

# Six models of argumentary oscillators

## Experimental results

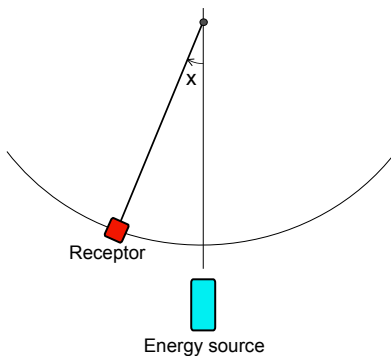
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DYNOLIN Conference, Paris (France)



## Definition.

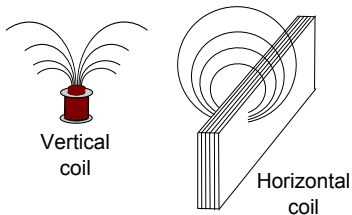


- History: Béthenod, Penner, Doubochinski.
- Oscillator of Duffing type.
- Submitted to an external periodic force.
- Modulation-coupling by spatial position.
- Equation (reduced time):

$$\frac{d^2x}{dt^2} + 2\beta \frac{dx}{dt} + x + \mu x^3 = A H(x) F(t)$$

with  $F(t)$ =periodic function of time.

## Experimental set-up: sources.

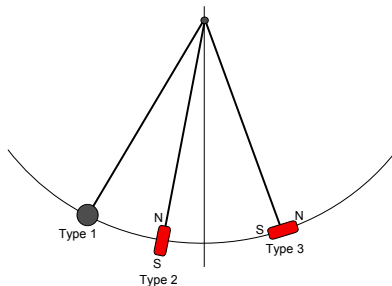


- Sources :
  - Horizontal coil (type I)
  - Vertical coil (type II), with or without iron core

## Experimental set-up: receptors.

- Receptors :

- Steel sphere (type 1)
- Vertical magnet (type 2)
- Horizontal magnet (type 3)



## Experimental set-up: sensors and generator.

- Angular-position sensor.  
This sensor gives the angular position of the pendulum's rod, with respect to the vertical reference. Its resolution is 100 k points/turn.
- Current sensor.  
This sensor gives a voltage-image of the current in the coil. It is a necessary sensor, to ensure that the current is maintained to a constant RMS value, and that no distortion is introduced in the waveform fedded to the coil. This sensor is included inside the generator.
- Generator.  
We use a B&K generator which practically generates an harmonic current into the coils. A current generator is necessary, to ensure that no magnetic feedback from the source-receptor interaction gets into the loop.

## Experimental set-up: software.

The software used to build the application is LabVIEW 2012.

The application allows the user to select and monitor:

- The source frequency
- The source intensity
- The expected frequency ratio between the source and the pendulum.

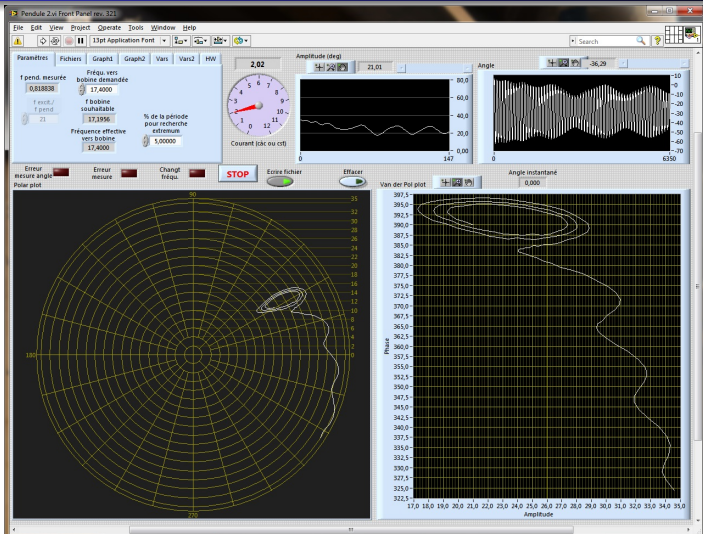
The user can visualize in real-time:

- The pendulum's motion
- The pendulum's amplitude
- The van der Pol diagram in rectangular coordinates
- The van der Pol diagram in polar coordinates

The argumentary oscillator.  
Experimental set-up.  
Modelling.  
Experimental results.  
Future work.

