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### Nonlinear modulation of $O_3$ and CO induced by mountain waves in the upper troposphere and lower stratosphere during terrain-induced rotor experiment

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[1] We analyze and explain distributions of trace gases  $(O_3, CO)$  and potential temperature observed in aircraft measurements in the upper troposphere and lower stratosphere during the terrain-induced rotor experiment. These distributions show fluctuations induced by mountain waves, with phases and amplitudes that are modulated and correlations that reverse sign along legs of the aircraft at constant altitudes. It is demonstrated that the observed correlations and distributions of gas traces can be explained by reversible processes induced by interactions between mountain waves with different wavelengths evolving on top of a mean vertical profile of ozone that is perturbed by synoptic scale motion. A wave with a large wavelength displaces the air column, causing horizontal variations in the vertical mean gradients. The short waves evolving in these modulated gradients induce wave signatures in O3 and CO, with amplitudes and phase relationships that depend on the vertical gradients encountered along the path of the aircraft. The proposed explanation is confirmed by reconstructed tracer variations deduced under this dynamical process. This is further supported by nonlinear analytical calculations that use background mean profiles from the observations, where the tracer variations induced by mutual wave-wave interaction are investigated.

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### 1. Introduction

[2] Gravity waves have major contributions to atmospheric circulation, structure, and variability in the middle atmosphere [*Fritts and Alexander*, 2003]. Spectra of tracer and temperature variabilities observed in aircraft measurements in the upper troposphere and lower stratosphere (UTLS) have shown the existence of waves in a broad spectrum with horizontal wavelengths extending from tens to thousands kilometers [*Nastrom et al.*, 1986; *Nastrom and Gage*, 1985; *Cho et al.*, 1999; *Bacmeister et al.*, 1996]. Several previous studies have provided evidence that vertical displacements induced by gravity waves may generate variations in atmospheric tracers [*Teitelbaum et al.*, 1994, 1996; *Moustaoui et al.*, 1999; *Holton*, 1987; *Chane-Ming et al.*, 2000; *Sofieva et al.*, 2004; *Alexander and Pfister*, 1995]. The tracer mixing ratio and the potential temperature are conserved under the effect of

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adiabatic gravity waves. The characteristics of an air parcel displaced by a wave are different from those of the surrounding parcels. This creates fluctuations with wave signatures and correlations that depend on the spatial gradients of the tracers.

[3] Nonlinear wave-wave interactions between waves with different characteristics may further complicate and increase irregularities in tracer distributions, and may result in local irreversible mixing. Nonlinear interaction has not been, however, often analyzed and identified from observations in the atmosphere. Danielsen et al. [1991] analyzed the effect of low- and high-frequency waves on tracer distributions during the Stratosphere-Troposphere Exchange Project experiments. Their results show that the low frequency, large amplitude waves can produce significant transport of tracers. The high-frequency waves with short vertical wavelengths have small amplitudes and are ineffective transporters. Their role is to render irreversible the transports induced by the lower-frequency waves. Moustaoui et al. [1999] analyzed aircraft measurements during the North Atlantic Flight Corridor experiment (POLINAT) and showed that nonlinear wave-wave interaction and generation of a secondary wave may account for tracer variability. They suggested that the nonlinear wave interaction may promote the tropospherestratosphere irreversible exchange. Waves may also cause mixing in the UTLS through local turbulence and instabilities

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that can be triggered during gravity wave breaking events [*Wang*, 2003; *Moustaoui et al.*, 2004; *Lane and Sharman*, 2006].

[4] The objective of this paper is to report and explain a nonlinear behavior of ozone  $(O_3)$  and carbon monoxide (CO) distributions observed in aircraft measurements in the UTLS region during terrain-induced rotor experiment (TREX). We demonstrate that the observed distributions of gas traces can be explained by reversible processes induced by nonlinear interactions between mountain waves with relatively short wavelengths modulated by waves with larger wavelengths. We mean by "*reversible*" that air is not irreversibly mixed, and that the distribution of a tracer could be reconstructed if one knows the locations where air is originating from, and the concentrations of the tracer at those locations.

[5] Observations have also reported that quasi horizontal isentropic transport of air masses from different locations can contribute to the variability of a tracer (ozone) [i.e., Reid et al., 1993; Reid and Vaughan, 1991; Legras et al., 2003; Orsolini, 1995; Bird et al., 1997; Mariotti et al., 1997; Moustaoui et al., 2003; Dobson, 1973; Deshler et al., 1990; Gibson-Wilde et al., 1997]. The quasi-horizontal transport is induced by synoptic scale motion in the presence of isentropic gradients of the tracer, and is sometimes associated with Rossby wave breaking events and filament generation. In the lower stratosphere, the dynamically unperturbed ozone profile generally increases with the vertical. Intrusions and excursions of air masses with poor (or rich) ozone can result in local layers within the ozone profile where the vertical gradient of ozone is negative [Moustaoui et al., 2003; Mariotti et al., 1997; Lemoine, 2004; and more recently, Pan et al., 2009].

[6] There are differences between tracer variability produced by gravity waves and those induced by synoptic scale motion. In the first case, the tracer variability is highly correlated with potential temperature fluctuations and can be attributed to vertical displacements of material surfaces induced by gravity waves. It does not require the presence of isentropic gradients of the tracer. In the second one, the variations of the tracer are not associated with significant potential temperature oscillations because the air parcel trajectories induced by synoptic scale motion are quasi horizontal, and nearly coincide with the mean isentropes. Isentropic gradient of the tracer is, however, required to produce the tracer variations in this case.

[7] Recent observational studies using aircraft measurements have provided evidence of irreversible mixing induced by synoptic scale motion near the tropopause [Randel et al., 2007; Fischer et al., 2000; Zahn et al., 2000; Hoor et al., 2002; Pan et al., 2004, 2006; James and Legras, 2009]. Ozone and carbon monoxide are usually used as stratospheric and tropospheric tracers to identify mixing; and correlations between these two tracers are frequently used as observational evidence of mixing in the UTLS. In this paper, we analyze aircraft measurements of O3 and CO obtained during TREX. The observed mean profile of ozone shows a decrease above the tropopause due to isentropic intrusion induced by synoptic scale motion. We will show that mountain waves evolving on top of this perturbed ozone profile induce tracer distributions with modulated phases and amplitudes; and complex correlations that reverse sign along the horizontal path of the aircraft. We will demonstrate using observations, theoretical arguments, and analytical investigations that these

behaviors in amplitudes and correlations can be explained by nonlinear interactions between waves with different wavelengths; without the need to include any mixing. After the introduction, we give in section 2 a brief description of the TREX campaign and we present real case numerical simulations demonstrating that the variabilities observed in ozone, CO and potential temperature are induced by mountain waves. In section 3, we present observations of O<sub>3</sub> and CO along several flight legs. Reconstruction and explanation of O<sub>3</sub> and CO variations; and the role of nonlinear interactions is given in section 4. Analytical arguments explaining the O<sub>3</sub> and CO variations based on nonlinear wave-wave mutual interactions are presented in section 5. Finally, the results are summarized in section 6.

