Mesopotamian Metrological Lists And Tables:Forgotten Sources

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From the outset of Mesopotamian archaeology, the archaeologists have constantly been excavating school tablets from the major sites of the Near East; these tablets were found incorporated in walls, in filling material, in pavements or abandoned in buildings which housed a scribal school. The majority of the tablets date from the Old Babylonian period, *i.e*. the beginning of the second millennium B.C. Today these tablets are spread all over the world, kept in the reserve collections of several important archaeology museums of the Near-East, of Europe and of the United States. Ten to twenty percent of these tablets are mathematical tablets. Some of the school mathematical texts have drawn the attention of the historians of science, in particular the numerical tables (multiplication tables, tables of reciprocals, tables of squares, etc.); but others remained in the dark. The latter were the metrological tablets, *i.e*. tablets containing enumerations of measures of various types (capacities, weights, surfaces, lengths) either in the form of simple lists, or in the form of correspondence tables. Why have these metrological texts been studied so little? What do they tell us about our comprehension of cuneiform mathematics? These are the questions this article intends to answer.

First, I shall give a short description of the Old-Babylonian metrological lists and tables. In the second section, I shall present the way the metrological lists and tables have been used by historians, from the publication of the earliest examples at the end of the nineteenth century, to the present "archival studies". In the third section, I shall concentrate on the sources from Nippur and shall study in detail the various processes of selection that have shaped the lots of school tablets discovered by the archaeologists, the tablet collections set up in museums, and finally the corpora assembled by the historians. The aim of this analysis is an attempt to reconstruct a set of tablets which constitutes the best representation of the mathematical activities of the Old Babylonian schools in Nippur. The last section will show the purpose of replacing the metrological lists and tables in their original "archives" *i.e*. the lots of school tablets.

1. Description of the sources

Before pursuing this investigation, a precise description of the metrological lists and tables in the Old Babylonian period should be given. These texts, which were standard school exercises, are attested not only throughout Mesopotamia, but also in Elam (Susa), and in Anatolia in slightly different forms[1]. The same sequence is often reproduced on numerous tablets. In Mesopotamia, these duplicates contain minor variants, but they are sufficiently uniform to allow a general description of the unique text they contain. This text, completed by possible variants, constitutes a “composite text”; it assemblesthe list of items – from at least one source – arranged the most accurately possible according to the epigraphic data[2]. The composite text relative to the metrological lists contains, in increasing order, an enumeration of measures of capacities, of weights, of surfaces, or of lengths. Concerning the metrological tables, the composite text contains the same items as the corresponding lists, but adds a sexagesimal number written in place value notation to the right of each measure. As regards the material evidence, large tablets have been recovered from Nippur and other sites, which give either the integrality of the metrological lists or the integrality of the metrological tables. These recapitulative texts always display the components in the same order: capacities, weights, surfaces, lengths. However, in most cases, our sources are pupils’ rough work containing brief excerpts of the lists or tables. This is the case in the following two tablets, of which a reproduction is given below (figures 1 and 2). The first is fragmentary, it contains an excerpt of a metrological **list** of length measures, the second is an excerpt of a metrological **table** of length measures (see also metrological list of capacity measures in figure 3).

[1] (Michel 2008).
[2] I shall come back to the question of the item order below. I have reconstructed a composite text from the mathematical school tablets from Nippur, which are of a remarkable stability, in (Proust 2007b, 311-324).

Metrological list of lengths (Ist Ni 3352, Istanbul)[3]

obverse
[4] danna
[4 1/2 danna]
[5] danna
5 1/2 [danna]
6 danna
6 1/2 danna
7 danna

reverse lexical (list of signs)

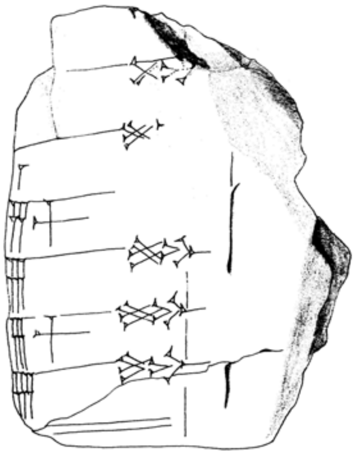

Figure 1

Metrological table of lengths
(HS 241, 7 × 5,1 × 2,4 cm, Jena)[4]

obverse

1 šu-si	10
2 šu-si	20
3 šu-si	30
4 šu-si	40
5 šu-si	50
6 šu-si	1
7 šu-si	1.10
8 šu-si	1.20
9 šu-si	1.30
1/3 kuš$_3$	1.40
½ kuš$_3$	2.30
2/3 kuš$_3$	3.20
5/6 kuš$_3$	4.10
1 kuš$_3$	5

reverse

1 1/3 kuš$_3$	6.40
1 ½ kuš$_3$	7.30
1 2/3 kuš$_3$	8.20
2 kuš$_3$	10

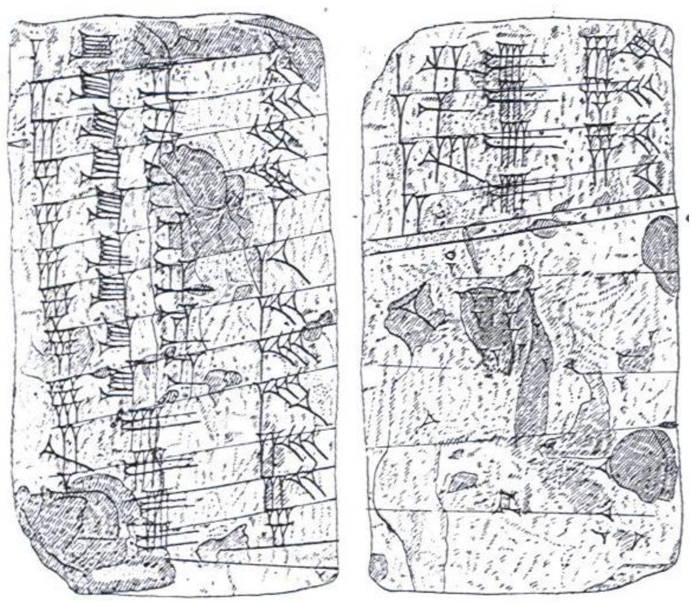

Figure 2

In order to express a measure (for example 6 1/2 danna), the scribes wrote count graphemes (here 6 1/2) followed by a unit grapheme (here danna; 1 danna ≈ 10 km)[5]. The difficulty for the scribes must have resided in the fact that the numerical systems varied according to the chosen units of measure. Furthermore, the factors between units are variable (except for weights where the factor 60 dominates). The metrological lists must have served

3 Copy (Proust 2007, pl.XI).
4 Copy (Hilprecht 1906, n°42, pl. 27).
5 All the mathematical tablets from Nippur kept in Istanbul (Ist Ni) and Jena (HS) have been integrated into CDLI database (http://cdli.ucla.edu/).

as an aid to help make sense of all these systems. They describe extensively not only the different units and factors between them, but also the numerations that are associated with them. A modern synthetic presentation of the metrological systems is given below in the form of factor diagrams: the names of the units of measure are given along with the multiplying factors defining each unit with respect to its multiples and submultiples (see table 1 below)[6].

The metrological tables are of a different nature. Although they are composed of the same entries, they do not have the linear structure of the lists, since they introduce a second dimension by giving for each measure its equivalent as a sexagesimal number in place value notation. I have placed these equivalents under the corresponding units in the factor diagrams below. As regards lengths, there are in fact two tables of correspondences between measures and numbers in place value notation: the one for horizontal dimensions (lengths, widths, diagonals) and the other for vertical dimensions (heights, depths). This table of supplementary heights was used for the calculation of volumes (in the following, this point will be considered in connection with F. Thureau-Dangin's work).

Capacities (1 sila$_3$ ≈ 1 litre)									
	gur	←5~	bariga	←6~	ban$_2$	←10~	sila$_3$	←60~	gin$_2$
	5		1		10		1		1
Weights (1 gu$_2$ ≈ 30 kg)									
	gu$_2$	←60~	ma-na	←60~	gin$_2$	←180~	še		
	1		1		1		20		
Surfaces (1 sar ≈ 36 m²)									
	GAN$_2$	←100~	sar	←60~	gin$_2$	←180~	še		
	1.40		1		1		20		
Lengths (1 ninda ≈ 6 m)									
	danna	←30~	UŠ	←60~	ninda	←12~	kuš$_3$	←30~	šu-si
	30		1		1		5		10
Heights									
	danna	←30~	UŠ	←60~	ninda	←12~	kuš$_3$	←30~	šu-si
	6		12		12		1		2

Table 1: metrological systems

Let us end this brief presentation of the metrological texts with some details regarding the notation of measures and the notation of the numbers they contain. The numerical values used to express measures belong to different systems, as indicated above. It is not the aim of the

[6] This clear representation was proposed by J. Friberg (1993: 387). A complete description of the numeration systems associated with each unit of measure can be found in many publications, for example (Powell 1987-1990; Friberg 1993; Nissen, Damerow & Englund 1993; Proust 2007b, 2009).

present article to describe all the peculiarities of these notations[7]. The essential point, which I would like to stress here, is the common characteristic of these systems: they all use an additive principle[8]. On the other hand, the numbers written opposite each measure in the metrological tables are sexagesimal and use a positional principle (or place value notation): the digits are written using the same signs whatever their position might be; each sign is defined by its position in the number; each position represents sixty times the one preceding it (*i.e*. the position on its right). The 59 digits of this numeration of base sixty are written using signs representing 1 (𒁹) or 10 (𒌋) that are repeated as many times as necessary. The example given below is a number with three sexagesimal places containing the successive "digits" 40, 26 and 44:

44.26.40

The cuneiform place value notation has yet another particularity, the importance of which will appear in the course of this article: there is no graphical means giving the position of units proper. For example, the numbers, which we write 1, 60, 1/60, are all represented on the tablets by the same sign 𒁹. Thus the magnitude of the numbers in place value notation is not specified. This property contrasts the numbers in place value notation with the numerical values employed to write measures, since the latter are written using an additive principle, and therefore represent well defined quantities. F. Thureau-Dangin adopted the term "abstract number" to designate sexagesimal numbers in place value notation, the adjective "abstract" referring to the fact that their order of magnitude is unspecified; I have used this term for the same reason, but also to underscore another property which can be seen in the metrological tables, although a general property in the mathematical and school documentation: these numbers are never followed by a unit of measure.

