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# Experimental characterization and numerical modeling of damage at the mesoscopic scale of woven composites

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#### Abstract

The mechanical behaviour of woven composites is modeled at the mesoscopic scale, taking into account the influence of the dry fabric preforming before resin injection, the relative shift and nesting between fabric layers, and the effect of yarn cracking and micro-decohesions at the yarn surface. The strain distributions observed at the composite surface are strongly influenced by the layer shift. A good agreement with strain distributions observed by Digital Image Correlation is obtained only if the layer shifts are similar to those observed in the real specimen. Mesoscopic damage is modeled by introducing discrete cracks in the Finite Element mesh of the Representative Unit Cell of the composite, similarly to the experimental observations. Therefore, the effects of mesoscopic scale damage on the macroscopic mechanical properties are evaluated. The numerical results show similar trends as the experimental data. However, in order to reach a quantitative agreement, the effect of micro-decohesions around the crack tips at the yarn surfaces have to be taken into account.

#### **1. Introduction**

Composite materials manufactured using textile architectures are receiving a growing interest in the field of advanced structural applications. The main reasons are related to the fact that the microstructure of fiber preforms can be tailored to satisfy specific needs in terms of mechanical performance. Complex reinforcement shapes may be directly woven with modern looms [1], which reduces the number of parts needed to assemble a composite structure, hence limiting the use of joints, which are classical weak points, and reducing the manufacturing costs.

In recent years, Onera has developed macroscopic models that successfully predict damage evolution and failure in textile composites [2;3]. Nevertheless, if the constituents or the reinforcement architecture of the composite change, expensive and time consuming experiments are necessary to identify the parameters of the models. Therefore, predictive modeling tools that take into account the reinforcement architecture are needed in the design phase. This modeling is carried out at the mesoscopic scale, which lies in between the macroscopic scale of the structure and the microscopic scale at which each individual fiber is modeled [4]. At the mesoscopic scale, yarns composed of several thousands of fibers are treated as homogeneous material embedded in the matrix. The Representative Unit Cell (RUC) is a part of the textile reinforcement describing the interlacing of warp and weft yarns of the composite.

Most textile composites are manufactured using matrix injection/infusion, or thermoforming. In order to increase the fiber volume fraction in the composite to improve its mechanical performances, the preform is usually compacted before adding the matrix. It has a significant influence on the yarn shape and paths and must therefore be taken into account in the mesoscopic RUC. Indeed, complex yarn shapes can be generated during the manufacturing process, including (i) local variations of both the section and the fiber volume fraction in the yarns [5;6], (ii) random shifts and nesting between the layers [5;7;8], and/or (iii) complex contact regions between the yarns. The shape and orientation of the yarns and their interaction have an important influence on the local strain/stress distributions of the composite under mechanical loading, as well as on damage initiation and propagation [9-11]. For instance, it has been shown in different studies that the nesting between the layers of a multilayered textile composite has to be taken into account since it can lead to different damage initiation conditions, and different patterns of damage progression [11-14]. Therefore, at the mesoscopic scale, it is extremely important to generate geometries with yarn shapes as close as possible to reality.

Several automated tools have been proposed throughout the last decade to generate geometrical models of fabric reinforcements, and covering a huge variety of weaving patterns [15-18]. However, the deformation occurring during the preforming step of the fabric is often neglected. Therefore, an

idealized geometric model of the fabric is commonly adopted, resulting in resin rich areas that are bigger than those observed experimentally. As a consequence, in order to preserve the overall fiber volume fraction in the composite (typically between 50 and 60%), the fiber volume fraction in the yarns must be set to higher values than observed in reality (sometimes 90%, or higher [19]). Yarn shapes closer to those of the real composite can be obtained from Finite Element (FE) modeling of the dry fabric preforming [20-24]. These methods are used in this work to create geometries of a slightly unbalanced four-layer plain weave fabric with different types of nesting between the layers.

In this work, an algorithm recently developed at ONERA [25], able to mesh any kind of deformed preform as well as the injected matrix complement (section 2.1), is used to generate RUC meshes that are consistent at the contact zones (section 2.2). The algorithm avoids the need for introduction of artificial matrix layers to avoid meshing problems close to the contact regions, as done for instance in [4;13;26-28].

