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In this paper, the effect of non-ideal actuators on the performance of reactive control for a heaving wave energy converter is studied. The concept of the control is to cancel all or part of the reactive terms in the equation of motion. The proposed control is causal, thus it may be applied in practice. Actuators efficiencies from 50 to 100% are considered.

The methodology used in the study relies on mathematical and numerical modeling. Control performance is investigated in regular waves and in irregular waves, and also from the perspective of the annual mean absorbed power at a typical Western Atlantic site. Motion constraints are not taken into account in the analysis for sake of simplicity.

As already shown in previous work, it is found that reactive control can increase the mean annual power absorption at the considered site by a factor 10 in case of ideal actuators. However, it is shown that actuators efficiency is critical to control performance, because of the large amount of reactive power involved in the control strategy. Thus, for low efficiencies actuators (<80%), control performance is a fraction of what it can be with ideal actuators (approximately 10%). Even with 90% efficiency, control performance is less than 30% of the ideal case. In the range 90–100%, every percent of increase in efficiency leads to significant increase in control performance.

1. Introduction

During the last decades, control strategies have been developed in order to increase the energy performance of wave energy converters (WECs) [1,2]. Model predictive control [3,4], latching [5,6], declutching [7], phase and amplitude control [8], and reactive control [9] are examples of the diversity of control strategies which have been studied.

In theory, reactive control is the best control as it allows reaching the theoretical maximum energy absorption [10–12] by bringing artificially the buoy to resonance [2]. However, it comes with difficult issues and drawbacks: the optimal reactive control is known to be anti-causal (meaning that it requires prediction of the future of the incident wave and wave excitation force [13]); its practical implementation goes with stability problems; last but not least, it involves to deal with large reactive power flux (as bringing the system to resonance requires canceling the reactive terms in the equation of motion). It was shown in [9] that the maximum of reactive power could be larger than ten times the average power. This is a difficult issue when implementing this control strategy to non-ideal actuators, as energy losses in these components might be equivalent or even larger than the energy gain obtained thanks to control.

In [14], a two-dimensional oscillating water column with optimal reactive control was considered. It was shown that optimal reactive control provides maximum energy absorption, but not maximum energy production due to power take-off losses. A 'phase-lag reduction' technique was proposed in order to maximize energy production. However, it cannot be applied in practice because of causality and stability issues. More recently, it was shown in [15] that a trade-off can be obtained between high average power absorption and actuators limitations by applying power saturation techniques. In [16], results for absorbed energy and produced energy are shown for the Wavestar WEC prototype in Denmark. Reactive control consists of applying a force proportional to the motion with a coefficient optimized for each sea state. It is shown that greater power generation is achieved when the coefficient is optimized with respect to the energy production instead of wave energy absorption. In [17], a heaving WEC with reactive control is considered with 90% energy efficiency for the control actuators. It is shown that reactive control still allows an increase in the performance of the WEC. However, the optimal control with taking into account 90% efficiency is different from the optimal control with ideal actuators (100% efficient). In [18], it is shown how

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ϵ	reactive part of equation of motion at ω frequency (Ns/m)
ϵ_0	reactive part of equation of motion at ω_0 frequency (Ns/m)
η	actuator efficiency [0; 1]
κ	control coefficient [0; 1]
λ	efficiency coefficient ($\mathbb{R}^+$)
μ_∞	added mass for infinite frequency (N/m)
ω_0	natural frequency (rad/s)
A	added mass (Ns/m)
A_s, B_s	coefficients of the Bretschneider energy spectrum ($\mathbb{R}^+$)
A_W	water plane area (m^2)
B	radiation damping coefficient (Ns/m)
B_{PTO}	power take-off damping coefficient (Ns/m)
$F_{control}$	control force (N)
F_{ex}	wave excitation force (N)
F_{PTO}	power take-off force (N)
$H_{1/3}$	significant wave height (m)
J	wave energy flux per unit wave crest (kW/m)
K	impulse response function of radiation force (N/m)
k	wave-number (m^{-1})
K_H	hydrostatic stiffness (N/m)
M	physical mass (kg)
$P_{control}$	control power (W)
P_{grid}	power flow at grid connection point (W)
P_{PTO}	PTO absorbed power (W)
S	energy spectrum (m^{2}s)
V	velocity of the buoy (m/s)
$X, \dot{X}, \ddot{X}$	position, velocity and acceleration (m, m/s, m/s^2)

the force and amplitude limitations affect the energy absorption under a reactive causal control.

In this paper, we address the effect of energy efficiency on the performance of a heaving WEC with *partial* reactive control [15]. The aim is to investigate what is the actual benefit of partial reactive control as a function of the energy efficiency of the actuators. Partial reactive control is considered because it has a high potential for performance improvement and because it is a practical option at present since it does not require prediction of the incident wave elevation or wave excitation force. It is shown that optimal power take-off and control coefficients are highly dependent on the energy efficiency of the control system and the control performance (defined as the ratio of averaged power delivered to the grid to the grid averaged power with ideal actuators) decreases rapidly with decreasing efficiency.