# 2. The TREX Campaign and Simulations of Mountain Waves

[8] The Terrain-Induced Rotor Experiment (TREX) is a coordinated observational field campaign and research program focused on the investigation of atmospheric rotors and closely related phenomena, such as turbulent dynamics of wave breaking associated with terrain-induced gravity waves. Rotors are related to lee wave activity and represent potential hazards to aviation. They are turbulent vortices that are generated around the first trough in the lee side of the terrain that triggers them. One of the key objectives of TREX was to provide a comprehensive set of in situ and remotely sensed meteorological observations and advance our understanding of mountain wave induced turbulence from the ground to the UTLS altitudes [*Grubisic et al.*, 2008].

[9] The TREX field campaign took place in March and April 2006 in Owens Valley in the lee of southern Sierra Nevada in eastern California. The Sierra Nevada is a nearly two-dimensional mountain range that is about 640 km long and 60–130 km wide with gentle upwind and steep ( $\approx$ 30 degrees) lee side slopes. The southern part of the range is the tallest and steepest; the ridge line there has an average altitude of 3,500 m and a number of peaks above 4 km, including the highest peak in the 48 contiguous states (Mt. Whitney 4,418 m). The TREX target region was located in the central part of Owens Valley, near town of Independence, where a dense cluster of ground-based in situ and remote sensing instruments was installed.

[10] In addition to the ground-based systems, three research aircraft were deployed in TREX in order to document the kinematic and thermodynamic structure of airflow upstream and downstream of the Sierra Nevada at a range of altitudes. The three aircrafts covered the altitudes from 150 m AGL in Owens Valley to about 14 km MSL in the UTLS region. The aircraft covering the UTLS region was the NSF/ National Center for Atmospheric Research (NCAR) G-V (HIAPER), which flew eleven research missions during the 2 month field campaign, in the range of altitudes between 9 and 14 km MSL over Owens Valley and the surrounding mountain ranges.

[11] For this study, we analyze in situ measurements performed by the NSF/NCAR G-V (HIAPER) aircraft on 25 March 2006 during TREX IOP 6. Figure 1 shows topography and the geographical location of the flight. The black curve (shaped like an ellipse) superimposed in Figure 1 shows the path of the NSF/NCAR G-V research aircraft. The



**Figure 1.** The domain used for the microscale nest simulation. This domain is nested to the finest WRF nest and uses 182 vertical levels. Topography (shaded) and the wind (vector) field simulated at 12 km on 25 March 2006, 20 UTC. The solid line indicates the track of the aircraft, and the dashed line indicates the location of the cross sections shown in Figure 3.

region where the flight took flight is known for strong generation of mountain waves. High activity of mountain waves was observed in the UTLS during TREX [*Smith et al.*, 2008; *Mahalov and Moustaoui*, 2009]. Figure 2a shows the altitude of the aircraft and the potential temperature measured along a large portion of the flight. It shows variations of potential temperature that are associated with mountain waves, and abrupt changes due to the changing altitude of the aircraft.

[12] To demonstrate that the variations observed in the potential temperature are indeed caused by mountain waves, we conducted real case simulations using the Weather and Research Forecasting (WRF) model and a nested microscale model. The WRF simulation was conducted for the period from 24 March 2006, 12 UTC to 26 March 2006, 12 UTC. Three nested domains were used for WRF with a horizontal grid spacing of 27 km, 9 km and 3 km. All the three WRF domains use 61 vertical levels and are centered over the location (36.49 N, 118.8 W). The largest WRF domain uses global data from ECMWF T799L91 for initial and boundary conditions. The outputs from the finest WRF nest are then used as inputs for a microscale simulation. The microscale model is described in details in the work of Mahalov and Moustaoui [2009]. The domain corresponding to the microscale nest is shows in Figure 1. It is nested to the finest WRF domain, with a horizontal grid spacing of 1 km, and a refined vertical grid using 181 levels (three times the number of vertical levels used in WRF). Figure 3a shows a vertical cross section for potential temperature and eastward wind along the



**Figure 2.** Potential temperature (solid line) (a) measured by HIAPER aircraft and (b) simulated by the microscale nest. The altitude of the aircraft is superimposed in Figure 2a (dotted line). The time is relative to 25 March 2006, 15 UTC. The time reflects the spatial variation along the path of the aircraft.



**Figure 3.** Vertical cross sections of potential temperature and eastward wind simulated by the microscale simulations on 25 March, 20 UTC (a) with and (b) without the mountains. The cross section is taken along the dashed line superimposed in Figure 1.

dashed line superimposed in Figure 1. It shows that the dynamics in the troposphere and the lower stratosphere is dominated by mountain waves, with high-wave activity dominating the potential temperature and the wind fields above the valley and in the downstream side of the highest elevation. The vertical distribution of potential temperature contours shows an increase in stability indicating that the tropopause height is on the average located around 11.5 km and that the value of the potential temperature at the tropopause is about 330 K. This average height of the tropopause, which is based on stability, is further confirmed by vertical profiles of potential temperature from dropsondes. These profiles will be presented in Figure 10. Figure 2b shows the simulated potential temperature variations along the aircraft flight after interpolating the microscale simulations in time

and space to the position of the aircraft. The simulated variations of potential temperature agree well with those found in the observations. This demonstrates that the observed variations are induced by mountain waves. To further confirm that the dynamics was dominated by mountain waves, another simulation was conducted using the same setup as in the previous one, except that the mountains were removed from both the WRF and the microscale models. Figure 3b shows the vertical cross section of potential temperature and the wind fields obtained after removing the mountains. Contrary to the simulation including mountains, the potential temperature contours are flat and the structures associated with the mountain waves are not present. This further demonstrates that the variations seen in the observations are generated by mountain waves.