[7] See publications quoted in preceding note.
[8] See (Proust 2008a).

2. Historiographical survey

Archives, lots, corpora and collections

The metrological lists and tables can be tackled in different ways that can be partly linked to different historiographical periods. The first approach is to consider only the text. This means led to many important results in the early stages of Assyriology. At that time, the discovery of a large corpus of texts implied an urgent need to characterize the main principles of Mesopotamian metrology; this knowledge was essential to understand the administrative and mathematical texts. The origin, the archaeological context and the material aspect of the tablets containing them, could be considered in these circumstances as relatively secondary. This standpoint was reinforced by the fact that during the Old Babylonian period, the metrological system was highly standardized over a large geographical area which enabled a reconstruction of the system using sources from different provenances. The second approach, which is described by some authors as an "archival"[9] approach, consists in considering the tablets as well as the texts. Thus the emphasis is placed on lots of epigraphic documents discovered in the same archaeological locus and therefore displaying a certain coherence. The most common term used to refer to a lot of school tablets found in a same *locus* by excavators is "school archives" though this is a language misuse in the sense that the ancient scribes had no intention of creating archives of school drafts[10]. Nevertheless such an approach requires precise knowledge of the circumstances of the discovery of the tablets by the archaeologists, but this clear archaeological context is often lacking. In this respect, let us recall the two principal means by which collections of cuneiform tablets have been set up, the one resulting from systematic excavations, the other resulting from clandestine excavations and from the market of antiquities[11]. The metrological lists and tables may have followed either one of

[9] See the work of the *30e Rencontre Assyriologique Internationale* published in 1986 (Veenhof 1986a). In his introduction, K.R. Veenhof draws a general picture of the changes in perspective that were introduced in Assyriology by the archival approach. He shows the relevance of paying great attention to the provenance of the tablets, to their material history and to the conditions of the creation of museum collections for the development of an economic, a social and a political history of the ancient Near-East (Veenhof 1986b). However, the work of this *Rencontre* essentially concentrated on administrative archives and did not consider much the lots that came from libraries and schools.

[10] The school tablets were not meant to be archived, on the contrary they were meant to be thrown away after having been used (see section 3). P. Clancier's article, in this volume, gives more information concerning the cuneiform documentation that can appropriately be named "archives". See also (Veenhof 1986a).

[11] On this subject see the beginning of P. Clancier's article in this volume. The way that the market of antiquities shaped the collections, on which the Assyriologists base their work, has not been studied much; the antiques business has always been opaque and, in recent periods it has been more the concern of investigative journalism than the one of academic research (Brodie 2006, 12). On the damage caused to the archaeological heritage by the trafficking of antiquities and the question of the relations between researchers and collectors, see (Brodie 2006;

these routes. But the tablets - often school tablets which had been reused as building material – are mostly of poor appearance and therefore have been of no interest to collectors. The majority of the tablets come from systematic excavations, they have been accumulated and often forgotten in museum reserves. The Assyriologists did not take an interest in these texts once the units of measures had been identified, nor did the historians of mathematics. The latter, with a few exceptions, did not consider these tablets as mathematical texts proper[12]. And it is only recently that these texts have become a subject of interest.

The different approaches rapidly mentioned above have a heavy impact on the studies of cuneiform mathematical texts. Historians choose certain criteria when they select their corpora of texts. These criteria can be archaeological (tablet lots of same provenance and dating), thematic (corpora of texts dealing with an identical subject) or based on museological considerations (museum or private collections); let us note that these criteria are not exclusive[13]. For reasons of accuracy, in the following investigation I shall distinguish the groups of texts or tablets according to their mode of selection. I shall use the term "lot" along with P. Clancier to designate the tablets that were stored together for various reasons in the same archaeological deposit; in the case of the school texts, I shall also use the standard and evocative expression "school archives", even though it is inappropriate (this fact is evoked by the quotation marks). I shall save the word "collection" to designate the groups of tablets assembled by curators or collectors in view of their keeping in museums or private collections. "Corpus" will refer to the gathering of texts made by modern historians in order

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Brodie, Kersel, Luke & Tubb 2006). More precisely concerning the impact of illegal excavations on the study of cuneiform archives, see for example (Veenhof 1986b, 35).

12 *« C'est un type de tablettes assez fréquent dans les musées, mais qui n'a pas attiré suffisamment l'attention. Les historiens des mathématiques les trouvant apparemment trop simples, une étude d'ensemble manque encore* » (Civil 1985, 77). Other authors have made similar remarks, for example (Michel 1998, 253; Robson 2004, 12). It must however be stressed that in some investigations, in particular in J. Friberg's publications, the metrological tables are studied as mathematical texts (see section 2).

13 Considering the first category (studies of corpora that were made on archaeological criteria), let us mention the work of H. Hilprecht (1906), and more recent examples like the studies of the mathematical tablets from Tell Harmal (Baqir 1950a, 1950b, 1951), from Susa (Bruins & Rutten 1961), from Ur (Friberg 2000), from Nippur (Robson 2001; Proust 2007b). Considering the second category (studies of corpora made on museological criteria), let us mention *Mathematische Keilschrifttexte* (Neugebauer 1935), the chapters of which correspond to the various European and American museum collections; among the publications of mathematical texts based on public or private collections, generally of unknown provenance, let us cite : (Robson 2000, 2004, 2005, Friberg 2005, 2007). The archaeological criteria may coincide with the museological criteria: the tablets from Susa published in (Bruins & Rutten 1961) are all kept in the Louvre Museum, the ones from Nippur published in (Proust 2008b) are all kept in Jena. Considering the third category (studies of corpora made on thematic criteria), let us mention *Mathematical Cuneiform Texts* (Neugebauer & Sachs 1945), the chapters of which are organized according to a thematic classification and no longer a museological one as it was the case in *Mathematische Keilschrifttexte*. The texts which have been studied by J. Høyrup (2002) are also grouped according to thematic criteria, different sections of the same tablet can be found in different chapters (for example, this is the case with the tablet BM 13901, the sections of which are analysed in four different parts of the book).

to study a particular topic; besides, these groupings are often taken from "lots" or from "collections".

It is of interest to compare the historians of mathematics' approach to the study of metrological texts with the one chosen by Assyriologists with respect to lexical lists. The lexical lists were first used to establish the Sumerian vocabulary[14]. Subsequently, the attention focused on the tablets themselves, and on the fact that they were school tablets[15]. In a similar way, the metrological lists and tables were first used to establish the Mesopotamian metrology. It is only recently that they have been considered as evidence of the mathematical activities in the scribal schools.

Periodisation

The historians' change of approach to the metrological lists and tables is manifest if one considers the various metrological lists and tables, which have been published[16] since the beginning of Assyriology. This chronology reveals three major periods:

- At the outset of Assyriology, the metrological lists and tables were the subject matter of important publishing work; in particular let us mention the contributions made by H. Hilprecht and F. Thureau-Dangin. The lists and tables allowed the comprehension of the major characteristics of Mesopotamian metrology: the notation of the units of measure, their relative values, and the notation of the associated numerical values.

-During the 1930-1975 period[17] – *i.e.* the period during which the most important part of the corpus of cuneiform mathematical texts known to us today was deciphered, translated and interpreted – the metrological lists and tables hold a marginal position among the published

[14] This is the purpose of the large series called "MSL" (*Materialen zum sumerischen Lexikon*, and then *Materials for the Sumerian Lexicon*), which provides Sumerologists with the major part of their lexicographical material. MSL is the systematic publication of the Mesopotamian lexical lists. It is composed of 18 volumes; the publication of volumes 1-14, 16-17 and SS1 spreads over years 1937 to 1986: volume 15 was published in 2004.
[15] This change of approach, which is characterized by the attention paid not only to the texts but also to the tablets, has been nicely shown by N. Velhuis in his study of the lexical lists from Nippur (Velhuis 1997, 3).
[16] The publication of the cuneiform texts does not obey stable or fixed rules from one period to another, nor from one author to the other. Here, the term "publication" of a tablet refers to a publication that gives a minimum of information on the tablet (inventory number, provenance and dating, when this information is known) and at least a copy or a transcription – even partial - of the tablet.
[17] This corresponds to the periods that Jens Høyrup calls the "heroic era" (1930-1940) and the "triumph of translation" (1940-1975) (Høyrup 1996)

texts. An analogous observation could be made with respect to the proportion of articles containing commentaries on the metrological lists and tables.

