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## **The switching between zonal and blocked mid-latitude atmospheric circulation: a dynamical system perspective**

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1 **The switching between zonal and blocked mid-latitude atmospheric**  
2 **circulation: a dynamical system perspective**

3 **Davide Faranda · Giacomo Masato · Nicholas**  
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6  
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8 **Abstract** Atmospheric mid-latitude circulation is dominated by a zonal, westerly flow. Such a flow is  
9 generally symmetric, but it can be occasionally broken up by blocking anticyclones. The subsequent  
10 asymmetric flow can persist for several days. In this paper, we apply new mathematical tools based  
11 on the computation of an extremal index in order to reexamine the dynamical mechanisms responsible  
12 for the transitions between zonal and blocked flows. We discard the claim that mid-latitude circulation  
13 features two distinct stable equilibria or chaotic regimes, in favor of a simpler mechanism that is well  
14 understood in dynamical systems theory: we identify the blocked flow as an unstable fixed point (or saddle  
15 point) of a single basin chaotic attractor, dominated by the westerlies regime. We also analyze the North  
16 Atlantic Oscillation and the Arctic Oscillation atmospheric indices, whose behavior is often associated  
17 with the transition between the two circulation regimes, and investigate analogies and differences with  
18 the bidimensional blocking indices. We find that the Arctic Oscillation index, which can be thought as  
19 a proxy for a hemispheric average of the Tibaldi-Molteni blocking index, keeps track of the presence of

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20 unstable fixed points. On the other hand, the North Atlantic Oscillation, representative only of local  
21 properties of the North Atlantic blocking dynamics, does not show any trace of the presence of unstable  
22 fixed points of the dynamics.

23 **Keywords** Blocking Index, Mid-latitude circulation, Extremal index

## 24 1 Introduction

25 In the time range of 2 – 8 days, the mid-latitude large-scale circulation is mainly driven by the destabi-  
26 lization of a westerly sheared jet, associated with a meridional temperature gradient (Holton and Hakim  
27 2013). The destabilizing mechanism is referred to as the baroclinic instability and it consists in the appear-  
28 ance of three dimensional wave structures (extra-tropical cyclones and anticyclones) normally embedded  
29 in the mid-latitude westerlies. The minimal model for such an instability is known as the Charney-Eady  
30 model (Charney 1947). Such a model is based on the stability analysis of the quasi-geostrophic potential  
31 vorticity equation coupled with a thermodynamic equation. The stability parameter is the Burger num-  
32 ber, i.e. the ratio between stratification (a quantity linked to the meridional temperature gradient) and  
33 rotational effects.

34  
35 Cyclones and anticyclones are generally embedded in the mid-latitude jet, and have average lifetimes  
36 of a few days that depend on their size, longitudinal asymmetry and interaction with the topography  
37 (Emanuel 2005; Rudeva 2008). However, a few times per year and with higher frequency in the winter  
38 season, large high-pressure structures may form and persist for several days, breaking up the westerlies  
39 circulation and forcing the jet to move towards higher latitudes or even split up into two branches, hence  
40 breaking the longitudinal symmetry. This kind of circulation is referred to as blocked flow (Charney and  
41 DeVore 1979). During blocking conditions a few extreme climate events have occurred like the December  
42 2010 cold spell in northern and central Europe or the summer 2003 heatwave over western Europe (Schär  
43 and Jendritzky 2004; Beniston 2004) or the 2010 heatwave over Russia (Dole et al 2011; Barriopedro  
44 et al 2011). It is therefore crucial to get a deeper understanding of the blocked flow and of the mechanism  
45 which regulates the transition to the westerlies regime.

46 Besides the anticyclones embedded in the mid-latitude jet, there is a large fraction of subtropical anticy-  
47 clones such as the Azores high that are located further south and are quasi-stationary. Since it is necessary  
48 to discern between typical blocking highs in the mid to high latitudes and those at lower latitudes, in this  
49 paper, we will focus only on the blocking occurring between mid to high latitudes, deferring to another  
50 publication the study of lower latitudes for which other robust blocking detection algorithms have been  
51 devised (Barriopedro et al 2006, 2010; Lupo and Smith 1995; Scherrer et al 2006).

52  
53 In order to understand the transition between those flow regimes, many studies of mid-latitude dynamics  
54 have been conducted both theoretically and experimentally. Legras and Ghil (1985) and Ghil (1987) have  
55 shown the intricacy of such circulation by studying an intermediate complexity model of a barotropic flow  
56 with dissipation forcing and topography. They observed two distinct equilibria which can be associated  
57 with either the westerlies or the blocked flows. Similar conclusions appear in (Mo and Ghil 1988) and are  
58 supported by experimental laboratory studies (Weeks et al 2000). A vast amount of literature points to  
59 very different mechanisms. Some of these theories relate blocking to resonant amplification of free quasi-  
60 stationary Rossby waves (Tung and Lindzen 1979) whereas others have considered barotropic (Simmons  
61 et al 1983) or baroclinic instability (Frederiksen 1982) as the main mechanism. The development of non-  
62 linear theories has led some authors to regard blocking episodes as a manifestation of multiple equilibria

63 in asymmetrically forced flows (Hansen 1986) or as a result of soliton-modon structures (McWilliams et al  
64 1981). A significant number of works invoke non-linear interactions, either between zonal flow and eddies  
65 (Charney and DeVore 1979), or between planetary waves (Egger 1978; Kung et al 1990; Christensen and  
66 Wiin-Nielsen 1996). Other authors have analyzed the relationship between blocking anticyclones and low-  
67 frequency planetary waves at the hemispheric scale (Wallace and Blackmon 1983; Dole 1986). A number  
68 of authors have pointed to the decisive contribution from high frequency cyclones to develop (Colucci  
69 and Alberta 1996; Nakamura et al 1997) and sustain (Shutts 1986) the typical low-frequency anomalies  
70 associated with blocking episodes. Conceptual models linking blocking anticyclones and transient eddies  
71 have been proposed (Shutts 1983). On the basis of observations some other works see blocking appearing  
72 as a distinctive dynamical feature with respect to westerlies circulation, but it consists of several multi-  
73 stable patterns (Vautard 1990; Masato et al 2009, 2012).

74  
75 In this paper we reanalyse data over the past decades to detect whether the dynamics of blocking is  
76 compatible with the existence of an unstable fixed point of the atmospheric mid-latitude circulation.  
77 This evidence comes from dynamical systems theory and is supported by the common experience that,  
78 within the blocked flow, atmospheric variables follow a highly predictable dynamics (persistence of the  
79 same weather conditions for several days), whereas in the zonal flow they mostly have a chaotic behavior  
80 (irregular alternation between cyclonic and anticyclonic phases). Such kind of dynamical features are also  
81 encountered for several dynamical systems ranging from toy models as the Pomeau-Manneville  
82 map, the Hénon map or the Lorenz equations (Benzi et al 1985) to intermediate complexity models  
83 (Payne and Sattinger 1975; Kaplan and Yorke 1979). The dynamics of all these systems is generally  
84 chaotic and takes place on a single basin chaotic attractor, but is sometimes trapped near an unstable  
85 fixed point. When this happens, an orbit stays in the vicinity of the fixed point for an amount of time  
86 that depends on the distance from the fixed point and its dynamical properties. As a result, the system  
87 experiences a temporary suppression of chaos.