Firstly, a partial sub-optimal reactive control is applied on a floating heaving WEC under regular wave excitation. Theoretical optimal values are derived for the maximization of the energy absorption from a WEC with non-ideal actuators. Eventually, irregular waves are considered in order to determine the impact of the conversion efficiency on the energy absorption in a more realistic environment.

2. Methods

2.1. Equation of motion of a heaving wave energy converter with ideal control

The wave energy converter under consideration is a floating buoy restrained to move in the heave degree of freedom only. The buoy has a cylindrical shape of 10 m diameter and 10 m draft. The water depth is supposed to be infinite. The control elements are split in two parts: a passive power take-off (PTO) modeled by a linear damper and an active actuator used to bring the system close to resonance; modeled as a motor.

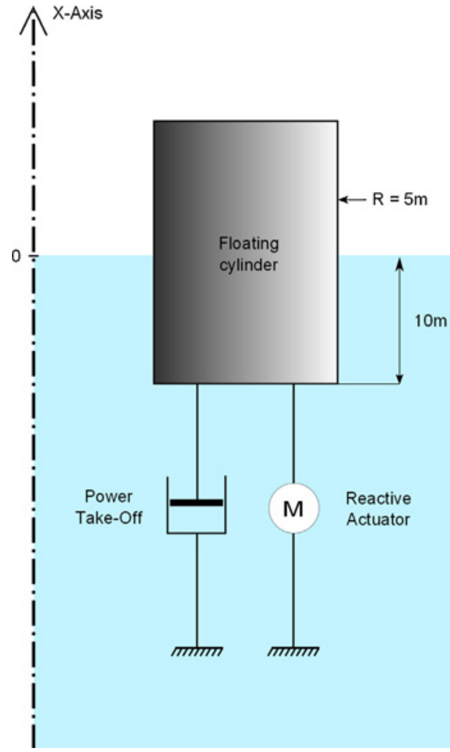

Fig. 1. Schematic of the considered heaving wave energy converter.

Let us assume the fluid to be incompressible, inviscid and the flow to be irrotationnal. The amplitude of motion and waves are considered small enough so that linearized potential theory may be used. Thus, the equation of motion of the wave energy converter can be written as follow (Fig. 1):

$$(M+\mu_\infty)\ddot{X}(t)+\int_0^t K(t-\tau)\dot{X}(\tau)d\tau+K_H X(t) = F_{ex}(t)+F_{PTO}(t)+F_{control}(t) \qquad (1)$$

with

- $X, \dot{X}, \ddot{X}$ are respectively the heave motion, velocity and acceleration of the buoy.
- M is the physical mass.
- $-\mu_\infty\ddot{X}-\int_0^t K(t-\tau)\dot{X}(\tau)d\tau$ corresponds to the radiation force in which μ_∞ is the infinite frequency added mass and K is the radiation velocity impulse response. According to the classical Cummins' decomposition [19], these two terms correspond to the effect of wave radiated by the body after an impulsive velocity at $t=0$. One can further approximate this function by a sum of N complex functions such as $K\simeq\sum_{j=1}^{N}\alpha_j e^{i\beta_j t}$ whose complex coefficients (α_j, β_j) can be obtained by using Prony's method [20]. Thus, after this approximation, one can show that the convolution product can be replaced by a sum of N additional radiative complex states $\int_0^t K(t-\tau)\dot{X}(\tau)d\tau=\sum_{j=1}^{N}I_j$, each I_j given by a simple ordinary differential equation $\dot{I}_j=\beta_j I_j+\alpha_j\dot{X}$. More details on the method can be found in [6].
- $K_H=\rho g A_W$ is the hydrostatic stiffness with A_W the water plane area.

- F_{ex} is the wave excitation force. It is related to a given incident energy spectrum $S(f)$, through the relation $F_{ex}(t) = \Re\left(\sum_j \sqrt{2S(f_j)\Delta f}\tilde{F}_{ex}(f_j)e^{-i(2\pi f_j t+\Phi_j)}\right)$ in which Δf is an adequate frequency step, Φ_j are a set of random phases and $\tilde{F}_{ex}$ are complex vectors of wave excitation force per unit wave amplitude in the frequency domain.
- F_{PTO} is the power take-off (PTO) force applied to the floating body. In this study, it is assumed to behave as a linear damper, i.e. it is simply proportional to the velocity $F_{PTO} = -B_{PTO}\dot{X}$.
- $F_{control}$ is a particular control force applied to the floating body as defined in the next section.

In this study, the hydrodynamic function K and the coefficients $\tilde{F}_{ex}$ and μ_∞ were calculated using the BEM code Aquaplus [21], dedicated to seakeeping computation.