Sources.  
Receptors.  
Sensors and generator.  
Software.

## Experimental set-up: application's main screen.



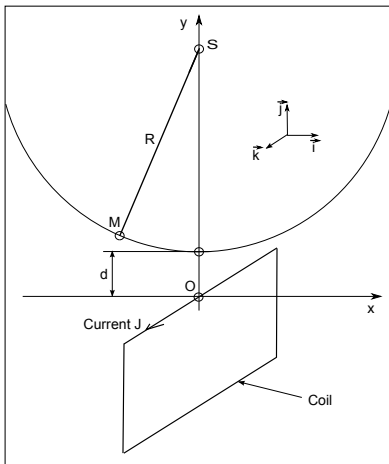
## Modelling Type I source.

The Type I source can be modeled as an infinite-length wire. The magnetic field at point M is:

$$\mathbf{B} = \frac{B_0 d}{OM} \mathbf{v} \quad (1)$$

where

- $B_0 = \frac{\mu_0 NJ}{2\pi}$
- $\mu_0$ =permeability of void
- N=number of turns in the coil
- J=value of the current in the coil
- $\mathbf{v}$ =unitary vector perpendicular to  $\mathbf{OM}$ .





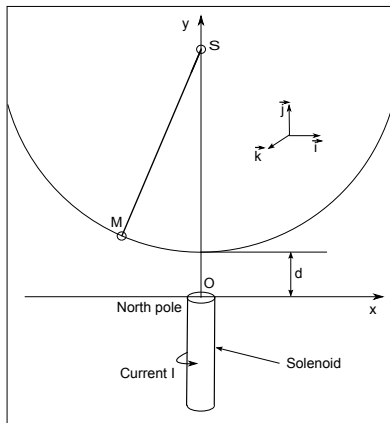
## Modelling Type II source.

The Type II source can be modeled as a magnetic dipole. The magnetic field at point M is:

$$\mathbf{B} = \frac{B_0}{OM^2} \mathbf{u}$$

where

- $B_0 = \frac{\mu_0 M_n}{4 \pi d^2}$
- $\mu_0 =$  permittivity of void ( $4\pi 10^{-7}$ )
- $M_n = \mu_0 N S I$  is the magnetic moment of the solenoid, with  $N =$  number of turns,  $S =$  area of one turn,  $I =$  current.
- $d =$  minimum distance between M and the solenoid.
- $\mathbf{u} =$  unitary vector along  $\mathbf{OM}$ .









## Modelling Types 1, 2 and 3 receptors.

- The Type 1 receptor is a steel sphere. It behaves like a magnetic dipole oriented along the magnetic field  $B$ , in opposite direction to  $B$ .
- The Type 2 receptor is a small magnet whose moment is along the pendulum's rod.
- The Type 3 receptor is a small magnet whose moment is perpendicular to the pendulum's rod.

## Modelling source-receptor pairs.

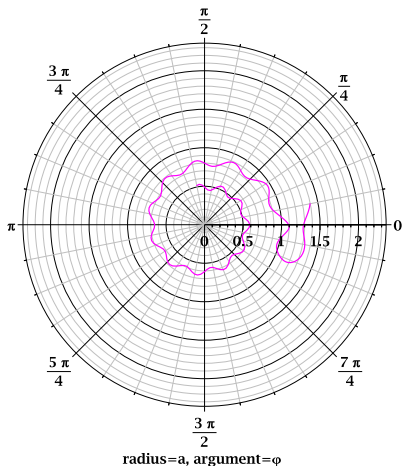
- The interaction between a Type 1 receptor and a Type I or II source is a force.
- The interaction between a Type 2 or 3 receptor and a Type I or Type II source is the sum of a force and a torque.

