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Abstract

In relation to the industrial need and to the progress of technology, LNE would like to improve the measurement of its primary pressure, spherical and flick standards. The spherical and flick standards are respectively used to calibrate the spindle motion error and the probe which equips commercial conventional cylindricity measuring machines. The primary pressure standards are obtained using pressure balances equipped with rotary pistons with an uncertainty of 5 nm for a piston diameter of 10 mm. Conventional machines are not able to reach such an uncertainty level. That is why the development of a new machine is necessary. To ensure such a level of uncertainty, both stability and performance of the machine are not sufficient and the data processing should also be done with an accuracy less than the nanometre.

In this paper, the new method based on the Small Displacement Screw (SDS) model is proposed. A first validation of this method is proposed on a theoretical dataset published by the European Community Bureau of Reference (BCR) in report n°3327. Then, an experiment is prepared in order to validate the new method on real datasets. Specific environment conditions are taken into account and many precautions are considered. The new method is applied to analyze the least squares circle, minimum zone circle, maximum inscribed circle and minimum circumscribed circle. The results are compared to those done by the reference Chebyshev best-fit method and reveal a perfect agreement. The sensibility of SDS and Chebyshev methodologies are investigated, and it is revealed that results remain unchanged when the value of the diameter exceeds 700 times the form error.

1. Introduction and literature review

This work is part of a project whose objective is to develop a new ultra-high precision cylindrical measurement machine [1-2]. The equipment is mainly dedicated to measure standards, such as flick standards and spheres, which are used for the calibration of industrial form measuring machines, and piston-cylinder [3] with an uncertainty of nanometers level.

However, the performance and stability of the new equipment alone cannot satisfy such requirements. There is therefore an absolute need to develop analysis methods of the form of the datasets, which may ensure a similar nanometric level of accuracy.

The International Organization for Standardization described the most common methods used to determine form errors, especially roundness errors [4]: Least Squares circle/cylinder method (LSC), Minimum Zone tolerance circle/cylinder method (MZC), Maximum Inscribed circle/cylinder method (MIC) and Minimum Circumscribed circle/cylinder method (MCC).

The LSC method is the most common approach to evaluate approximated roundness [5], and is mainly used in dimensional metrology for the simplicity of its application and to the uniqueness of its solution. In practice, the least squares method is appropriate where random measurement errors predominate. For cylindrical artefacts, the LSC method denotes the circle fitting the roundness profile. Usually the centre of that circle is used to fit the smallest circumscribed and the largest inscribed circles or cylinders to the roundness or cylindricity profiles. The radial separation between the circumscribed and inscribed circles represents the roundness error.

The LSC method is based on the mathematical principles that minimize the sum of the squared deviations of the measured points from the fitted feature [6]. This robust method does not guarantee the minimum zone solution specified in the standards. The deviation values and geometric tolerances are generally larger than the actual ones and lead to an over-estimation of the form error of the target. A modified least square method is developed in [7], which takes the best geometrical estimation of orthogonal distances by measuring the deviational errors in sampled data. The normal least squares fit is developed in [8], and requires to solve the equations of normal least-squares fit.

The MIC, MCC and MZC circle methods are presented in detail in [9]. For the MIC and the MCC methods, the radial distance represents the maximum inscribed, minimum circumscribed, respectively. The MZC method corresponds to the two concentric circles with minimum radial separation that contain the roundness profile. The radial separation between the inner and outer reference circles is the roundness error. The MZC method is appropriate in most cases where random measurement errors are small compared to form errors. Basically, the MZC method generates an optimal solution and fewer out-of-tolerance parts compared to the LSC, MIC and MCC methods, due to the minimum radial separation distance between the reference circles [4]. These methods require to solve a non-linear problem which needs an implementation applying optimization techniques. Various techniques for optimization and mathematical calculations were developed in previous works in order to evaluate roundness errors [10-15].

This paper details the mathematical description of the new method based on the small displacement screw (SDS) model for LSC, MIC, MCC and MZC analysis methods for roundness evaluation. An experiment is carried out using a conventional machine to measure roundness, and the developed SDS method will result in evaluating form errors. Results will be compared with those obtained using the reference Chebyshev best-fit algorithm.

2. General context: design of the new geometric measuring machine

Currently, LNE is developing a new ultra-high precision machine dedicated to the measurement of roundness, straightness and cylindricity. The aim is to achieve form metrology with an uncertainty of less than 10 nm for both roundness and straightness and less than 20 nm for cylindricity.

The concept of the machine applies the dissociated metrological structure principle which consists in dissociating the metrology frame from the supporting structure. The architecture of the machine is based on the comparison of two surfaces: a reference cylinder and a cylindrical artifact. This approach gets rid of errors due to the motion of the mechanical guiding elements (spindle and linear guiding systems). The test cylinder is located inside the hollow reference cylinder. Eight or more capacitance sensors ([16]) are focused on the reference cylinder and up to four probes are focused on the artifact. The concept of the machine is completely symmetric and perfectly respects the Abbe principle. The metrology loop goes only through reference and probing elements. As a consequence, measurements are never influenced by the quality of motion of mechanical guiding elements and are only affected by both the performance of probing elements and the stability of reference elements. The calibration of all probes of the machine is automatic and carried out in-situ over a 60 µm travel range using the nanometric piezoelectric actuators. It is based on the use of a modified multi-step form error separation technique allowing separation between the form errors of both

the reference cylinder and the cylindrical artifact. More details concerning the operation, architecture and design of the new geometric measuring machine can be found in [1-2].
To ensure measurement with nanometer level of accuracy, the machine should be stable at the nanometer level, but also the program should ensure evaluation at the same order of accuracy, and this is the aim of this paper.

3. General geometric surface identification method

Form error measurement on a mechanical target leads to determining both the position and parameter values of the surface model which fits best the measured dataset. Such an operation absolutely requires achieving two steps. The first one consists in acquiring data and the second one consists in defining the geometric surface model. The resolution of this problem is known as "solving an inverse problem".

3.1. Principle of the SDS method

Usually, an ideal geometric surface is defined by a set of data (x_i, y_i, z_i, a_i, b_i, c_i), such that (x_i, y_i, z_i) correspond to the Cartesian coordinates of the theoretical point-set $M_{th(i)}$ and (a_i, b_i, c_i) correspond to the cosine parameters of the normal vectors $\vec{n}_i$ at each point [17].