- During the recent period, the publication of school texts, which includes the metrological lists and tables, has enjoyed renewed favor in a spectacular way.

In this section, I shall particularly insist on the first period, which isn't usually considered in the historiography of cuneiform mathematics. Concerning the 1930-1975 period, I shall limit myself to pointing out some important studies for the present investigation, although probably of minor importance as regards the extent of the publishing work of the time; on this subject let us refer to the work of J. Høyrup and J. Friberg[18]. The last period is discussed at length in sections 3 and 4 of this article.

Characterization of Mesopotamian metrology

The first metrological tablet coming from the excavations in Iraq was published in 1861; it is a large table from Larsa (modern Senkereh) that is kept in the British Museum (Museum number BM 92698[19]). It was at once considered of major importance by F. Thureau-Dangin[20]. Thanks to this tablet, G. Smith partially identified the metrological system of lengths in 1872[21]. But neither the school character, nor the mathematical character of the text had been recognized at this date. Therefore the first cuneiform mathematical texts historians came across were the metrological tables, and in some ways without them knowing it. Following the discoveries made by the Babylonian Expedition, H. Hilprecht - the scientific director of the excavations - revealed the existence of scribal schools and cuneiform mathematics to the scientific world and to the general public, on the one hand thanks to the monumental account of his expeditions in 1903 and, on the other hand, by the publication of

[18] (Friberg 1982; Høyrup 1996). J. Høyrup's historiography, which is today accepted as the authority on this matter (1996), focuses on the work relative to the great erudite mathematical texts of the Old Babylonian and Neo-Babylonian periods, and thus begins in 1930. J. Friberg's notes do not contain a historiographic study proper, but rather give a chronological presentation of all cuneiform mathematical publications since 1854 *i. e.* the beginning of Assyriology - until 1980; this annotated bibliography is a basic working tool for the specialists, but unfortunately it has never been published.

[19] This tablet named "Table of Senkereh" by the Assyriologists of the time, has been published in several stages by (Rawlinson & Smith 1861; Lenormant 1873; Lepsius 1877; Thureau-Dangin 1930a; Neugebauer 1935-7).

[20] For him it is « depuis le début de l'assyriologie la *crux interpretatum* » (Thureau-Dangin 1909).

[21] (Smith 1872 ; Friberg 1982).

the mathematical tablets found in the "Temple Library" of Nippur in 1906[22]. It is interesting to point out that H. Hilprecht who discovered at the same time the schools and the mathematics, did not dissociate the metrological texts from the mathematical texts. A division line only appears later, when attention focuses on erudite mathematics. With few exceptions[23], the work of H. Hilprecht published in 1906 remains without any equivalent until the recent publication of the school tablets from Ur, from Nippur and from Sippar etc. (see the end of this section). As noted by E. Robson, Hilprecht's observations on the school tablets are remarkable, in particular those made with reference to the material aspect of the tablets[24]. Furthermore, the lines of Hilprecht's copies are extremely precise and depict not only the cuneiform signs but also the asperities of the clay surface; these copies reveal both his interest in the texts and in the tablets (see figure 2 above and figure 3 below).

[22] (Hilprecht 1903, 1906). In fact Hilprecht's discovery was violently contested by some members of the expedition (see below).
[23] A notable exception is E. Chiera's study of lexical lists of names published in 1916. This study contains one of the first systematic descriptions of types of school tablets (Chiera 1916; Veldhuis 1997, 5).
[24] (Robson 2002, 238-239).

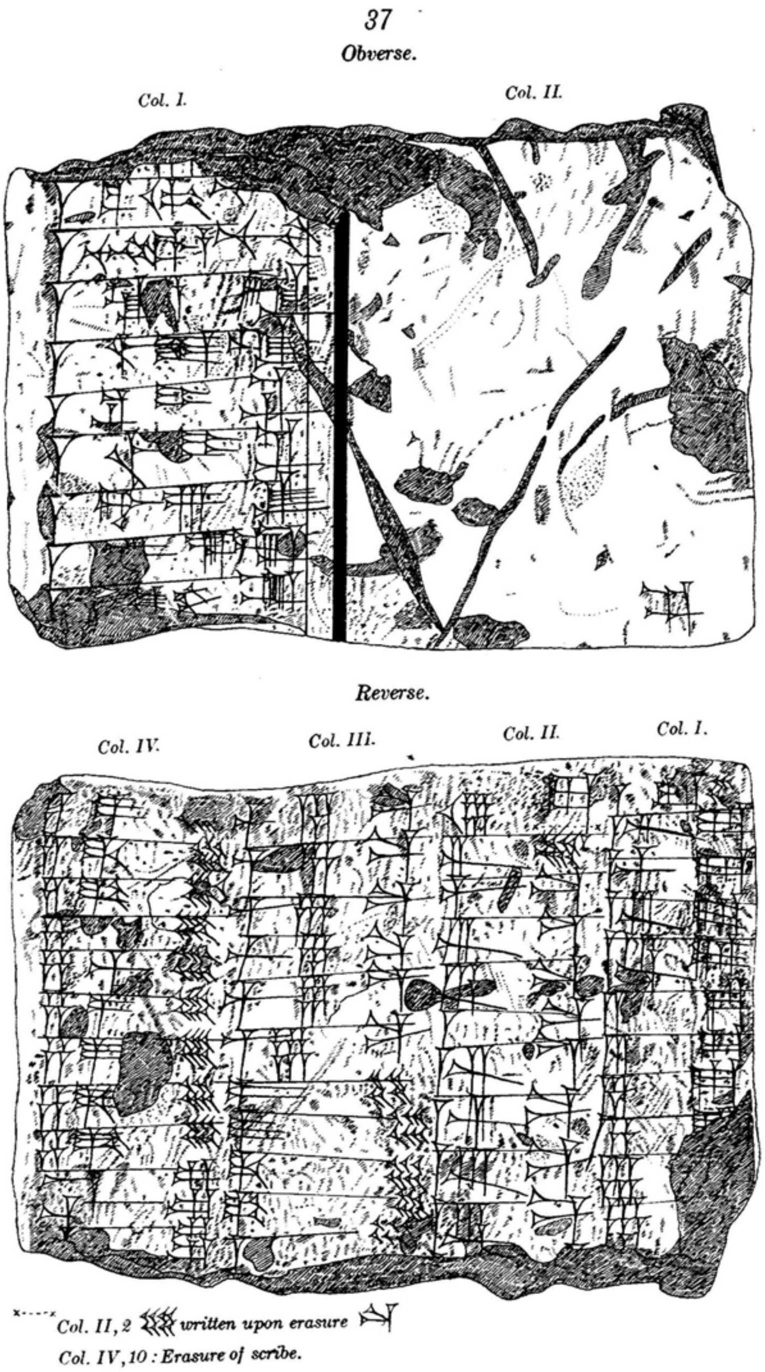

Figure 3 : type II tablet from Nippur – Copy by H. Hilprecht[25]

His description of "type II" tablets, made in terms hardly to be amended today, is given below; note that the above terminology (*i.e.* type II) was only adopted recently by the

[25] Tablet kept in Jena (HS 239), published by H. Hilprecht (1906, n°37); two other fragments of the same tablet (HS 250+HS 256) were found in Jena's collection by J. Oelsner (Proust 2008b, n°4). The complete tablet should have measured approximately 18 cm high and 8.5 cm wide. On the obverse, one can see a master's model giving a list of signs (Proto-Ea), and remnant signs and traces of erasure can be distinguished on the surface meant to contain the copies; a metrological list of capacity measures is inscribed on the reverse.

Assyriologists, who defined a typology of the texts produced in a school context, more than 70 years after Hilprecht's description:

On the Obverse of Nos. 20, 24, 37 (cf. Pls. IV and XIII) the priest in charge of the class wrote the left column with his own hand as a model for the pupil, who copied the text in the right column. When the exercise was satisfactory, the teacher removed the pupil's writing by scraping the upper layer of clay off the right column. Frequently, however, before destroying the pupil's exercise, the teacher turned the tablet over and inscribed the Reverse with a similar or an entirely different text, sometimes writing his model twice or three times, after the manner of our Schulvorschriften*. On some of the tablets examined the right column has been inscribed and scraped off so frequently that it is considerably thinner than the left column. They are even specimens where the right column has been cut off entirely. In other cases the pupil's exercise has been removed so superficially that, like a Greek palimpsest, the traces left aid in deciphering the contents of the preserved but frequently damaged left column* (Hilprecht 1906, p. x-xi[26]).

The importance of all these details for the understanding of the school context will subsequently be largely underestimated, until a recent renewal in the studies of school texts occurred.