### 3. Ozone and CO Fluctuations in the UTLS

[13] Figure 4a shows O<sub>3</sub>, CO mixing ratios and potential temperature along a leg (Leg1) of NCAR/HIAPER research aircraft flight on 25 March. The altitude of the aircraft during this leg was in the UTLS region at z = 12 km, and the potential temperature has values greater than 330 K. Strong fluctuations in the horizontal are present in both tracers and potential temperature. These fluctuations exhibit irregularities with different scales suggesting high activity of gravity waves with different wavelengths in the UTLS. O<sub>3</sub> (CO) and potential temperature are strongly and positively (negatively) correlated. Irregularities with small peaks and larger wavelengths are present in all parameters, indicating that the fluctuations are produced by the same waves. Positive correlations between the fluctuations of  $O_3$  and temperature are expected when these fluctuations are produced by wave-induced vertical motions. Air parcels displaced upward (downward) from below (above) to a given altitude bring smaller (higher)  $O_3$ and potential temperature values compared to the surrounding air. As the aircraft flies horizontally through these displaced air masses, it encounters air coming from different vertical levels, causing fluctuations in O<sub>3</sub> and temperature that are positively correlated, and with values that depend on the origins of the air masses encountered. The strong negative correlation between CO and potential temperature reflects the observed decrease of CO in the vertical, since it is produced mainly in the troposphere. Figure 4b shows O<sub>3</sub> and potential temperature variations along a second leg (Leg2) of the aircraft track. The altitude of the aircraft was in the stratosphere at z = 12.3 km. The variations of ozone exhibit striking behavior with reversal of  $O_3$  correlations. The correlations between O<sub>3</sub> and potential temperature fluctuations are negative between times of 2.20 h and 2.31 h of the flight; then they become positive thereafter along the same altitude. In Figure 4b, the fluctuations of CO show small amplitude between 2.20 h and 2.31 h, with correlations that are not clearly defined, and regions where CO and temperature are in phase (2.24–2.27 h). Beyond flight times of 2.31 h, the observed fluctuations of CO have larger amplitudes, and opposite phases with respect to the potential temperature. Negative correlations between O<sub>3</sub> and potential temperature are unusual in the stratosphere. We will demonstrate in the next section that modulation of vertical gradients in the mean distributions caused by variations with larger wavelengths



**Figure 4.** Potential temperature (black dotted line),  $O_3$  (blue solid line), and CO (red dash-dotted line) mixing ratio as a function of time measured by HIAPER aircraft for (a) Leg1 and (b) Leg2. The tick curves are the filtered variations (see text). The time is relative to 25 March 2006, 15 UTC. The time reflects the spatial variation along the path of the aircraft.

combined with vertical motions induced by short gravity waves can explain the fluctuations presented in Figure 4.

## 4. Reconstruction and Explanation of O<sub>3</sub> and CO Variations and the Role of Nonlinear Interactions

[14] When the fluctuations in tracers are dominated by vertical motions, it is possible to link these fluctuations with the vertical mean gradients. The altitude of the aircraft was constant during the legs presented in Figure 4. The vertical gradients cannot be directly calculated. Nevertheless, one can still recover the vertical gradients from the horizontal variations as follow. Let's assume an idealized situation where unperturbed  $O_3$  and potential temperature depend on the vertical coordinate only. Under the effect of waves, the surfaces of  $O_3$  and potential temperature, initially horizontal, experience complex deformations depending on the number and the characteristics of the waves involved. An aircraft crossing these perturbed material surfaces at constant altitude will encounter air masses that were initially located at different vertical levels. Thus, crossing horizontally perturbed

material surfaces is equivalent to sampling different vertical layers in the unperturbed situation. Plotting  $O_3$  as a function of potential temperature will result in partial cancellations of the fluctuations since they are both Lagrangian and are produced by the same wave-induced displacements; and only the mean variations will remain. Figures 5a and 5b show O<sub>3</sub> and CO as a function of potential temperature. The curves in Figures 5a and 5b are obtained by using each individual value of ozone, CO and potential temperature from the observations during the entire flight, and plotting these data in the order they were taken. Most of the curves of ozone and CO follow the same mean shape. The mean profiles superimposed in Figures 5a and 5b are constructed as follows. First, O<sub>3</sub>, CO and potential temperature values are collected during the entire flight including Leg1 and Leg2. The potential temperature axis is subdivided into values  $\theta_i$  ranging form 311 K to 370 k with an interval of  $\delta\theta = 0.2$  K. When an observed value of potential temperature falls into the interval  $\left[\theta_i - \delta\theta/2\right]$ ,  $\theta_i + \delta \theta/2$ ], the O<sub>3</sub> and CO values are affected to that interval. For each interval centered at  $\theta_i$ , the collected values of O<sub>3</sub> and CO are averaged within that interval. The resulting O<sub>3</sub> and



**Figure 5.** Vertical profiles of (a)  $O_3$  (blue thin line) and the averaged  $O_3$  (black thick line) from the aircraft measurements and (b) CO (red thin line) as a function of potential temperature from the aircraft measurements and the averaged vertical profile of CO (black thick line).

CO profiles are then smoothed using a running mean to obtain the mean  $O_3$  and CO profiles shown in Figure 5.

[15] In the UTLS, the mean O3 increases between 330 K and 342 K. For values of potential temperature above 342 K. the mean vertical profile of the O<sub>3</sub> shows a decrease within a layer between 342 K and 356 K in the lower stratosphere. The decrease of O<sub>3</sub> in the lower stratosphere with respect to altitude is further confirmed in Figure 6. Figure 6 shows filtered  $O_3$  and potential temperature variations and the altitude of the flight between times of 4.50 h and 5.10 h. The filter applied is a low-pass nonrecursive filter that uses Kaizer window to avoid Gibbs phenomenon [Hamming, 1983]. It removes all the fluctuations with periods less than 0.135 h, which corresponds to a cutoff wavelength of 97.2 km, since the speed of the aircraft is about 200 ms<sup>-1</sup>. This leg of the flight was chosen because the aircraft changed the altitude of its path. We note that as the altitude of the flight increases between 12 km and 13.2 km, the mean potential temperature increases from 340 K to 360 K. In contrast, the mean O<sub>3</sub> decreases between these levels.

[16] The filtered variations of O<sub>3</sub>, CO and temperature are also superimposed in Figures 4a and 4b (tick curves). The filter used is a low pass filter that removes the fluctuations with periods less than 0.075 h, which corresponds to wavelengths below 54 km. The filtered variations exhibit modulations with larger scales. In Leg1, the modulated potential temperature has values between 330 K and 340 K. Thus, the modulated mean O<sub>3</sub> (CO) gradients seen by the aircraft are positive (negative) along Leg1 (Figures 5a and 5b). The vertical displacements induced by the small waves are then expected to produce fluctuations with positive (negative) correlations between ozone (CO) and potential temperature along the entire leg. This explains the relationships between the fluctuations observed in Leg1 (Figure 4a). In contrast, the modulated mean potential temperature for Leg2 has values between 340 K and 350 K between 2.20 h and 2.31 h times of the flight. The modulated gradients sampled by the aircraft in this part of Leg2 have negative values for O<sub>3</sub> and small values for CO (Figures 5a and 5b). The wave signature induced by the short waves (smaller wavelength) is expected to have fluctuations with negative correlations for O<sub>3</sub>, and small amplitudes for CO. Beyond 2.31 h, the modulated potential temperature drops to values lower than 340 K where the modulated O<sub>3</sub> gradient is positive and the modulated CO gradient has larger values. At this time, the short waves are expected to produce fluctuations with reversal of correlations for O<sub>3</sub> and negative correlations with larger amplitudes for CO compared to the first part of Leg2. This behavior is clearly seen in Leg2 (Figure 4b). Thus, our explanation for the changes in the phases and amplitudes of the fluctuations produced by the short waves and the role of the modulation induced by the large variations is supported by the observations.

