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Optimal coordinated lateral control of a UAV formation using a virtual structure approach

Horri N.M.¹, Dahia K.²

¹Coventry university, Department of aerospace and electrical engineering, Coventry, UK.
²ONERA (Office National d’Etudes et de Recherches Aérospatiales), Palaiseau, France.

Abstract: The ability to fly formations of multiple Unmanned Aerial Vehicles (UAVs) is increasingly being considered as a way of enabling a wealth of new applications, from multi observer unambiguous digital elevation mapping to interferometric imaging and receiver localization, to name a few. The main challenges in formation flying are associated with the need for fine coordinated guidance, with fine navigation and control of the absolute and relative positions and orientations of each vehicle. This paper addresses the coordinated lateral control problem of achieving an optimal tradeoff between formation keeping accuracy and energy consumption. The control approach uses a semi-autonomous virtual structure approach for formation control, where the absolute station keeping objective of the formation is ground planned from mission requirements and the UAVs autonomously control their two dimensional relative lateral positions and orientations to maintain a virtual structure. The virtual structure is chosen to be a triangular formation of three UAVs, without loss of generality. Two formation control strategies are compared. The first approach uses a nonlinear Lyapunov stabilizing controller. The second one uses feedback linearization laws, to which linear model predictive (MPC) controllers are superposed to optimize the formation keeping autopilots, in the presence of controller constraints. The feedback linearized MPC controller is easy to implement and outperforms the nonlinear controller in terms of settling time, under the same controller constraints.

Key words: Control, virtual structure, UAV formation, optimal, MPC.

I. INTRODUCTION

Formation flying of Unmanned Aerial Vehicles (UAVS) is creating new prospects and enhancing mission efficiency and by adding new synchronized and multi-observer imaging capabilities to a wide range of applications from digital elevation mapping for land surveys, search and rescue operations, remote sensing, disaster monitoring to security, defense and localization applications. Recent research is directed towards developing autonomous formation flying methods to reduce the amount of ground processing. [1], [2], [3] [4] Autonomy often implies decentralized or consensus based methods, without a hierarchy in the formation. [5] However most attempts to demonstrate formation flying assume that there is a formation or some form of hierarchy through a nearest neighbor tracking approach, but this makes the flight control performance of a UAV dependent on the others.

The approach being considered here consists of a virtual structure approach which originates from spacecraft formation flying applications. [5], [6] [7] The virtual structure is taken to be a triangular formation in hover mode. The guidance, navigation and control system uses of a
centralised guidance approach for stationkeeping. Formation keeping is however autonomously performed onboard the UAVs, which control their relative motions with respect to the desired formation configuration and trajectory. The formation can then conceptually be ground operated as a virtual rigid structure, as if it consisted of a single aircraft structure. The station keeping objective under consideration here is the lateral trajectory control of multiple concentric desired trajectories for a formation of three UAVs. Target encirclement is indeed a common objective with single UAVs [8] and is extended here to a triangular UAV formation scenario. Collision risks are reduced by design by increasing the separations between the concentric orbits. Reconfiguration maneuvers would be required in practice to enable various exploration radii, but are beyond the scope of this paper. The individual UAVs control their lateral positions with respect to the centre of gravity of the formation, with a synchronized heading control objective. The relative positions are coupled with the relative orientations, particularly in roll, which directly affect the heading using bank angle controlled autopilots.

The mathematical model of the UAV formation is nonlinear. Two different approaches are considered for the coordinated feedback control. The first one uses conventional Lyapunov design to synthesize the nonlinear trajectory controllers. Nonlinear control is often considered in formation flying due to the need for robustness to model uncertainty and other perturbations. [9] A second controller is then introduced, consisting of a feedback linearizing term [10] which is superposed to a model predictive controller (MPC) for relative motion control (formation keeping). MPC control is known to be an efficient implementation of optimal trajectory tracking control under controller constraints, where the deviation with respect to the desired trajectory is continuously predicted and corrected over shifting time intervals until the full maneuver is completed. [11] [12] [13]. The comparative simulation analysis shows that the feedback linearizing MPC controller outperforms standard nonlinear controllers after incorporating controller constraints.