As for F. Thureau-Dangin, he took a particular interest in metrology, probably because of the great variety of genres of texts that he studied (literary, administrative, mathematical). His remarks on the subject pepper the "notes" of the *Revue d'Assyriologie* from 1893 to 1947. When reading these notes one is struck by the diversity of exploited sources: tablets of different origin, often unknown, and dating of periods that extend from Sargon of Akkad (approx. 2300) to the Seleucid era (approx. 300), administrative texts, mathematics, monumental inscriptions, bricks, various artefacts, stories of Herodotus. The metrological tables only represent a small portion of these sources, but he considered them as crucial. F. Thureau-Dangin's deep understanding of numbers and of Babylonian calculation can possibly be explained by his familiarity with the administrative, literary and mathematical sources. The importance he attached to the tables enabled him to grasp essential aspects of the practice of numbers. I shall only mention two of these aspects here: his discovery of the definition of units of volume in the mathematical texts, and his analysis of the particularities of numbers in place value notation.

[26] See a similar description of "type II" tablets in section 3.

As early as 1900, F. Thureau-Dangin revealed the singular fact that the units of volume have the same name as the units of surface[27]. In 1930 – and in a more developed manner in his *Textes Mathématiques Babyloniens*[28] – he detailed the scribes' construction of volume units: each unit of volume is equal to a unit of surface associated with a depth of 1 k u š 3 (50 cm) and bares the same name. The sources he used to establish this definition were the mathematical texts and the "Table of Senkereh" BM 92698[29].

More importantly, F. Thureau-Dangin drew attention to the fact that there are two different correspondences with respect to the linear measures in the metrological tables. The existence of two metrological tables of length measures had been noted by H. Hilprecht, who had published examples of these two types as early as 1906[30]. But it was F. Thureau-Dangin, who showed the importance of these two correlations for the understanding of volumes. And yet this major point was not really exploited in later works. It is not mentioned by O. Neugebauer, who doesn't recognize it in his publication of a tablet from Nippur (2N-T 530)[31]. The role, in volume calculations, of these two correspondence tables, the one used for horizontal dimensions and the other for vertical dimensions, was in some ways "rediscovered" and developed much later by J. Friberg[32].

Another essential aspect of F. Thureau-Dangin's work concerns the sexagesimal numbers in place value notation that are used in the mathematical texts. He was especially interested in the lack of indication of magnitude order in the cuneiform writing; this particularity has already been mentioned above:

Il est important de faire observer que, dans cette notation, il n'existait aucun moyen d'exprimer l'ordre absolu de grandeur d'une unité donnée. Hilprecht a soutenu que dans les tables de division qu'il a publiées, le nombre à diviser est 60^4 (c'est-à-dire 12 960 000, qui serait, selon lui, le "nombre de Platon"). En réalité, la question ne se pose même pas, car l'unité considérée peut être d'un ordre quelconque et représenter aussi bien l'unité simple

[27] (Thureau-Dangin 1900, 112)
[28] (Thureau-Dangin 1930b; Thureau-Dangin 1938, xvi-xvii).
[29] (Thureau-Dangin 1938, xiv, xvii). F. Thureau-Dangin (1930b) mentions that H. Waschow independently reached the same results; the latter's conclusions were published slightly later (Waschow 1932).
[30] Hilprecht noticed the two correspondences, but didn't try to explain them (Hilprecht 1906, 35, 66, pl. 27 n° 41, 42).
[31] (Neugebauer & Sachs 1984, 248-250). See (Friberg 1993, 387) who points this out.
[32] J. Friberg uses the school tablets from Ur (UET 7-114 and UET 7-115) that give metrological tables, in which notes specify "for lengths and widths" at the end of the tables containing one of the correspondences and "for heights and depths" at the end of the tables containing the other of the correspondences, and in a colophon of the "tablet of Senkereh" (Friberg 1987-90, 543; Friberg 1993, 387; Friberg 2000, 156). For this reason, I have called the first tables "table of lengths", and the second tables "table of heights" (Proust 2007b, 107-111).

que 60 ou une puissance de 60, ou encore une unité fractionnaire, 1/60, 1/60² etc.(Thureau-Dangin 1921, p. 124).

Even more significantly, F. Thureau-Dangin showed that there existed a link between this particular way of noting numbers and the manner of calculating; this connection, as we shall see below, is fundamental:

Si la grandeur absolue pouvait sans inconvénient rester indéterminée, c'est parce que le système n'était destiné qu'à servir d'instrument de calcul. […] *Le système savant offrait le grand avantage d'éviter les rompus. Tout nombre, dans ce système, présente la forme d'un entier* **(**Thureau-Dangin 1938, p. x).

Thus,

L'expression du nombre atteint dans le système savant un degré de simplicité, d'homogénéité et d'abstraction qui n'a jamais été dépassé (Thureau-Dangin 1938, p. xix).

The important point underlined by F. Thureau-Dangin is that the absence of information relative to the order of magnitude is not a deficiency of cuneiform writing. On the contrary it is a property linked to the use of numbers: the numbers in place value notation are tools for calculation, and the power of this instrument lies precisely in the fact that the indetermination of the position of the units allows to avoid "*les rompus" i.e.* fractions. But if the scribes did not see a drawback in using a system, in which the "absolute magnitude (of numbers) could remain indeterminate with no inconveniences", why then did F. Thureau-Dangin feel the need to recreate an "absolute magnitude" in his translations? Let us illustrate this with an example:

Toi, dénoue l'inverse de 32, tu trouveras 1'52''30'''. Porte1'52''30''' à 36, la hauteur, tu trouveras 1°7'30'' (Thureau-Dangin 1938, p. 35).

Here F. Thureau-Dangin gives the symbols degree (°), minute (′), second (″) etc., although the cuneiform text does not contain these marks. This habit of restoring the order of magnitude of numbers written in place value notation is not peculiar to Thureau-Dangin, it is a feature of all the authors studying cuneiform mathematical texts. Why does a way of writing numbers, which did not present any disadvantages to the scribes, prove to constitute a problem for historians from Thureau-Dangin up to now? The question that is raised here concerns the scribal calculation practices. As a matter of fact, the elementary school texts permit us to shed some light on this question and, hence, to understand why it is pointless to substitute our system by the scribes' one in the texts. It is here, we shall see, that lies the interest of considering the school texts and, more precisely, the importance of replacing the metrological tables in their original context. On the basis of the reconstituted collection of mathematical tablets from Nippur, I shall show how the metrological tables establish a relation between the measure systems and the numbers written in place value notation with no indication of magnitude. I shall also show that an analysis of the entire set of school texts permits to grasp how this relation was actually used in calculations.

Grounding his work on a large variety of sources, including the known metrological tables of the time, T. Thureau-Dangin was able to derive some essential characteristics from the metrological and numerical systems in use in the administrative and mathematical texts. But, curiously, his point of view was only partially adopted (as one can see with the example given above *i.e*. the disfavor of his discoveries on the "table of heights"). Furthermore, he did not deal with the metrological tables as forming part of school tablet lots and consequently, the significance of this context, and in particular the question tackling the way the scribes themselves learnt metrology, remained unknowned to him.

Metrological lists and tables in the studies of cuneiform mathematics

From the end of the 1920s onwards, the historians' interest turned to the large erudite mathematical texts. These texts, which came from clandestine excavations and were bought from antique dealers, were starting to flow into the European and American museums. This new wave contrasted with the previous one in every respect: these tablets, of beautiful appearance but of unknown provenance, displayed an elaborate content revealing to the historians the very ancient roots of mathematics. The cuneiform mathematical texts were first published by F. Thureau-Dangin in the *Revue d'Assyriologie*, followed by O. Neugebauer in the 1930s. These researchers edited most of the texts and defined the terminology that still today represents the core of cuneiform mathematics[33]. The primary sources were mainly gathered in the following books: *Mathematische Keilschrifttexte I-III* published in 1935 and

[33] Among the notable additions to this initial corpus, let us mention the documentation coming from the systematic excavations after WWII, principally the documentation from Susa and Tell Harmal.

1937 by O. Neugebauer; *Textes mathématiques babyloniens* published in 1938 by F. Thureau-Dangin; *Mathematical Cuneiform Texts* published by O. Neugebauer and A. Sachs in 1945.

Correlatively the texts, which held the historians attention, were selected for their mathematical content and independently of the contexts they came from. The tablets produced by the schools were generally not studied much for their own sake and among these, some even less than others. Numerical tables were studied (allowing to calculate reciprocals, products, squares, roots), whereas the metrological lists and tables were largely ignored. Many numerical tables can be found among the school texts published by O. Neugebauer in both *Mathematische Keilschrifttexte* and *Mathematical Cuneiform Texts*, but very few metrological lists and tables (only 9 out of the 175 school texts) and no recent edition. The metrological texts, even more than the other school texts, were thought to be too simple, as M. Civil has pointed out (see citation above). This conception of metrological tables is illustrated by O. Neugebauer in the following passage of *Mathematical Cuneiform Texts*, the only passage where the latter mentions the subject at all:

The teaching of metrological rules was undoubtedly the purpose of many examples in our texts which are very simple from the mathematical point of view but require the mastery of the ratios between various units (Neugebauer & Sachs 1945, p. 4).

This does not mean that O. Neugebauer was not concerned with metrological problems. Actually, he was the author of a major discovery in this respect *i.e.* the use of the brick as a unit of volume[34].