## Summary Table: the six H-functions.

	Type I source: Wire along Oz	Type II source: Solenoid along y'y
Type 1 receptor	$\frac{T}{T_0} = -\frac{\alpha}{(1+\eta^2\alpha^2)^2} \text{ with } T_0 = \kappa B_0^2 \eta^2$  $T_{max} = 0.325 \frac{T_0}{\eta} \text{ at } \alpha = \frac{1}{\eta\sqrt{3}}; L_{0.5} \approx \frac{1.214}{\eta}$	$\frac{T}{T_0} = \frac{\alpha}{(1+\eta^2\alpha^2)^3} \text{ with } T_0 = 2\kappa B_0^2 \eta^2$  $T_{max} = 0.259 \frac{T_0}{\eta} \text{ at } \alpha = \frac{1}{\eta\sqrt{5}}; L_{0.5} \approx \frac{0.8458}{\eta}$
Type 2 receptor	$\frac{T}{T_0} = \frac{1-\eta^2\alpha^2}{(1+\eta^2\alpha^2)^2} \text{ with } T_0 = -(1+\eta)m_0 B_0$  $T_{max} = T_0 \text{ at } \alpha = 0; L_{0.5} \approx \frac{0.972}{\eta}$	$\frac{T}{T_0} = \frac{1-\frac{\eta}{6}\alpha^2}{(1+\eta^2\alpha^2)^{\frac{5}{2}}} \alpha \text{ with } T_0 = 3\eta^2 m_0 B_0$  $T_{max} = 0.286 \frac{T_0}{\eta} \text{ at } \alpha = \frac{1}{2\eta}; L_{0.5} \approx \frac{0.984}{\eta}$
Type 3 receptor	$\frac{T}{T_0} = \frac{\alpha}{(1+\eta^2\alpha^2)^2}, T_0 = (1+\eta)(1+2\eta)m_0 B_0$  $T_{max} = 0.325 \frac{T_0}{\eta} \text{ at } \alpha = \frac{1}{\eta\sqrt{3}}; L_{0.5} \approx \frac{1.214}{\eta}$	$\frac{T}{T_0} = \frac{1-2\eta^2\alpha^2}{(1+\eta^2\alpha^2)^{\frac{5}{2}}}, T_0 = (1+\eta)m_0 B_0$  $T_{max} = T_0 \text{ at } \alpha = 0; L_{0.5} \approx \frac{0.768}{\eta}$

## Van der Pol amplitude/phase representation.

$\nu = 63.0, \lambda = 10.0357, \beta = 0.0050, \mu = -0.1667, A = 2.100, n = 11.$



- The averaging method searches an approximate solution  $x = a(t) \sin(\omega t + \varphi(t))$ , where  $a(t)$  and  $\varphi(t)$  are slowly-varying functions of time.
- $(a, \varphi) \rightarrow$  Natural representation: van der Pol.
- Polar form is better, due to invariance vs  $\frac{2\pi}{n}$  rotation. Radius= $a$ , argument= $\varphi$ .
- Example of integral curve winding up around the origin (numeric calculus from averaged equations).
- Initial conditions:  $a_0 = 1.45, \varphi_0 = 0.2$
- Parameters:  $\lambda = \frac{\nu}{\omega_0} = 10.0357, \eta = 100.$

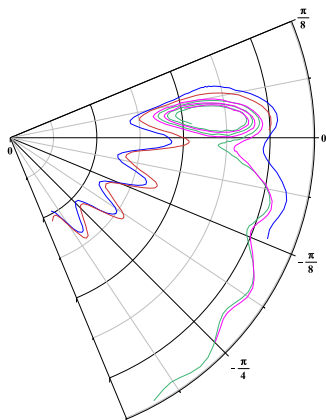
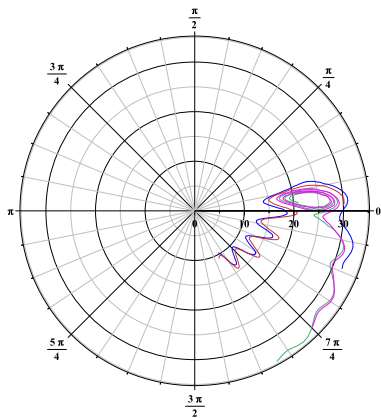
## Source I with Receptor 1.

No experimental results available as of yet, due to the weakness of the magnetic interaction. Construction of a new coil and enhanced receptor are ongoing.

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Experimental results.  
Future work.

Representation.  
Experimental results.