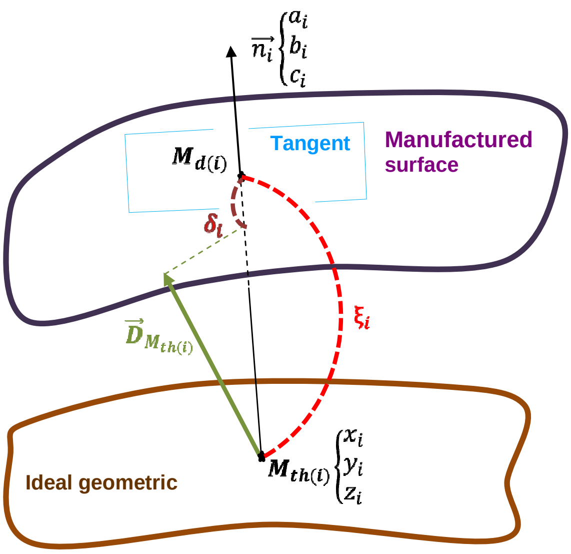


Fig. 1: Schema of the general geometric identification method of surface.

Consider the manufactured surface represented by the measured dataset $M_{d(i)}$. Independently from the geometric model attributed to the sought surface, the method is based on matching N measured points $M_{d(i)}$ with N theoretical points $M_{th(i)}$ of the ideal geometrical surface. The calculation of the variation in measurement parameters ξ_i is to be completed for each couple of sets ($M_{d(i)}$; $M_{th(i)}$) with the normal vector $\vec{n}_i$. The variation in measurement parameters ξ_i corresponds to the distance $M_{d(i)}M_{th(i)}$ between both sensed and theoretical datasets (Fig.1). The manufactured surface is identified by an ideal geometric surface defined by the theoretical points $M_{th(i)}$ with a normal vector $\vec{n}_i$ and a variation in measurement parameters ξ_i, sensed along the normal vector $\vec{n}_i$.

3.2. Description of the model

The method presented below is based on the SDS model. If we consider the formula of the small displacement screw model applied at point *A*, the corresponding equation may be presented as follows (Eq.1):

$$[T_A] = \begin{Bmatrix} \vec{R} \\ \vec{D} \end{Bmatrix} = \begin{Bmatrix} \alpha\vec{x} & u\vec{x} \\ \beta\vec{y} & v\vec{y} \\ \gamma\vec{z} & w\vec{z} \end{Bmatrix} \qquad \text{Eq.1}$$

where $\vec{R}$ is the small rotation vector $\left(|\vec{R}| \leq 5°\right)$ and $\vec{D}$ is the small displacement vector. The surface of the artefact never corresponds perfectly to the theoretical surface. To minimize the distance between points M_d and M_{th}, it is important to apply a small displacement $\vec{D}_{Mth(i)}$ that brings $M_{th(i)}$ and $M_{d(i)}$ to be as close as possible (Eq.2).

$$\delta_i = \xi_i - \left(\vec{D}_A . \vec{n}_i + \left(\overrightarrow{AM_{th(i)}} \wedge \vec{n}_i \right) . \vec{R} \right) = \xi_i - \left([P_i]_A . [T_A] \right) \qquad \text{Eq.2}$$

If we consider both the small displacement screw model $[T_A]$ and the Plücker coordinates $[P_i]_A$ screw theory, we obtain a formula that contains *p* linear equations depending on *6* unknown parameters *(α, β, γ, u, v and w)*. These parameters represent the components of the small displacement screw model as defined in Eq.3.

$$[P_i]_A = \begin{Bmatrix} \vec{n}_i \\ \overrightarrow{AM_{th(i)}} \wedge \vec{n}_i \end{Bmatrix} \qquad \text{Eq.3}$$

If the number of datasets *N* representing the manufactured surface is equal to 6, this leads to a system of 6 independent linear equations. Then, the solution is easily obtained as: $\delta_{j=1...6} = 0$. However, in dimensional metrology, *N* number of measured points is greater than *p* number of independent unknown parameters. This configuration requires to determine the optimal value of the small displacement screw model following the criterion of distance minimization between the theoretical model and the measured points.

Two methods solve the linear system of equation:

- The least-squares method which gives a statistical distribution of variation δ_i around the theoretical geometric surface.
- The method based on linear programming allowing to minimize and/or maximize the function.

A/ Least-squares method

If we consider the function *W* in Eq.4, its minimization leads to solve the problem as presented in Eq.5. The Plücker coordinates $[P_i]_A$ are shown in Eq.6

$$W = \sum_{i=1}^{p} \left(\xi_i - \left([P_i]_A . [T_A]\right)\right)^2 \qquad \text{Eq.4}$$

$$\frac{\partial W}{\partial(\alpha, \beta, \gamma, u, v, w)} = 0 \qquad \text{Eq.5}$$

, with ∂ representing the partial derive.

$$[P_i]_A = \begin{Bmatrix} a_i\vec{x} & (y_i c_i - z_i b_i)\vec{x} \\ b_i\vec{y} & (z_i a_i - x_i c_i)\vec{y} \\ c_i\vec{z} & (x_i b_i - y_i a_i)\vec{z} \end{Bmatrix} = \begin{Bmatrix} a_i\vec{x} & L_i\vec{x} \\ b_i\vec{y} & M_i\vec{y} \\ c_i\vec{z} & N_i\vec{z} \end{Bmatrix} \qquad \text{Eq.6}$$

The optimization of the function *W* can be done by solving Eq.7. Hence, we obtain the values of the *6* unknown parameters *(α, β, γ, u, v* and *w)* which characterize the position of the measured surface with respect to the theoretical surface.

$$\begin{bmatrix} \sum a_i^2 & \sum b_i a_i & \sum c_i a_i & \sum L_i a_i & \sum M_i a_i & \sum N_i a_i \\ \sum a_i b_i & \sum b_i^2 & \sum c_i b_i & \sum L_i b_i & \sum M_i b_i & \sum N_i b_i \\ \sum a_i c_i & \sum b_i c_i & \sum c_i^2 & \sum L_i c_i & \sum M_i c_i & \sum N_i c_i \\ \sum a_i L_i & \sum b_i L_i & \sum c_i L_i & \sum L_i^2 & \sum M_i L_i & \sum N_i L_i \\ \sum a_i M_i & \sum b_i M_i & \sum c_i M_i & \sum L_i M_i & \sum M_i^2 & \sum N_i M_i \\ \sum a_i N_i & \sum b_i N_i & \sum c_i N_i & \sum L_i N_i & \sum M_i N_i & \sum N_i^2 \end{bmatrix} \times \begin{bmatrix} u \\ v \\ w \\ \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} \sum a_i \xi_i \\ \sum b_i \xi_i \\ \sum c_i \xi_i \\ \sum L_i \xi_i \\ \sum M_i \xi_i \\ \sum N_i \xi_i \end{bmatrix} \qquad \text{Eq.7}$$

B/ Linear programming method

The linear programming method allows finding the minimum of a general problem using as example the routine of linear programming on MatLab software.