The numerical tables were the subject of careful investigation by O. Neugebauer, who devotes to them the entire chapter I of his *Mathematische Keilschrifttexte*. He examines the tablets' aspect and names the tablets "simple" when they contain only one table, "combined" when they contain several tables; he distinguishes different types of multiplication tables depending on the way they are presented (types A, B, or C); he indicates the formulation of the *incipit*, the presence and content of the colophons, the graphic singularities (*e.g.* writing 19 as 20-1), the terminology, the dating, the provenance. In the plates of *Mathematische*

[34] In some of the mathematical texts, volumes are expressed in terms of number of bricks of standard format, and not according to the units of volume of the standardized Mesopotamian system (Neugebauer 1935-7, I: 399; Neugebauer & Sachs 1945, 94-97).

Keilschrifttexte II (plates 61 to 69), O. Neugebauer reconstructs the general structure of about thirty large recapitulative tablets containing the complete list of standard reciprocal tables and multiplication tables.

Therefore there exists a sharp contrast between the manner O. Neugebauer considers the numerical tables and the way he deals with the metrological lists and tables. The latter do not seem to belong entirely to what O. Neugebauer considers as the mathematical corpus proper. However the metrological and numerical school tablets were found together in the same lots, in comparable quantities, and they are kept in the same museums. As we shall see below, the metrological and numerical school tablets from Nippur are an integral part of the same puzzle, and they shed light on each other.

Metrological lists and tables in the "school archives"

The interest of the historians of mathematics in metrological systems was revived by the work of M. Powell from 1971 onwards, followed by J. Friberg's work on Archaic and Neo- Babylonian metrologies[35]. Several particularly important aspects of the metrological tables were brought to the fore by J. Friberg, as for example, the existence of a specific table for measures of height and depth already mentioned above[36]. So as to integrate these various results into a homogenous and coherent interpretation, it was necessary to replace the metrological tables in the archaeological lots, to which they belonged; this became possible thanks to the extremely important publication work under way.

For the past twenty years, historians have been publishing more and more actively complete "school archives": these archives assemble as far as possible all the texts found in a given place, and in particular the literary and mathematical texts. Some of these publications are "*catalogues raisonnés*" placing the new material at the researchers' disposal[37]. Others reconstitute collections of homogenous provenance and dating on the basis of previously published texts, associated with commentaries and interpretations[38]. Finally, others give new

[35] (Powell 1971, 1979, 1987-1990; Friberg 1979, 1993, 1994, 1999).
[36] (Friberg 1987-90, § 5.1; Friberg, Hunger & Al-Rawi 1990, 509; Friberg 2000, 156). Besides, J. Friberg underscored the important fact that the metrological table of surfaces can also be used as a table of volumes (Friberg 1987-90, 543).
[37] See for example the school tablets from Uruk (Cavigneaux 1982, 1996) and from Tell Harmal (Al-Fouadi 1979).
[38] See for example the mathematical school tablets from Ur (Friberg 2000).

material accompanied by an archival approach[39]. In 2002, less than one hundred metrological lists and tables had been published since the outset of Assyriology. Soon, with the current publication of the school tablets from Nippur and from Mari, the available documentation will exceed 500 tablets and fragments containing metrological lists or tables.

3. The mathematical tablets from Nippur

In the preceding sections, we examined the way the metrological lists and tables – taken as texts but not as archaeological artifacts – allowed the understanding of Mesopotamian metrology. However, this approach does not answer the questions relative to scribal calculation practices, notably in the school and erudite contexts. We shall now look at these texts from a different angle giving prominence to the material history of the clay tablets which contain the texts. This leads us to the site of Nippur, the place of discovery of the most important "school archives", and hence to the most extensive lot of metrological lists and tables.

The tablets excavated by the Babylonian Expedition

More than 800 tablets containing mathematical texts have been found in these "school archives" which were excavated at the end of the 19th century by the American mission of the University of Pennsylvania, the Babylonian Expedition[40]. Among them, almost half are metrological lists and tables (more or less 350 tablets and fragments),

The Babylonian Expedition campaigns were managed by a Committee assembling the major institutional (University of Pennsylvania) and private subscribers. This Committee seems to have put constant pressure on the excavators to supply the newly created museum with antiquities and tablets. This context partly explains why the excavators indulged in a genuine "tablet hunt" that had disastrous consequences on the quality of the archaeological work: deep tunnels were dug without consideration for the superficial constructions and surface layers,

[39] See for example the school tablets from Ur (Charpin 1986), from Tell Haddad (Cavigneaux 1999), from Sippar (Tanret 2002), from House F in Nippur (Robson 2001) or from Babylon in the Neo-Babylonian period (Cavigneaux 1981). See also the study of the lexical lists from Nippur made by N. Veldhuis, who has been in a way the promoter of the new approach to the school texts (Veldhuis 1997).

[40] The school tablets excavated by the Joint Expedition from 1948 to 1990 are less numerous, and the ones kept in Baghdad are not accessible. Nevertheless, they have the big advantage of presenting a clear archaeological context in comparison with the tablets excavated by the Babylonian Expedition. A complete study of the collections kept in Philadelphia and in Chicago has been published by E. Robson (2001).

and with no prior topographical survey; the excavations were led by an important workforce without scientific supervision[41]. Therefore the tablets excavated by the Babylonian Expedition do not benefit from any archaeological context, and it is impossible to know the precise place and the stratigraphic level, in which they were discovered.

Thousands of school tablets were excavated in this way from the deep strata of the site of Nippur. The finds were then brought to Istanbul, where they were split between the American excavators and the Ottoman authorities. The way the tablets were shared out was the outcome of agreements between the two parties within the framework of the laws on the Ottoman heritage, which were promulgated in 1883, permitting a strict control over the movement of antiquities[42]. Hence some of the tablets stayed in Istanbul, the others were sent to Philadelphia. Subsequently, a number of the tablets in Philadelphia were transferred to Jena, where they still are today. This episode, was the result of a conflict between the American excavators. In fact, H. Hilprecht, scientific director of the excavations, was excluded from the University of Pennsylvania because of a difference of opinion with certain of his colleagues of the Babylonian Expedition such as J. Peters, director of the first two excavation campaigns. After his death, his personnal collection was bequeathed to the University of Jena. We shall not enter here into the details of the turbulent history of the Babylonian Expedition[43]. But some aspects of the controversy are interesting to mention in the context of this article. First, a polemic crystallized over the existence of a school in Nippur, this thesis was defended by H. Hilprecht and contested by his adversaries; astronomical and mathematical tablets, in particular examples of multiplication tables, were produced by H. Hilprecht as exhibit in his favor[44]. These events had important repercussions on the conditions of preservation of the tablets, on the fragmentation of the collection into three parts which are kept in different countries, and also on the content of each of them. This controversy was also a major driving force in the diffusion of Hilprecht’s discoveries since,

[41] Concerning the analysis of the consequences of the Committee’s politics for the excavation methods in Nippur, see (Westenholz 1992). See also the account of H. Hilprecht (Hilprecht 1903, 332, 334, 328-329, 339-340).
[42] (Lafont 1984, 179; Hilprecht 1903, 570, 572-574).
[43] B. Kuklick has recently published a book devoted to what may be called the “Hilprecht-Peters controversy”, its intellectual context and its consequences for the development of American Assyriology (Kuklick 1996). The two main protagonists have abundantly expressed their point of view in different publications and articles; see in particular (Hilprecht 1903, 1904, 1908; Peters 1905).
[44] See for example, the letters addressed in 1905 to the Trustees of the University of Pennsylvania by J. Peters and by H. Hilprecht (Hilprecht 1908, 14, 35). J. Peters accused H. Hilprecht of lying about the provenance of these tablets.

according to E. Robson's analysis[45], the publication of the mathematical tablets from Nippur were a response to the numerous attacks on his work.

With the aim of shedding light on some of the selection principles relative to the school tablets, I shall consider two key periods in the history of the sources from Nippur: the sorting processes made by the ancient scribes themselves and the recent sorting operations leading to the formation of the collections in Philadelphia, Istanbul and Jena, in which H. Hilprecht played a major part.

The processes of selection

The collections of school tablets from Nippur which reached us are the result of various kinds of operations of selection, ancient and modern. These selection processes might pertain to the entire set of school tablets, or might concern more specifically the elementary mathematical texts or the metrological texts proper. The protagonists involved in these operations are quite varied: the ancient scribes themselves, the pioneers of Mesopotamian archaeology, the antique dealers, the people in charge of the Ottoman heritage, Americans and Europeans, the historians of mathematics, the curators and collectors, and of course the part played by chance and the ravage of time. The corpus of metrological lists and tables from Nippur bares the mark of the different selection phenomena, to which the cuneiform sources are often subject, and it shows the importance of their impact on historiography.

It is rarely possible, when studying ancient sources, to get an idea – even approximate – of the proportion of tablets that were actually produced at a given time, in a given location. In the case of the school tablets from Nippur, it is nevertheless possible not only to pinpoint some of the selection processes, but also to estimate the way certain categories of tablets may have been filtered.

The first protagonists of these selection processes were the scribes themselves. Close observation of the school tablets shows that the tablets contain many marks of recycling operations: they could be kneaded again, voluntarily broken, agglomerated together in clay containers, incorporated in walls or floors; these tablets were not intended to be kept, but to be destroyed or thrown away. The tablets that have come down to us were found incorporated in

[45] (Robson 2002, 237).

building materials (this has in a certain way contributed to their preservation). In some instances, the tablets were abandoned in schools which activities suddenly stopped after a fire or a similar disaster[46]. Further the scribes did not deal with their daily exercises in the same way as certain of the master pieces, which may have been kept and could have circulated from one school to another[47].

