

Experimental and Numerical Study of Interfacial Fracture Parameters of a Brazed Joints

Ahlem Baazaoui, Thibaut Fourcade, Olivier Dalverny, Joël Alexis, Moussa Karama

▶ To cite this version:

Ahlem Baazaoui, Thibaut Fourcade, Olivier Dalverny, Joël Alexis, Moussa Karama. Experimental and Numerical Study of Interfacial Fracture Parameters of a Brazed Joints. Advanced Materials Research, 2015, vol. 1099, pp. 9-16. 10.4028/www.scientific.net/AMR.1099.9 . hal-01135046

HAL Id: hal-01135046

https://hal.science/hal-01135046

Submitted on 24 Mar 2015

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers. L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Open Archive Toulouse Archive Ouverte (OATAO)

OATAO is an open access repository that collects the work of Toulouse researchers and makes it freely available over the web where possible.

This is an author-deposited version published in: http://oatao.univ-toulouse.fr/
Eprints ID: 13549

Identification number: DOI: 10.4028/www.scientific.net/AMR.1099.9 Official URL: http://dx.doi.org/10.4028/www.scientific.net/AMR.1099.9

To cite this version:

Baazaoui, Ahlem and Fourcade, Thibaut and Dalverny, Olivier and Alexis, Joël and Karama, Moussa *Experimental and Numerical Study of Interfacial Fracture Parameters of a Brazed Joints.* (2015) Advanced Materials Research, vol. 1099. pp. 9-16. ISSN 1022-6680

Experimental and Numerical Study of Interfacial Fracture Parameters of a Brazed Joints

A. Baazaoui^{1,a}, T. Fourcade^{1,b}, O. Dalverny^{1,c}, J. Alexis^{1,d} and M. Karama^{1,e}

¹University of Toulouse; INP/ENIT; LGP; 47, avenue d'Azereix; F-65013 Tarbes, France

^aahlem.baazaoui@enit.fr, ^bthibaut.fourcade@enit.fr, ^colivier.dalverny@enit.fr, ^djoel.alexis@enit.fr,

^emoussa.karama@enit.fr

Keywords: AuGe solder, shear test, fractography, thermomechanical, interfacial fracture, cohesive zone model, finite element analysis.

Abstract. This paper deals with an identification methodology of the interfacial fracture parameters to predict the lifetime of a metallic brazed joint. The methodology is based on an experimental-numerical study whereby the optimal parameters are obtained. The experimental data, using the scanning electron microscope analysis, allowed approving that failure of the assembly based AuGe solder seems first to appear near the interfaces. These results were confirmed by micrographs analysis of the solder/insert and solder/substrate interfaces. Then, using shear test results and parametric identification coupled with a finite elements model (FEM) simulation, the damage constitutive law of the interfacial fracture based on a bilinear cohesive zone model are identified. The agreement between the numerical results and the experimental data shows the applicability of the cohesive zone model to fatigue crack growth analysis and life estimation of brazed joints.

Introduction

Solder joint failure, either due to thermal loads or mechanical loads, is a significant reliability concern in power electronic packaging. The majority of the research show that failure seems first to appear near the solder/die and/or solder/ceramic substrate interfaces (Ren et al., 2013) (Baazaoui et al., SF2M2013) (Baazaoui et al., 2013) (Müge, 2007). These failure modes are mainly due to the presence of a high porosity ratio (Msolli et al., 2012) and/or brittle intermetallic compounds (Müge, 2007) (Baazaoui et al., CFM2013) (Yao et al., 2008). To understand the interfacial cracking phenomena in adhesive joints and thus to optimize the design of high temperature packaging, cohesive zone models (CZM) (Crisfield & Alfano , 2001) (Davila & Camanho, 2002) (Li et al., 2005) have been widely used.