II. UAV FORMATION CONFIGURATION

All UAVs have a desired position and orientation requirement with respect to a virtual structure reference frame, of which the centre is the centre of gravity of the formation and the unit vectors are defined as follows:
The axes X, Y, Z in figure 1 denote the principal axes of the virtual structure. The relative orientations of the UAVs consist of the orientations of their body axes with respect to the axes X, Y and Z. The number of UAVs that can be chosen is arbitrary, but in the numerical simulation, a triangular formation of 3 UAVs will be considered.

### III. EQUATIONS OF MOTION

The lateral motion of the UAVs can be described by the set of equations

\[
\begin{align*}
    \dot{x}_i &= U_i \cos \psi_i \\
    \dot{y}_i &= U_i \sin \psi_i \\
    \dot{\psi}_i &= \frac{g_0}{U_i} \tan \Phi_i \\
\end{align*}
\]  

(`eq.1`)  

where \( x_i, y_i \) denote the Cartesian coordinates and \( \psi_i \) denotes the heading angle of UAV \( i \).

The variables \( U_i \) and \( \Phi_i \), respectively representing the aircraft longitudinal speed and the bank angle, can be seen as virtual inputs to this lateral aircraft control problem. Both \( U \) and \( \Phi \) are obtained from standard inner loop autopilots: The auto-throttle and bank angle autopilots, respectively, using the aircraft’s engine thrust and ailerons. In the simulation analysis, perfect tracking of the speed and bank angle commands is assumed. The UAVs are assumed to fly at the same altitude using the same standard altitude hold autopilot. The dynamics for the position and heading of the fleet of fixed wing UAVs are given by:

\[
\begin{align*}
    \dot{x}_{ci} &= U_{ci} \cos \psi_{ci} \\
    \dot{y}_{ci} &= U_{ci} \sin \psi_{ci} \\
    \dot{\psi}_{ci} &= \Omega_i \\
\end{align*}
\]  

(`eq.2`)  

\[
\begin{align*}
    x_e = x_i - x_{ci} \\
    y_e = y_i - y_{ci}
\end{align*}
\]  

(`eq.3`)
The aircraft formation guidance is managed by imposing an initial phase shift of 120 degrees between the three UAVs such that $\psi_{c1} = 0$, $\psi_{c2} = 120$ degrees, $\psi_{c3} = 240$ degrees.

For each UAV, a heading error angle variable is introduced as follows:

$$\psi_{ei} = \psi - \psi_{ci}$$

### IV. Controller designs

#### A. Lyapunov stabilizing controller design

We consider the following expression of the Lyapunov function:

$$V_i = V_{i1} + V_{i2}$$

$$V_{i1} = \frac{1}{2}(x_{ei}^2 + y_{ei}^2)$$

$$V_{i2} = \frac{1}{2} \psi_{ei}^2$$

The time derivative of this Lyapunov function can be decomposed into two terms. The first one depends on the positions and can be expanded as follows:

$$\dot{V}_{i1} = x_{ei}\dot{x}_{ei} + y_{ei}\dot{y}_{ei}$$

The latter expression can be rearranged as follows:

$$= U(x_e \cos \psi + y_e \sin \psi) - U_c(x_e \cos \psi_c + y_e \sin \psi_c)$$

For the contribution to the Lyapunov function due to the heading angle, we use a backstepping approach, take $\psi = \psi_c$ (controlled using $\Phi$) and a condition on $U$ to track the desired $x_c, y_c$ trajectories exponentially is given by the control law:

$$U = -K(x_e \cos \psi + y_e \sin \psi) + U_c$$

$$\dot{V}_{i2} = \psi_{ei}\dot{\psi}_{ei} = \psi_{e} \left( \frac{q}{U} \tan \Phi - \Omega \right)$$