[17] Nonlinear wave-wave interactions between the shorter and the larger waves are necessary to explain the measurements of  $O_3$  and CO. In the following, we reconstruct the behaviors of phases and amplitudes found in the variations



**Figure 6.** Filtered potential temperature (black dotted line),  $O_3$  (blue solid line), and the altitude of the aircraft (dashed black line) as a function of time for a portion of the flight where the aircraft changed its altitude.



**Figure 7.** Variations of  $O_3$  (blue solid line) and CO (red dash-dotted line) as a function of time reconstructed from potential temperature (black dotted line) for (a) Leg1 and (b) Leg2.

of  $O_3$  and CO using two different methods. In the first method, the short fluctuations of  $O_3$  and CO with periods smaller than 0.075 h (wavelength lower than 54 km) are removed from the observations using the same filter as in Figure 4. The larger fluctuations of  $O_3$  (CO) and potential temperature are however kept in the variations. The short fluctuations of  $O_3$  and CO are then recalculated from those of potential temperature after partial linearization where the short waves are linearized with respect to both the mean and the larger variations using the following relation:

$$q = \tilde{q}(\tilde{\theta}) + \frac{\partial \tilde{q}}{\partial \tilde{\theta}}(\tilde{\theta}) \times [\theta - \tilde{\theta}].$$

Here, q is O<sub>3</sub> (CO) mixing ratio,  $\theta$  is the potential temperature; and  $\tilde{q} = \bar{q} + q_1$  and  $\tilde{\theta} = \bar{\theta} + \theta_1$  are O<sub>3</sub> (CO) and potential temperature variations after removing the short fluctuations only (the thick curves in Figures 4a and 4b).  $q_1$  and  $\theta_1$  denotes the contributions from the larger wave, and  $\bar{q}$  and  $\bar{\theta}$  are the mean profiles after removing both the small and the larger fluctuations. Because the potential temperature and the tracer are conserved under the effect of the larger wave, the shapes of the mean curves do not change in the ( $\theta - q$ ) space. The vertical gradients are then derived from the mean curves in Figures 5a and 5b  $\left(\frac{\partial \bar{q}}{\partial \bar{\theta}}(\tilde{\theta}) = \frac{\partial \bar{q}}{\partial \bar{\theta}}(\tilde{\theta})\right)$ ; but these gradients are taken at  $\tilde{\theta} = \bar{\theta} + \theta_1$  instead of  $\bar{\theta}$ . In another word, the vertical gradients on top of which the short waves evolve are modified by the larger wave. We note that the above relation used to recalculate the variations of the O<sub>3</sub> and CO is not completely linear because the linearization is applied with respect to the mean plus the larger variations. Thus, the small fluctuations still interact with the larger variations, and the product of the perturbations between the two waves is implicitly involved in the vertical gradients used for the tracer reconstruction:

$$\frac{\partial \overline{q}}{\partial \overline{\theta}} (\overline{\theta} + \theta_1) \times [\theta - \widetilde{\theta}] = \left( \frac{\partial \overline{q}}{\partial \overline{\theta}} + \frac{\partial^2 \overline{q}}{\partial \overline{\theta}^2} \theta_1 + \dots \right) \\ \times \theta_2 = \frac{\partial \overline{q}}{\partial \overline{\theta}} \theta_2 + \frac{\partial^2 \overline{q}}{\partial \overline{\theta}^2} \theta_1 \theta_2 + \dots,$$

where  $\theta_2 = \theta - \hat{\theta}$  is the contributions from the short waves.

[18] The reconstructed  $O_3$  and CO are shown in Figures 7a and 7b for Leg1 and Leg2. The short fluctuations deduced from the above relation agree very well with the observations. The correlation between the reconstructed and observed  $O_3$  is



**Figure 8.** (a) Variations of  $O_3$  (blue solid line) and CO (red dash-dotted line) as a function of time reconstructed from the vertical displacements (see text) for Leg2 in the nonlinear case. Potential temperature is also superimposed (black dotted line). (b) As in Figure 8a, but for the linear case.

0.98 for Leg1 and 0.84 for Leg2; and the correlation for CO is 0.95 and 0.92 for Leg1 and Leg2, respectively. Not only the correlations are high, but also the amplitudes of the reconstructed variations are comparable to those found in the observations. This confirms the dynamical explanation proposed in this paper. If the fields were completely linearized with respect to the mean profiles, both shorter and larger variations would be removed. The relation that would be used is:  $q = \overline{q} + \frac{\partial \overline{q}}{\partial \theta}\theta_1 + \frac{\partial \overline{q}}{\partial \theta}\theta_2$ . This relation is just a linear superposition of the two waves and does not involve the product of perturbations. In the following, we will show that linear superposition of both waves does not explain the observed variations in O<sub>3</sub> and CO.

[19] To further confirm that the changes in amplitude and the reversal of phase with respect to the potential temperature observed during Leg2 are induced by nonlinear effects, we reconstruct the tracer variations using a second method. In this method, we calculate the vertical displacements induced by all waves, and we use these displacements to reconstruct the tracers in both linear and nonlinear cases. For this purpose, we use the vertical mean profiles as a function of altitude of the potential temperature  $(\overline{\theta}(z))$ , ozone and CO  $(\overline{q}(z))$  that will be presented in Figure 10. The method used to derive these mean profiles will be explained in section 5. The vertical displacements are given by  $\delta = z - z_o$ , where z =12.3 km is the altitude of the flight during Leg2 and  $z_o$  is the altitude where a parcel is originating from. The origin of the parcel  $z_o$  is obtained from the mean vertical profile and the observations of the potential temperature in Leg2; and solving the Lagrangian relation:  $\theta(z) = \overline{\theta}(z_o)$ . The variations of ozone and CO (q(z)) are then reconstructed from these displacements together with the mean vertical profiles of the tracers ( $\overline{q}(z)$ ) by using the relation:  $q(z) = \overline{q}(z - \delta)$ . Figure 8a shows the variations of ozone and CO reconstructed by this method for Leg2. They agree well with the observations. Again, the reconstructed variations have amplitudes that are comparable with the observations; and the correlations between these variations and the observed ones are 0.81 and 0.97 for ozone and CO, respectively. In the linear case, the ozone and CO are calculated using the linearized relation:  $q(z) = \overline{q}(z) - \frac{\partial q}{\partial z}\delta$ . We note that  $\delta = \delta_1 + \delta_2$  includes the effect of both the large and short waves. The resulting variations are shown in Figure 8b. These variations are very different



**Figure 9.** Reconstructed  $O_3$  using the CAS method at (a) 342 K and (b) 355 K on 25 March. Gray, blue, green, and yellow correspond to values of 100, 150, 200, and 250 ppb. The red dot indicates the location of the TREX observations. The CAS is initialized by ozone data from HIRDLS on 20 March.

from the observations and have small amplitudes. They are negatively correlated with  $\theta$  along the entire leg. This occurs because the mean ozone and CO gradients have small magnitude with negative values at z = 12.3 km.