The instantaneous PTO power is given by:

$$P_{PTO} = -F_{PTO}V = B_{PTO}V^2 \tag{2}$$

with $V = \dot{X}$. Assuming that all force terms are linear in the right hand side of Eq. (1), it can be rewritten in the frequency domain:

$$\left[\frac{iK_H}{\omega} + B(\omega) - i\omega(M + A(\omega))\right]\tilde{V}(\omega) = \tilde{F}_{ex}(\omega) + \tilde{F}_{PTO}(\omega) + \tilde{F}_{control}(\omega) \tag{3}$$

in which $\tilde{f}(\omega)$ denotes the complex amplitude resulting from the Fourier transform of $f(t)$, and A and B are the radiation coefficients in frequency domain, such as,

$$\tilde{K}(\omega) = B(\omega) + i\omega(A(\omega) - \mu_\infty) \tag{4}$$

In regular waves, one can show [22] that the time average $\overline{P}_{PTO}$ of the PTO power is given by:

$$\overline{P}_{PTO} = \frac{1}{2}B_{PTO}|\tilde{V}|^2 \tag{5}$$

2.1.1. A particular realizable reactive control

For an axisymmetric heaving WEC, it is well known that there is a maximum for the power absorption [2]. It is equal to the wave energy flux per unit wave crest length J divided by the wave-number k in regular waves:

$$P_{\max} = \frac{J}{k} \tag{6}$$

It is obtained if the velocity is in phase with the excitation force, and if the PTO damping coefficient is set equal to the radiation damping coefficient. The first condition is automatically fulfilled when the system is at resonance. When it is not the case, reactive terms (masses and stiffness terms) in the equation of motion become dominant and the power absorption drops usually far from the theoretical maximum.

To overcome this issue, one can introduce a control force aiming at canceling the reactive terms in the equation of motion, thus bringing the system close to resonance. In this study, it is defined as:

$$F_{control} = \kappa((M + A(\omega_0))\ddot{X} + K_H X) \tag{7}$$

with ω_0 the natural frequency of the system. κ is a coefficient whose value is taken between 0 and 1. If $\kappa = 0$, the control force is 0 whereas if $\kappa = 1$, masses and stiffness terms are totally canceled in Eq. (3) when $\omega = \omega_0$. Note that κ strictly equal to 1 is not a practical option because the system becomes unstable, even in the numerical simulations. κ can be close but must be kept smaller than 1.

Taking into account the equation for the control (7) in the equation of motion, one obtains the assembled equation of motion, in frequency domain:

$$\left[\frac{iK_H(1-\kappa)}{\omega} + (B(\omega) + B_{PTO}) - i\omega(M(1-\kappa) + A(\omega) - \kappa A(\omega_0))\right] \times \tilde{V}(\omega) = \tilde{F}_{ex} \tag{8}$$

In time domain:

$$(M + \mu_\infty - \kappa(M + A(\omega_0)))\ddot{X} + \int_0^t K(t-\tau)\dot{X}(\tau)d\tau + B_{PTO}\dot{X} + (1-\kappa)K_H X = F_{ex} \tag{9}$$

2.1.2. Control power and grid power with ideal control system

Let us consider the energy flow from the incoming waves to the electrical grid (Fig. 2).

The instantaneous power that control has to provide to the buoy is given by:

$$P_{control} = \kappa((M + A(\omega_0))\ddot{X} + K_H X)\dot{X} \tag{10}$$

The power flow P_{grid} at the grid can be defined as the difference between the PTO power, or the absorbed power, and the control power defined above:

$$P_{grid} = P_{PTO} - P_{control} \tag{11}$$

Over a cycle, the sign of the instantaneous control power $P_{control}$ is not constant. It varies, being positive for some part of the cycle and negative for the other part. When it is positive, it means that the control system actually takes energy from the electricity grid and converts it into mechanical energy of the WEC. Assuming that there are no energy losses in the actuators of the control system, the invested energy is fully recovered during the other part of the cycle. Consequently, the time average of the control power $\overline{P}_{control}$ is equal to 0. Thus, with an ideal reactive control system, the time average of the grid power is equal to the mean PTO power $\overline{P}_{grid} = \overline{P}_{PTO}$.

2.2. Control power and grid power with non-ideal control system (efficiency smaller than 100%)

In a more realistic configuration, the efficiency of the actuator will be smaller than the one, because of energy losses. Moreover, with reactive control, the amplification of the motion is very large. Thus, practical motion constraints should be taken into account. In this study only the effect of actuator's efficiency is investigated for sake of simplicity.

Let us defined a more realistic actuator with an efficiency smaller than 100%, generating energy losses. This assumption implies that when the control power flow goes from the grid to the WEC, i.e. the actuator is working as a motor, it needs more electrical power supply than the mechanical power that it can actually deliver to the WEC. In the other way, when the power flow goes from the WEC to the grid, i.e. the actuator is working as a damper, the power at the grid is smaller than the absorbed power by the actuator from the WEC. Mathematically, it can be written as:

$$P_{grid} = \eta_3 P_{PTO} - \begin{cases} \frac{1}{\eta_1}P_{control} & \text{if} \quad P_{control} \geq 0 \\ \eta_2 P_{control} & \text{if} \quad P_{control} < 0 \end{cases} \tag{12}$$

with $\eta_1 \in [0, 1]$ the control system efficiency when the actuator provides energy to the WEC, $\eta_2 \in [0, 1]$ the control system efficiency when it absorbs energy from the WEC and $\eta_3 \in [0, 1]$ the power take-off efficiency.