The manufactured surface is defined by the measured points $M_d(i)$. This method aims at covering all datasets M_d between two surfaces ΔS and ΔI ,as shown in Fig.2. This operation can be completed by applying both the SDS and the linear programming methods described below, as shown in the following equations, Eq.8 and Eq.9.

minimize: $Z = \Delta S - \Delta I$ Eq.8

subject to: $\begin{cases} \Delta S - (\xi_i - ([P_i]_A \cdot [T_A])) \geq 0 \\ \Delta I - (\xi_i - ([P_i]_A \cdot [T_A])) \leq 0 \end{cases}$ Eq.9

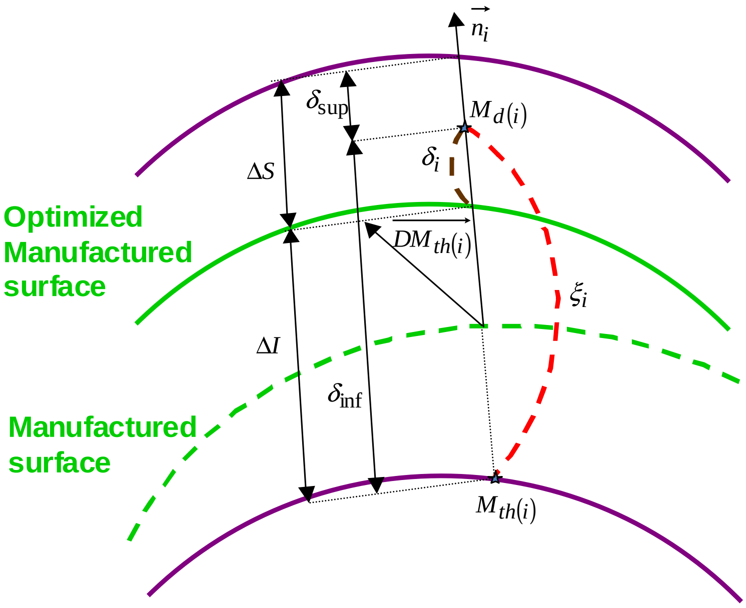


Fig. 2: General specification of the linear programming of the form error problem

Since the proposed SDS method is based on a linear programming method, the accuracy and stability of the solution depend on the number of both undulation and measured points. To reduce the effect of any of these parameters on the results, the processing of a dataset is programmed to be completed 10 times after which the results are compared.

However, when using any high-precision machine for cylindricity measurement, the number of data points is usually more than 3,600, which improves the quality of processing. In addition, for dimensional metrology applications, the cylindrical artefact presents an ultra-high quality of surface finish, and typically does not contain any geometry defect. These characteristics allow to find very small variations of the artefact's topology, which reduces considerably the risk to have bad results.

4. Application of the small displacement screw method to evaluate 2D-roundness

Regarding the 2D-roundness evaluation (theoretical circle with radius *R*), the normal vector $\vec{n}_i$, the variation in measurement parameters ξ_i, the spatial coordinates of the point $M_{th}(i)$, the Plücker coordinates screw $[P_i]_A$ and the small displacement screw $[T_A]$ are respectively presented in Eq.10 to Eq.13.

$$\overrightarrow{n_i} = \begin{bmatrix} \cos(\theta_i) = \dfrac{x_i}{x_i^{\ 2} + y_i^{\ 2}} \\ \sin(\theta_i) = \dfrac{y_i}{x_i^{\ 2} + y_i^{\ 2}} \\ 0 \end{bmatrix}$$, with θ_i : angular position step Eq.10

$$\xi_i = \sqrt{x_i^{\ 2} + y_i^{\ 2}} - R$$ Eq.11

$$[P_i]_A = \begin{Bmatrix} cos(\theta_i)\vec{x} & 0 \\ sin(\theta_i)\vec{y} & 0 \\ 0 & 0 \end{Bmatrix}$$ Eq.12

$$T_A = \begin{Bmatrix} 0 & u\vec{x} \\ 0 & v\vec{y} \\ 0 & 0 \end{Bmatrix}$$ Eq.13

4.1. Least-squares circle (LSC)

To obtain the LSC circle passing through the maximum number of datasets, we have to solve the problem minimizing the variation δ_{ic} presented in Eq.14.

$$\delta_{ic} = \xi_i - \left(u\cos(\theta_i) + v\sin(\theta_i) + \Delta r\right)$$, Δr being the radius variation Eq.14

Optimising function $\delta_{ic} \Rightarrow \partial\delta_{ic} / \partial(u, v, \Delta r) = 0$ allows to determine the three unknown parameters *(u, v* and Δr *)* and to solve the system of independent equations described by the matrix formula in Eq.15.

$$\begin{bmatrix} \sum cos(\theta_i)^2 & \sum cos(\theta_i)sin(\theta_i) & \sum cos(\theta_i) \\ \sum cos(\theta_i)sin(\theta_i) & \sum sin(\theta_i)^2 & \sum sin(\theta_i) \\ \sum cos(\theta_i) & \sum sin(\theta_i) & p \end{bmatrix} \times \begin{bmatrix} u \\ v \\ \Delta r \end{bmatrix} = \begin{bmatrix} \sum \xi_i \ cos(\theta_i) \\ \sum \xi_i \ sin(\theta_i) \\ \sum \xi_i \end{bmatrix}$$ Eq.15

4.2. Minimum circumscribed and maximum inscribed circle

To obtain both MCC and MIC in 2-D including or excluding all the datasets M_d, the linear programming method (simplex method) can be applied. The expression of variation δ_{ic} is similar to the formula presented in Eq.14. Δr represents the increase in the minimum radius of the theoretical circle $(\Delta r \leq 0)$ in the case of MIC, and inversely in the case of MCC. The function Z to be optimised is: $Z = \Delta r$.