The CZM can be considered as an improvement of the Griffith theory (linear fracture mechanics). Indeed, the CZM is able to describe the entire fracture process including crack initiation and propagation and taking into account the presence of a process zone upstream developing the crack tip. Since the developing works by Barenblatt (Barenblatt, 1962) and Dugdale (Dugdale, 1996), many cohesive zone models have been proposed in the literature (Tvergaard & Hutchinson, 1992) (Needleman & Xu, 1994) (Ortiz & Camacho, 1996) (Zavattieri & Espinosab, 2001). All of them start from the assumption that one or more interfaces can be defined, where crack growth is allowed by the introduction of a possible discontinuity in the displacement field. This interfacial fracture has been modeled successfully in many applications, including the power electronic packaging, using the cohesive zone approach based on a traction-separation law (Needleman, 1987). This law is defined the relation between the traction load exerted by the joining layer as a function of the separation between the fracture surfaces or across the entire layer. For pure-mode, three important parameters governing a traction-separation law are the cohesive element stiffness K, the maximum traction stress t^0 and the energy dissipation at failure G^C . The governing law for a specific interface is often found by assuming a relation of a certain form and then matching a finite element simulation with experimental observations.

In this work, we propose to study and characterize the interfacial fracture behaviour of a metallic brazed joint. The first part of the study deals with the experimental characterisation performing the

connection process of copper inserts using Au88Ge12 eutectic solder alloy, then the shear test based on the image correlation followed by a fractography analysis. In the second part, the constitutive law of the bilinear cohesive zone model and the finite element modelling of the assembly are presented. Finally, the estimation of the bilinear CZM parameters were performed based on an experimental-numerical methodology.

Experimental Characterizations of Brazed Joints

Experimental Procedure. The assembly is constituted by copper inserts of dimensions $4.7 \times 4.7 \times 2$ mm which are bonded on copper substrates of dimensions $30 \times 10 \times 2$ mm at a temperature of 420° C using the eutectic solder alloy Au88Ge12. The multilayer structure are presented in the Fig 1.a. More details in specimen elaboration has been demonstrated in elsewhere (Baazaoui et al., SF2M2013) (Baazaoui et al., CFM2013). Notably, reference mark with dark color, which represented by the points A and B an shown in Fig. 1.b, is bonded to measure the displacement of the assembly and the displacement here is defined as the variation of non-contact measurement reference marks along the top surface of the inserts under shear loads.

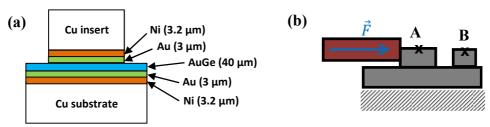


Fig. 1. Sample preparation, a) multilayer structure of the bonding process, b) testing configuration of the AuGe specimen.

To determine the load-displacement behaviour of the assembly, displacement-controlled shear tests with a constant rate of 1 mm/min were operated at room temperature by INSTRON apparatus dedicated to perform shear tests on electronic devices. Meanwhile, an innovative non-contact detecting system using the correlation camera was introduced to measure displacement of the assembly. The database of the experimental results obtained is used for identification of the interfacial fracture parameters of the brazed joint.

Experimental result analysis. Fig.2 shows the shear test curves obtained by INSTRON apparatus (Fig. 2.a) and video displacement measurement (Fig. 2.b). For both load-displacement results, the maximal applied load exceeds 1050 N. The crosshead and real displacement at failure reaches 1.27 mm and 0.136 mm, respectively. The latter result (Fig. 2.b) defines the accumulation displacements of the AuGe brazed joint and the two Cu/solder interfaces and the extra displacement in the first curve (Fig. 2.a) is induced by the load frame, load cell and load clips.

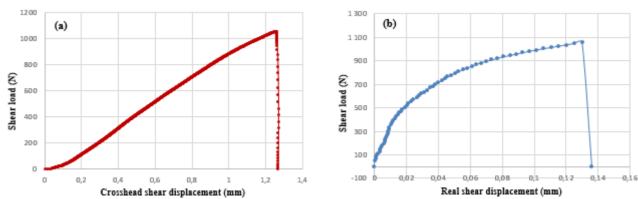


Fig. 2. Shear test results for 1 mm/min of a) effective apparatus results and b) correlation camera results.

