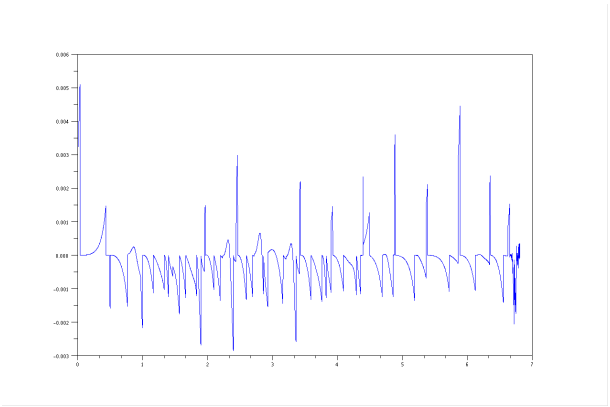

Figure 4 : Normalised unstable component of the error during 7 years around a quasi-halo orbit (L1 of the Sun Earth system)

The second one, developed by Howell and Pernika [11], is the Target Points method: a set of points are chosen on the nominal orbit and each of them are compared with a predicted state computed from the previous one. By this way, the satellite tends to keep close to the nominal path. It boils down to cancel the error vector, whereas the Floquet modes approach was designed to cancel the unstable component of this error.

Correction manoeuvres are calculated linearly by propagating the transition matrix state Φ along the nominal orbit. δr_i and δv_i stand for the position and velocity error at the time of the manoeuvre and ti+1 is the time of the next impulse :

$$\phi(t_i, t_{i+1}) \begin{pmatrix} \delta r_i \\ \delta v_i + \delta V \end{pmatrix} = 0 \quad (5)$$

The frequency of the control depends on the stability and the accuracy of the nominal orbit. But it has to be more than eight manoeuvres per year.

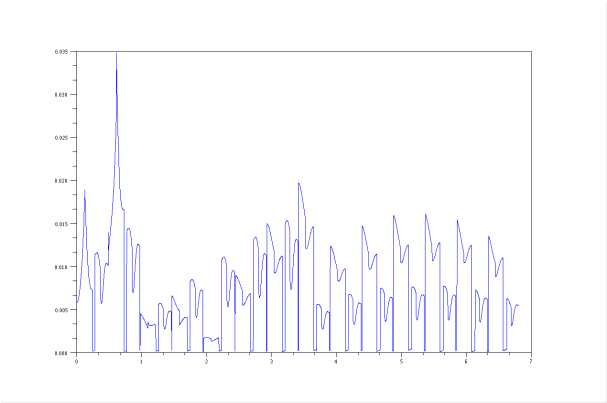

Figure 5 : Module of the error vector between nominal and current path. Same orbit as Figure 2.

Finally, we include the effect of a tracking error in the execution of the manoeuvres and also in the state restitution. Typically we use error of 10 km for position and 10 cm/s for velocities. These errors allows to take account the sensors and actuators drawbacks.

The following table sum up the results obtained for the specified tracking error and an error of 10 km in the initial conditions (quasi halo around L1):

	Floquet modes	Taget points
Cost per year (m/s)	5,1	11,2
Mean error (with the nominal orbit)	0.02	0.005
Number of manoeuvres per year	9	10

Table 1 : Outputs from the station-keeping procedure

In the case of Target Points method, the cost is higher but the error between the nominal and the current path is reduced.

5. TRANSFERS

5.1. Stable and unstable manifolds

The set of all points in phase space which are attracted to an invariant orbit in positive time admit a manifolds structure and called stable manifold (unstable manifold for negative time). This manifolds are also invariant under the Hamiltonian flow (see [14].

PRIAM toolbox is able to compute stable and unstable manifold associated to a nominal orbit. Theses manifolds are very useful since they provide optimal path to reach or leave an orbit around a libration point.

The eigenvectors of the variational matrix after one revolution give the directions of the tangent spaces to theses invariant manifolds at a fixed point.

From a selected point Xo on the orbit, the linear approximation of the manifold is given by :

$$Xm = Xo + \varepsilon\, Vw \quad (6)$$

where ε is a small distance (about 200 km) and Vw is the scaled eigenvector (stable or unstable).

In order to obtain the entire manifold, we propagate the initial conditions Xm backwards in time if we want to compute the stable manifold and forward in time if we deal with unstable one.