88  
89 We propose to detect the existence of unstable fixed points in the mid-latitude atmospheric circulations  
90 by using recent results obtained for recurrences of dynamical systems (Freitas et al 2010; Lucarini et al  
91 2012; Freitas et al 2012; Faranda et al 2013). These results have opened a new branch of research where  
92 recurrences of a certain observation in an orbit are treated via the statistics of extreme events. The nov-  
93 elty of this approach lies in the fact that classical extreme value laws can be found for such recurrences  
94 for almost every point of chaotic attractors (Freitas et al 2010). In (Faranda and Vienti 2013) we have  
95 exploited this technique to study instrumental temperature records, and check that temperature recur-  
96 rences obey one of the three classical extreme values, i.e. the atmosphere behaves as a chaotic system. Via  
97 this analysis, a map of European temperatures can be constructed whose recurrence is likely or unlikely  
98 within a certain time scale of interest.

99  
100 In order to detect the possible unstable fixed points of the atmospheric dynamics, we will analyze several  
101 blocking indices. In general, a blocking index is defined in terms of the difference of pressure (or conju-  
102 gated fields) between two different locations at the same longitude. When the flow is zonal, this difference  
103 always has the same sign because anticyclones are generally located at lower latitudes. Conversely, when  
104 the flow is blocked, low pressure systems tend to move to low latitudes and anticyclones to high latitudes,  
105 reversing the meridional gradient in pressure. A blocking event is identified as the persistence of such  
106 conditions for several days.

We will begin our analysis with the Tibaldi-Molteni index (Tibaldi and Molteni 1990), defined at each longitude by differences of geopotential heights, and compare the results with those obtained for the bidimensional blocking index introduced by Pelly and Hoskins (2003), where differences are taken over a potential vorticity surface closer to the tropopause and to the core of the jet stream. After collecting evidence for the existence of unstable fixed points and their spatial distribution, we will perform the analysis on one-dimensional indices of atmospheric circulation to see whether they keep any trace of the existence of unstable fixed points. In particular, we will focus on the Artic Oscillation (AO) index, which is roughly a hemispheric average of the Tibaldi-Molteni index, and on the North Atlantic Oscillation (NAO) index, representative of the North-Atlantic/European regions only (NOAA 2015; Hurrell et al 2003).

The paper is organized as follows: in section 2, we give an overview of the method and explain the analogy between unstable fixed points and blocked circulation via dynamical systems toy models. In section 3, we present evidence for the existence of unstable fixed points by using the Tibaldi-Molteni and the Pelly blocking indices. In section 4, we analyze the role of one-dimensional indices, such as the AO and the NAO. Finally, we discuss how to improve the modeling of the mid-latitudes circulation on the basis of the results obtained.

## 2 Method and results from dynamical systems

In this section we A) show how to detect unstable fixed points in dynamical systems and time series, B) explain the inference procedures, and C) discuss the estimation of the parameters describing the statistical properties of the unstable point.

### 2.1 Detection of unstable fixed points with dynamical systems techniques

Let us consider a discrete-time dynamical system. This is a relevant hypothesis for an atmospheric system (Lucarini et al 2013), as both models and observations are made at discrete times. The dynamics is governed by the map  $T$ , which iterates the variables of the system  $x$  according to

$$x_{t+1} = T(x_t). \quad (1)$$

We assume that, by starting from a random initial condition, the dynamics follows a chaotic trajectory on a well-defined surface of the phase space, i.e. the attractor. We fix a point  $\zeta$  on the attractor and measure the time series of the distances between  $\zeta$  and the subsequent iterations of the orbit:

$$w(t; \zeta) = -\log(d(T(x_t), \zeta)),$$

where  $d$  is a distance function between two vectors. We are interested in the high extremes of  $w(t; \zeta)$ , for all  $t$ . By construction, such extremes define the recurrences of the system. To identify the extremes, we apply the block-maxima approach. It consists of dividing the time series  $T(x_t)$  into intervals of length  $m$ . Every  $m$  observations, the closest recurrence to the point  $\zeta$  is taken. If  $n$  intervals of length  $m$  are available in the series, one obtains  $n$  closest recurrences. If the system is chaotic, the logarithmic weight forces the asymptotic extreme value distribution to follow a Gumbel law. A detailed explanation for this can be found in Faranda and Vaienti (2013), but the reason is intuitive: the Gumbel law is of the form

144  $G(x) = \exp(-\exp(-x))$ . One of the exponential functions comes from the exponential recurrence statis-  
 145 tics, the other from the inverse of the logarithm. Other choices for  $w$ , typically power laws, constrain the  
 146 asymptotic extreme distribution to be either a Weibull or a Fréchet distribution.

147

This theoretical framework applies to almost all the points of a chaotic attractor except at the unstable  
 fixed points. A fixed point of the system in Eq. (1) is one that repeats itself under iteration, i.e.  $T(x) = x$ .  
 An unstable or repelling fixed point is one for which the distance between itself and any point in a  
 surrounding neighborhood increases under iterations (Katok and Hasselblatt 1997). A theoretical result  
 from dynamical systems, obtained by Freitas et al (2012), states that when  $\zeta$  is an unstable fixed point  
 of the recurrence map  $T$ , then the distribution of  $w(t; \zeta)$  follows a modified Gumbel law:

$$G(x, \theta) = \exp(-\exp(-\theta x)),$$

148 where  $\theta$  is a parameter known as the extremal index.

149 The concept of the extremal index originally appears in extreme value theory (Leadbetter et al 1983),  
 150 where  $\theta$  gives a measure of ‘clustering’, i.e. the tendency of a random process to exceed a threshold at  
 151 consecutive times. If a threshold  $u$  is applied to a series of observations  $x_1, x_2, \dots, x_s$ , the exceedances  
 152 are those for which  $x_i > u$ . Heuristically, the extremal index can then be thought of as  $\theta = 1/\ell$ , where  
 153  $\ell$  is the mean duration of consecutive exceedances (clusters), i.e. the average of the time intervals spent  
 154 above  $u$ .

155

156 In the dynamical systems context,  $\theta$  can vary across the phase space depending on the  $\zeta$  from which  
 157 recurrences are measured (Freitas et al 2012). Clustering occurs when consecutive iterates of the orbit  
 158 are observed near a point of the attractor. For almost all the nonsingular points there is no clustering.  
 159 This means that, on average, an orbit enters the neighborhood of  $\zeta$  once at time, which gives  $\theta = 1$ .  
 160 However, when  $\zeta$  is close to an unstable fixed point,  $\theta < 1$ . The smaller the  $\theta$ , the larger the cluster size,  
 161 i.e. the longer the time the orbit stays in the vicinity of  $\zeta$ . In dynamical systems, the cluster extends in  
 162 both time and space: we observe a time cluster in the minimum distances between the unstable fixed  
 163 point and the orbit. This in turn effectively corresponds to a spatial cluster because the orbits are held  
 164 within the vicinity of the unstable fixed point.

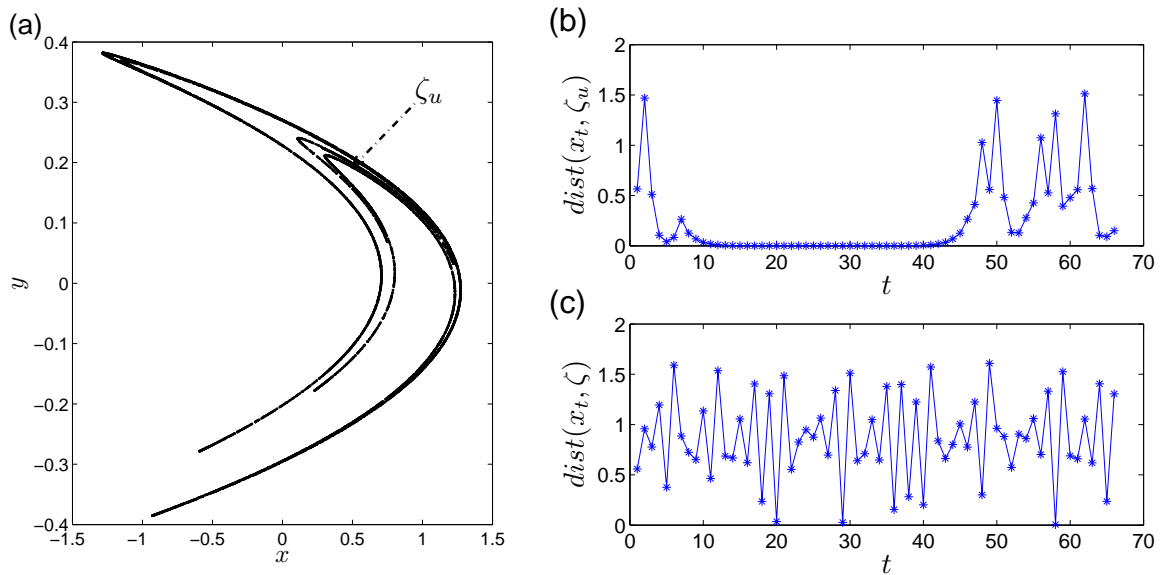
165

166 As an example, we consider the Hénon system (Hénon 1976) whose attractor is shown in Fig.1a, obtained  
 167 by iterating the following set of equations:

$$\begin{aligned} x_{t+1} &= y_t + 1 - 1.4x_t^2, \\ y_{t+1} &= 0.3x_t. \end{aligned} \tag{2}$$

168 In this picture, the presence of the unstable fixed point  $\zeta_u$  is not obvious. Its existence can be proved  
 169 analytically by solving  $T(x, y) = (x, y)$ , with approximate solution  $\zeta_u = (0.63, 0.19)$ , as indicated in  
 170 Fig.1a. The dynamics around this point is different from that of a generic point, and this can be captured  
 171 by computing the recurrences. Fig.1b and Fig.1c show distances of iterates of the orbit measured from  
 172  $\zeta_u$  and a generic point  $\zeta$  of the attractor. In the former case, a long cluster is visible and its length can  
 173 be determined via the extremal index  $\theta$ , whose estimation we describe in the next section (for the Hénon  
 174 map  $\theta$  can also be computed analytically Freitas et al (2012)).

175 Although the Hénon dynamics is not representative of the atmospheric circulation, it is helpful to illus-  
 176 trate the general dynamical behavior around an unstable fixed point. In chaotic dynamical systems we  
 177 can only observe clustering at unstable fixed points (see Freitas et al (2012) , Theorem 1), independently



**Fig. 1** **a**: Hénon attractor, obtained by iterating Eq. (2). The location of the unstable fixed point  $\zeta_u$  is indicated. **b**: Series of distances from the unstable fixed point  $\zeta_u$ . Clustering occurs when the trajectory gets close to  $\zeta_u$ . **c**: Series of distances from a generic point of the attractor  $\zeta$ . No clustering occurs.

178 of the complexity of the system. If we consider the time series of observations as the output of a dynamical system, then we may hope to track the presence of unstable fixed points by measuring  $\theta < 1$  for some reference values  $\zeta$ .

## 181 2.2 Algorithm and inference of the extremal index

182 In order to compute the extremal index  $\theta$  for time series we use the following algorithm:

- 183 1. Consider a time series consisting of  $s$  observations:  $\{x_t : t = 1, 2, \dots, s\}$ .
- 184 2. Fix  $\zeta$  to be a point of the series itself.
- 185 3. Compute the series  $w_t(\zeta) = -\log(d(x_t, \zeta))$ .
- 186 4. Take a very high quantile  $q$  of the  $w_t$  distribution, in order to consider only the closest recurrences (when  $d(x, \zeta)$  is small,  $w(x, \zeta)$  is large).
- 187 5. Compute the extremal index.

189 A number of estimators exist for  $\theta$ . The simplest of these, the so-called “runs” or “blocks” methods, approximate  $\theta$  by dividing the number of recurrent clusters by the total number of recurrences (Embrechts et al 1997). The cluster could be defined as a consecutive sequence of recurrent observations, terminating when an observation is no longer recurrent (by falling outside the small interval that defines recurrences). Depending on the context, different definitions of clusters may be appropriate. Another estimator, (Ferro and Segers 2003), is constructed out of the second moment of times between recurrences. For the analysis presented in this paper, we adopt another estimator by Süveges (2007), and we have checked that our results are robust across extremal index estimators. For fixed quantile  $q$ , Süveges’ estimator reads:

$$\theta = \frac{\sum_i^{N_c} (1-q) S_i + N + N_c - \sqrt{\left(\sum_i^{N_c} (1-q) S_i + N + N_c\right)^2 - 8N_c \sum_i^{N_c} (1-q) S_i}}{2 \sum_i^{N_c} (1-q) S_i},$$

197 where  $N$  is the number of  $w(t, \zeta)$  above the chosen quantile (i.e. recurrences),  $N_c$  the number of obser-  
 198 vations which form a cluster of at least two consecutive recurrences, and  $S_i$  the length of each cluster  $i$ .  
 199 For details of the derivation of this estimator, see Süveges (2007).

### 200 2.3 Finite-size effects

201 Faranda et al (2011) discussed some of the problems related to the finiteness of datasets when studying  
 202 the recurrences around a certain value in a time series. The results  $\theta = 1$  at generic points and  $\theta < 1$   
 203 at unstable fixed points hold only in the limit  $q \rightarrow 1$ , i.e. for the closest recurrences. When dealing with  
 204 finite data sets, the limit  $q \rightarrow 1$  is unattainable and, depending on the marginal distribution, some points  
 205  $\zeta$  may be associated with an extremal index  $\theta < 1$  even when they are not unstable fixed points.

206 To illustrate this effect, we analyse a time series generated by an auto-regressive process of order 1  
 207 (AR(1)),  $x_t = \phi x_{t-1} + \sigma \epsilon_t$ , where  $|\phi| < 1$  is the magnitude of the auto-regressive coefficient and  $\epsilon_t$   
 208 is a random variable drawn from a normal distribution,  $\sigma = 0.1$  is a constant. It is known that the  
 209 extremes of this process do not cluster in the limit  $q \rightarrow 1$ , so that  $\theta = 1$  for all  $\zeta$  and all  $\phi$ . However,  
 210 for finite datasets and fixed  $q$ , clustering will be observed among the weakly (exponentially) correlated  
 211 exceedances all the while they have not exited an interval around  $\zeta$  whose size is a function of  $\phi$  and the  
 212 underlying marginal density: for larger  $\phi$  and  $\zeta$  chosen in the wings of the marginal density, there is a  
 213 greater ‘finite-size’ clustering.

214 In our numerical experiment, we take  $\phi = 0.5$  and synthetic datasets of lengths 30000 (similar to the  
 215 real time series we later analyse) and 300000. We plot the empirical marginal densities in Fig. 2a , and  
 216 the computed  $\theta$  as a function of  $\zeta$  in Fig. 2b. Note that  $\theta < 1$  in the wings of the distribution, even  
 217 though the model has no inherent tendency to cluster extremes. Increasing the length of the datasets  
 218 sharpens the estimate of the theoretical quantile of the marginal density. But it does not change the size  
 219 of the interval that defines recurrences. For a fixed quantile (i.e. not increasing with sample size), the  
 220 extremal index is therefore relatively insensitive to the sample size. For the AR(1) process it is possible  
 221 to estimate  $\theta$  analytically by calculating the mean sojourn time inside the recurrence interval. This time  
 222 represents the length of a cluster — after which observables leave the interval, thereby terminating it.  
 223 The inverse of the mean sojourn time thus provides an estimate of  $\theta$ . The analytical curve obtained is  
 224 shown in Fig. 2b.