The load-real displacement behaviour obtained by the camera correlation show an important and non-linear plastic deformation. Knowing the maximal load and the insert surface, the shear strength

of this assembly is 47.5 MPa approximately. Another studies of the AuGe solder was performed by Hosking et al. (Hosking et al., 1999) and they show that the AuGe tensile strength is about 200 MPa which corresponds to a shear strength of $\frac{200}{\sqrt{3}} = 115.47$ MPa. Compared with this latter result, the obtained shear strength of 47.5 MPa in our study is very low. Indeed, the failure seems first to appear in the Cu/substrate interfaces. To verify these assumptions, the failure zones for the AuGe specimen were analyzed using the scanning electron microscope (Fig. 3). The EDX analysis of the specimen failure shows the presence of the AuGe and Ni. The micrographs and EDX analysis of AuGe broken specimen illustrate a mixed fracture: adhesive and brittle failure in the two Cu/solder interfaces and cohesive and ductile failure in the joint.

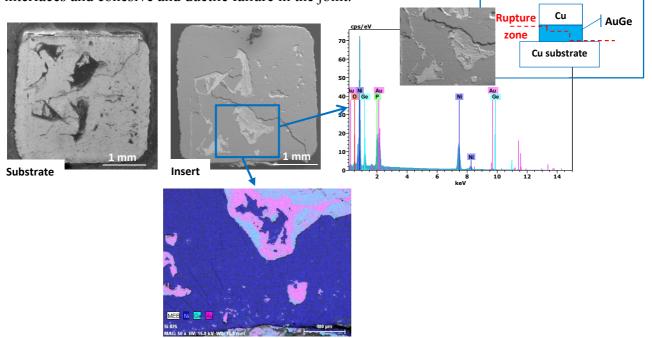


Fig. 3. SEM Micrographs of AuGe specimen failure and EDX analysis of the same zone.

Numerical simulations of the interfacial fracture

To simulate the behaviour and failure mechanism of solder interconnects under the shear loading, a fracture mechanics criteria based cohesive zone model was employed. The behaviour laws of cohesive elements are usually described in terms of traction versus separation $t_i(\delta_i)$ which related the interface's relative displacement δ_i to the traction vector t_i (Fig. 4). Generally, the cohesive elements behave elastically until damage initiation and obey a softening behaviour afterwards. The specific energy dissipated by the cohesive element G_i can be calculated from the area under the t_i - δ_i curve. Different constitutive laws of the cohesive zone model were proposed in the literature (Needleman, 1987) (Tvergaard & Hutchinson, 1992) (Needleman & Xu, 1994) (Ortiz & Camacho, 1996) (Zavattieri & Espinosab, 2001) (Hutchinson & Tvergaard, 2002). Since the shape of the function $t_i(\delta_i)$ may influence the results of the simulation (Alfano, 2006) (Scheider, 2009), it is crucial to identify a law that is suitable for capturing the interfacial fracture extension behaviour of the cohesive layers. According to the nature of the material (ductile, brittle,...), the type of loading considered (monotonic or cyclic loadings), the cohesive zone model can be defined. The fractography results of the AuGe solder based assembly carried out in the first section have been shown a brittle damage in the Cu/solder interfaces. To predict the interfacial and brittle damage behaviour observed experimentally, the irreversible and bilinear cohesive zone model (Alfano & Criesfield, 2001) (Camanho & Davila, 2002) may apply. This bilinear cohesive zone model is currently the most widely used for interfacial fracture behaviour simulation due to its availability in ABAQUS® software. Thus, it has been widely used to characterize the interfacial damage behaviour in power electronic packaging (Ren et al., 2013) (Yao & Keer, 2013). These studies have shown the performance of the bilinear cohesive zone methods for describing and predicting fracture

initiation and propagation at the interfaces of electronic assemblies. They also approved the convergence between the numerical simulation and experimental results.