4.3. Minimum zone circle

The 2D-MZC covers all the datasets M_d. Using this method requires the application of the linear programming method. The expression of variation δ_{ic} is similar to the formula presented in Eq.14. For all datasets, we need to solve the system of independent equations (Eq.16) and minimize Eq.17:

$$\begin{cases} \Delta S - (u\cos(\theta_i) + v\sin(\theta_i) + \Delta r) = \xi_i \\ \Delta I - (u\cos(\theta_i) + v\sin(\theta_i) + \Delta r) = \xi_i \end{cases} \quad \text{Eq.16}$$

$$Z = \Delta S - \Delta I \quad \text{Eq.17}$$

5. Theoretical evaluation of the SDS method

The above methodology was implemented and applied to a perfect theoretical dataset (without noise) published by the Commission of the European Community Bureau of Reference (BCR) in report n°3327 [18]. The perfect regular 20 data coordinates are illustrated in Table 1 and present a known solution. The data are analyzed using the SDS method in order to evaluate the LSC, MIC, MCC and MZC. Results are presented in Fig.3 and corresponding roundness values are presented in Table 2. All the results obtained here reveal a perfect similarity with the published results, and provide evidence of the high performance and accuracy of the developed methodology.

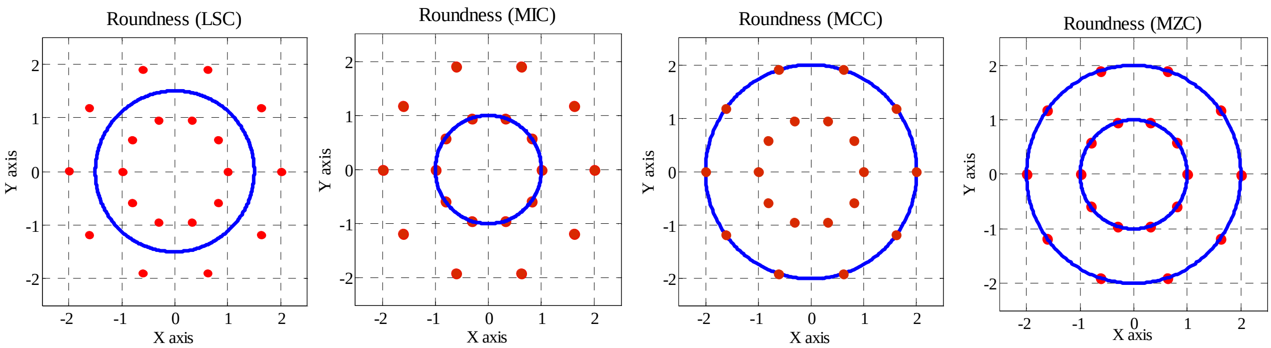


Fig. 3: LSC, MIC, MCC and MZC analyses of the numerical datasets published in the BCR report n°3327 [19], using the SDS method (● theoretical datasets, ▬▬ SDS analysis). The analysed datasets are presented in the Table 1. (a): evolution of the least squares circle, (b): evolution of the maximum, (c): evolution of the minimum circumscribed circle, (d): evolution of the minimum zone circle.

No.	***X***	**Y**
1	0.6283	1.00
2	1.2566	1.00
3	1.885	1.00
4	2.5133	1.00
5	3.1416	1.00
6	-2.5133	1.00
7	-1.885	1.00
8	-1.2566	1.00
9	-0.6283	1.00
10	0	1.00
11	0.6283	2.00
12	1.2566	2.00
13	1.885	2.00
14	2.5133	2.00
15	3.1416	2.00
16	-2.5133	2.00
17	-1.885	2.00
18	-1.2566	2.00
19	-0.6283	2.00
20	0	2.00

Table 1: Theoretical datasets coordinates published in the BCR report n°3327 [19].

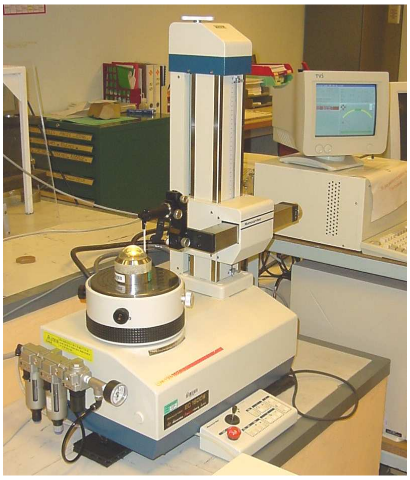
Theoretical test	**Small displacement screw method**	**Reference Chebyshev best-fit**	**Form error variation**
	Form error ΔF_1 (µm)	Form error ΔF_2 (µm)	$\lvert\Delta F_2 - \Delta F_1\rvert$ (nm)
MIC	1.0000	1.0000	0
MCC	1.0000	1.0000	0
MZC	1.0000	1.0000	0
LSC	1.0000	1.0000	0

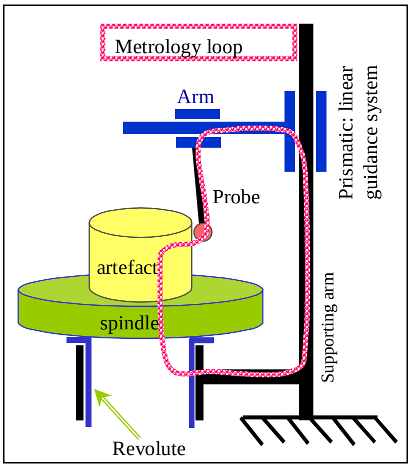
Table 2: LSC, MIC, MCC and MZC analyses of the theoretical dataset in Table 1 and published in the BCR report n°3327 [19], by applying the developed SDS method. The obtained values of the LSC, MIC, MCC and MZC are identical to those published in the BCR report and to those obtained when applying the reference Chebyschev best-fit method.