[20] The existing satellite measurements for global ozone distributions are too coarse to resolve the negative ozone gradient observed during TREX. We used the technique of the contour advection with surgery (CAS) [Mariotti et al., 1997; Waugh and Plumb, 1994] to reconstruct the finescale structure of  $O_3$ . The initial  $O_3$  distribution is obtained on 20 March 2006 from the High Resolution Dynamics Limb Sounder (HIRDLS) instrument. O<sub>3</sub> field is discretized as a set of contours with different values. The contours are transported for 5 days up to 25 March 2006 at 18 UTC by global winds provided by the GFS analyses. Figures 9a and 9b show the fine-scale distributions of O<sub>3</sub> reconstructed on 25 March 2006 using the CAS method at 342 K and 355 K, respectively. At 342 K isentropic level, a filament with high ozone with values between 200 and 250 ppb is found within the TREX region on 25 March. In contrast, the ozone values found at 355 K on the same day are lower (100–150 ppb). The reconstructed ozone distribution shows values that agree with the ones observed by the aircraft at both levels (Figure 5). Thus, the reconstructed ozone fields are able to resolve the decrease of ozone observed during TREX, and suggest that the ozone increase at 342 K is due to isentropic horizontal advection.

## 5. Analytical Study of Wave-Wave Mutual Interaction

[21] Next, we show analytical results using the observed mean vertical profiles of ozone and CO; where two upward

propagating stationary waves with different wavelengths are generated by topography. The effect of these waves on the variations of the tracers in the UTLS region is then analyzed for the same levels as the altitudes of the flight during Leg1 (12 km) and Leg2 (12.3 km) for both linear and nonlinear cases. We do not intend to reproduce the observations in this section. Our goal is to demonstrate that the nonlinear interaction between two waves can explain the modulation in amplitude and the reversal of phase found in the tracers. The mean vertical profiles of  $O_3$ , CO and potential temperature as a function of altitude are needed in this case. First, a simple analytical profile of potential temperature is needed for the analytical calculations. This profile is constructed in a such way that it is close to the profiles observed by the dropsondes launched from the same aircraft; and also close to the averaged potential temperature values at Leg1, Leg2 and two other legs above them (Figure 10). Then, for each altitude, the mean O<sub>3</sub> and CO values are found by interpolating the averaged O<sub>3</sub> and CO curves measured by the aircraft (Figure 5) to the mean potential temperature value at that altitude. The resulting O<sub>3</sub> and CO profiles as function of altitude are superimposed in Figure 10. The analytical profile of the potential temperature is also superimposed in Figure 10; it is given by:

$$\begin{split} \overline{\theta}(z) &= \theta_0 \exp\left\{\frac{N_1^2}{g}z\right\} \text{ for } z \leq z_1, \\ \overline{\theta}(z) &= \overline{\theta}(z_1) \exp\left\{\frac{N_1^2}{g}(z-z_1) + \frac{\Delta N^2}{2g\Delta z}(z-z_1)^2\right\} \text{ for } z_1 < z \leq z_2, \\ \overline{\theta}(z) &= \overline{\theta}(z_2) \exp\left\{\frac{N_2^2}{g}(z-z_2)\right\} \text{ for } z > z_2, \end{split}$$



**Figure 10.** Vertical profiles of potential temperature as a function of altitude from dropsondes (black thin dashed line) and the analytical profile of potential temperature (black tick solid line). The circles indicate the averaged potential temperature values at four different altitudes of the aircraft (see text). The profiles of the mean  $O_3$  (blue thin solid line) and CO (red dash-dotted line) are also superimposed.

where  $\theta_0 = 300$  K is the potential temperature at the ground,  $N_1^2 = 0.78 \times 10^{-4} s^{-2}$  and  $N_2^2 = 4.9 \times 10^{-4} s^{-2}$  are the square of the Brunt Vaisala frequencies in the troposphere and in the stratosphere, respectively,  $\Delta N^2 = N_2^2 - N_1^2$  represents the jump in the stability at the tropopause;  $N^2$  (z) increases linearly between  $z_1 = 11$  km and  $z_2 = 12$  km,  $\Delta z = z_2 - z_1$ , and g is the acceleration of gravity. The mean profiles are then perturbed by vertical displacements induced by two waves. The equations of motion for a steady case in the Boussinesq approximation are:

$$u_0 \partial_x \eta + \frac{g}{\overline{\theta}} \partial_x \theta = -\{\partial_z \psi \partial_x \eta - \partial_x \psi \partial_z \eta\}$$
(1)

$$u_0 \partial_x \theta - \overline{\theta}_z \partial_x \psi = -\{\partial_z \psi \partial_x \theta - \partial_x \psi \partial_z \theta\}$$
(2)

$$u_0 \partial_x q - \overline{q}_z \partial_x \psi = -\{\partial_z \psi \partial_x q - \partial_x \psi \partial_z q\},\tag{3}$$

where,  $\psi$  is the stream function,  $\eta = u_z - w_x = \Delta \psi$ ,  $u = \psi_z$ ,  $w = -\psi_x$ ,  $\theta$  and q are the fluctuations of potential temperature and the tracer, respectively. All the fields are separated into a linear ( $\psi^l$ ) and a nonlinear ( $\psi^n$ ) contributions (i.e.,  $\psi = \psi^l + \psi^n$ ).

### 5.1. Linear Contribution

[22] The linear solution is obtained by ignoring the nonlinear terms in equations (1), (2) and (3). These equations, after integrating with respect to x, become  $u_0\eta^l + \theta^l (g/\overline{\theta}) = 0$ ,  $u_0 \theta^l - \overline{\theta}_z \psi^l = 0$  and  $u_0q^l - \overline{q}_z \psi^l = 0$ ; which gives:  $\theta^l = (\overline{\theta}_z/u_0)\psi^l$ ,  $\eta^l = -(N^2/u_0^2)\psi^l$  and  $q^l = (\overline{q}_z/u_0)\psi^l$ . After using Fourier series  $(\psi^l (x, z) = \sum_{\psi k}^l (z)e^{ikx})$  and  $\eta = \Delta \psi$ , the following equation for  $\hat{\psi}$  can be obtained:

$$\hat{\psi}_{k,zz}^{l} + \left(\frac{N^{2}(z)}{u_{0}^{2}} - k^{2}\right)\hat{\psi}_{k}^{l} = 0.$$
(4)