Constitutive law of the cohesive zone model. For pure-mode I and pure-mode II or mode III loading, the shape of the constitutive bilinear CZM law is shown in Fig. 4.b. This available traction-separation law (TSL) is to assume the linear elastic behaviour followed by the initiation and evolution of damage. A high initial stiffness K is used to hold the top and bottom faces of the decohesion element together in the linear elastic range. Once the peak values of the normal or shear nominal stress are attained, the stiffnesses are gradually reduced to zero. The analytical expression of the bilinear cohesive model law is as follows (Camanho & Davila, 2002) (Alfano, 2006):

$$t_{i} = \begin{cases} K \delta_{i} & \text{if } \delta_{i} \leq \delta_{i}^{0} \\ (1 - D_{i}) K \delta_{i} & \text{if } \delta_{i}^{0} < \delta_{i} < \delta_{i}^{f} \\ 0 & \text{if } \delta_{i} \geq \delta_{i}^{f} \end{cases}$$
(Eq. 1)

$$D_i = \frac{\delta_i^f (\delta_i^{max} - \delta_i^0)}{\delta_i^{max} (\delta_i^f - \delta_i^0)}$$
(Eq. 2)

Where δ_i^0 and δ_i^f represent the relative displacement at damage initiation and complete failure as shown in Fig. 5, respectively. δ_i^{max} refers to maximum value of the relative displacement attained during the loading history. δ_i^0 and δ_i^f are given by (Alfano, 2006):

during the loading history.
$$\delta_i^0$$
 and δ_i^f are given by (Alfano, 2006):

$$\delta_i^0 = \frac{t_i^0}{K} \quad \text{and } \delta_i^f = \frac{2 G^C}{t_i^0}$$
(Eq. 3)

 G^{C} represents the fracture energy of the cohesive elements.

A fundamental aspect in the formulation of the cohesive constitutive model is the requirement that the energy dissipated at a fracture propagation must be equal to the fracture energy, the following relation (Eq. 3) must be satisfied:

$$G_i = \int_0^{\delta_i^f} t_i d\delta_i = G^C$$
 (Eq. 4)

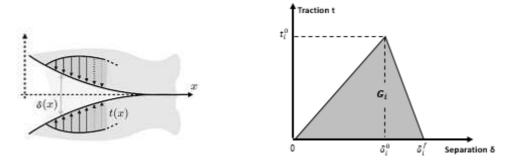


Fig. 4. a) Configuration of the cohesive fracture, b) TSL for bilinear cohesive zone model.

In this paper, only pure-mode II debonding problem is considered so that the analysis is simplified by focusing the attention on scalar interface relationships relating one relative-displacement component δ to the dual stress component t. In this case, the CZM characteristic parameters are the cohesive element stiffness K or the cohesive layer modulus E, the fracture energy G^C and the strength t^0 .

Cohesive zone modeling of the interfacial fracture. To simulate the behaviour and failure mechanism of AuGe solder interconnects under shear loading, a 2D finite element model has been developed incorporating the cohesive zone model which defined in the previous section and using the commercial finite element software ABAQUS®. The geometrical and boundary conditions are shown in Fig. 5. This model is composed of copper insert, copper substrate which are bonded together with the eutectic solder alloy Au88Ge12, two cohesive layers and a rigid shear tool. The two cohesive zone layers are placed in the solder/insert and solder/substrate interfaces, where

fatigue failure is typically observed. The thickness of the cohesive and joint layers, measured experimentally, is about 4 and 50 μ m, respectively. The two cohesive layers were modeled using a 4-nodes bilinear quadrilateral element of type 2D cohesive (COH2D4). The length of the cohesive elements was 100 μ m. In the thickness, the cohesive zone must be discretized with a single layer of cohesive elements (Needleman & Xu, Numerical simulations of fast crack growth in brittle solids, 1994). In this case, the path of the cracking is naturally determined by the overall field and the cohesive response. The solder, the insert and the substrate layers were meshed with a 4-node bilinear 2D plane stress quadrilateral elements (CPS4R). The size of the elements in the solder is the same as the cohesive elements (100 μ m) in the length and 10 μ m in the thickness.