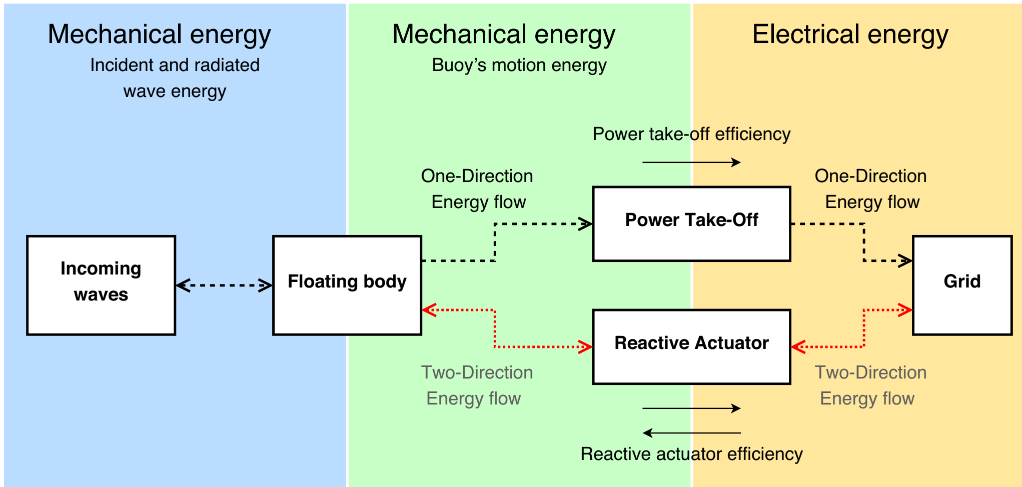

Fig. 2. Energy flow from the waves to the grid.

In regular waves, it can be shown that the control power fluctuates with twice the frequency of the incident wave excitation force F_{ex}.

$$P_{control}(t) = \frac{\kappa \epsilon_0 |\tilde{V}|^2}{2} \sin(2\omega t + 2\phi) \tag{13}$$

with

$$\epsilon_0 = \frac{K_H}{\omega} - \omega(M + A(\omega_0)) \tag{14}$$

$$\tilde{V} = |\tilde{V}| e^{i\phi} \tag{15}$$

Let us consider the time average of the grid power:

$$\overline{P}_{grid} = \frac{1}{T} \int_0^T P_{grid}(t) dt \tag{16}$$

Using Eq. (12) and considering the sign of $\epsilon_0 \sin(2\omega t + 2\phi)$ in Eq. (13), one can show,

$$\overline{P}_{grid} = \eta_3 \overline{P}_{PTO} - \frac{\kappa |\epsilon_0| |\tilde{V}|^2}{2\pi} \left(\frac{1}{\eta_1} - \eta_2 \right) \tag{17}$$

Thus, one obtains the expression for the grid power:

$$\overline{P}_{grid} = \overline{P}_{PTO} \left(\eta_3 - \frac{\kappa |\epsilon_0| \lambda}{B_{PTO}} \right) \tag{18}$$

with

$$\overline{P}_{PTO} = \frac{1}{2} B_{PTO} |\tilde{V}|^2 \tag{19}$$

$$\lambda = \frac{1}{\pi} \left(\frac{1}{\eta_1} - \eta_2 \right) \tag{20}$$

These two last equations show that the grid power depends on the parameter B_{PTO}. Therefore it is of interest to determine what is the optimal value B_{PTO}^{opt} for this coefficient, i.e. the one which maximizes the grid power $\overline{P}_{grid}$. It is obtained by differentiating equation (18) with respect to B_{PTO}:

$$B_{PTO}^{opt} = \frac{\lambda |\epsilon_0| \kappa}{\eta_3} + \sqrt{\left(B + \frac{\lambda |\epsilon_0| \kappa}{\eta_3} \right)^2 + (\epsilon - \kappa \epsilon_0)^2} \tag{21}$$

with

$$\epsilon = \frac{K_H}{\omega} - \omega(M + A) \tag{22}$$

Back to the expression for the grid power, Eq. (18), one can see that, if $\eta_1 = \eta_2 = 1$ (and so $\lambda = 0$), thus the grid power is equal to the wave power absorbed through the PTO. If it is not the case, it can differ significantly. It may even reach positive values if $\kappa |\epsilon_0| \lambda / B_{PTO} > \eta_3$ which would mean that the WEC would absorb energy from the grid instead of delivering energy to it.

Using Eq. (21) in Eq. (18), one can show:

$$\overline{P}_{grid} = \frac{1}{4} \frac{|F_{ex}|^2}{B + B_{PTO}^{opt}} \eta_3 \tag{23}$$

In order to maximize the grid power, B_{PTO}^{opt} should be as small as possible. According to Eq. (21), it depends on the partial reactive control coefficient κ. Looking for the optimal κ^{opt} minimizing the PTO damping coefficient B_{PTO}^{opt} leads to:

$$\kappa^{opt} = \max \left\{ 0, \frac{\epsilon}{\epsilon_0} \frac{1 - 2\lambda B / (\eta_3 |\epsilon|) - (\lambda/\eta_3)^2}{1 + (\lambda/\eta_3)^2} \right\} \tag{24}$$

Note that minimizing the PTO damping coefficient B_{PTO}^{opt} may lead to unrealistically large amplitude motion. In practice motion constraints may limit the minimum value for the B_{PTO}^{opt} coefficient.