6. Experiment set-up

6.1. Conventional machine for cylindrical measurement

To evaluate the developed methodology based on the SDS method on real datasets, an experiment is developed using conventional high-precision machines (“KOSAKA” machine) for roundness assessment. Measurements are performed by comparing the form of the measured part with a high quality movement of the air-bearing spindle. The roundness of a part is measured by subjecting it to a high quality rotational movement and by monitoring its surface with a fixed probe. These machines typically have a series of loop structures, which are made of a succession of solids joined by customizable linkages able to generate relative positions or movements between two solids [1]. Fig.4(a) shows a picture of the conventional and industrial geometry measurement machine used here to achieve the experiment. Fig.4(b) describes the kinematic scheme of this type of machine and shows the metrology loop that reflects its metrological performance. Therefore, the recorded measurement combines both form and motion errors [2].





(a) ***(b)***

Fig. 4: Photo and kinematics diagram of the Kosaka conventional high precision machine and identification of the metrology loop which passes through the supporting frame, the metrology frames and the sensing element (tactile probe) (Expanded interval estimate $U_{95\%}$ = ±42 nm, confidence interval: 95 %).

In the case of cylindrical artefacts, the surface to be measured is scanned using three serial linkages which represent the essential components of the metrology loop: a revolute joint between the precision air-bearing spindle and the supporting frame, a mechanical guiding element between the column and the carriage and a mechanical guiding element between the carriage and the arm. A coder and rulers are used to determine the coordinates along the scanning axes. The measuring probe, in contact with the artefact to be measured, is the last component of the metrology loop.

6.2. Experiment conditions

Tests are carried out inside the LNE cleanroom. The temperature and hygrometry are respectively controlled at 20° ±0.2 and 50% ±5. The whole experiment is installed on an optical table with advanced vibration isolation features to avoid low frequency vibrations.

Three measurements are completed separately on a cylinder standard, a flick standard and a cylindrical artefact with different undulations per revolution (UPR). These standards represent the most employed artefacts in industrial applications to respectively calibrate the rotation error of the air-bearing spindle, the linearity/behaviour of the tactile probe and to evaluate the filtering function (longwave-pass and shortwave-pass filters) incorporated in all the software that equip the industrial conventional cylindrical machine, as described in the European standard EN ISO 12180-2.

For a best use of the roundness machine, the first step consists in centring and tilting, as much as possible, the cylindrical artefact axis along the vertical rotating z-axis of the machine. Then 3,600 points over the cylindrical target surface are recorded. The developed routine and methodology are applied in order to evaluate the form error (roundness) of the cylindrical artefact.

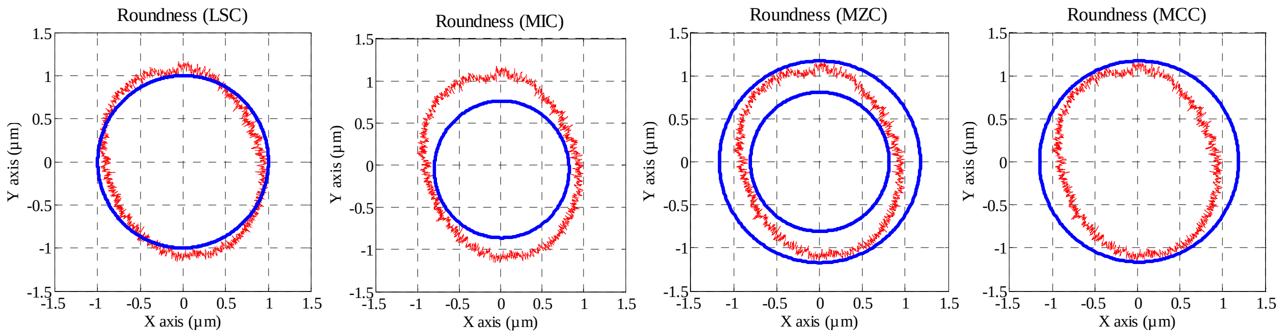
6.3. Results

The first experimental test is carried-out with the cylindrical artefact of 75 mm diameter. 3,600 points are recorded and the evaluation of the results is done using the developed SDS method. The recorded data combine form errors of the artefact, motion errors of the air-bearing spindle, noise of the measuring tactile probe and nonlinear residuals of the same probe. The form errors (roundness here) vary according to the angular positions of the artefact, the error of the air-bearing spindle vary according to its angular positions and the probe errors vary according to its working range. The error motions of the air-bearing spindle include two aspects: repeatable (systematic) and unrepeatable errors. The repeatable error motions can be identified using any technique from reversal, multi-step and multi-probe error separation methods [2, 19]. The unrepeatable error motions are random errors and can not be identified. Nevertheless they can be reduced by applying the temporal redundancy which consists in increasing and averaging the number of measurements, or by applying a shortwave-pass filter.

Therefore the repeatable error motions of the air-bearing spindle are identified here when applying the multi-step separation error method. 18 equidistant angular-step positions of the artefacts are generated and the processing of the recorded data gives the evolution of the repeatable error motions of the air-bearing spindle according to the angular positions. The maximum repeatable error motion's value is less than 120 nm.

After a compensation of the repeatable error motions only roundness, unrepeatable error motions, noise error of the measuring probe and its nonlinear residuals are kept. The unrepeatable error motions, respecting a Gaussian distribution, are of 25 nm and are small in comparison to the repeatable error motions. In addition, both noise and nonlinear residuals of the tactile probe are evaluated to less than 20 nm, when a working range of 20 µm is considered. The obtained data after compensation of the repeatable error motions of the air-bearing spindle are dominated by the roundness form error.

The budget of uncertainty of the Kosaka machine of cylindricity assessment evaluates to $u = \pm 21$ nm, which leads to the expanded uncertainty (interval estimate) of $U_{95\%} = \pm 42$ nm when considering the confidence interval of 95 %. We noticed that the established uncertainty does not take into account any error relating to the processing of the recorded dataset.