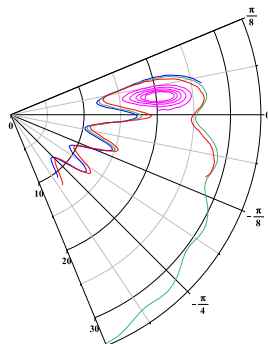
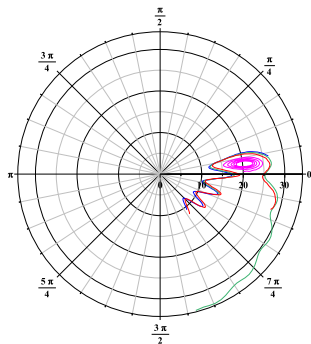
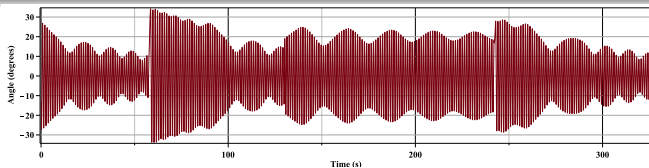
## Source 1 with Receptor 2.



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## Source 1 with Receptor 3.

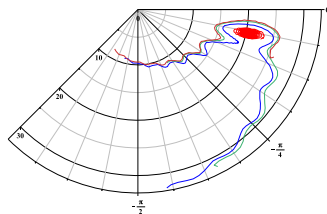
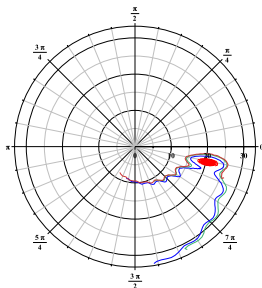




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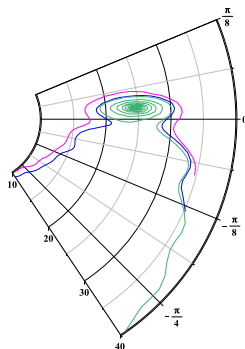
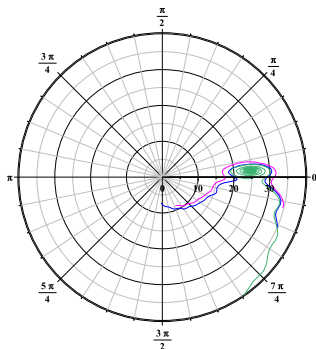
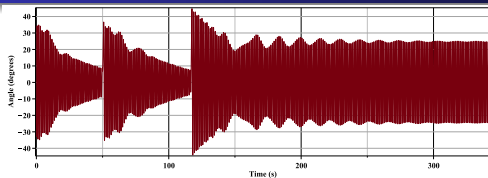
## Source II with Receptor 1.



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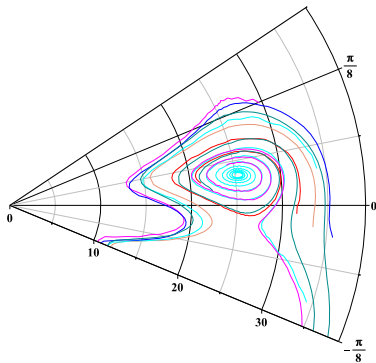
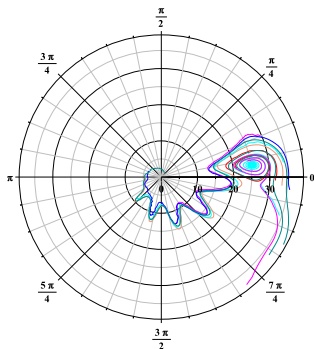
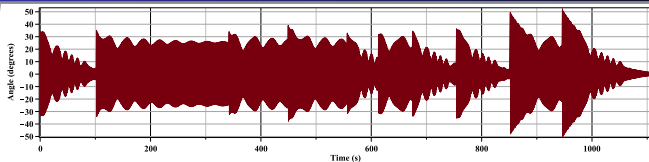
## Source II with Receptor 2.



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## Source II with Receptor 3.



## Future work

- Establishment of a symbolic assessment of the error due to the averaging method.
- Development of other models (1 DOF).
- More than 1 DOF.

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Thank you for your attention.