At the ground we impose that  $\psi^{l}(x, 0) = -u_0 h(x)$ , where h(x) = $h_1 cos(k_1 x) + h_2 cos(k_2 x)$  is the elevation at the ground, and  $u_0 = 14 \ ms^{-1}$  is the mean wind. The elevation has a contribution from two waves with amplitudes of  $h_1 = 0.5$  km and  $h_2 = 0.3$  km.  $k_1$  and  $k_2$  are the wavenumbers that correspond to a horizontal wavelengths of  $\lambda_1 = 300$  km and  $\lambda_2 = 30$  km, respectively. At the top we impose for each wave that  $d\hat{\psi}_k^l/dz = j\hat{m}\hat{\psi}_k^l$ , where  $\hat{m}^2 = (N_2^2/u_0^2 - k^2)$  is the square of the vertical wavenumber *m* at the top, so that the wave energy is radiated upward. For upward propagating mountain waves, the phases of the waves tilt upstream. Thus, for  $u_0 > 0$ , the horizontal (k) and the vertical (m) wavenumbers have the same sign. Here, we chose to solve for k > 0. The vertical wavenumber m at the top is then positive. The vertical wavelengths for the larger wave is 9.9 km in the troposphere and 3.9 km in the stratosphere. For the shorter wave, the vertical wavelengths are 10.5 km and 4 km in the troposphere and in the stratosphere, respectively. Equation (4) is solved separately for the two waves. The details of the analytical derivation is given in Appendix A. The total variations of potential temperature and the tracers are then given by a linear superposition of the two waves as:

$$\theta = \overline{\theta} + \frac{\theta_z}{u_0} (\psi_1^l + \psi_2^l)$$
$$q = \overline{q} + \frac{\overline{q}_z}{u_0} (\psi_1^l + \psi_2^l)$$

[23] Figure 11 shows the variations of potential temperature,  $O_3$  and CO calculated from linear superposition of the two waves at the same level as in Leg2 (z = 12.3 km). In this case, the modulation of phases and amplitudes of the short fluctuations and the reversal of  $O_3$  correlations are not present. The correlation between ozone and potential temperature does not change. The amplitude of the short fluctuations are small and the correlation between ozone (CO) and potential temperature is negative. This occurs because the mean vertical gradients of ozone and CO are negative and have small values at z = 12.3 km.

### 5.2. Nonlinear Contribution

[24] By replacing all the fields by linear and nonlinear contributions (i.e.,  $\alpha = \alpha^{l} + \alpha^{n}$ ), equations (1), (2) and (3) could be written for the nonlinear contributions as:

$$u_0 \partial_x \eta^n + \frac{g}{\overline{\theta}} \partial_x \theta^n = -\{\partial_z \psi \partial_x \eta - \partial_x \psi \partial_z \eta\}^l$$
(5)

$$u_0 \partial_x \theta^n - \overline{\theta}_z \partial_x \psi^n = -\{\partial_z \psi \partial_x \theta - \partial_x \psi \partial_z \theta\}^l \tag{6}$$

$$u_0 \partial_x q^n - \overline{q}_z \partial_x \psi^n = -\{\partial_z \psi \partial_x q - \partial_x \psi \partial_z q\}^l.$$
(7)

Here, the nonlinear forcing is obtained from the linear solution. After replacing  $\eta^l$ ,  $\theta^l$  and  $q^l$  in the right hand side



**Figure 11.** Variations of O<sub>3</sub> (blue solid line), CO (red dash-dotted line), and potential temperature (black dotted line) as a function of distance from analytical calculations at z = 12.3 km for the linear contributions. The variation of potential temperature induced by the larger wave (tick dotted line) is superimposed. The horizontal dashed line corresponds to  $\theta = 342$  K.

of (5)–(7) by their linear expressions  $((\overline{\theta}_z/u_0)\psi^l, -(N^2/u_0^2)\psi^l, (\overline{q}_z/u_0)\psi^l)$ ; and integrating over x we obtain:

$$u_0\eta^n + \frac{g}{\overline{\theta}}\theta^n = -\frac{N_z^2}{2u_0^2}(\psi^l)^2 \tag{8}$$

$$u_0\theta^n - \overline{\theta}_z\psi^n = \frac{\overline{\theta}_{zz}}{2u_0}(\psi^l)^2 \tag{9}$$

$$u_0 q^n - \overline{q}_z \psi^n = \frac{\overline{q}_{zz}}{2u_0} (\psi^l)^2 \tag{10}$$

where 
$$(\psi^l)^2 = \sum_k \sum_p \hat{\psi}_k^l \hat{\psi}_p^l e^{j(k+p)x} = \sum_K \left( \sum_{K=k+p} \hat{\psi}_k^l \hat{\psi}_p^l \right) e^{jKx}.$$

By combining equations (8)–(9), and using Fourier expansion we obtain the following equation for  $\hat{\psi}^n$ :

$$\hat{\psi}_{K,zz}^{n} + \left(\frac{N^{2}(z)}{u_{0}^{2}} - K^{2}\right)\hat{\psi}_{K}^{n} = -\frac{1}{2u_{0}^{3}}\left(N_{z}^{2} + \frac{g}{\overline{\theta}}\overline{\theta}_{zz}\right)\sum_{K=k+p}\hat{\psi}_{k}^{j}\hat{\psi}_{p}^{j}.$$
(11)

[25] The wavenumbers that are excited by the nonlinear interaction between the two waves are  $K = k_1 + k_2$ ,  $k_2 - k_1$ ,  $2k_1$  and  $2k_2$ . To study the effect of large wave on the short one, we consider the mutual wave-wave interaction only ( $K = k_1 + k_2$ ,  $k_2 - k_1$ ). Equation (11) is solved with the boundary conditions  $\hat{\psi}_K^n = 0$  at the ground and  $d\hat{\psi}_K^n/dz = jM\hat{\psi}_K^n$  at the top so that the wave energy is radiated upward, where  $M^2 = (N_2^2/u_0^2 - K^2)$ . The analytical solution for equation (11) is given in Appendix A. The nonlinear contributions for the potential temperature and the tracer are then given by:  $\theta^n = \sum_K \hat{\theta}_K^n e^{jKx}$  and  $q^n = \sum_K \hat{q}_K^n e^{jKx}$  where

$$\hat{\theta}_{K}^{n} = \frac{\overline{\theta}_{z}}{u_{0}} \hat{\psi}_{K}^{n} + \frac{\overline{\theta}_{zz}}{2u_{0}^{2}} \sum_{K=k+p} \hat{\psi}_{k}^{l} \hat{\psi}_{p}^{l}$$
(12)

$$\hat{q}_{K}^{n} = \frac{\overline{q}_{z}}{u_{0}}\hat{\psi}_{K}^{n} + \frac{\overline{q}_{zz}}{2u_{0}^{2}}\sum_{K=k+p}\hat{\psi}_{k}^{l}\hat{\psi}_{p}^{l}$$
(13)

The total variations which include the effects of both linear superposition of the two waves and the nonlinear contributions are given by:

$$egin{aligned} & heta = \overline{ heta} + rac{\overline{ heta}_z}{u_0}(m{\psi}_1^l + m{\psi}_2^l) + m{ heta}^n \ & extsf{q} = \overline{ extsf{q}} + rac{\overline{ extsf{q}}_z}{u_0}(m{\psi}_1^l + m{\psi}_2^l) + m{ extsf{q}}^n \ . \end{aligned}$$