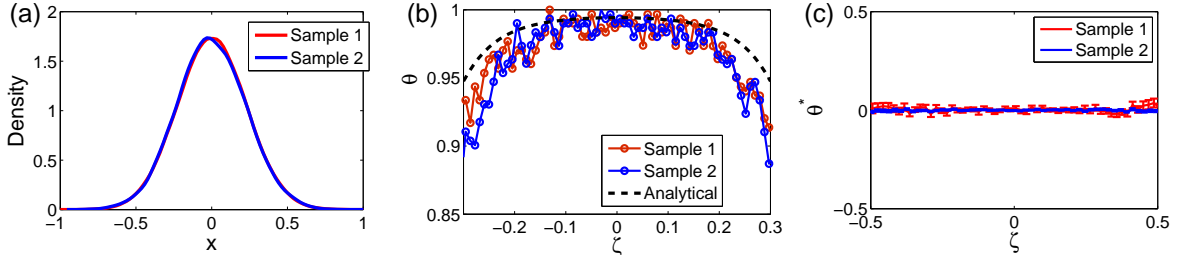
225 Our method to deal with such finite-size effects is based on the following observations: the extremal index  
 226 obtained for some fixed  $q < 1$  depends on i) the shape of the marginal density, ii) the spectral properties  
 227 of the process (i.e.  $\phi$  in our above example), iii) the choice of  $\zeta$ , since clustering is a *local* property.

228  
 229 We have tested i) and ii) by changing both the marginal and the magnitude of the coefficient  $\phi$ , which  
 230 is directly linked to the spectrum for the process (Brockwell and Davis 2002). We can go further by  
 231 generating surrogates of the original dataset that have identical marginal distributions and, to within a  
 232 very low tolerance, the same spectral properties. In this way we can ‘subtract’ the effects of i) and ii)  
 233 in the computation of  $\theta$ . To perform this in practice, we use the Iterative Amplitude Adjusted Fourier  
 234 Transform (IAAFT) of Schreiber and Schmitz (1996). In order to compute the residual extremal indices,

235 which we will denote by  $\theta^*(\zeta)$ , it is sufficient to average over several surrogate estimates of  $\theta$ , such that:

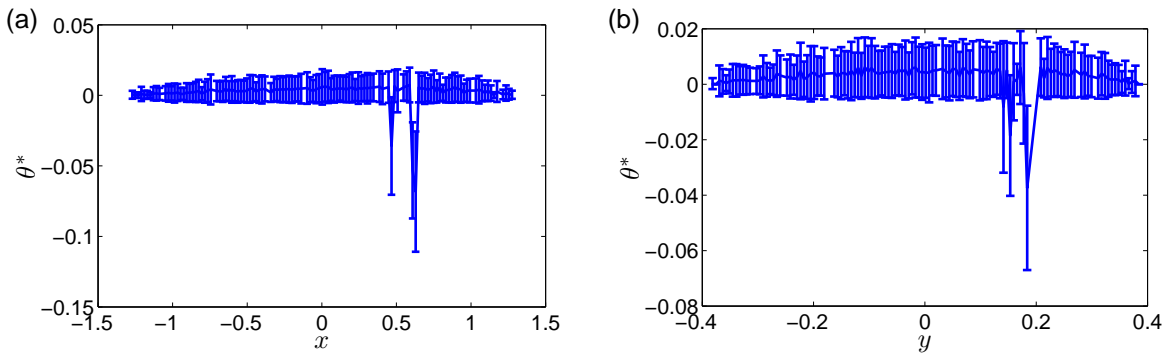
$$\theta^*(\zeta) = \langle \theta(\zeta, x_{\text{SURR}}) \rangle - \theta(\zeta, x), \quad (3)$$

236 where the  $\langle \rangle$  are averages over realizations of surrogate data. This procedure has been tested on the  
 237 auto-regressive samples previously analysed. The residual  $\theta^*$  is plotted in Fig. 2c and shows no local  
 238 clustering.



**Fig. 2** **a** Empirical density for data generated by an autoregressive process. **b**: extremal indices  $\theta$  computed at 100 reference points  $\zeta$ , together with its analytical estimate (dashed line). **c**: residual extremal index  $\theta^*$  computed at the same  $\zeta$  as in the central panel. Sample 1 contains 30000 data. Sample 2 contains 300000 data.

239 An analysis of the Hénon attractor provides a test of property iii). Here, we can check whether we are  
 240 able to recover the known location of the unstable fixed point via a computation of  $\theta^*$  for the time series  
 241 of  $x$  and  $y$ . Results are shown in Fig. 3 for 30000 observations. The location of  $\zeta_u$  is picked out by the  
 242 two peaks of  $\theta^*$  at  $x = 0.63$  (Fig. 3a) and  $y = 0.19$  (Fig. 3b). Secondary peaks are also visible in these  
 243 plots, which are related to the influence of  $\zeta_u$  in nearby locations of the attractor.



**Fig. 3** Residual extremal index  $\theta^*$ , based on 30000 iterations of the Hénon map. **a**:  $x$  observable. **b**:  $y$  observable.

### 244 3 Analysis of Multidimensional blocking indices

245 The first bidimensional blocking index was introduced by Scherrer et al (2006) and it is based on the  
 246 original definition given by Tibaldi and Molteni (1990); Tibaldi et al (1994). This index determines

247 the longitudinal asymmetry of the atmospheric flow between  $40^\circ\text{N}$  and  $80^\circ\text{N}$ , by comparing meridional  
 248 gradients of geopotential height at 500hPa ( $Z$ ). For each longitude in the northern extra-tropics, a  
 249 southern gradient  $B_S$  and a northern gradient  $B_N$  of  $Z$  are computed as follows:

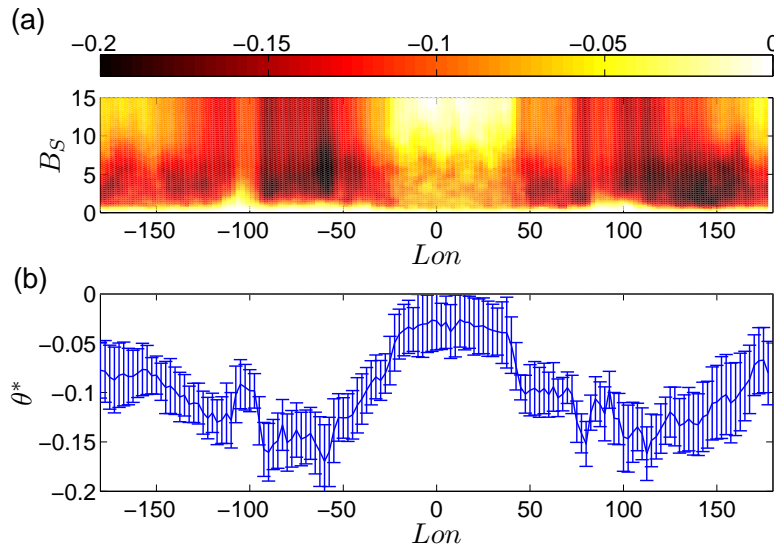
$$B_S = \frac{Z(\phi_o) - Z(\phi_s)}{\phi_o - \phi_s}$$

$$B_N = \frac{Z(\phi_n) - Z(\phi_o)}{\phi_n - \phi_o}$$

250 where  $\phi_n = 80^\circ + \delta$ ,  $\phi_o = 60^\circ + \delta$ ,  $\phi_s = 40^\circ + \delta$ ,  $\delta = -5^\circ, 0^\circ, 5^\circ$ . A given longitude is considered to be  
 251 *blocked* at a given time if the following two conditions are satisfied for at least one value of  $\delta$ :

$$(1): B_S > 0, \quad (2): B_N < -10 \text{ m/degree}$$

252 Here, we analyze daily time series of  $B_S(t)$  computed for the  $Z$  field of the National Centers for Envi-  
 253 ronmental Prediction (NCEP) daily reanalysis (Kalnay et al 1996), which represent the strength of the  
 254 Tibaldi-Molteni blocking index, under the condition that  $B_S > 0$  and  $B_N < -10$ . Blocking events mainly  
 255 occur around  $180^\circ$  (in the Pacific) and at  $0^\circ$  (in the Eastern North Atlantic) longitude (d'Andrea et al  
 256 1998). For our analysis, positive values of  $B_S(t)$  also satisfying condition (2) are considered. The values  
 257 of  $\theta^*$  plotted in Fig. 4 are computed using the technique described in the previous section, with 100 refer-  
 258 ence points of  $\zeta = B_S$ . In the following analysis we generate 100 surrogates to compute  $\langle \theta(\zeta, x_{\text{SURR}}) \rangle$ ,  
 259 and take  $q = 0.995$ . We find that our results are robust with respect to quantile provided  $q \geq 0.99$ .