If the control system is ideal (i.e. $\eta_1 = \eta_2 = \eta_3 = 1$), $\lambda = 0$. Thus, κ^{opt} is equal to ϵ/ϵ_0 which means that full reactive control is optimal. However, if the control system is not ideal, the optimal reactive control coefficient κ^{opt} decreases as a function of the square of λ/η_3.

The grid power with optimal values for B_{PTO}^{opt} and κ^{opt} can be written as follow,

$$\overline{P}_{grid} = \begin{cases} \frac{|F_{ex}|^2}{8B} \frac{1 + (\lambda/\eta_3)^2}{1 + (\lambda/\eta_3)|\epsilon|/B} \eta_3, & \text{if } \kappa^{opt} > 0 \\ \frac{|F_{ex}|^2}{4(B + \sqrt{B^2 + \epsilon^2})} \eta_3, & \text{if } \kappa^{opt} = 0 \end{cases} \tag{25}$$

3. Results

3.1. Regular waves

Fig. 3 shows, for each period, the efficiency limit from which reactive control would become useless. For comparison, reactive control coefficient κ is set to $\epsilon/\epsilon_0 \approx 1$ and to its optimal value $\kappa = \kappa^{opt}$ (calculated according to Eq. (24)). It is assumed for sake of simplicity that all efficiencies share the same value, i.e. $\eta_1 = \eta_2 = \eta_3 = \eta$. $\kappa = \epsilon/\epsilon_0$ corresponds to full reactive control whereas $\kappa = \kappa^{opt}$ corresponds to optimal partial reactive control. One can see that with full reactive control, the actuators efficiency limit is 0.63, except for periods close to resonance (no reactive control is needed close to resonance since the reactive part in the equation motion naturally vanishes).

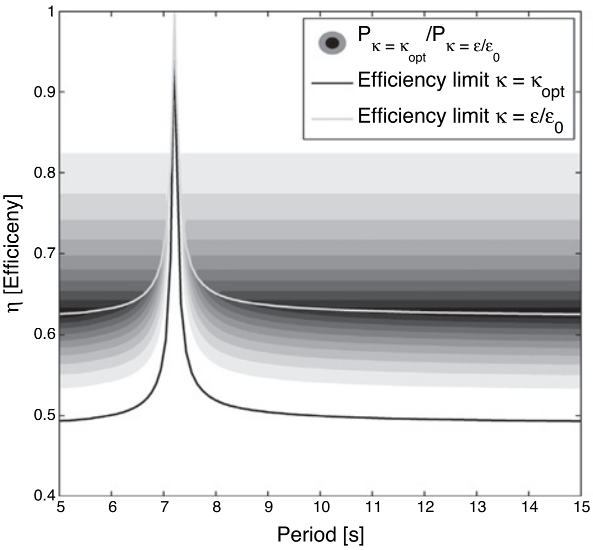

Fig. 3. Ratio between grid power with an optimal κ and grid power with $\kappa = \epsilon/\epsilon_0$.

In other words, for lower efficiencies, the increase in absorbed wave power does not compensate the large losses in the power used for control. Therefore, grid power will be larger without control than with full reactive control. If optimal partial reactive control is selected ($\kappa = \kappa^{opt}$), one can see that the efficiency limit is lowered to 0.5. It means that partial reactive control allows increasing grid power in comparison with no control for efficiencies between 0.5 and 0.63. Moreover, the ratio of the grid power with optimal partial reactive control and full reactive control is shown as well on this graph. It can be seen that optimal partial reactive control always leads to greater grid power than full reactive control, up to 20% for efficiency close to 0.6. Even if power absorbed from the waves is greater with full reactive control than with partial reactive control, grid power is actually greater with partial reactive control than with full reactive control when taking into energy losses in the actuators.

Numerical results are shown in Fig. 4 for a 9-s regular wave with 1 m amplitude. Top figure shows the normalized averaged power (ratio of averaged power delivered to the grid averaged power with ideal actuator; or control performance) as a function of the actuators efficiency. The behavior appears to be non-linear with a steeper as the efficiency increases. For low efficiencies actuators (50–70%), absorbed power with reactive control is a small fraction of what it can be with ideal actuators. It varies from 7 to 15%. From 70% to 80%, the control performance is doubled from 9 to 18%. 10 more percents of efficiency allows another doubling of control performance, raising to 35%. The last 55% of control performance is achieved within the range 90–100% efficiencies. In this range, every percent of efficiency leads to a significant increase in power absorption.