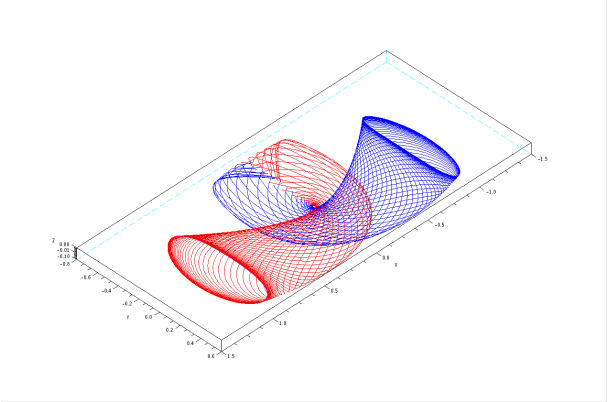

Figure 6 : Stable and unstable manifolds around halo orbits of L1 and L2.

5.2. Transfer from Earth to libration points

The way to obtain a low-energy transfer from the Earth to an halo or quasi-halo orbit is to insert the spacecraft into the stable manifold of this orbit (Manifold Orbit Insertion strategy).
Then theoretically, no manoeuvres have to be done at

the time of arrival near the libration point.

We suggest a transfer with two manoeuvres, one to leave a circular parking orbit near the Earth (of radius rp) (ΔV_1) and the other to inject the spacecraft into the stable manifold at a radius ri (ΔV_2).

By a way of optimisation, we are able to compute the best connection orbit between the times of the two manoeuvres. This orbit is determined with its Keplerian parameters by minimizing the total fuel consumption and satisfying the boundary conditions.

We proceeds as follow :

- First we obtain the trajectory of the manifold that minimize the distance from the Earth.
- We select the point of radius ri in the manifold path.
- We look for an optimal conic orbit whose perigee is rp and which pass through the selected point of the manifold. Two parameters as Argument of perigee and Right Ascension are previously sorted and then covered. We obtain finally the remainder parameters, especially the corresponding inclination.
- From the initial parking orbit the first manoeuvre is deduced. The second one is also evaluated.
- By mean of numerical integration, we refine the second manoeuvre in order to be exactly at the selected point on the manifold.

If the last manoeuvre is done far away from the Earth (large value of ri) , the total cost is reduced but the injection error might increase, due to the fact that a long time separate the two velocity increments.

For the simulations, a suitable value of ri has been taken between 30000 and 60000 km.

An example of transfer is given as follow :

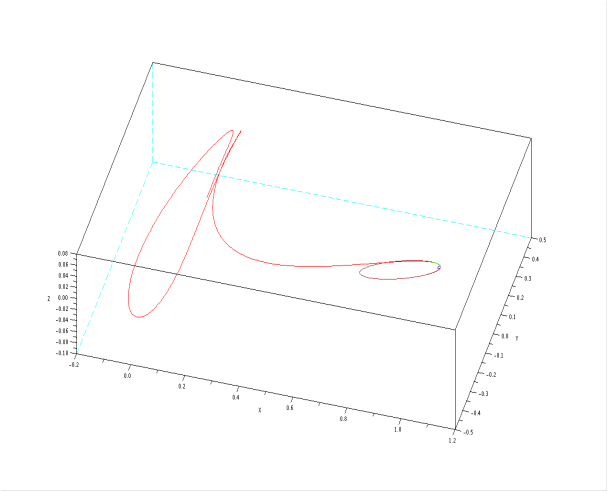

Figure 7 : Transfer to the L1 from the Earth

Departure inclination (with respect to the ecliptic frame)	0.1040
Departure radius (km)	7200
Semi-major axis of the connection orbit (km)	139600
Eccentricity of the connection orbit	0.9484
Injection Manoeuvre ΔV_1 (m/s)	2957
Insertion Manoeuvre ΔV_2 (m/s)	400
Radius ri of ΔV_2 (km)	58880
Time between the two manoeuvres	3 h 45
Total time of the transfer	3,4 mts

Table 2 : Example of transfer between Earth and L1 from a 800 km orbit parking

5.3. Transfers between Lissajous orbits of the same libration point

A methodology for the transfer between two Lissajous orbits around the same libration point, making use the linear geometry of the phase space, has been drown from the work of University of Catalunya [4].

These kind of transfers were first motivated by the missions Planck and Gaia of the ESA Space Program, which planned to change their orbit amplitudes.