**Fig. 4** **a**: Extremal indices  $\theta^*$  (color scale) as a function of longitude  $Lon$  and the Tibaldi-Molteni blocking index  $B_S(t)$ . **b**: extremal index averaged over all  $B_S$  vs longitude. Negative values indicate the presence of unstable fixed points. All the values are significant at a 95% level.

Some areas attain negative values of  $\theta^*$  and, over all, the average extremal index is negative at all longitudes (Fig. 4b). Therefore, for these areas the dynamics of blocking is compatible with the existence of unstable fixed points. It is quite surprising to observe that at  $0^\circ$  longitude  $\theta^*$  is close to zero (although the values of  $\theta^*$  are significant also for this area, implying only moderate clustering. Although the European region is affected by strong blocking events, these area is not the one attaining the strongest clustering of  $B_S$ ).

In order to have a detailed geographical description of the local clustering features, we will consider the local blocking index introduced by Pelly and Hoskins (2003) and Tyrlis and Hoskins (2008). This blocking index  $B$  is a macroscopic measure of the strength of the meridional gradient of potential temperature on the isopotential vorticity surface with value 2 PVU (also called the PV2 surface; 1 PVU [ $1 \times 10^{-6}$  K  $m^2$   $kg^{-1}$   $s^{-1}$ ]). This surface corresponds approximately to the tropopause, as described by Hoskins et al (1983).  $B$  is computed every  $5^\circ$  of longitude as the difference of the average of potential temperature for regions  $15^\circ$  of latitude. Whenever  $B$  takes positive values, it is referred to as local instantaneous blocking. Negative values of  $B$  correspond instead to the westerlies mid-latitude circulation associated with the zonal flow. The more negative  $B$ , the stronger such a circulation.

We computed  $\theta^*$  at each grid point for 100 values of  $\zeta$ , uniformly spaced between  $\min(B)$  and  $\max(B)$ . We then averaged  $\theta^*$  for three  $B$  intervals corresponding respectively to blocked regimes  $B > 0$  (Fig. 5a) moderate zonal flows  $-10 < B < 0$  (Fig. 5b), and strong zonal flow  $B < -10$  (Fig. 5c). Only grid points with sufficient statistics to obtain reliable values of  $\theta^*$  have been considered. For  $B > 0$  (blocked flows),  $\theta^* \neq 0$  appears almost everywhere. Strong geographical differences appear in the distribution of  $\theta^*$ : negative values are concentrated at higher latitudes, especially over Canada and central Asia where  $\theta^* \simeq -0.15$ . For the North-Atlantic sector, the dynamics over the Canadian region has been recognized by Vautard (1990) as a driver of the development of blocking structures. In the same paper, it has been argued that the precursor of transitions to the blocked circulation can be found by considering the weather evolution over this region. It is worth noticing that the unstable fixed points tend to be located in proximity or just downstream of the two major mountain chains (i.e. the Rocky Mountains and the Tibetan Plateau). Not surprisingly, these are a necessary ingredient to foster stationary waves (Brayshaw et al 2008). In low dimensional dynamical systems only the neighborhood of unstable fixed points is affected by clustering. this is not necessarily the case for spatially extended systems: the effect of the unstable fixed points can be transported in space by the dynamics. Unstable fixed points found with our analysis may be the preferred breaking points for the amplification of Rossby waves, or for the development of non-linear interactions. At a mathematical level, there are indeed evidence of the relation between unstable fixed points and the destabilization of systems governed by partial differential equations (see for example the work by Memory (1991) Example 2 for the delayed logistic equation with diffusion and reference therein). Evidences that this behavior is consistent with the atmospheric dynamics and the blocking phenomena can be found in (Christensen and Wiin-Nielsen 1996).

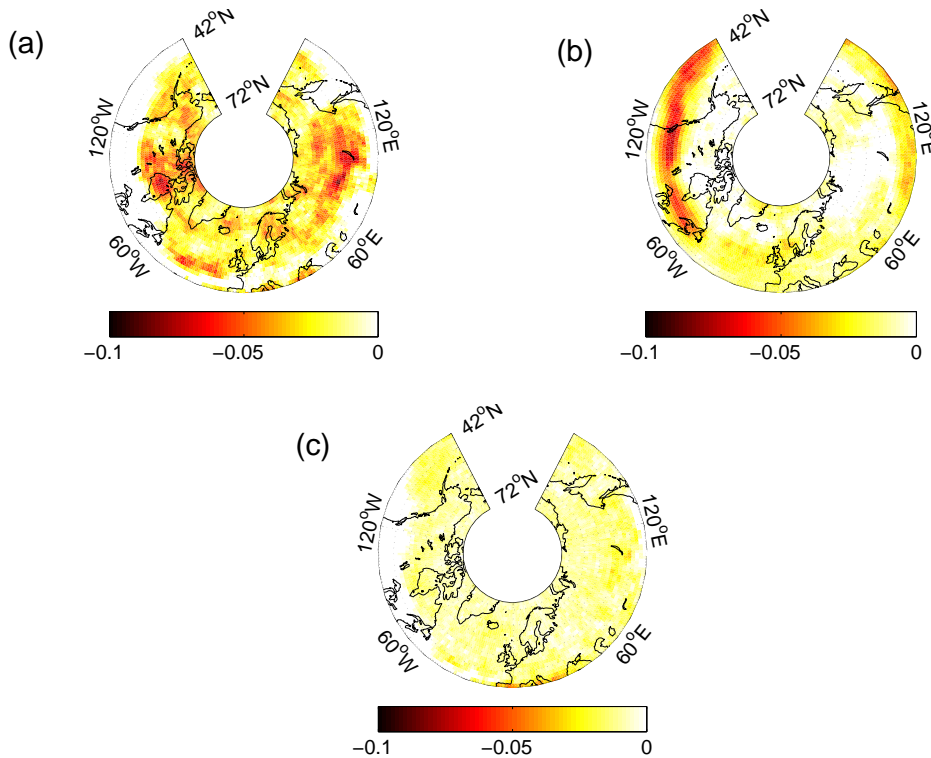
For the North Atlantic, the location of unstable fixed points is consistent with the one obtained by (Buehler et al 2011) -Fig 1a), where the blocking frequency is computed with a dynamical technique called the Lagrangian method which consists in incorporating in the blocking definition the contribution of cyclonic activity to reduce artificial maxima. The use of the surrogate acts similarly to this additional condition by removing the mean dynamical component, this might explains why the longitudinal profiles are different with respect to the ones obtained by Tibaldi and Molteni (1990) and other authors.