Table 1. Properties of the AuGe and copper materials used for assembly (Hosking et al., 1999) (Msolli, 2011).

Properties	AuGe	Copper
Elastic modulus (GPa)	69.2	128
Poisson's ratio	0.32	0.36
Yield strength (MPa)	160	233
Tensile strength at failure (MPa)	200	268
Thermal conductivity (W.m ⁻¹ .K ⁻¹)	44.4	398
Mass density (Kg.m ⁻³)	14670	8850
Specific heat capacity (J.Kg ⁻¹ .K ⁻¹)	134	380
Thermal expansion coefficient (µm.m ⁻¹ .K ⁻¹)	12.3	17.3

In the current model, we supposed that the two cohesive layers have the same interfacial properties and an isotropic mechanical behaviour. The procedure used to determine the CZM law's parameters is explained in the next section. The AuGe solder and the copper insert and substrate were considered to have elastoplastic behaviour. All mechanical proporties for AuGe and copper materials were respectively listed in the Table 1. The plastic behaviour of the AuGe solder and the copper substrates was taken from the experimental results of Hosking et al. and S. Msolli, respectively (Hosking et al., 1999) (Msolli, 2011). This assembly is subjected to shear loading with a displacement rate of 1 mm/min and at a temperature room of 20°C. The displacement rate is applied in a reference point located in the rigid shear tool.

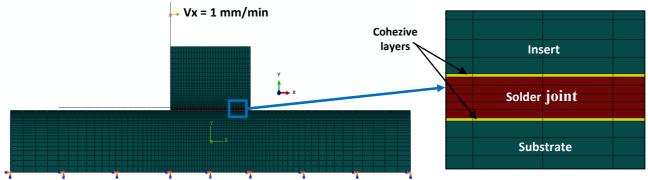


Fig. 5. Finite element model of sandwiched assembly joined by AuGe solder.

Identification procedure method of the interfacial constitutive law

Identification algorithm. One of the biggest challenges faced by researchers who work with cohesive zone models is to determine the model parameters because standard experimental tests do not exist (Bhate et al., 2007). As a matter of fact, the CZM links the microstructural failure mechanisms to the displacement fields governing bulk deformations. Thus and as mentioned before, a cohesive zone is characterized by the properties of the material, the damage initiation condition and the damage evolution function. As defined in the previous section, the considered constitutive law is the bilinear cohesive zone model. In this case, the properties of the interfacial behaviour required to be identified are the cohesive element stiffness K or the cohesive layer modulus E for the elastic behaviour and the strength t^0 and the fracture thoughness G^C for the damage constitutive

law. In this study, we considered the modulus of elasticity as known during the identification procedure and it is supposed to be the same as the joint layer (69.2 GPa). Thus, the number of the CZM parameters to identify is 2; the fracture energy and the strength of the interfaces layer. The identification of this two interfacial fracture parameters was achieved using the results from the shear test and corresponds to the resolution of an inverse identification problem. A usual methods of resolving such a problem is summarized in Fig. 6 (Fourcade et al., 2014).

The input data from which identification is done are the experimental shear test results, the behavior law of the solder/insert and solder/substrate interfaces associated with a set of initial parameter and the numerical model of the test. For a set of parameters \underline{x} , a difference vector $\underline{e}(\underline{x})$ is then calculated as:

$$\underline{e(\underline{x})} = \{\alpha_1(\underline{x}), \dots, \alpha_i(\underline{x}), \alpha_n(\underline{x})\}$$
 (Eq. 5)

Where $\alpha_i(\underline{x}) = \omega_e \frac{P_i^{EF}(\underline{x}) - P_i^{exp}}{P_i^{exp}}$. P^{exp} is the mean experimental shear test curve, P^{EF} is the simulated one and ω_e is the weight and have a value of 1 in order to maximize the weight of the value at maximum depth, which is the one with the smallest uncertainty. The cost function is then defined as the Euclidean norm of the difference vector (Eq. 6).

$$J(\underline{x}) = \|\underline{e}(\underline{x})\| \tag{Eq. 6}$$

The identification is led by minimizing the cost function using the Levenberg-Marquardt algorithm from the *scipy.optimize* python library. The unknown parameters t^0 and G^C were initially set to 47 MPa and 4.8 N/mm, respectively, and the domain of these two parameters is given in Table. 2. The initial values were chosen over the experimental range. The maximal value of the cohesive elements strength t^0 is set to the value of the AuGe joint strength at failure.