Recycling practices, which consisted of separating the pupils' rough work from the teachers' texts and incorporating this rough work in the building materials - led to the unintentional creation of "school archives". The lots of school tablets, which were formed in this manner, are probably representative of the ordinary activity of the school; since one cannot see why the scribes would have gone to the trouble to sort out their rough work before throwing it away. As 10 to 20% of the rough work are mathematical tablets, this proportion should represent an approximate indication of the number of written texts devoted to mathematics.

The work carried out by the Babylonian Expedition also acted like a filter. A first selection was made by H. Hilprecht and the Ottoman commissaries at the time of the tablet share-out in the court of the new archaeological museum in Istanbul. The allotments were not made at random. In H. Hilprecht's own words[48], he took advantage of his knowledge of the cuneiform script to direct the way the tablets were shared out, reserving for the American share the tablets that were of most interest to him, namely the literary and historical texts, and leaving for Istanbul the tablets which seemed to him to be repetitious, namely the administrative texts; a few of the latter would suffice to satisfy the curiosity of the American philologists.

A second process of selection occurred when H.Hilprecht, who kept the tablets that he thought to have the highest value, made up his own collection. And as a matter of fact, the mathematical tablets kept in Jena are in an exceptional state of preservation[49], whereas most of the exemplars in Istanbul are quite disheartening fragments. Of course, the esthetical aspect is not the only criterion of selection that guided H. Hilprecht. For example, a sharp

[46] Concerning the practices of tablet recycling and the cases of sudden cessation of the schools' activities, see for example (McCown & Haines 1967, 64; Civil, Green & Lambert 1979, 7-8; Faivre 1995; Tanret 2002; Gasche & de Meyer 2006; Robson 2001).
[47] (Civil, Green & Lambert 1979: 7-8; Veenhof 1986b).
[48] (Hilprecht 1903). Also see (Kuklick 1996: 64).
[49] Concerning the copies and photos see (Proust 2008b).

discrepancy can be noted in the share-out of certain types of tablets. One category composed of small "oblong tablets"[50] represents 38% of the tablets in Jena, against 8% of the tablets in Istanbul. It is therefore possible to recognize Hilprecht's personal preferences, which can be related to the collectors' craze for multiplication tables and to the fact that the tables had become the stake of the polemic opposing Hilprecht to Peters[51]. Therefore the categories of texts which are considered important to the modern protagonists can be overrepresented in some museum collections. For this reason, in my investigation into the mathematical tablets from Nippur[52], it seemed necessary to take into consideration the three collections, which are the result of the different allotments made under Hilprecht's supervision. In this way we obtain a corpus that is as close as possible to the initial lot excavated in Nippur by the Babylonian Expedition.

The American and local excavators also played an active part. What happened to the objects they excavated? In principle everything was sent to Istanbul, where it was to be shared out and inventoried. But the teams of excavators worked in great isolation, lacked scientific supervision and the temptations of the market of antiquities were strong; these circumstances may have had an influence on a specific part of the finds: the most spectacular and the best preserved tablets. Under these conditions, the fact that only a few erudite mathematical texts from Nippur[53] are known to us does not means that only a few exemplars were found on this site. One cannot exclude the possibility that the excavators might have found mathematical tablets; subsequently these tablets may have disappeared in the clandestine networks and joined the stream of tablets of "unknown origin" which was to supply the museums of the Occident. On the other hand, the market value of daily exercises was certainly low. This rough work - probably because of its use as building materials - is most often in a fragmentary state. Thus the latter were of little interest to both the collectors and the researchers; many of these tablets are kept in collections containing objects from systematic excavations and forgotten in museum reserve collections.

One can see how at every stage of their history, the mathematical tablets were valued differently by the ancient and modern protagonists. The tablets did not circulate in the same

[50] These tablets - known as "type III tablets" by Assyriologists - were named "oblong tablets" (im-gid$_2$-da) by the scribes themselves. They often contained multiplication tables.
[51] (Robson 2002, 234, 237)
[52] (Proust 2007b)
[53] Out of more than 800 mathematical tablets from Nippur, only three contain erudite texts: CBS 11681, CBS 12648, Ni 5175 + CBS 19761 (Proust 2007b, chap. 7).

manner depending on whether they were elementary or erudite, fine-looking or fragmentary. The composition of the various collections, which provide our sources, bears the mark of this history.

The last filter is the one made by the editors. After the first period of discovery, in which H. Hilprecht was particularly active, the historians lost interest in the school tablets. The fact is clearly expressed by H. de Genouillac, the epigraphist of the French mission of Kish, who did not bother with the publication of the tablets and merely mentioned their existence, while apologizing for giving a "long inventory" (very useful to us today) of "documents that are chiefly of little interest in themselves"[54]. Furthermore, as I have shown at the beginning of this section, the metrological lists and tables have interested historians mainly as a source enabling to establish Mesopotamian metrology, and very little as mathematical or school texts.

Therefore the third section of this article will focus on the school character of the tablets from Nippur. The chosen corpus for this study is composed of the three collections of tablets excavated by the Babylonian Expedition: the ones of Istanbul, Jena and Philadelphia[55]. As it can be seen from the analysis of the various operations of selection which marked the sources from Nippur, this set minimizes the distortion effects due to the modern protagonists. As we shall see, the metrological lists and tables give much more than a definition of measure units. They shed new light on two aspects: the curricula of the scribe's education and the practices of calculation in Nippur.

4. The status of school metrology in Nippur

Curriculum

Some attempts to consider the school tablets as such - *i.e.* not only for the lexical, philological and metrological information they contain - were made in the early stages of

[54] (Genouillac (de) 1924 PRAK II: 45; Genouillac (de) 1925)

[55] I have made a complete study of these three collections in (Proust 2007b). I personally studied all the mathematical tablets in Istanbul (which have been published in the above book with the contribution of Antoine Cavigneaux for the lexical texts) and in Jena, published in (Proust 2008b) with the contribution of Manfred Krebernik for the lexical texts. E. Robson very generously put at my disposal her digital photographs and her database relative to the Philadelphia sources. Concerning certain statistical data, I also took into account E. Robson's work on the tablets excavated by the Joint Expedition in the second half of the 19th century (Robson 2001).

Assyriology. Let us recall how H. Hilprecht paid particular attention to the material aspect of the tablets in his book of 1906, and how he used these observations to show the educational nature of the lexical and mathematical tablets that were discovered by the Babylonian Expedition. Nevertheless, the school tablets as archaeological objects became subsequently of secondary importance. The material aspect of the tablets came back to the fore when M. Civil established a typology of the school tablets on the basis of the sources from Nippur; this typology is widely followed today. It is based on an analysis of the material properties of the tablets and on the existing links between these aspects and the tablets' use in teaching practices[56]. Notably the tablets, known as "type II" tablets, especially numerous in Nippur and already noticed by H. Hilprecht, played an important part in the reconstruction of the scribal curricula[57].

Consideration of the texts' organization gives another point of view on the "school archives". Assyriologists have revealed structuring elements in the gigantic Sumerian lexical lists which cover the school tablets: catch lines, existence of a doxology[58] at the end of some series, standardized format of items. Let us also mention an Old Babylonian catalogue[59] of unknown origin, which contains an inventory of the lexical lists named by their incipit. The various lexical lists are structured in autonomous units of text, whether the lists are enumerations of cuneiform signs or thematic vocabularies. They have a beginning (the *incipit* given in the catalogue), an end (often marked with a double line, and sometimes with a doxology), and contain a more or less fixed sequence of lemmata[60]. The various lists are autonomous entities, but this doesn't mean that they are completely independent of each other. The Assyriologists who have studied them have shown that they are written in a specific order; this order can be established thanks to some of the structuring features (catch lines and catalogue). N. Veldhuis systematically used the correlations between the texts on the obverse and on the reverse of type II tablets to show that this order follows the teaching curriculum. Finally it appears that the instruction in Nippur was divided in two distinct levels:

[56] (Civil, Green & Lambert 1979, 5; Veldhuis 1999).
[57] In type II tablets, the texts on the obverse and the ones on the reverse are independent from each other and often belong to different categories (for example, a lexical text on the obverse and a mathematical text on the reverse). The text on the obverse consists of the master's model text (left column), and of one or two copies made by the pupil. The text on the reverse is a list which had previously been studied by the same pupil and was reproduced from memory (see figures 1 and 3).
[58] In Nippur, this praise formula addresses Nisaba, the goddess of the scribes: "Praise Nisaba".
[59] Tablet YBC 13617, kept in the University of Yale (Hallo 1982).
[60] The composite text, which was reconstructed from the Old Babylonian sources from Nippur, can be found in the various volumes of the series *Materials for the Sumerian Lexicon*.

a first level called "elementary" by historians, in which the apprentice scribes studied the lexical lists and were expected to be able to render in writing the lists that they had learnt by heart; a second level known as "advanced", in which grammar and Sumerian literature were studied. The "composite text" of the lexical lists, organized according to the convergent evidence mentioned above, could be a rather good representation of what a young scribe from Nippur had to have memorized by the end of his elementary education. The arrangement of the entire set of texts, which was revealed by the reconstructed curricula, points to the possible existence of an institution[61]. Let us therefore examine how the mathematical tablets fit into this set. We shall see that, considered from this angle, the metrological lists and tables prove to be important, and this is the first reason to reinsert them within the group of mathematical texts.