Fig. 5-b and c show the values of  $\theta^*$  respectively when  $-10 < B < 0$  and  $B < -10$ . The more zonal the flow, the more negative  $B$  and the less negative values of  $\theta^*$  are observed. This is compatible with the

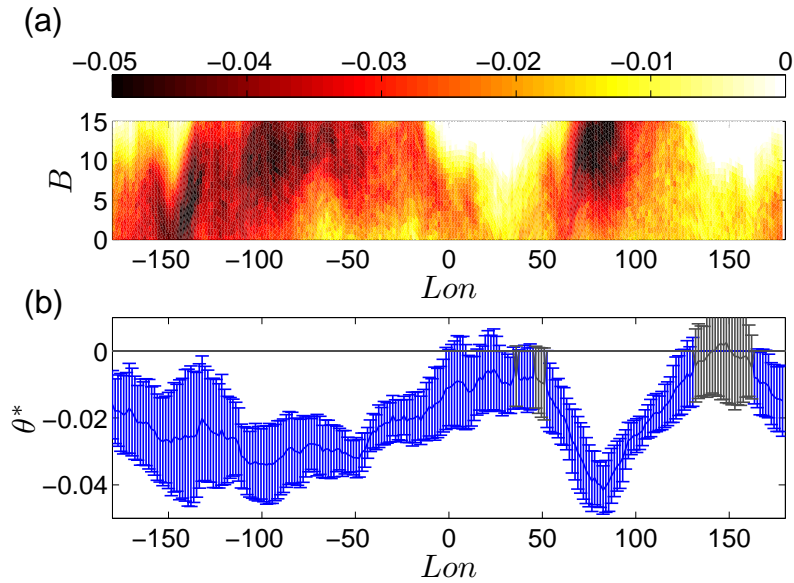
306 idea that unstable fixed points are associated to the blocked flow, as it will be confirmed by the analysis  
 307 of the AO index is the next section.

308

309 The meridional averages of the  $B > 0$  are reported in Fig. 6. As for the Tibaldi-Molteni index we observe  
 310 a complex dependence on both the longitude and the values of  $B$ . Almost everywhere the averages are  
 311 negative and significant at a 95% level. It is important to compare the  $\theta^*$  for  $B$  with the ones obtained  
 312 for the Tibaldi-Molteni index. We observe that the scaling is different: although significant, the values  
 313 of  $\theta^*$  for  $B$  are almost 4 times smaller in magnitude than the value obtained for  $B_S$ . We conjecture that  
 314 the Pelly index has smaller  $\theta^*$  because it is computed by averaging over 15 degrees of latitude whereas  
 315 the Tibaldi Molteni over 50 degrees of latitude (35N to 85N). If we plot together the results for the  
 316 two indexes by rescaling them with the factor 10/3 (equivalent to the latitude ratio), we obtain good  
 317 agreement as shown in Fig. 7. Tests performed on low dimensional lattice systems indicate that the  
 318 averaging window have an impact on the magnitude of  $\theta^*$  but this will need to be further investigated to  
 319 get a complete understanding of the results. For the moment, we just note that by doing such a rescaling,  
 320 results are consistent over a wide range of longitudes except the Asian area.



**Fig. 5** Residual extremal index  $\theta^*$  for three different ranges of the bidimensional blocking index  $B$  defined by Pelly et al. Pelly and Hoskins (2003). **a:**  $\theta^*$  averaged over  $B > 0$  values corresponding to blocked flow regimes. **b:**  $\theta^*$  averaged over  $-10 < B < 0$  values corresponding to weak zonal flow regime. **c:**  $\theta^*$  averaged over  $B < -10$  values corresponding to strong zonal flow regime. See text for descriptions.



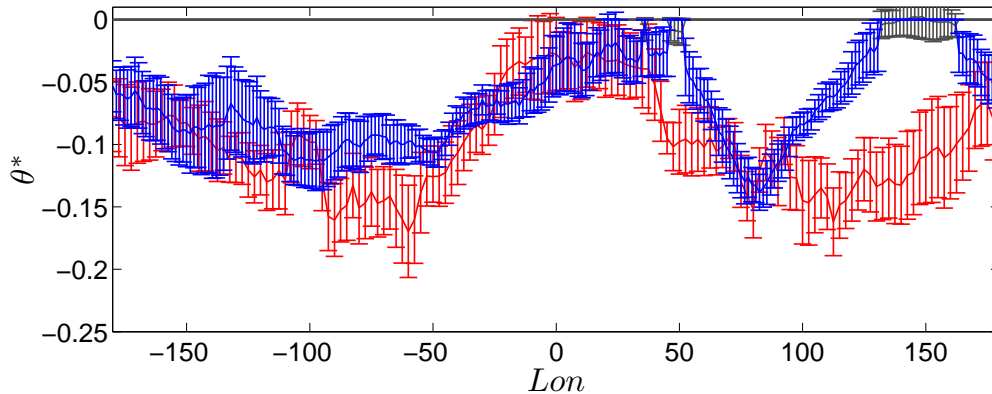
**Fig. 6 a:** Residual Extremal indices  $\theta^*$  versus longitude for the blocking index  $B$ . Negative values indicate the presence of unstable fixed points. **b:** Meridional average of  $\theta^*$  between 35N and 70N. The blue errorbars show the significant results at the 95% level and represent the standard deviation of the mean over different latitudes.

#### 321 4 Analysis of NAO and AO Atmospheric indices

322 Although local indices provide comprehensive geographical information about the atmospheric circula-  
 323 tion, one-dimensional indicators have been extensively used to characterize and forecast specific phenom-  
 324 ena (see Hurrell and Deser (2010) and references therein).

325 Over Europe, the transition between zonal and blocked atmospheric dynamics has been historically  
 326 associated to the North Atlantic Oscillation (NAO) index, originally defined as the difference in pressure  
 327 between Lisbon and Reykjavik (Hurrell 1995). Here we use the NOAA version of such index available via  
 328 (NOAA 2015) computed by using the Rotated Principal Component Analysis (RPCA) used by Barnston  
 329 and Livezey (1987). The positive phase of the NAO reflects below-normal heights and pressure across  
 330 the high latitudes of the North Atlantic and above-normal heights and pressure over the central North  
 331 Atlantic, the eastern United States and western Europe. The negative phase reflects an opposite pattern  
 332 of height and pressure anomalies over these regions (Hurrell et al 2003).

333 The Arctic Oscillation index (AO) is more representative of the blocking dynamics over the entire north-  
 334 ern hemisphere: it is constructed by projecting the daily 0 UTC 1000mb height anomalies pole-ward  
 335 of 20°N onto the leading mode of the Empirical Orthogonal Function (EOF) analysis of monthly mean  
 336 1000mb height during the years 1950-2014 (Thompson and Wallace 1998). Hence, the AO index behaves  
 337 like a zonal average of the Tibaldi-Molteni index. In the negative phase, the polar low pressure system  
 338 (also known as the polar vortex) over the Arctic is weaker, which results in weaker zonal flow. When the  
 339 AO is positive the polar circulation is stronger and forces cold air and storms to remain farther north.  
 340 The NAO and AO indices exhibit considerable inter-seasonal and inter-annual variability, and prolonged



**Fig. 7** Comparison between  $\theta^*$  obtained for the Tibaldi-Molteni index (red errorbars) and  $\frac{10}{3}\theta^*$  for the  $B$  index (blue errorbars for significant values at 95% and grey non significant values). The errorbars for the  $B$  index have been multiplied by a factor  $\sqrt{10/3}$ . The factor  $10/3$  is the ratio between the area considered to compute the Tibaldi Molteni index with respect to the  $B$  index.

341 periods of both positive and negative phases of the pattern are not rare (Thompson and Wallace 1998).  
 342 The daily NAO and AO data used in this paper are maintained by the US Climate Prediction Center  
 343 and can be downloaded via the NOAA website (NOAA 2015).