There are two ways to model the damage at the mesoscopic scale. First, approaches based on continuum damage mechanics have been often used to simulate the damage mechanisms [4;9;26;27]. The advantage of these methods is that they are based on damage variables able to predict the degraded stiffness. Being usually combined with well established failure criteria, continuum damage models show, in general, robust damage detection. However, when it comes to damage propagation the calculations can show non-physical behaviours [4]. In this case, the second approach that consists in introducing cracks directly into the mesh seems to be necessary to predict the behaviour of a woven composite structure.

The purpose of this work is to present the FE tools developed at ONERA for the evaluation of the effects of mesoscopic scale damage on the mechanical performances of a woven composite part. The generation and meshing of a compacted RUC is presented in Section 2. Then, the local strain distributions obtained using mesoscopic FE simulations are compared with those obtained experimentally using stereo digital image correlation (section 3). Finally, the sequence of intra-yarn transverse damage has been characterized thanks to microscopic observations on the tested specimens. Based on these experimental observations, discrete intra-yarn transverse cracks are inserted into the yarns of the RUC that are perpendicular to the loading direction (section 4). This is done using the crack insertion algorithm developed by Chiaruttini et al. [29]. The effects of mesoscopic scale transverse damage on the macroscopic mechanical properties of the composite are evaluated and compared with the experimental data (section 5).

#### 2. Generation of a compacted Representative Unit Cell

#### 2.1. Modeling of fabric compaction and nesting

The material studied in this work is a four-layer plain weave fabric of glass fiber and epoxy matrix. The advantage of modeling such architecture is that it has a relatively simple geometry with a small RUC. Nevertheless, due to compaction and nesting, the compacted reinforcement exhibits multiple contacts zones between yarns that are significantly deformed with respect to their initial shape. Modeling the preforming step of the dry reinforcement allows for generating such reinforcement geometries. First, four layers of a plain weave unit cell are generated and stacked on top of each other. The Hivet and Boisse procedure [30] is used and ensures no interpenetrations between yarns. Second, the layers are shifted with respect to one another in the fabric plane. The relative layer shifts are determined from experimental observations. They are then cut at the boundaries of the unit cell box, such that each layer is limited to the same unit cell. Figure 1a show the FE mesh of such a lay up in the case of maximum nesting. The dry fabric is then compacted using the method of Nguyen et al. [23]. A simplified mechanical behaviour is used for the yarns [22], which in the case of compaction modeling yields qualitatively good yarn shapes. Figure 1b shows the results of the preforming step on a four layered plain weave RUC having maximum nesting, compacted from an initial thickness of 2.9mm to a final thickness of 1.65mm. The local yarn orientations are obtained by projection of the yarn center line, and assigned to each integration point. The yarn volume fraction (total occupied by yarns divided by the total volume of the unit cell) is 52.0% in the non compacted state and 87.3% in the compacted one. Therefore, in order to obtain a total fiber volume fraction in the composite of 60% (a typical value for compacted fabrics), the fiber volume fraction in the yarns would reach an unphysical value of 115% if a non-compacted geometry is used, as done in [11;13;26].



**Figure 1.** FE simulation of the compaction of four layers of plain weave fabric with maximum nesting. The colors in (b) represent the magnitude of the vertical displacement.

Ivanov [19] has encountered the same problem for braided fabrics. Using the compacted unit cell shown in Fig. 1b, the fiber volume fraction in the yarns has to be 68.7% so as to reach 60% of total fiber volume fraction in the composite.

#### 2.2. Generation of consistent Finite Element mesh of the Representative Unit Cell

It is quite easy to generate FE meshes of yarns by using automated meshing tools. However, it poses a challenge to obtain meshes that are consistent where yarns are in contact. A consistent mesh is obtained if the surface nodes of each yarn are at the same locations in the contact zone. During the compaction of the dry fabric, the yarn surfaces may slide on each other, leading to non consistent meshes, even if the initial non-preformed meshes were generated with consistent contact zones. Therefore, an algorithm has been recently developed at Onera to generate consistent meshes of yarns of arbitrary shapes that are consistent at the contact zones. Small interpenetrations and voids that are sometimes generated by the contact modeling used in the preforming step modeling are eliminated by the algorithm. More details about this algorithm can be found in the work of Grail et al [25]. Figure 2 shows the resulting FE mesh of a compacted lay-up of four layers of plain weave with the layer shifts taken from experimental observations.



**Figure 2.** Optimized FE mesh of the compacted RUC with the experimentally observed shift between the fabric layers. (a) Total RUC with matrix complement. (b) Mesh of the yarns only.