A condition on the bank angle command $\Phi$ to track the desired heading exponentially is to adopt the control law:

$$\tan \Phi = -K_2 \psi_{e} + \Omega \Rightarrow \Phi = \arctan(-K_2 \psi_{e} + \Omega)$$

#### B. Feedback linearization to enable linear MPC control

For each UAV, the lateral dynamics can be written compactly as:

$$\dot{x}_1 = u_1 \sin \psi$$

$$\dot{x}_2 = u_1 \cos \psi$$

$$\dot{\psi} = u_2$$

(eq.10)
where $u_2$ is a newly defined variable:
\[ u_2 = \frac{g_0}{\bar{u}} \tan \phi_i \]  
(eq.11)

With $\mathbf{x} = [x_1, x_2, \psi]^T = [x, y, \psi]^T$

The dynamic model can be written in vector form as:
\[ \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \cos \psi & 0 \\ \sin \psi & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \]  
(eq.12)

\[ x_3 = u_1, \dot{x}_3 = u_3 \]

By taking the second derivative of the position vector, the acceleration commands appear in the right hand side of the equation:
\[ \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} \cos \psi & -u_1 \sin \psi \\ \sin \psi & u_1 \cos \psi \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ u_2 \end{bmatrix} \]  
(eq.13)

The following change of variables is used for feedback linearization of the system:
\[ \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} = \begin{bmatrix} \cos \psi & -u_1 \sin \psi \\ \sin \psi & u_1 \cos \psi \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \]  
(eq.14)

The closed loop model then consists of two double integrators:
\[ \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} \]
\[ \begin{bmatrix} \ddot{u}_2 \\ \ddot{u}_3 \end{bmatrix} = H^{-1} \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} \]  
(eq.15)

Where $H = \begin{bmatrix} \cos \psi & -u_1 \sin \psi \\ \sin \psi & u_1 \cos \psi \end{bmatrix}$ is non singular for $u_1 \neq 0$

A conventional output feedback controller could consist of a PD feedback of $y_1, y_2$ but the approach under consideration here

**C. Linear MPC control methodology**

The optimization problem under consideration is the minimization of a quadratic cost function under control effort limitations:
\[ J_i = \int_0^\infty \left( x_i^T Q x_i + u_i^T R u_i \right) dt \]
\[ \text{st.} \left| u_i \right| < u_{i,\text{MAX}} \]  
(eq.16)

MPC also enables the incorporation of dynamical constraints, but this is beyond the scope of the paper. The predictions of the control and control correction vectors over the control horizon $N_c$ are given by:
\[ U_{\alpha}(k) = [u(k, k) \ldots u(k + N_c - 1/k)]^T, \Delta U_{\alpha}(k) = [\Delta u(k, k) \ldots \Delta u(k + N_c - 1/k)]^T, \]
The predicted states at time can then be obtained from the predicted control inputs $u(k+i/k)$, with $i=0,Nc-1$:

$$\mathbf{x}(k + j/k) = A_j^{\top}\mathbf{x}(k) + \left[A^j A^{j+1} \cdots A^{j+N} \right]B_c U_{c1}(k), \quad j=1,N$$

$$U_{c1}(k) = U_{c1}(k-1) + \Delta U_{c1}(k) \quad \text{(eq.17)}$$

The predicted state vector $\mathbf{X}_p(k) = [\mathbf{x}(k+1,k) \cdots \mathbf{x}(k+N/k)]^\top$ over the prediction horizon $N$ is given by:

$$\mathbf{X}_p(k) = \Phi \mathbf{x}(k) + \Gamma u(k-1) + G \Delta u_c(k) \quad \text{(eq.18)}$$

where the full predictive state and control matrices are given by:

$$\Phi(j) = A^j, \Gamma(j) = \sum_{i=0}^{j-1} A^i B, \quad j=1,N$$

$$G(j) = \left[\sum_{i=0}^{j-1} A^i B \cdots \sum_{i=0}^{N_c-1} A^i B\right], \quad j = N_c, N$$