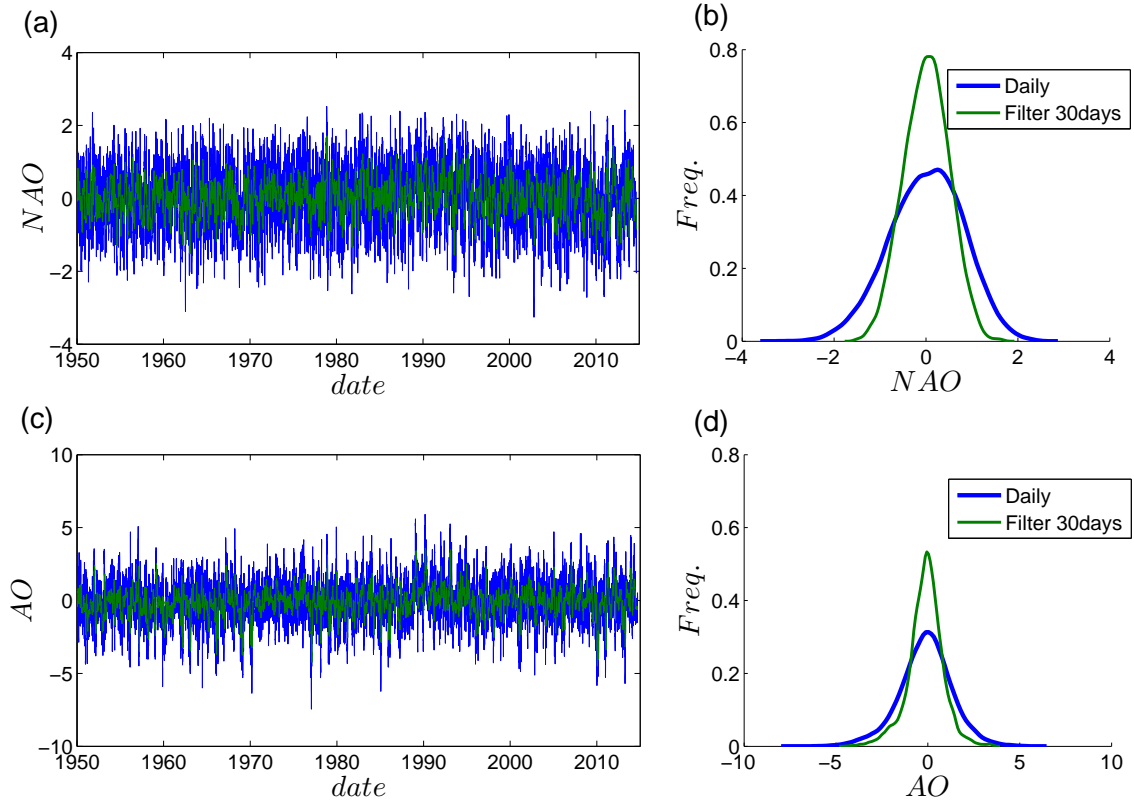
344 With the analysis of a one-dimensional index, we can explore the link between complex atmospheric dy-  
 345 namics and simple one-dimensional observables. A priori, there is no reason why the two indices should  
 346 provide the same information. The AO is a hemispheric average, the NAO a regional index. Even though  
 347 they are both one-dimensional time series, we will keep in mind their different origin.

348

349 We start by analysing the time series and the histograms shown in Fig. 8. The distributions of both  
 350 NAO and AO are unimodal and peaked around zero, roughly similar to a Gaussian. The indices seem to  
 351 spend most of their time around zero values with noisy fluctuations superimposed. A thirty-day moving  
 352 average filter (green curves) reproduces a unimodal histogram, and there is no evidence that the time  
 353 series oscillates between two states. It is therefore hard to recognize in the data any trace of bistability  
 354 or multistability.

355

356 Even if we look at single episodes, a dynamical structure compatible with the existence of an unstable  
 357 fixed point remains unclear: let us consider two examples of negative NAO and AO phases (Fig. 9a-d)  
 358 and one positive phase (Fig. 9e and Fig. 9f) recorded respectively for September 2002, December 2010  
 359 and January 1988. For the 2002, the negative phases of NAO and AO seem to be comparable with the  
 360 dynamics of the Hénon attractor around the fixed point  $\zeta_u$  (Fig. 1b)), but the duration and intensity of  
 361 the negative phases are different. In December 2010, Europe experienced a severe cold spell. Although the  
 362 NAO and the AO indices settle to negative values, the NAO oscillates over several values resembling that  
 363 of a chaotic variable, whereas the AO seems to cluster for consecutive days around values of  $-4$  and  $-2$ .  
 364 For January 1988, the behavior of the indices look much more chaotic with oscillations associated with  
 365 the mean lifetime of baroclinic structures (a few days). This latter regime can be compared to Fig.1c),

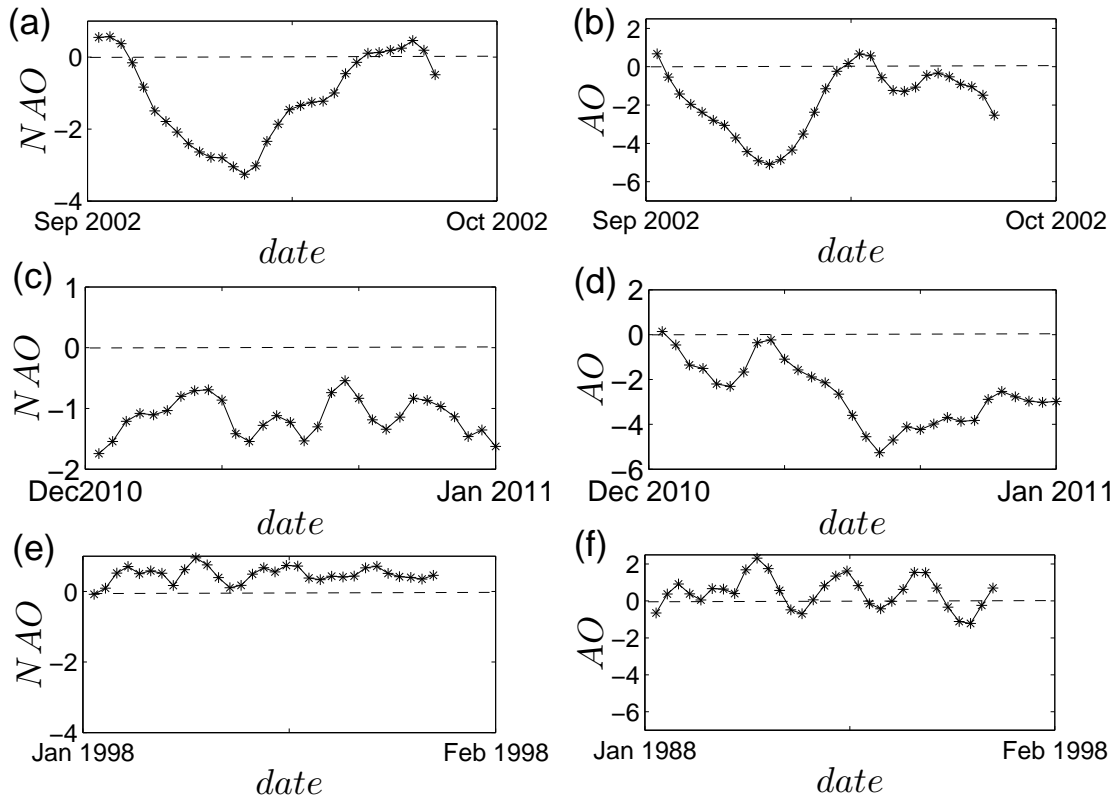


**Fig. 8** NAO (a) and AO (c) daily time series and their empirical density functions (b and d respectively). Blue: original dataset, green: moving average filter over a 30 day window.

366 i.e. with a typical point of the Hénon attractor. The contradicting results obtained for single episodes  
 367 imply that the existence of an unstable fixed point must be assessed statistically via the computation of  
 368  $\theta^*$ .