Studies relative to the typology of school tablets and to the organization of lexical lists developed independently of the research carried out into the mathematical texts[62]. And yet the mathematical school tablets present exactly the same material and textual characteristics as the ones described above with respect to the lexical lists: the same typology, the same structuring elements[63]. Therefore the mathematical texts are indissolubly linked to the lexical texts, and it appears that they must be studied in close connection with the latter[64]. The importance of the metrological lists and tables can be immediately deduced from the fact that they amount to almost half of the mathematical tablets excavated by the Babylonian Expedition. Thus, the status of metrology in the scribal curricula already appears clearly from a sole quantitative standpoint. But as we shall see more deeply, this status is a functional one.

The school sources show that the metrological lists and tables belong to a large group of structured texts written as enumerations, which constitutes the core of elementary

[61] The issue of the existence of an educational "institution" such as a school is a debate among scholars. The picture is probably very different depending on the city under consideration. The schools of Nippur are perhaps the ones for which the term "institution" is the most suitable.
[62] In order to qualify this affirmation, it is necessary to insist on some studies. As mentioned above (p. XX), O. Neugebauer had identified several types of multiplication tables but, to my knowledge, he doesn't seem to have put his observations in connexion with the ones of H. Hilprecht and E. Chiera. Powell's work lies on the border between mathematics and lexicography, since his interest focused on numerical and metrological notations in the lexical lists (Powell 1971). However, it is a purely philological study, relatively independent from the context of the scribal schools.
[63] This fundamental fact was shown by E. Robson in her study of House F in Nippur (Robson 2001). I then myself systematically pursued this similarity in my study of the sources from Nippur excavated by the Babylonian Expedition (Proust 2007b).
[64] This is what E. Robson has done in her study of 'House F' in Nippur; this study considers the "school archives" as they were found by the archaeologists - without separating the texts according to the various modern disciplines – by investigating jointly the lexical, literary and mathematical texts (Robson 2001).

education in the scribal schools of Nippur. The mathematical school texts just as the lexical ones contain an assemblage of these lists including excerpts – of various lengths – of one or more lists. The writing modes of these excerpts reveal, on their scale, how they integrate into the overall architecture, for example in the case of the multiplication tables where catch lines refer one section to another. An analysis of these structural elements along with a statistical study carrying on the correlation between texts showing different features on the obverse and reverse sides of type II tablets permit to reconstruct the curriculum of elementary education *i.e.* the order in which the lists were written and taught; this analysis is similar to the one made by N. Veldhuis[65]. The apprentice scribe started by learning syllabaries, simple names and then lists of Sumerian vocabulary. It was probably at this point that the pupils began their training in metrological lists and tables, followed by the study of complex cuneiform sign lists and numerical tables, most likely simultaneously. At the end of the elementary level, the scribes studied small texts written in Sumerian (proverbs and legal texts) and probably began learning calculation methods. The possible sequence of mathematical texts at the elementary level is summed up in the following table 2[66]:

Metrological lists	list C (capacities) list P (weights) list S (surfaces) list L (lengths)
Metrological tables	table C (capacities) table P (weights) table S (surfaces) table L (lengths) table Lh (heights)
Ordinary numerical tables	table of reciprocals 38 tables of multiplication table of squares
tables of roots	table of square roots table of cube roots

Table 2: Elementary mathematical texts

The mathematical curriculum is nevertheless not entirely clear, and some uncertainties remain. First, although a chronological order can be defined, nothing can be said concerning

65 (Veldhuis 1997; Robson 2001; Proust 2007b).

66 Concerning the details relative to the textual and statistical studies which enabled me to establish this table, see (Proust 2007b, chap. 5).

the duration of the studies or the ages of the pupils at the various stages. Further, the respective position of metrological lists and tables within the curriculum has not been elucidated. I have suggested that these two categories of texts were not intended for the same students. The tables probably belonged to a curriculum more specifically directed towards mathematics. Thus, there would have been not one, but several teaching curricula[67].

Whatever uncertainties remain on certain aspects of instruction in Nippur, we have a sufficient number of elements to reconstruct fairly precisely a picture of the organization of mathematical education. I have attempted in the following diagram to represent the quantitative and qualitative aspects of these observations[68].

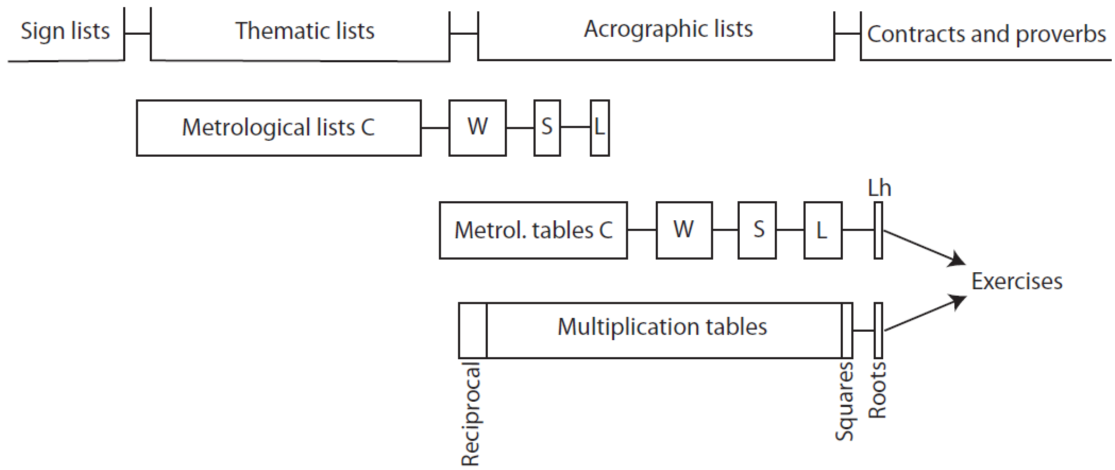

Figure 4: the elementary mathematical curriculum in Nippur

In the above diagram, the different types of lists are represented by rectangles, the surface of which is approximately proportional to the number of tablets containing the texts – or part of the texts – in question. This diagram gives an idea of the way the mathematical and lexical texts are linked together. The layout renders the chronological sequence of the mathematical and lexical training (*i.e.* lists and tables) and the connections between the various texts that are mentioned by our sources, in particular the type II tablets. The three series of mathematical tables (metrological tables, ordinary numerical tables, tables of roots) form a comprehensive group, but a consideration of their statistical frequency indicates that only a relatively small proportion of the scribes from Nippur actually had the opportunity to study and assimilate the entire group of texts. As for the lexical lists, the “composite text” formed by these tables is a good representation of the set of structured results that the scribes

[67] (Proust 2007a)
[68] I here reproduce a diagram published in (Proust 2007b, 152).

of Nippur had to memorize, at least the ones who had attended the elementary mathematical curriculum to the end.

Elementary level mathematics was followed by an advanced level, just as it was the case for the instruction in writing and in Sumerian. The advanced level in mathematics mainly consisted of calculation exercises in multiplication of numbers in place value notation, in finding reciprocals, and in surface calculations of squares and rectangles of given side. All these exercises were done on tablets known as “type IV” tablets, *i.e*. small square tablets, strongly convex on the reverse. Now what is the relation between the tables assimilated at the elementary level and these advanced level exercises?

We can presently draw a first conclusion from the texts studied in this article. The typology of the tablets, the organization of the texts and the statistical data provide information permitting a reconstruction of the scribal education: a possible specialized curriculum, the learning order of the lists, the interrelation between the different subjects, the teaching methods. Therefore, one can see how the “archival” approach to the lots of tablets found by the archaeologists shows their coherence. More importantly for us here, it also reveals the central and fundamental part played by the metrological lists and tables in this context. Let us now consider how the interpretation of other mathematical texts may benefit from this new understanding, in particular the comprehension of the advanced exercises. This is the second reason to reinsert the lists and tables in their original context.

Practices of calculation and conception of numbers

Let us examine more closely how the mathematical school tablets from Nippur allow to grasp the calculation practices and the conception of numbers that were taught to the scribes. Concerning the elementary level, this content can be approached in two different ways. First, the “composite text” can be considered; it informs us on the data memorized by the scribes. Second, one can examine the manner in which the various parts of the memorized text were inscribed by the pupils on the clay tablets; this last approach, as we have seen, informs us more particularly on the way the tables were taught. Concerning the advanced level, the situation is different because the texts are not standardized. Therefore, we are unable to reconstruct a “composite text”, which could help us understand how the texts interrelate. These calculation exercises are all different and this indicates that the authors were more autonomous. In the following lines, I shall show how the elementary mathematical tables were used to learn calculation techniques given in the exercises at the advanced level.

As we have already mentioned, the first stage of the mathematical education was based on metrological lists with the aim of transmitting the principles of writing measures. The lists show how to represent practical quantities in writing. This knowledge was indispensable for the ordinary operations of accounting and administration, in particular balance sheets, and therefore must have been taught to all the apprentice scribes. This is substantiated by the large proportion of lists of capacity measures in the lots of tablets from Nippur.