(a) (b) (c) (d)

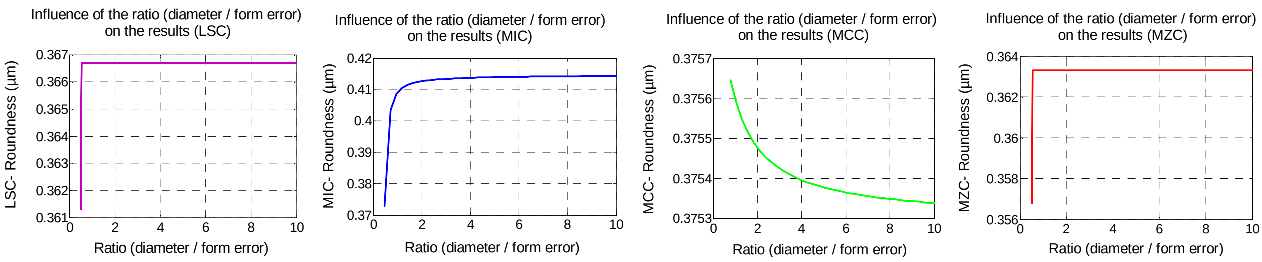
Fig. 5: LSC, MIC, MCC and MZC analyses of the experimental datasets measured on the cylindrical artefact, using the SDS method (experimental datasets, SDS analysis) (Expanded interval estimate $U_{95\%}$ = ±42 nm, confidence interval: 95 %). The analyses are performed on the datasets after compensation of the repeatable error motions of the air-bearing spindle (a): evolution of the least squares circle, (b): evolution of the maximum inscribed circle, (c): evolution of the minimum circumscribed circle, (d): evolution of the minimum zone circle.

For the first test on the cylindrical artefact of 75 mm diameter, LSC, MIC, MCC and MZC analyses of the roundness are obtained and presented in Fig.5. Only for a graphical visualization need, the diameter of the artefact is considered as being equal to one micrometer; otherwise it would be impractical to graphically visualize form error of the artefact which is very small (around 0.4 µm). However, when processing the recorded dataset, the considered value of the diameter is equal to the real value of the test cylindrical artefact's diameter (75 mm). The corresponding values of roundness are presented in Table 3.

	Small displacement screw method	**Reference Chebyshev best-fit**	**Form error variation** $\lvert \Delta F_2 - \Delta F_1 \rvert$ (nm)
Cylindrical artefact (diameter of 75 mm)	Form error ΔF_1 (µm) (Expanded interval estimate $U_{95\%}$ = ±42 nm, confidence interval: 95 %)	Form error ΔF_2 (µm) (Expanded interval estimate $U_{95\%}$ = ±42 nm, confidence interval: 95 %)	
MIC	0.4139 $^{\pm 0.042}$	0.4139 $^{\pm 0.042}$	0
MCC	0.3753 $^{\pm 0.042}$	0.3753 $^{\pm 0.042}$	0
MZC	0.3633 $^{\pm 0.042}$	0.3633 $^{\pm 0.042}$	0
LSC	0.3667 $^{\pm 0.042}$	0.3677 $^{\pm 0.042}$	1

Table 3: LSC, MIC, MCC and MZC analyses of the experimental dataset which are obtained when measuring the cylindrical artefact of 75 mm diameter and by applying the developed SDS method. The results are compared to those done by the reference Chebyschev method which are identical.

To investigate the influence of the value of the diameter on the roundness and check whether the value of the diameter should be absolutely bigger than the value of the form error, the processing is done again with many values of the ratio diameter/form-error between 0.37 and 10. According to Fig. 6, when the ratio diameter/form-error is less than 1, the values of roundness can considerably change which leads to consider that the SDS method can not be applicable for very small holes (the value of the diameter of the hole is close to the value of the form error). For such a case (small diameter), a developed method based on the adjacent facets can be applied to evaluate roundness [20]. When the ratio (diameter/form error) increases, the roundness values become unchanging over the entire range.



(a) (b) (c) (d)

Fig. 6: Analysis of the LSC, MIC, MCC and MZC processing of the recorded dataset for the cylindrical artefact (roundness measurement) when changing the ratio between the considered value of the diameter and the form error of cylindrical artefacts (Expanded

interval estimate $U_{95\%}$ = ±42 nm, confidence interval: 95 %). The analyses are performed on the datasets after compensation of the repeatable error motions of the air-bearing spindle (a): evolution of the least squares circle according to the ratio variation (diameter / form error), (b): evolution of the maximum inscribed circle according to the ratio variation (diameter / form error), (c): evolution of the minimum circumscribed circle according to the ratio variation (diameter / form error), (d): evolution of the minimum zone circle according to the ratio variation (diameter / form error).

The second measurement is performed on the flick standard of 14.3 µm and of 50 mm diameter under the same conditions. 3,600 data points are recorded and the values of roundness (LSC, MIC, MCC and MZC) are presented in Table 4.

	Small displacement screw method	**Reference Chebyshev best-fit**	**Form error variation** $\left\|\Delta F_2 - \Delta F_1\right\|$ (nm)
Flick standard (14.3 µm)	Form error ΔF_1 (µm) (Expanded interval estimate $U_{95\%}$ = ±42 nm, confidence interval: 95%)	Form error ΔF_2 (µm) (Expanded interval estimate $U_{95\%}$ = ±42 nm, confidence interval: 95%)	
MIC	$14.3720^{\pm 0.042}$	$14.3720^{\pm 0.042}$	0
MCC	$14.3933^{\pm 0.042}$	$14.3933^{\pm 0.042}$	0
MZC	$14.3496^{\pm 0.042}$	$14.3496^{\pm 0.042}$	0
LSC	$14.3800^{\pm 0.042}$	$14.3800^{\pm 0.042}$	0

Table 4: LSC, MIC, MCC and MZC analyses of the experimental dataset which are obtained when measuring the flick standard of 14.3 µm and by applying the developed SDS method. The results are compared to those done by the reference Chebyschev method which are identical.

The third measurement is achieved on a cylindrical artefact of 75 mm diameter and with different undulations per revolution (UPR): 15 UPR, 50 UPR, 150 UPR and 500 UPR (European standard EN ISO 12180-2). When the number of undulations per revolution exceeds 500 UPR, the error is considered as being the roughness. 3,600 data points are recorded over one perimeter of the artefact and the Fast Fourier Transform (FFT) is applied as in Fig.7. It reveals that the form error amplitude values are quite similar for all undulations (between 0.5 and 0.6 µm). The values of roundness (LSC, MIC, MCC and MZC) are analyzed by applying the SDM method and the values are presented in Table 5.