#### 3. Experimental analyses and surface strain fields

Experimental tests were carried out at Onera for different four-layer plain weave fabric composites specimens under uni-axial static tensile load to (i) determine local strain distributions in the plies and (ii) characterize the onset and patterns of progressive damage mechanisms. Plain fabric architecture was chosen because it is the smallest pattern commonly used, which simplifies the comparison between experimental analyses and numerical simulations. The reinforcement is made of E-Glass fibers. The epoxy matrix is the Araldite LY564 produced by Huntsman. Different specimens have been tested under different kind of loadings, e.g. monotonic and incremental loadings. In contrast to unidirectional composites, the yarn interlacing pattern in textile composites causes heterogeneous strain fields with large strain gradients around yarn crimp regions. Classical electrical resistance strain gauges can thus not provide an adequate spatial resolution. Therefore, the local strains on the composite surface have been measured using Digital Images Correlation (see Figure 3). In addition, both acoustic emission and microscopic analyses have been used in order to understand the local damage behaviour.



**Figure 3.** Longitudinal  $(\varepsilon_{11})$  and transverse  $(\varepsilon_{22})$  strain fields obtained experimentally by digital image correlation on the top surface of a composite specimen with four layers of plain weave reinforcement under tensile load in direction of the blue arrows ( $\sigma_{11}$ = 90MPa) compared to FE simulations of four adjacent RUC with different layer shifts: (a) shifts of the specimen, (b) shifts to obtain maximum nesting, and (c) no shifts.

This instrumentation allows for detecting the damage onset in the composite in terms of yield stress and location. Moreover, the density and total length of the visible cracks on the composite edge are measured under an optical microscope. For all the tested specimens, the number of visible cracks transverse to the loading direction was measured. In the case of specimens subjected to incremental loadings, the number of cracks/mm<sup>2</sup>,  $\rho_{s_i}$  is calculated at each loading step. The damage observed just before failure is shown in Fig 4 (this picture was not taken at the failure location). Transverse yarn cracks are observed with micro-decohesions between yarns and matrix plastification and damage around the crack tips. The procedure presented in Section 2 is used to generate FE meshes of a RUC with different shifts between the fabric layers: no shift (and thus no nesting), the layer shift allowing for maximum nesting (see Figure 1), and the layer shifts observed experimentally (See Figure 2). Periodic boundary conditions are applied in the direction of the warp and the weft yarns. The top and bottom surfaces are stress free, as in the experiment. The elastic properties of the matrix provided by the manufacturer are  $E_m=2.3GPa$  and  $v_m=0.35$ . The properties of the E-glass fiber are  $E_f=72.4GPa$  and  $v_f=0.2$ . The homogenized properties of the yarns are calculated by performing a FE calculation at the microscopic scale, as done by Melro et al [26].



**Figure 4.** Damage pattern observed by optical microscopy on the specimen edge just before failure (the image was not taken at the failure location).

The fiber orientation at each integration point is given by projection on the yarn center line. The homogenized elastic properties are calculated from the average stress and strain over the RUC. There is a good agreement with the experimentally measured values, and the influence of the layer shift on the homogenized properties is negligible. The strain fields at the top surface of the RUC obtained for different nested RUC are compared to experimental observations in Fig. 3. The local strain distribution is strongly influenced by the shift between the fabric layers. The differences in the strain pattern are due to the relative position of the layers to one another and the difference of yarn shapes obtained after compaction modeling. The surface strain distributions agree well between numerical and experimental results only if the same relative layer positions as in the studied composite specimen are used to generate the RUC of the composite. The elastic properties obtained (i) numerically with different nesting between the layers and (ii) experimentally are presented in Tab 1. The nesting between the layers has a negligible influence on the macroscopic homogenized properties of the material.

Method	Nesting	E11 (GPa)	E22 (GPa)	G12 (GPa)	ν12
EF	Zero	21.5	22.7	6.63	0.14
EF	Max	21.3	22.6	6.60	0.14
EF	random	20.9	22.4	6.47	0.13
MT	random	21.2	22.6	6.46	0.12
Tests	/	21.48 <u>+</u> 0.58	24.18 <u>+</u> 1.14	6.25 <u>+</u> 0.35	0.127 + 0.014

**Table 1.** Macroscopic elastic properties of the four-layer plain architecture obtained (i) numerically (EF) by taking into account different types of nesting between the layers, (ii) analytically (MT) [30] and (iii) experimentally with confidence intervals of 95%.