[26] Figure 12 show O<sub>3</sub>, CO and potential temperature variations at z = 12.3 km. The variation of potential temperature induced by the large wave is superimposed in Figure 12. The phases and amplitudes of O<sub>3</sub> and CO vary along the horizontal with reversal of O3 correlations, depending on the values of the modulated potential temperature and the mean vertical gradients. For regions where the potential temperature induced by the large wave is above 342 K, the correlation between ozone and potential temperature fluctuations induced by the short wave is negative. When the larger variation of temperature drops below 342 K, the ozone correlation is positive. This occurs because the mean ozone gradients are negative (positive) above (below) 342 K (Figure 5a). The amplitude of the short variation of CO increases when the larger variation of temperature drops below 342 K. This happens because the mean vertical gradient of CO has negative values with larger magnitude below 342 K (Figure 5b). Thus, the larger wave modulates the vertical mean gradients of ozone and CO, and the short wave causes fluctuations in the tracers with amplitude and phases that depend on the locally modulated gradients. We note that since the primary waves have very different wavelengths, the secondary waves have wavelengths that are closer to that of the shorter wave, and produce beat waves as seen in Figure 11 [Teitelbaum and Vial, 1991; Moustaoui et al., 1999].

[27] The modulation of the phases and amplitudes of the short fluctuations, and the reversal of  $O_3$  correlations seen in Figure 12 are not present when the effect of both waves is



**Figure 12.** Variations of O<sub>3</sub> (blue solid line), CO (red dash-dotted line), and potential temperature (black dotted line) as a function of distance from analytical calculations at z = 12.3 km including the nonlinear contributions. The variation of potential temperature induced by the larger wave (tick dotted line) is superimposed. The horizontal dashed line corresponds to  $\theta = 342$  K.

simply linearly superposed (Figure 11). In this case,  $O_3$  correlation does not change. The fluctuations of  $O_3$  and CO have small amplitude and negative correlations because the mean gradients of  $O_3$  and CO are negative, and have small values. This occurs because, in the linear case, the short and the large waves evolve independently of each other, and the effect of the large wave is not seen by the short wave. Thus, the observed variations in the tracers are explained by non-linear wave-wave interaction. The variations of  $O_3$ , CO and potential temperature were also calculated at the same level as in Leg1 (z = 12 km; not shown). In this case, the modulated potential temperature is below 342 K, and the correlation between  $O_3$  (CO) and potential temperature is positive (negative) and do not change as in the observations.

#### 6. Conclusion

[28] In this paper we reported and explained modulations in ozone and CO fluctuations observed in aircraft measurements in the UTLS region during TREX. The horizontal fluctuations of ozone, CO and temperature show phases and amplitudes that vary along the path of the aircraft. The mean ozone profile shows decrease above the tropopause, in a layer located in the lower stratosphere. We demonstrated, using observations, simple analytical and theoretical arguments that the complex distributions in O<sub>3</sub>, CO and temperature observed by the aircraft are generated by interactions between waves with different wavelengths. Variations with larger wavelengths modulate the vertical mean gradients. Short waves evolving in these modulated gradients create tracer and temperature fluctuations with amplitudes and phases that depend on the locally modified gradients encountered by the aircraft. We showed that this phenomenon is nonlinear and cannot be explained under the linear assumption. Contour Advection with Surgery (CAS) calculations suggest that the formation of the observed inversion in the mean ozone gradient, on top of which the mountain waves evolve, is due to a filament induced by synoptic scale motion with relatively high ozone values passing over the TREX region at 342 K, and an intrusion with low ozone at 355 K.

[29] As stated in the introduction, examples of perturbed vertical profiles of ozone showing decreases similar to the one observed during TREX have been reported in many previous studies. What is new here is the effect of mountains waves evolving on top of these perturbed profiles and their impact on the distributions of the tracers. We showed that these variations cannot be reconstructed by considering linear contributions alone. This is because the vertical mean gradients are complex, and vary rapidly within a short distances that are comparable to the vertical displacements induced by the waves. We demonstrated that the complex variations in the amplitudes and phases induced by the short wave can be explained when mutual interaction between waves is taken into account. Our results are different from those presented in Danielsen et al. [1991]. These authors studied the effect of waves with short vertical wavelengths on the transport induced by larger waves. They concluded that the waves with short vertical wavelengths do not participate to the transport of the tracer. Their role is to irreversibly homogenize the variations induced by the large waves. In our case, we refer to short and large with respect to the horizontal wavelength. The analytical calculations use two waves with comparable vertical wavelengths. We show that both large and short waves are effective transporters of the tracers. The variations of the tracers are well reconstructed without including any mixing. This suggests that the observed complex variations in the tracers are due to reversible dynamical processes. Thus TREX observations suggest that the air layers within the UTLS including the tropopause may behave like material surfaces and are mostly reversible. Our results may have important implications. They show that some of complex variations in tracers with intricate correlations such as the ones presented in this paper can still be explained by reversible dynamical processes; and that the correlations between



**Figure A1.** Vertical cross section of normalized vertical velocity from analytical calculations of mountain wave including a stability jump at the tropopause (see text). The wave is generated by a bell shaped mountain.

tracers should be used with caution in order to identify the real mixing.

### Appendix A

[30] In this appendix we give the analytical solutions for the linear and the nonlinear cases (equations (4) and (11)).

### A1. Linear Solution

[31] After expending all the fields using Fourier series:  $\alpha^{l} = \sum_{k} \hat{\alpha}_{k}^{l} e^{ikx}$ , the spectral component of the stream function will satisfy the following equation:

$$\hat{\psi}_{k,zz}^{l} + \left(\frac{N^{2}(z)}{u_{0}^{2}} - k^{2}\right)\hat{\psi}_{k}^{l} = 0.$$
 (A1)

[32] By convention, this equation is solved for the positive wavenumbers only (k > 0). The physical fields being real, the spectral coefficients corresponding to the negative wavenumbers are deduced from the positive ones by taking the complex conjugate ( $\hat{\psi}_{-k} = \hat{\psi}_k^*$ ). The general solutions for  $z \le z_1$  and  $z > z_2$  where the stability is constant ( $N_1$  and  $N_2$ ) is straightforward:

$$\hat{\psi}_k^l(z) = C_1^b \exp(jm_1 z) + C_2^b \exp(-jm_1 z), \text{ for } z \le z_1$$
 (A2)

$$\hat{\psi}_k^l(z) = C_1^t \exp(jm_2 z) + C_2^t \exp(-jm_2 z), \text{ for } z > z_2,$$
 (A3)

where,  $m_1^2 = (N_1^2/u_0^2 - k^2)$  and  $m_2^2 = (N_2^2/u_0^2 - k^2)$  are the square of the vertical wavenumbers in the troposphere and the stratosphere, respectively. For  $z_1 < z \le z_2$ , the stability depend on z (N(z)), and the equation to be solved for  $\psi$  becomes:  $\hat{\psi}_{k,zz}^l + (A + Bz)\hat{\psi}_k^l = 0$ , where,

$$A = \frac{1}{u_0^2} \left\{ N_1^2 - \frac{\Delta N^2}{\Delta z} z_1 \right\} - k^2, \ B = \frac{1}{u_0^2} \frac{\Delta N^2}{\Delta z},$$
$$\Delta N^2 = N_2^2 - N_1^2 \text{ and } \Delta z = z_2 - z_1.$$