Standard with different undulations per revolution (diameter of 75 mm)	**Small displacement screw method**	**Reference Chebyshev best-fit**	**Form error variation**
	Form error ΔF_1 (µm) (Expanded interval estimate $U_{95\%}$ = ±42 nm, confidence interval: 95%)	Form error ΔF_2 (µm) (Expanded interval estimate $U_{95\%}$ = ±42 nm, confidence interval: 95%)	$\lvert \Delta F_2 - \Delta F_1 \rvert$ (nm)
MIC	$3.7263^{\pm 0.042}$	$3.7263^{\pm 0.042}$	0
MCC	$3.7258^{\pm 0.042}$	$3.7258^{\pm 0.042}$	0
MZC	$3.7136^{\pm 0.042}$	$3.7136^{\pm 0.042}$	0
LSC	$3.7253^{\pm 0.042}$	$3.7253^{\pm 0.042}$	0

Table 5: LSC, MIC, MCC and MZC analyses of the experimental dataset which are obtained when measuring the cylindrical artefact with different undulations per revolution (15 UPR, 50 UPR, 150 UPR and 500 UPR) and by applying the developed SDS method. The results are compared to those done by the reference Chebyschev method which are identical.

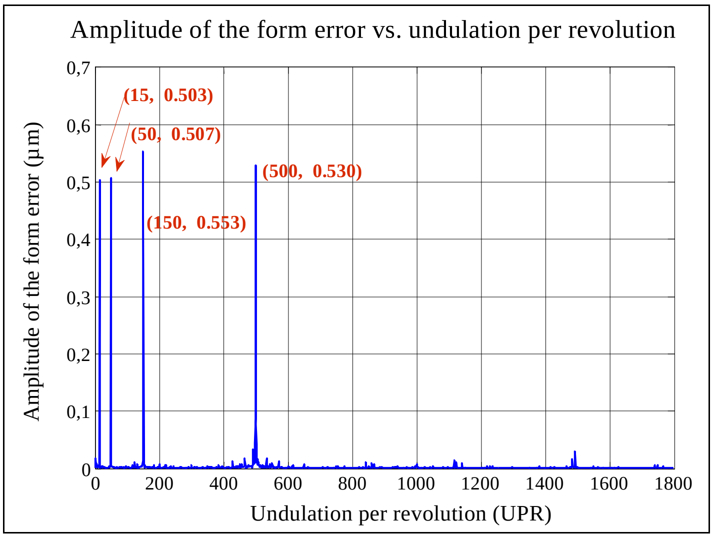


Fig. 7: Application of the FFT to the datasets representing the form of the artefact which contains four undulations per revolution (15 UPR, 50 UPR, 150 UPR and 500 UPR) (Expanded interval estimate $U_{95\%}$ = ±42 nm, confidence interval: 95 %).

6.3. Analysis and comparison with Chebyshev best-fit results

To evaluate the limitation of the proposed method, a comparison with existing methods should be done for LSC, MIC, MCC and MZC. The reference Chebyshev best-fit is known by its robust results.

A/ Description of the Reference Chebyshev algorithm for 2D-circles

The Chebyshev algorithm is used to calculate the circle that minimizes the maximum distance separating a data point from the surface of the element taken orthogonally. In mathematical terms, if the circle is described using a vector of parameters, then the Chebyshev best-fit element, with v based on the approach using the constrained optimization problems, can be defined as the following general formula (Eq.18):

$$\min_{v} G(v) \quad \text{subject to } c_i(v) \geq 0, \quad \forall\, i \in I \qquad \text{Eq.18}$$

where $c_i(v)$ is the constraints function, I denotes the indices of inequality constraints and $G(v)$ is the objective function. This expression (Eq.18) provides an important advantage because of the availability of considerable mathematical theories and algorithmic approaches for this form [12-13]. For both cases of a 2D-circle and minimum zone problem, if we suppose that the circle is specified by the parameters v and $d_i(v)$ (distance between the i^{th} measured point to the element defined by v which can be positive or negative according to the position of the point and the element) then the MZC can be identified by solving the following formula (Eq.19).

$$\min_{v} \max_{i} |d_i(v)| \text{ , with } \forall\, i \in I \qquad \text{Eq.19}$$

For both MCC and MIC 2D-circles problems, Anthony et al [11-12] assumed variable $r_i(v)$, which denotes the distance from the i^{th} measured point to the core of the element and is always a positive quantity (Eq.20 and 21).

$$\min_{v} \max_{i} |r_i(v)| \text{ , with } \forall\, i \in I \qquad \text{Eq.20}$$

$$\max_{v} \min_{i} |r_i(v)| \text{ , with } \forall\, i \in I \qquad \text{Eq.21}$$

For a 2D-circle whose centre has coordinates (a,b), the distance from the i^{th} measured point (x_i, y_i) to the core of the circle element can be described by the conventional equation of the circle. By applying the reference Chebyshev algorithm, the constraint function can be solved as a linear problem.

B/ Analysis and discussion

The performance of the proposed SDS method is evaluated by comparing the obtained values of roundness (LSC, MIC, MCC and MZC) measured on the cylindrical artefact of 75 mm diameter to those with the reference Chebyshev best-fit method. The developed SDS method is applied to analyze the roundness for the cylindrical artefact. The comparison between both the SDS and the Chebyshev best-fit methods is done and the values are reported in the Table 3. They illustrate a perfect agreement between both methods, except the for the LSC, which presents a variation of 1 nm.

The agreement between both methodologies is confirmed again for the flick standard of 14.3 µm and of 50 mm diameter. As previously, the repeatable error motions of the air-bearing spindle are compensated and both the SDS and the reference Chebyshev best-fit are applied to analyze the roundness. All roundness values (LSC, MIC, MCC and MZC) are presented in Table 4 and reveal identical results.

For the last test done on the cylindrical artefact of 75 mm diameter and with different undulations per revolution, and after a compensation of the repeatable error motions, the process is done again by applying the SDS and the reference Chebyshev best-fit methods. The results (LSC, MIC, MCC and MZC) concerning the last test are presented in Table 5 and again reveal a perfect agreement.