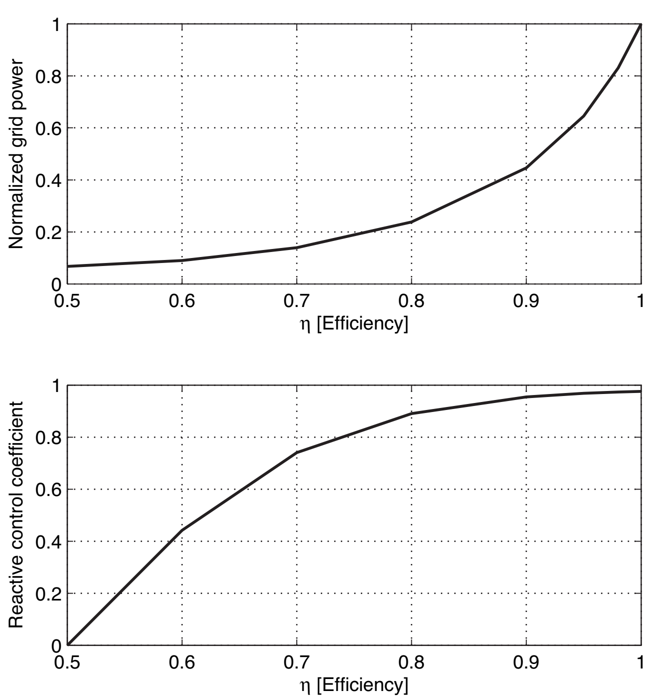

Fig. 4. Grid power and optimal reactive control coefficient as a function of actuators efficiency in a 9 s regular wave with 1 m wave amplitude.

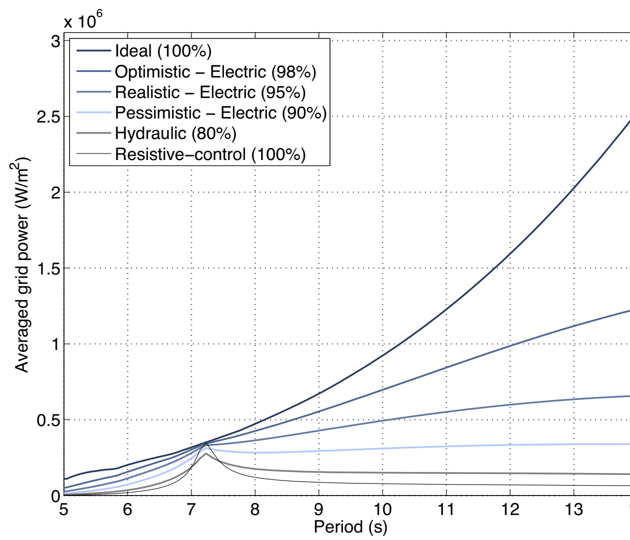

Fig. 5. Grid power as a function of wave period (regular waves) without control and with control with different actuators efficiencies.

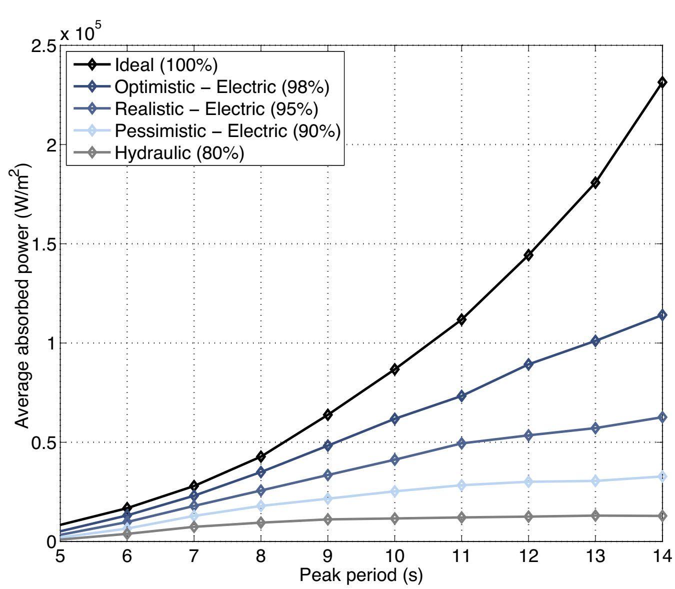

Fig. 6. Grid power as a function of spectrum peak period (irregular waves) without control and with control with different actuators efficiencies.

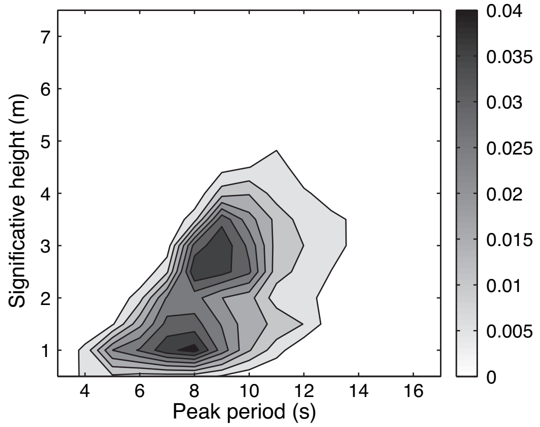

Fig. 7. Scatter diagram offshore the island of Yeu, France.

These results show how the actuator efficiency is critical to reactive control. Indeed, it can be seen that to achieve 50% control performance requires actuators efficiency as high as 94%. Such a high efficiency is a technological challenge. It is unlikely that it could be achieved with hydraulic components, for which 80% efficiency is generally considered [23]. It might be obtained with direct drive technologies.

However, it must be noticed that even with low efficiency actuators (60–80%), the reactive control considered in this study is still able to increase significantly the wave power absorbed by the device. Indeed, at 70% efficiency, the power is twice the absorbed power without control.