#### 4. Influence of damage on the homogenized elastic properties of the composite

In this work, mesoscopic damage is modeled by inserting cracks in the FE mesh of the RUC, using the algorithm of Chiaruttini et al [29]. This algorithm allows the simulation of complex cracks growth phenomena, and one of the advantages of this approach is that almost any kind of cracks can be introduced in very complex meshes. It is based on the modification of an initial undamaged mesh that is refined in the vicinity of the crack front. First, the crack surface is represented by a surface mesh made of linear triangular elements. Then, the undamaged zone of the volume mesh is adapted such that the crack can be inserted. A volume remeshing operation is finally applied to the complete mesh. The main advantage of introducing discrete damage in the RUC mesh is that the influence of damage on the local stress state can be directly observed. Moreover, the damage can be introduced in the RUC as it is observed in the real specimen as the cracks can be counted, measured and localized on the composite.



**Figure 5.** Representative Unit Cell of four layers of plain weave fabric with transverse cracks in the weft yarns and yarn/yarn and yarn/matrix decohesion around the crack tips: (a) one crack per weft yarn, (b) two cracks per weft yarn.

Experimental observations show that at the mesoscopic scale, damage starts by cracking of the yarns that are transversely oriented to the load direction. Decohesion at the yarn surface is observed at the crack's tip. To reproduce numerically this kind of damage, cracks are introduced one by one in the wefts yarns (perpendicular to the loading direction) of the RUC mesh, until each weft yarn contains

two cracks, located at one third and two third of the yarn width (See Fig. 5). A random nesting between the layers is used so that yarn position and shape is as close as possible to experimental observations. The cracks are supposed to be perpendicular to the load direction and to run through the whole width of the RUC. Micro-decohesions of different lengths are also generated around the crack tip. The homogenized elastic properties are calculated by FE simulations using the method described in Section 3. The decrease of elastic properties with growing crack density  $\rho_s$  is compared to the experimental results in Fig. 6. The results show that the trend of the evolution of Young's modulus and Poisson ratio with the crack density is well predicted by the numerical simulations. The reduction of the properties is slightly underestimated if micro-decohesions are not included. An average decohesion length around the crack tip of 0.1mm gives better results. It has to be noted that in reality not all cracks cover the whole width of the specimen. However, it is impossible to determine their actual length without a micro-CT analysis. Such results are not available for the time being, but a study is planned to take place in near future. Therefore, numerical simulations may overestimate the reduction of the elastic properties, since the total crack surface may be overestimated for a given number of cracks. This leads to the conclusion that the correct average micro-decohesion length is probably higher than 0.1mm.



**Figure 6.** Comparison between experimental and numerical results of the effect of transverse cracks in the weft yarns on Young's modulus (a) and Poisson ratio (b) in warp direction as function of the crack density  $\rho_s$  and the micro-decohesion length  $\mu$ .

#### Conclusions

A strategy for the numerical modeling of damage in woven composites at the mesoscopic scale has been presented. The complex geometry of the reinforcement in the RUC is correctly described as the influence of dry fabric preforming before resin injection is taken into account by modeling the fabric compaction. Consistent FE meshes are generated without the need for inserting a matrix layer between yarns. Several four-layered plain weave fabrics with different shifts between layers have been studied. Comparison with strain fields observed by DIC on the surface of the specimens shows that the numerical results are in good agreement with the experiments qualitatively and quantitatively only if random shifts between the fabric layers are included in the model. The macroscopic elastic constants calculated numerically by means of volumetric homogenization were also evaluated, and the results are in good agreement with the experimental data, whatever the layer shift is. Hence, the nesting between the layers seems to have a negligible influence on the calculation of the macroscopic elastic properties of the material. Due to a better correlation in terms of surface strain fields, damage has been introduced into the FE model of the RUC with random shifts between the fabric layers. This was done by inserting transverse yarn cracks and micro-decohesions at the yarn surfaces around the crack tips. The evolution of the elastic properties obtained from numerical simulations show the same trends as observed by experiments. To obtain a good quantitative agreement in the property reduction due to damage, micro-decohesions around the crack tips have to be included in the model. Despite the simplified crack geometry, the results are encouraging. Future work covers a more detailed modeling of the 3D crack geometry based on micro-CT observations, the inclusion of further damage mechanisms, such as matrix damage and fiber breaking with the aim of predicting the structural behaviour of woven composites until failure.

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