C/ Investigation of the SDS and **Chebyshev best-fit** *methods stabilities*

The roundness analyzes (MIC, MCC and MZC) using the SDS and Chebyshev best-fit methods illustrate a perfectly similar result when considering higher values of the diameters (>1mm). To understand the limitation the SDS method when scanning small holes, the SDS method stability was investigated by varying the value of the diameter between 20 µm and 50 mm. The investigation of the impact of the diameter's variation is completed when re-analyzing the MZC, MIC and MCC, corresponding to the second test performed on the flick standard. The results of MIC, MCC and MZC analysis are presented in Fig.8 and the values are recorded in Table 6. From Table 6, we note that when applying the SDS method, the MIC analysis is constant over the whole range of diameters between 20 µm and 50 mm. However, the MIC analysis based on the Chebyshev best-fit is stable only when the value of the diameter exceeds 10 mm. The maximum variation of the Chebyshev best-fit when analysing MCC can reach 10 nm. The analysis based on the SDS method is more stable than the reference Chebyshev method. For the MCC analysis, the SDS method also present results (variation of 2 nm for the diameter variation between 20 µm and 50 mm) more stable than those done by the reference Chebyshev best-fit method (variation of 4 nm). For the MZC analysis, the SDS method presents less stable results (variation of 6 nm for the diameter variation between 20 µm and 50 mm) than those done by the Chebyshev best-fit method (variation of 4 nm for the diameter variation between 20 µm and 50 mm).

Since these programs will be integrated in the newly ultra-high cylindrical measurement machine, with nanometric levels of accuracy, it is essential to be vigilant on such kind of issues in order to avoid introducing additional errors related to numerical processing. Following the different comparisons presented in Table 6, the diameter value of the part should be at least 700 times the value of the form error when using either the SDS or the reference Chebyshev best-fit method.

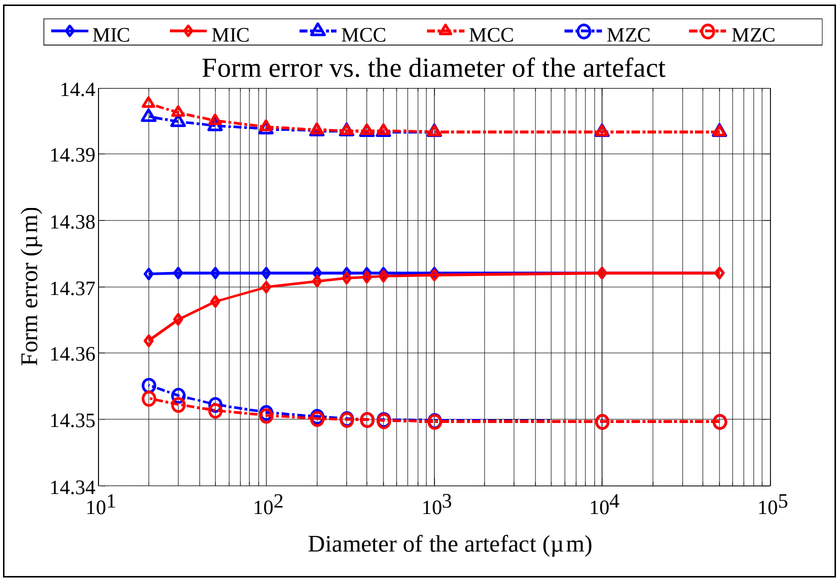


Fig. 8: Investigation of the stability of both SDS (blue curves) and Chebyshev (red curves) methodologies following the evolution of the cylindrical artefact diameter between 20 μm and 50 mm (Expanded interval estimate $U_{95\%}$ = ±42 nm, confidence interval: 95 %).

Diameter (μm)	**MIC** (Expanded interval estimate $U_{95\%}$ = ±42 nm, confidence interval: 95%)		**MCC** (Expanded interval estimate $U_{95\%}$ = ±42 nm, confidence interval: 95%)		**MZC** (Expanded interval estimate $U_{95\%}$ = ±42 nm, confidence interval: 95%)	
	SDS	**Cheb**	**SDS**	**Cheb**	**SDS**	**Cheb**
50000	14.3720	14.3720	14.3933	14.3933	14.3496	14.3496
10000	14.3720	14.3720	14.3933	14.3933	14.3496	14.3496
1000	14.3720	14.3718	14.3934	14.3934	14.3498	14.3497
500	14.3720	14.3716	14.3934	14.3935	14.3499	14.3498
400	14.3720	14.3715	14.3934	14.3935	14.3500	14.3499
300	14.3720	14.3713	14.3935	14.3936	14.3501	14.3500
200	14.3720	14.3709	14.3936	14.3937	14.3504	14.3501
100	14.3720	14.3699	14.3938	14.3942	14.3510	14.3506
50	14.3720	14.3678	14.3943	14.3950	14.3523	14.3514
30	14.3720	14.3651	14.3949	14.3962	14.3537	14.3523
20	14.3719	14.3619	14.3957	14.3976	14.3551	14.3531

Table 6: Investigation of the stability of both the developed SDS and reference Chebyshev best-fit (Cheb) methods when changing the value of the diameter of the flick standard of

14.3 µm. The change of the diameter is realized only when processing the data and the experimental datasets are the same for all analyses.

7. Conclusion

In this paper, the mathematical formulations of the Small Displacement Screw (SDS) method are presented and detailed. This method is developed and implemented in Matlab to analyze roundness: LSC, MIC, MCC and MZC. The developed SDS method is applied to analyze one theoretical dataset, published in the BCR report n°3327 and results are identical to those published.

To evaluate the SDS method on a real dataset, an experiment is prepared to measure the roundness of three cylindrical artefacts: cylinder standard of 75 mm, flick standard of 14.3 µm and the cylindrical artefact of 75 mm diameter, and with different undulations per revolution (15 UPR, 50 UPR, 150 UPR and 500 UPR). These targets are frequently used in mechanical industrial production to calibrate conventional high-precision machines, which are used to control manufactured parts. The experiment is completed inside the LNE clean-room under excellent environmental conditions: temperature, hygrometry, pressure and cleanliness. The SDS is applied to analyze LSC, MIC, MCC and MZC.

In order to investigate the limitation of the SDS method, the obtained results were compared to those obtained when applying the reference Chebyshev best-fit method. The comparison of the results reveals identical values of the MIC, MCC and MZC and confirms both the performance and accuracy of the SDS method.

The stability of both methodologies is investigated and reveals that results usually remain unchanged when the diameter value of the part is equal to 700 times than the value of the form error (roundness). Moreover, the SDS method is more stable than the Chebychev method when the diameter of the artefact is less than 500 µm, especially for the MIC and MCC analysis methods.

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