Bottom figure shows the optimal reactive control coefficient κ^{opt}, calculated according to Eq. (24), as a function of the efficiency. One can see that, contrary to control performance, it varies mostly in the low efficiency range. Indeed, κ^{opt} goes from 0 to 0.9 from 50 to 80%. It shows that in the low efficiency range, full reactive control ($\kappa = 1$) is not optimal. Intermediate reactive control ($0 < \kappa < 1$) will allow maximizing power absorption.

The same conclusions can be drawn for other wave periods. It can be seen in Fig. 5 which shows the averaged absorbed power in regular waves as a function of the wave period for different efficiencies. The considered efficiencies are ideal (100%), Optimistic, Intermediate and pessimistic direct drive technology (98%, 95%, 90%) and hydraulic (80%). The case without control is calculated with 100% efficiency (i.e. $\eta_1 = \eta_2 = \eta_3 = 1$) with optimal damping

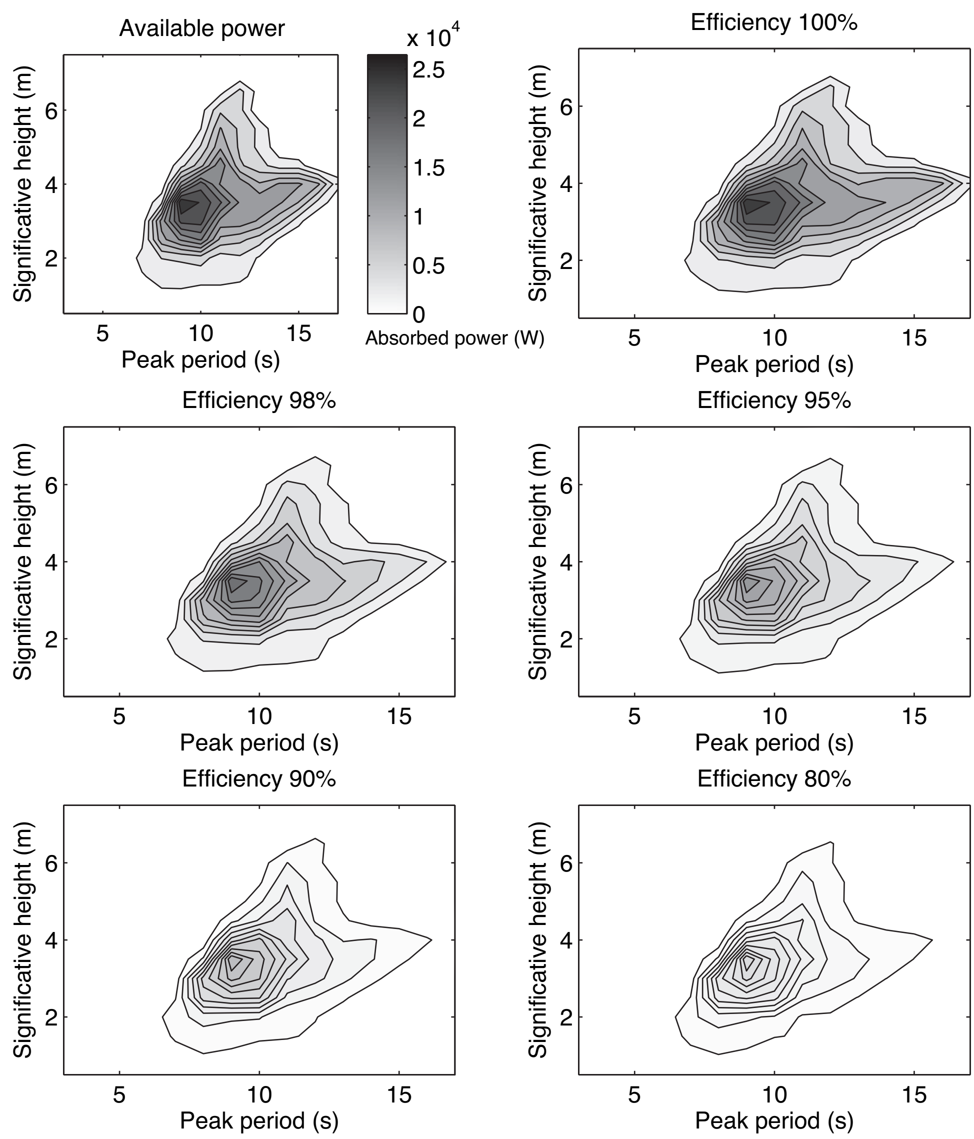

Fig. 8. Power matrices of the device with different actuators efficiencies.

coefficient. One can see that even with the smallest considered efficiency (80%), reactive control allows doubling to tripling the absorbed power in comparison with absorbed power without control. Although it is much less than what can be achieved with highly efficient actuators, it is still a large improvement in power absorption. Thus it may still be worth considering this reactive control in practice.

3.2. Irregular waves

Previous results were obtained for regular waves. In practice, ocean waves are irregular. In this study, irregular waves are synthetized using the Bretschneider energy spectrum defined as follow,

$$S(\omega) = \frac{A_s}{\omega^5} e^{-B_s/\omega^4} \tag{26}$$

with $A_s = (5/16)\omega_m^4 H_{1/3}^2$ and $B_s = (5/4)\omega_m^4$, ω_m being the peak frequency of the spectrum and $H_{1/3}$ being the significant wave height.