PRIAM toolbox has been designed in order to deal with these new requirements. It is able to compute the manoeuvres to change the in plane or out-plane

amplitudes of a Lissajous orbit. It also handles with change of orbit phases. This last case will be very useful for the design of eclipse avoidance strategies.

The approach is to insert the satellite into the stable manifold of the final Lissajous orbit starting at a point of the initial orbit. The transfer is not possible at any time, especially in the case of amplitude reduction. Therefore we have to compute the possible and optimal time of the manoeuvres.

For instance, the Hershel mission of the ESA Space Program have dropped its x-y excursion by a factor 2 (see [3]). The velocity increment is about 50 m/s.

5.4. Eclipse avoidance

A new mission requirement is to avoid some exclusion zones: for orbits near L1, the exclusion zone is three degrees about the solar disk since the radiation of the Sun in this area can disturb communications with Earth. For orbits near L2, the area to be avoided is the Earth half-shadow.

Halo orbits are attractive because satellites never cross exclusion zones, but the major drawback is that the y excursions is too large.

If Lissajous orbits are used for long missions, the spacecraft will enter the exclusions zone. The time before the first entrance depends on the initial phases of the orbit

Therefore a LOEWE strategy (Lissajous Orbit Ever Without Eclipse), studied by [4], has been implemented to deal with these exclusions zones. The idea is to find the best initial phases in such a way that the spacecraft remain the maximum time outside the exclusion zones.

For instance this time is about 4 to 6 years around the L1 of Sun-Earth system. If the mission is longer than the maximum possible time, an optimal manoeuvre is done to change the phase of the orbit.

The exclusions zone appears in the plan yz as a disk of radius $y^2+z^2 < R^2$
Radius R is about 90000 km for L1 and 14000 km for L2.

If we denote by Φ and Ψ the phases of the orbit at epoch t, of the in-plane component and the out-plane one respectively, the border of the disk will depends linearly of the excursions Ax and Az such that :

$$k_0 A_x^2 \sin(\phi) + A_z^2 \cos(\Psi) = R^2 \quad (7)$$

This equation define an smooth curve C isotopic to a point in the quotient space $T^2=R^2/$ (πR x πR). The phases variables ({(Φ,Ψ) $\in$ R^2}, p) are considered as the covering space of torus T^2 where p is the canonical projection.

When a Lissajous trajectory, following a straight line of constant speed in, intersect one of the curve belonging to $p^{-1}(C)$, it means that the spacecraft is entering a exclusion zone.

We can prove that if we choose initial phases that are tangential to one of these curve, the time without collisions will be longer.

For long term missions, the strategy consists in doing a jump at each collision to be tangential to the corresponding border curve.

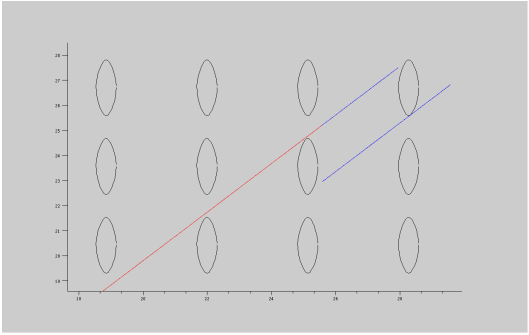

Figure 7 : Phases tangent to one motif of $p^{-1}(C)$ to avoid exclusions zones.

The cost of the LOEWE strategy is less than 50 m/s per year and decrease as amplitudes become larger.

5.5. Heteroclinic connections

Heteroclinic connections consist in matching the unstable manifold of an orbit around L1 (or L2) with the stable manifold of an orbit of L2 (or L1). This is a way to join to libration points of the same system.

The chosen cross-section is the plan orthogonal to the x-axis that split the secondary body (Earth).

Since a level of energy has been fixed, the toolbox scan several orbits and search for the intersection of there manifolds in the plane (y,ydot) then (z,zdot). When a suitable matching has been founded, we propagate the manifolds from the intersection and obtain one heteroclinic trajectory.

These connections could be very useful to realise interplanetary missions since they provide a zero-cost transfer between two distant areas of the solar system. The same approach could be used to link two planets looking for the intersection of two equilibrium points of different system - but the distance are so far that the manifolds do not coincide. However a technical procedure has been carried out by [18] to realise these kind of transfer. This method combines patched conic transfer and invariant manifolds trajectories.

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