The metrological tables introduce another dimension, which is expressed visually by two sub-columns on the tablets. To each measure written in the left sub-column corresponds a sexagesimal number in place value notation, written in the right sub-column. As we have already underlined in section 1, the tables reveal two types of numbers: the ones on the left, which are governed by an additive principle, are used to express measures; the ones on the right are given in place value notation with no specific order of magnitude, they are "abstract numbers" in the sense that they are not accompanied by units of measure.
Therefore the metrological tables have a double function: first, similarly to the lists, they contain all the information connected with the numerical and metrological writing systems in use during the Old Babylonian period in almost all of Mesopotamia; second they establish a relation between measures and abstract numbers.

The numerical tables follow the metrological tables in the curriculum. They contain a listing of many elementary operations (reciprocals, products, squares, square roots, cube roots). Let us stress two essential traits of these tables: 1) They are exclusively written using abstract numbers; 2) they only give operations of multiplicative nature (multiplication, calculation of reciprocals and derived operations mentioned above). Now there is a clear link between these two traits: the multiplicative operations do not require knowing the position of the units; and calculations of reciprocals are even greatly facilitated by a notation using numbers with "floating value".

The curriculum continued with series of exercises in multiplication using abstract numbers, followed by calculations of reciprocals of regular numbers (numbers, which reciprocal can be written in base 60 with a finite number of positions[69]), also using abstract numbers. These exercises clearly show a tight link between abstract numbers and multiplication/calculations of reciprocals.

The next stage was devoted to the calculation of surfaces. Six tablets of similar aspect and content found in Nippur[70] show the coherence of this pedagogic system. These tablets are square tablets that give in the lower right corner a small text written in Sumerian, and in the upper left corner a multiplication using abstract numbers. Figure 5 below shows an exemplar.

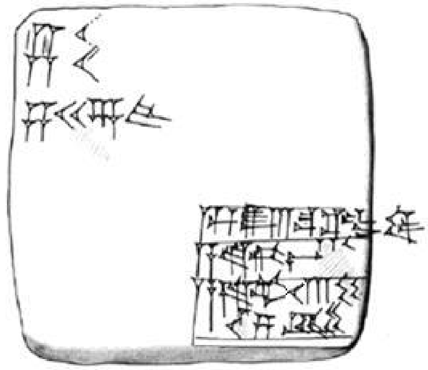	Translation 2.10 2.10 4. 26$^{\text{sic}}$.40 1/3 kuš$_3$ 3 šu-si the side (of the square). What is its surface? Its surface is 13 še 1/4$^{\text{sic}}$ še.

Figure 5 – Tablet Ni 18, Museum of Istanbul, copy by C. Proust

A first remark ensues from a superficial examination of the text: the measures given in the small text in the lower right corner are just as they appear in the metrological lists and in the left sub-column of the metrological tables (numerical values written using an additive system followed by a unit of measure); the numbers written in the upper left corner are as they appear in the right sub-column of the metrological tables and in the numerical tables (abstract numbers). If we examine more closely the numerical data of the text, we see that the relation between the measures (lower corner) and the abstract numbers (upper corner) is the one given in the metrological tables[71]. Therefore there is a fundamental link between the different elementary mathematical texts, and we understand the importance of reading the tables and the other texts together. Further, the layout of the text on the tablet reveals that the separation between the measures on one side and the abstract numbers on the other is analogous to the

[69] They are therefore products of powers of divisors of the base (2, 3 and 5).
[70] Five of them have been published in (Neugebauer & Sachs 1984, 246-251) ; the sixth Ni 18, which is represented here (figure 4), has been published in (Proust 2007b, 193, pl. I).
[71] This is true for the six tablets from Nippur, including the one giving a calculation mistake in the multiplication, indicated by "sic" in the translation.

one noticed in the metrological tables. The tables' structure induces us to pay attention to the layout of the exercises, which had never been considered in the publications connected with these tablets. Thus it is manifest that the metrological tablets constitute a tool for the calculation of surfaces. One can see the link between the data stored in the elementary tables (and memorized) and the calculation practices; the latter implying the need to constantly switch back and forth between the notation of measures and the multiplicative calculations with abstract numbers. The way the exercises of surface calculations are organized on the tablet clearly shows that the instruction particularly stressed the difference between these calculation stages. This method of calculation, which requires switching back and forth between the texts, is illustrated in the figure 6 below[72]:

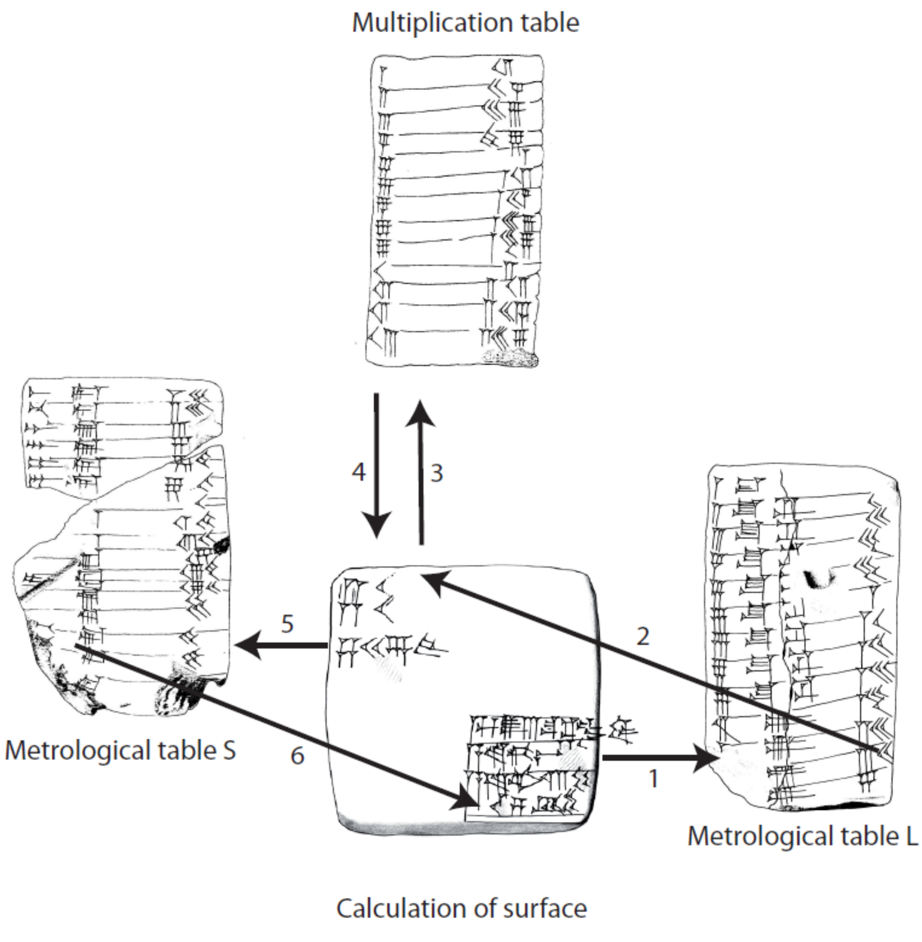

Figure 6: calculation of a surface

The lengths of the sides of the square written in the lower corner are transformed into abstract numbers using the metrological tables of lengths (1); the abstract number found is

[72] Figure taken from (Proust 2007b, 251).

written in the upper corner (2); the multiplication of the number by itself is made using the multiplication tables (3); the product found is placed below the two factors (4); this abstract number is transformed into a surface measure using the metrological tables of surfaces (5); the measure found constitutes the solution to the small exercise (6). Let us remark that reading the tables from left to right does not give rise to any difficulties, on the other hand the reading from right to left requires much mental control to deal with the orders of magnitude.

Figure 6 illustrates an essential aspect of the corpus of school texts from Nippur: all the texts are closely linked together, each one occupying a precise position in an elaborate mechanism of calculation. For the historians today, this represents a large puzzle, where each piece is necessary for the comprehension of the whole. Without the metrological tables, which constitute the key pieces, the puzzle could not reveal a discernible image.

As it has now been reconstructed, this mechanism allows us to answer a number of questions with respect to the calculation practices of the scribes. The mechanism is based on the separation between the different functions of numbers: the role of quantification is expressed by numerations using additive principles, and the role of multiplicative calculations is covered by a sexagesimal numeration in place value notation with no order of magnitude. Under these conditions, we understand that locating the units' position of abstract numbers is useless: numbers in place value notation are not quantities, they only serve as tools for multiplicative calculations and calculations of reciprocals. Calculation using numbers with "floating value" is no inconvenience, on the contrary it gives an extraordinary power to this tool. There is no point for the modern historians to complicate their calculations by substituting a modern system cluttered with zeros to the very effective ancient system.

We see how the metrological lists and tables, first used to establish the Mesopotamian metrology and then marginalized during the period of discovery of the great mathematical texts, finally show to be an essential part of the calculation mechanisms taught in Nippur. These mechanisms are based on the delimitation of two distinct numerical universes, the one devoted to measures and counting, the other devoted to calculation in the multiplicative field. The metrological tables permit the constant switching back and forth from the one to the other. Thus the mathematical school texts from Nippur form an extremely structured and functional group, of which no part should be neglected if we wish to understand its meaning. This structure gives us information not only on the scribal curricula, but more importantly on the fundamental notions that the master scribes taught to their pupils. These texts, when considered in a coherent way, shed light on several important aspects of the background of the erudite scribes.

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