It is discretized using 250 frequency bins from 0.2 rad/s to 3 rad/s. Numerical simulations are based on integrating equation (9) by a time stepping procedure. The simulated time interval is 250 s. For each sea state, results shown are the average of 8 simulations with different sets of random phases. B_{PTO} and κ coefficients are optimized to recover maximum energy from the incident waves for each sea state.

Fig. 6 shows the averaged absorbed power per meter of significant wave height square as a function of the spectrum peak period. As in Fig. 5, 80, 90, 95, 98 and 100% efficiencies were considered. The reactive control coefficient κ is optimized for each sea state and each efficiency. Absorbed power without control and 100% efficiency is also plotted for reference. One can see that contrarily to regular waves, the absorbed power with control steadily increases with the peak period. Indeed, even if the peak period is close to the natural period of the device, control is able to improve power absorption due to frequency spreading in the spectrum.

As in regular waves, one can see that high control performance relies on highly efficient actuators. Indeed, 5% energy loss in actuators efficiency halves the control performance. For the 80% efficiency case, the control performance is approximately 10% of the ideal case. However, one should notice that as in the regular wave case, it still allows increasing significantly the power absorbed by the device.

Let us consider the mean annual power performance of this device at a possible deployment site offshore the island of Yeu in France (46°43′30″ N, 2°20′50″ W). Its scatter diagram is shown in Fig. 7.

Power matrices of the device were computed for actuators efficiency of 80%, 90%, 95%, 98% and 100%. Results are shown in Fig. 8. Theoretical maximum power matrix is also in Fig. 8. It is obtained by using the fact that the maximum capture width for each frequency bin of the discretized spectrum is equal to the wavelength divided by 2π for an axisymmetric device [2].

By multiplying the power matrices and the scatter diagram, one can calculate the mean annual power absorption at the considered site. Mean annual absorbed power is shown in Fig. 9 for different actuators efficiencies. For reference, the theoretical maximum for absorbed power and the absorbed power without control are plotted on the same graph.

Again, one can see how control performance is related to actuators efficiency. For the smallest considered efficiency, 80%, the mean annual power is almost doubled. However, the factor can be 4 with a 90% efficiency. It can be almost 10 with an almost ideal actuator with 98% efficiency.

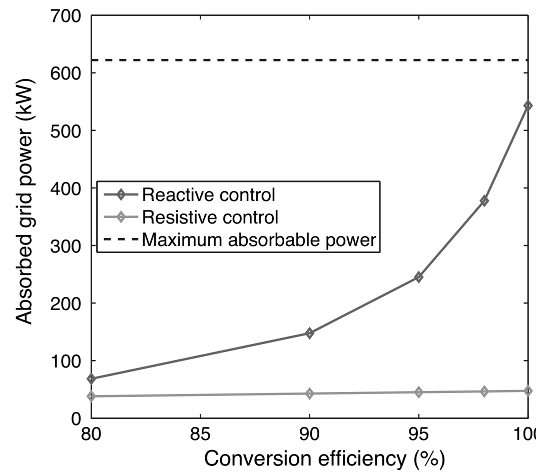

Fig. 9. Mean annual absorbed power at l'Ile d'Yeu under various conversion efficiency, France.

4. Conclusion

In this paper, the effect of non-ideal actuators on the performance of reactive control for a heaving wave energy converter is studied. The aim of the control is to cancel partly the reactive terms in the equation of motion. The proposed control is causal. The methodology relies on mathematical and numerical modeling. Control performance is investigated in regular waves and in irregular waves, and also from the perspective of the annual mean absorbed power at a typical Western Atlantic site.

It is shown that actuators efficiency is critical to power performance. With ideal actuators, control improves the mean annual power absorption of the device by a factor larger than 10. However, even with optimal parameters $\kappa = \kappa_{opt}$ and $B_{PTO} = B_{PTO}opt$, the factor is reduced to 4 for actuators with 90% efficiency. It is reduced to 2 for actuators with 80% efficiency. Below 50% efficiency, energy losses are so large that reactive control becomes useless. For efficiencies larger than 80%, full reactive control (κ coefficient close to 1) is close to optimal. For smaller efficiencies, it is beneficial to consider partial reactive control ($0 < \kappa < 1$).

Power performance relates directly to the revenue from the wave power plant. For the particular generic wave energy device considered in this paper, increasing the actuators efficiency from 90% to 91% would lead to an increase in revenue by as much as 10%. It demonstrates how efficiency is critical to control performance.

The method derived in this paper is applicable to most oscillating wave energy devices. Quantitative results are expected to vary from one device to another. However, results are expected to be qualitatively similar for wave energy devices featuring resonant behaviors (for example heaving buoys or oscillating water columns).

Impact on motion constraint is not treated in this study, but it remains an important technological aspect that might limit drastically the efficiency of a such reactive control, especially for linear wave energy converters. To conclude, it appears that reactive control could be used to increase significantly the power performance of wave energy converters, provided that highly efficient components (80–90%) are used. With that respect, direct drive PTO systems may be the most favorable technical solutions.

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