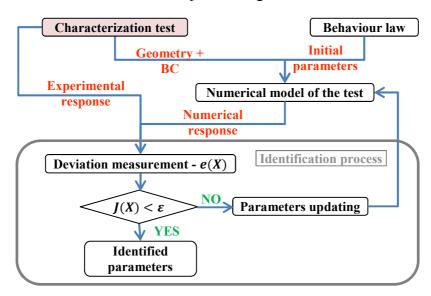


Fig. 6. Block diagram of the identification of CZM constitutive law from simple shear tests (Fourcade et al., 2014).

Table 2. Domain for cohesive zone model parameters during identification process.

	Min	Max
Strength t^0 (MPa)	XXX	200
Fracture energy G^{C} (N/mm)	XXX	XXX

Estimation of the interfacial fracture parameters and discussion. The values of the strength t^0 and the fracture energy G^C was determined using an iterative procedure to obtain the best match between model predictions and the experimental measurements of the load-displacement behaviour.....

Summary

To predict the lifetime of the AuGe brazed joint, a bilinear CZM parameters of Cu/AuGe interfaces were identified using inverse method with experimental shear test and finite element modelling. The experimental data allowed approving the load-displacement behaviour of the joint and the two Cu/solder interfaces. The SEM and EDX fractography analysis show a mixed fracture: adhesive in the two Cu/solder interfaces and cohesive in the joint. The interfacial fracture of these assembly was then simulated using 2D finite element modelling. The parameters of the bilinear CZM for the case of mode II loading were obtained based on inverse methodology and the results seem to be in good agreement with the experimental results.......

References (à changer le style)

- [1] A. Abdul-Baqi, P.J.G. Schreurs and M.G.D. Geers (2005). Fatigue damage modeling in solder interconnects using a cohesive zone approach. International Journal of Solids and Structures, 42:927-942.
- [2] G. Alfano (2006). On the influence of the shape of the interface law on the application of cohesive-zone models. Journal of Composites Science and Technology, 66: 723-730.
- [3] G. Alfano and M. Crisfield (2001). Finite Element Interface Models for the delamination analysis of laminated composites: mechanical and computational issues. International Journal for Numerical Methods in Engineering, 50, 50:1701-1736.
- [4] A. Baazaoui, J. Alexis, O. Dalverny and M. Karama (2013). Caractérisation expérimentale du comportement mécanique d'assemblage haute température pour l'électronique de puissance. MATEC Web of Conferences 7, Journées annuelles de la SF2M2013, Villeuve d'Ascq, France., 02013: p1-3.
- [5] A. Baazaoui, O. Dalverny, J. Alexis and M. Karama (2013). Study and mechanical characterization of high temperature power electronic packaging. 21ème Congrès Français de Mécanique, CFM2013, Bordeaux, France.
- [6] G. Barenblatt (1962). The mathematical theory of equilibrium cracks in brittle fracture. Advances in applied mechanics, 55-129.
- [7] D. Bhate, D. Chan, G. Subbarayan and L. Nguyen (2007). Fatigue crack growth and life descriptions of Sn3.8Ag0.7Cu solder joints: a computational and experimental study. Proceedings of 57th Electronic Components and Technology Conference, (pp. 558-565). Reno, USA.
- [8] P.P. Camanho and C.G. Dávila (2002). Mixed-Mode Decohesion Finite Elements for the Simulation of Delamination in Composite Materials. NASA/TM, 1-37.
- [9] D. Dugdale (1996). Yielding of steel sheets containing slits. Mechanics and Physics of Solids, 100-104.
- [10] T. Fourcade, O. Dalverny, J. Alexis, C. Seguineau and J. Desmarres (2014). Parametric identification of elastic-plastic constitutive laws using spherical indentation. Proceedings of International Symposium on Aircraft Materials ACMA2014. Marocco.
- [11] F. Hosking, J. Stephens and J. Rejent (1999). Intermediate Temperature Joining of Dissimilar Metals. Welding Journal.