The cost function can now be written as:

$$J = \sum_{j=0}^{N_c-1} \|X(k + j/k) - X_{ref}(k)\|^2_Q + \sum_{j=0}^{N_c-1} \|\Delta U(k + j/k)\|^2_R \quad \text{(eq.19)}$$

where $Q$, $R$ matrices are weighting matrices for the norms of the state, control and control variation vectors. The proposed MPC controller uses Rossiter’s toolbox, which implements the quadratic programming task.

V. NUMERICAL SIMULATION ANALYSIS

In this numerical simulation section, the desired trajectories are taken to be concentric orbits for three miniature fixed wing aircraft, without loss of generality. The concentric trajectories are a simple way of reducing collision risk by design. The assumed separations are taken to be small, but large enough to theoretically avoid collision assuming wing spans of 0.5m. The concentric orbits can easily be scaled for a given imaging application requirement. It is also noteworthy that collision, which is not the focus of this investigation, is not only theoretically avoided through the separations but also because of the phase shifts when motion is synchronised. In a practical scenario, an application dependent margin should be added to increase the separation distances. The focus of the paper was on developing a controller design for a hovering formation scenario for observations from multiple directions of a target.

The desired formation configuration generated by applying the guidance equations of eq.2 is a triangular formation that sweeps a full rotation in 2D space as shown in figures 2 and 3 and 4. Figure 2
shows that the positions of the UAVs are oscillating about a region of interest because the desired orbits are concentric. Using the relative motion requirements that UAV1 is shifted by +0.5m on the X axis and +0.6m on the Y axis with respect to the CoG of the formation, UAV2 and UAV3 are respectively shifted by 120 and 240 degrees on an equilateral triangle. This is similar to the conditions used in reference [11]. The phase was successfully tracked and the plot not shown due to space limitations.

The UAV formation configuration obtained by time propagation of the trajectories is a triangular structure, as shown in figure 3. The actuator constraints are applied such that \( |u_1| < 1, |u_2| < 8 \), approximately corresponding to \( |u_1| < 10\left(\frac{m}{s}\right), |u_2| < 8 \) (after conversion to degrees) in the MPC optimal control problem formulation. The same constraint is then imposed to the nonlinear controller of equations (9), (10). The time responses of the controllers are shown for UAV-1 to meet paper length requirements, but the simulations and their analysis for UAVs 2 and 3 is very similar. The MPC control inputs are shown in figure (4).

The trajectory tracking using MPC under these constraints leads to an overshoot but the trajectory tracking is successful. Using nonlinear control, the inherent robustness of Lyapunov design also ensures trajectory tracking under the same constraints. However, the tracking performance is deteriorated as the system goes through higher level oscillations for a longer time before stabilising, under the same constraints as MPC. Linear MPC together with feedback linearisation therefore constitutes a simple way to enhance tracking efficiency compared to conventional nonlinear tracking control.

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**Fig. 2: Concentric formation trajectories of 3 UAVs**
Fig. 3: Propagated hovering virtual structure triangular formation trajectories

Fig. 4: Actuator control inputs of UAV-1 (Speed and bank angle)
Fig. 5: Constrained nonlinear controller trajectories of UAV-1

Fig. 6: MPC controlled trajectories of UAV-1
VI. CONCLUSION

A virtual structure approach was used to plan the lateral motion of a UAV formation to enable a hover mode with three observers forming a triangular structure, without loss of generality. A Lyapunov stabilizing controller was successfully designed and implemented for trajectory tracking. The tracking performance was subsequently improved under the same controller constraints, by superposing a linear MPC controller to a nonlinear feedback linearizing controller. A nonlinear extension of the proposed MPC controller is needed to allow for a more optimal response with larger initial deviations from the desired trajectory.

VII. REFERENCES