369 After applying the procedure described in Section 2-B, we obtain, for each value of the NAO and AO,  
 370 the residual extreme value index  $\theta^*$ , as shown in Fig. 10. We recall that for NAO and AO values around  
 371 zero there is no additional clustering, i.e. no unstable fixed points can be detected. The behavior of the  
 372 two indices is indeed different.  $\theta^*$  is negative for negative AO, following the core hypothesis of this paper  
 373 that blocked circulation can be associated with the existence of unstable fixed points. In contrast,  $\theta^*$  is  
 374 positive for positive NAO.

375 The relation between negative  $\theta^*$  and blocking for the AO can be explained if we look at the average  
 376 sea level pressure fields for the days such that  $AO < -4$ , i.e.  $\theta^* \simeq -0.15$ . This average is shown in Fig.  
 377 11a is compared against the average over the remaining days represented in Fig. 11b. They reproduce  
 378 respectively the blocked flow with an high over Iceland and a low pressure systems over the Azore Islands,  
 379 and the usual zonal flow with the Azore anticyclone and a low pressure located between Greenland and  
 380 Iceland.

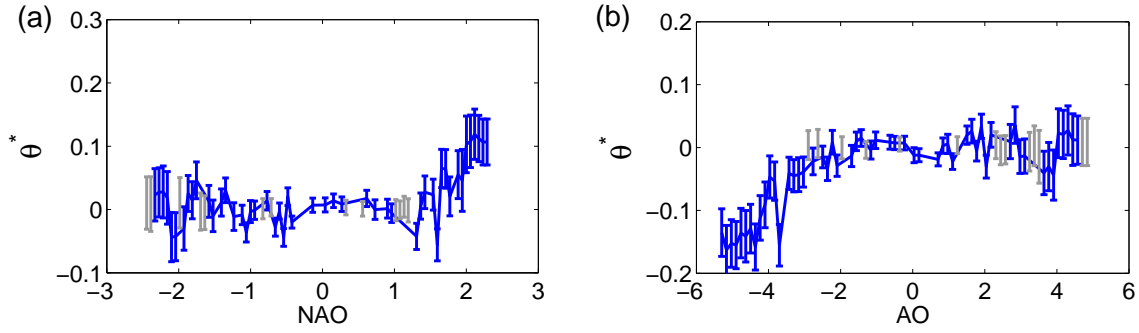


**Fig. 9** NAO (a-c-e) and AO (b-c-d) daily time series for some specific events. **a-b**: September 2002. **c-d**: December 2010. **e-f**: January 1988

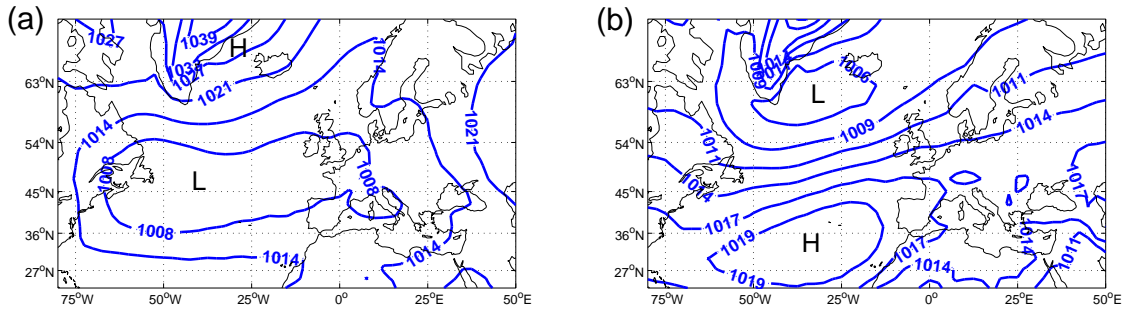
381 A priori, there is no reason why the two indices should follow the same behavior. Features of the NAO  
 382 are compatible with that observed for the Tibaldi-Molteni and the Pelly index around  $0^\circ$  longitude,  
 383 where the blocked flow was not associated with negative  $\theta^*$ . The disruption of clusters for positive NAO  
 384 corresponding to zonal flow is compatible with the presence of complex geography, which tends to destroy  
 385 the typical time scales of baroclinic instability (1.5 to 3 days). It is encouraging to find the trace of the  
 386 existence of unstable fixed points for the AO index, i.e., that the hemispheric average does not erase the  
 387 clustering properties found for  $B$  and  $B_S$ . This analysis indeed suggests that the  $AO$  is more sensitive  
 388 to blocking phenomena than the  $NAO$ . We thus can account for the empirical observation of (Ambaum  
 389 et al 2001).

## 390 5 Conclusion and Discussion

391 In this paper, we have adapted the concept of the extremal index, as applied in dynamical systems, to  
 392 the analysis of atmospheric indices which describe the switching between zonal flow to blocked flow in  
 393 the atmospheric mid-latitude circulation. We have presented evidence that the switching between atmo-  
 394 spheric and blocked circulation can be associated with the existence of an unstable (saddle) fixed point



**Fig. 10** Residual extremal index  $\theta^*$  for the NAO index (a) and the AO index (b). Error bars represent a standard deviation of the mean taken over the ensemble of 100 surrogates. Blue: significant values at 95% level. Grey: non significant values



**Fig. 11** a: Sea level pressure field averaged over all the days corresponding to the presence of unstable fixed point (AO < -4). b: sea level pressure field averaged over all remaining days.

395 of the atmospheric dynamics. The novelty of this approach lies in the use of observations, rather than  
 396 intermediate complexity models or GCM. Our results appear to be robust across blocking indices, and  
 397 are consistent with mid-latitude circulation mechanisms and local geography.

398

399 Such information is preserved in the AO time series, which is a sort of hemispheric average of the Tibaldi-  
 400 Molteni index. On the other hand, it shows that the NAO index does not keep track of the presence  
 401 of unstable fixed point. This result seems to contradict the fact that blocking events can be often be  
 402 tracked by a negative phase of the NAO index (Hurrell et al 2003). However, we remark that the operative  
 403 definition of blocking used in forecasts centers such as the NOAA strongly differs from the dynamical one  
 404 employed for the computation of unstable fixed points: the first definition requires both large negative  
 405 values of the NAO and the persistence for 5 days, the dynamical one requires the persistence of the same  
 406 negative values for several days, independently on its magnitude. Moreover, as detailed in (Memory  
 407 1991), when partial differential equations govern the dynamics of a system, the presence of an unstable  
 408 fixed point may trigger instabilities which propagate in space, thus creating long range interactions (tele-  
 409 connections) between the initial location of unstable fixed point and the region affected. Our analysis  
 410 seems to point to one of the mechanisms invoking non-linear interactions, either between zonal flow and  
 411 eddies or between planetary waves (Charney and DeVore 1979; Egger 1978; Kung et al 1990; Christensen

and Wiin-Nielsen 1996) as the driving mechanisms for the blocking phenomena. The complex spatial distribution of unstable fixed points let us discard the possibility that simple bi-stability mechanisms could explain the transitions between blocking and zonal flow, as proposed in (Tung and Lindzen 1979; Shutts 1983, 1986).

This paper also suggests a novel approach to statistical and dynamical modeling: in Masato et al (2009) the properties of blocking indices have been widely investigated and compared to the statistics of stationary red noise process. The authors argued that the statistical model was not sufficient to describe the characteristics of blocking and claimed that the persistence beyond that given by a red noise model is due to the self-sustaining nature of the blocking phenomenon. Here, we have shed light on this self-sustaining nature: When the circulation settles in a blocked regime, the presence of unstable fixed points gives rise to the persistence (the self-sustaining nature) of quasi-stationary conditions. In order to improve the statistical modeling of blocking phenomena, one has to account for local clustering effects in statistical models, which is a non-trivial challenge.

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