- [12] V. Hutchinson and J. Tvergaard (2002). Two mechanisms of ductile fracture: void by void growth versus multiple void interaction. International Journal of Solids and Structures, 39:3581-3597.
- [13] S. Li, M. Thouless, A. Waas, J. Schroeder and P. Zavettieri (2005). Use of mode-I cohesive-zone models to describe the fracture of an adhesively-bonded polymer-matrix composite. Composites Science and Technology, 65:281-293.
- [14] S. Msolli (2011). Modélisation thermomécanique de l'assemblage d'un composant diamant pour l'électronique de puissance haute température. Toulouse: PhD thesis of INP University.
- [15] S. Msolli, A. Baazaoui, O. Dalverny, J. Alexis and M. Karama (2012). Viscoplastic behavior of diamond die attach subjected to high temperature conditions. IEEE, 13th International conference on thermal, mechanical and multi-physics simulation and experiments.
- [16] E. Müge (2007). Thermomechanical fatigue failure of interfaces in lead-free solders. Netherlands: PhD thesis, Eindhoven University of Technology.
- [17] A. Needleman (1987). A continuum model for void nucleation by inclusion debonding. Journal of Applied Mechanics, 54:525-531.
- [18] A. Needleman and X. Xu (1994). Numerical simulations of fast crack growth in brittle solids. Journal of the Mechanics and Physics of Solids, 42(9): 1397-1434.
- [19]O. Nguyen, E.A. Repetto and M. Ortiz (2001). A cohesive model of fatigue crack growth. International Journal of Fracture, 110:351-369.
- [20] G. Ortiz and M. Camacho (1996). Computational modelling of impact damage in brittle materials. International Journal of Solids and Structures, 33(20-22): 2899-2938.
- [21] Q.D. Yang, D.J. Shim and S.M. Spearing (2004). A cohesive zone model for low cycle fatigue life prediction of solder joints. Microelectronic Engineering, 75:85-95.
- [22] H.H. Ren, X.S. Wang and S. Jia (2013). Fracture analysis on die attach adhesives for stacked packages based on in-situ testing and cohesive zone model. Microelectronics Reliability, 1-8.
- [23] I. Scheider (2009). Micromechanical based derivation of traction-separation laws for cohesive model simulations. Journal of Procedia Engineering, 1:17-21.
- [24] K. R. Siegmund (2003). An irreversible cohesive zone model for interface fatigue crack growth simulation. Engineering Fracture Mechanics, 70:209-232.
- [25] V. Tvergaard (1990). Effect of fibre debonding in a whisker-reinforced metal. Materials Science and Engineering, A125, 203-213.
- [26] V. Tvergaard and J. Hutchinson (1992). The relation between crack growth resistance and fracture process parameters in elastic-plastic solids. Journal of the Mechanics and Physics of Solids, 40, 1377-1397.
- [27] Y. Yao and L. Keer (2013). Cohesive fracture mechanics based numerical analysis to BGA packaging. Microelectronics Reliability, 53:629-637.
- [28] Y. Yao, S. Vaynman, L. Keer and M. Fine (2008). Energy based micromechanics analysis on fatigue crack propagation behavior in Sn-Ag eutectic solder. Journal of Electronic Materials, 37(3):339-346.
- [29] P. Zavattieri and H. Espinosa (2001). Grain level analysis of crack initiation and propagation in brittle materials. Acta Materialia, 49(20): 4291-4311.