[33] The above equation can be written as:

$$rac{\partial^2 \hat{\psi}_k^l}{\partial \xi^2} - \xi \hat{\psi}_k^l = 0, ext{ where, } \xi = -B^{1/3} \left( z + rac{A}{B} 
ight).$$

[34] This equation is called the Airy equation and the general solution is:

$$\hat{\psi}_{k}^{l}(z) = C_{1}A_{i}(\xi) + C_{2}B_{i}(\xi), \text{ for } z_{1} < z \le z_{2},$$
 (A4)

where  $A_i(\xi)$  and  $B_i(\xi)$  are the Airy functions. For a flow  $u_0 > 0$ over a topography (h(x)), we impose at the ground that  $\hat{\psi}_k^l(0) = -u_0 \hat{h}_k$ . At the top, we impose that the phases of waves tilt upstream so that energy is radiated upward. For  $u_0 > 0$ , the horizontal (k) and the vertical (m) wavenumbers have the same sign. Here, we chose to solve for k > 0, and the vertical wavenumber *m* is then positive  $(C'_2 = 0)$ . The solution is completely specified by imposing the above conditions and the continuity of  $\hat{\psi}$  and  $d\hat{\psi}/dz$  at  $z_1$  and  $z_2$ . That is:

$$\begin{split} C_1^b + C_2^b &= \hat{\psi}_0 \\ C_1 A_i(\xi_1) + C_2 B_i(\xi_1) &= C_1^b \exp(jm_1 z_1) + C_2^b \exp(-jm_1 z_1) \\ &- B^{1/3} \Big\{ C_1 A_i^{'}(\xi_1) + C_2 B_i^{'}(\xi_1) \Big\} = jm_1 (C_1^b \exp(jm_1 z_1) \\ &- C_2^b \exp(-jm_1 z_1)) \\ C_1 A_i(\xi_2) + C_2 B_i(\xi_2) &= C_1^t \exp(jm_2 z_2) \\ &- B^{1/3} \Big\{ C_1 A_i^{'}(\xi_2) + C_2 B_i^{'}(\xi_2) \Big\} = jm_2 C_1^t \exp(jm_2 z_2). \end{split}$$

The prime in the above equations  $(A'_i \text{ and } B'_i)$  denotes the derivative with respect to  $\xi$ .  $\xi_1 = -B^{1/3} (z_1 + \frac{A}{B})$  and  $\xi_2 = -B^{1/3} (z_2 + \frac{A}{B})$ . These five equations are easily solved to determine  $C_1^b$ ,  $C_2^b$ ,  $C_1$ ,  $C_2$  and  $C_1^t$ . Figure A1 shows vertical cross section of vertical velocity for a flow over a bell shaped mountain:  $h = h_0/[1 + (x/L)^2]$ , where  $h_0 = 3$  km and L = 40 km. The mean wind is  $u_0 = 20 \text{ ms}^{-1}$ . The behavior of the solution is similar to the one expected for mountain waves. We note that the vertical wavelength decreases from the troposphere into the stratosphere as the stability increases, and that the mountain wave is perfectly radiated at the upper boundary  $(z_t = 30 \text{ km})$ .

#### A2. Nonlinear Solution

[35] The equation to be solved for the nonlinear contribution is:

$$\hat{\psi}_{zz}^{n} + \left(\frac{N^{2}(z)}{u_{0}^{2}} - K^{2}\right)\hat{\psi}^{n} = Q_{K}(z)$$
(A5)

where

$$Q_K(z) = -\frac{1}{2u_0^3} \left( N_z^2 + \frac{g}{\overline{\theta}} \overline{\theta}_{zz} \right) \sum_{K=k+p} \hat{\psi}_k^I \hat{\psi}_p^I.$$
(A6)

[36] This equation is solved with the boundary conditions  $\hat{\psi} = 0$  at the ground so that the total solution satisfy the specified condition:  $\psi = \psi^l + \psi^n = -u_0h$ . At the top, we impose  $d\hat{\psi}^n/dz = jM\hat{\psi}^n$  so that the wave energy is radiated upward, where  $M^2 = (N_2^2/u_0^2 - K^2)$ .

[37] The general solution of this equation is:

$$\begin{split} \hat{\psi}_{K}^{n} &= (f^{tb}(z) + C_{1}^{b}) \exp(jM_{1}z) + (g^{b}(z) + C_{2}^{b}) \exp(-jM_{1}z), \\ \text{for } z \leq z_{1} \\ \hat{\psi}_{K}^{n} &= (f(z) + C_{1})A_{i}(\xi) + (g(z) + C_{2})B_{i}(\xi), \text{ for } z_{1} < z \leq z_{2} \\ \hat{\psi}_{K}^{n} &= (f^{t}(z) + C_{1}^{t}) \exp(jM_{2}z) + g^{t}(z) \exp(-jM_{2}z), \text{ for } z > z_{2} \end{split}$$

where,

$$f^{b}(z) = \frac{1}{2jM_{1}} \int_{0}^{z} Q_{k}(\eta) \exp(-jM_{1}\eta) d\eta,$$
  

$$g^{b}(z) = -\frac{1}{2jM_{1}} \int_{0}^{z} Q_{k}(\eta) \exp(jM_{1}\eta) d\eta,$$
  

$$f^{t}(z) = \frac{1}{2jM_{2}} \int_{z_{t}}^{z} Q_{k}(\eta) \exp(-jM_{2}\eta) d\eta,$$
  

$$g^{t}(z) = -\frac{1}{2jM_{2}} \int_{z_{t}}^{z} Q_{k}(\eta) \exp(jM_{2}\eta) d\eta,$$

$$f(z) = \frac{\pi}{B^{1/3}} \int_{z_1}^z Q_k(\eta) B_i(\xi) d\eta,$$
  
$$g(z) = -\frac{\pi}{B^{1/3}} \int_{z_1}^z Q_k(\eta) A_i(\xi) d\eta.$$

[38] The constants  $C_1^b$ ,  $C_2^b$ ,  $C_1$ ,  $C_2$  and  $C_1^t$  are specified by imposing  $C_1^b + C_2^b = 0$  and the continuity of  $\psi$  and its vertical derivative at  $z_1$  and  $z_2$ .

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