

Argumentary Duffing oscillators-VISHNO 2014

Stable-regime probability

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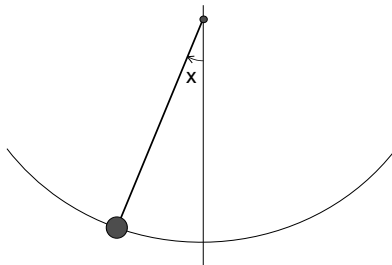
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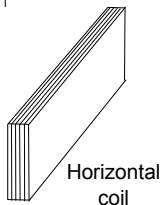
XIXth SYMPOSIUM VISHNO

Vibrations, SHocks and NOise

Definition, type, equation.



Vertical coil

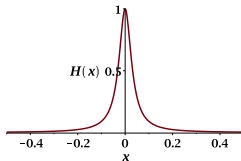
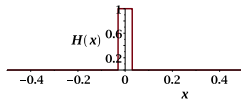
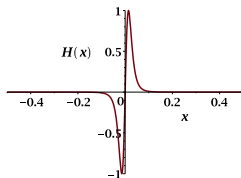


Horizontal coil

- History: Béthenod, Penner, Doubochinski.
- Oscillator of Duffing type.
- Submitted to an external harmonic force. Coupling modulation by the spatial position.
- Equation (reduced time):

$$\frac{d^2x}{dt^2} + 2\beta \frac{dx}{dt} + x + \mu x^3 = A H(x) \sin\left(\frac{\nu t}{\omega_0}\right)$$

The H function for various physical models.



These types are linked to the model of the physical interaction between the oscillator and the external force.

- $H(x) = \frac{x}{(1 + \gamma x^2)^3}$ for type I (Béthenod),
- $H(x) = 1$ if $|x| < x_0$ and $H(x) = 0$ otherwise for type IIa (Doubochinski),
- $H(x) = \frac{1}{1 + \gamma x^2}$ for type IIb (our model).

The averaging method.

Starting from
$$\frac{d^2x}{dt^2} + 2\beta \frac{dx}{dt} + x + \mu x^3 = A H(x) \sin\left(\frac{\nu t}{\omega_0}\right),$$

observing that amplitude and phase of $x(t)$ are slowly variable $\rightarrow x(t) = a(t) \sin(\rho t + \varphi(t))$ with $\rho = \text{constant}$.

Obtaining a standard system, with $\theta(t) = \rho t + \varphi(t)$:

$$\begin{cases} \dot{a} = \frac{\cos(\theta)}{\rho} \left(-2\beta a \rho \cos(\theta) - \mu a^3 \sin^3(\theta) + A H(a \sin(\theta)) \sin\left(\frac{\nu}{\omega_0} t\right) + a \sin(\theta)(\rho^2 - 1) \right) \\ \dot{\varphi} = \frac{\sin(\theta)}{a \rho} \left(-a \sin(\theta)(\rho^2 - 1) + 2\beta a \rho \cos(\theta) + \mu a^3 \sin^3(\theta) - A H(a \sin(\theta)) \sin\left(\frac{\nu}{\omega_0} t\right) \right), \end{cases}$$

with $a(t)$ and $\varphi(t)$ = amplitude and phase of the oscillator.

Averaging versus time (type II oscillators):

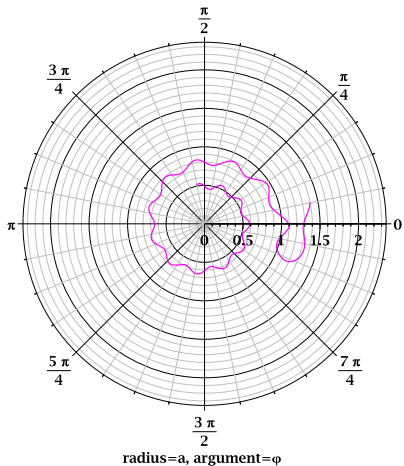
$$\begin{cases} \dot{a} = -\frac{A}{4\rho} S(a) \sin(n\varphi) - \beta a \\ \dot{\varphi} = \frac{3\mu}{8\rho} (a^2 - a_n^2) - \frac{A}{4a\rho} \cos(n\varphi) D(a). \end{cases}$$

with $n = \text{odd integer}$, $\rho = \frac{\nu}{n \omega_0}$, $3\mu \frac{a_n^2}{4} = \rho^2$, $S(a)$ and $D(a)$ = functions depending only on the form of function H.

Problem is invariant by a $\frac{2\pi}{n}$ rotation.

Van der Pol amplitude/phase representation.

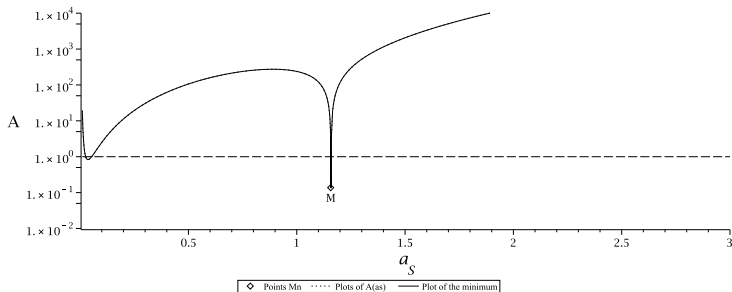
$\nu = 63.0, \lambda = 10.0357, \beta = 0.0050, \mu = -0.1667, A = 2.100, n = 11.$



- $(a, \varphi) \rightarrow$ Natural representation: van der Pol, polar, with radius= a , argument= φ .
- Polar form is better, due to invariance vs $\frac{2\pi}{n}$ rotation.
- Example of integral curve winding up around the origin (numeric calculus from averaged equations).
- Initial conditions: $a_0 = 1.45, \varphi_0 = 0.2$
- Parameters: $\lambda = \frac{\nu}{\omega_0} = 10.0357, \gamma = 10100.$

The A function.

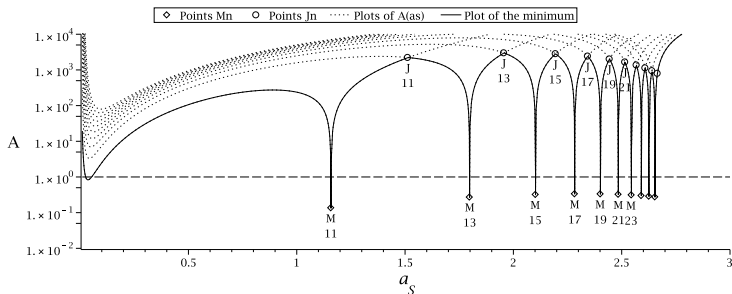
Recalling
$$\begin{cases} \dot{a} = -\frac{A}{4\rho} S(a) \sin(n\varphi) - \beta a \\ \dot{\varphi} = \frac{3\mu}{8\rho} (a^2 - a_n^2) - \frac{A}{4a\rho} \cos(n\varphi) D(a) \end{cases} \quad \text{with} \quad \begin{cases} \dot{a} = 0 \\ \dot{\varphi} = 0 \end{cases}$$



External force amplitude (A) as a function of the oscillation amplitude at the stationary solution (a_S).

The A function - Contiguous regions.

One value for n for each region located between contiguous J points.

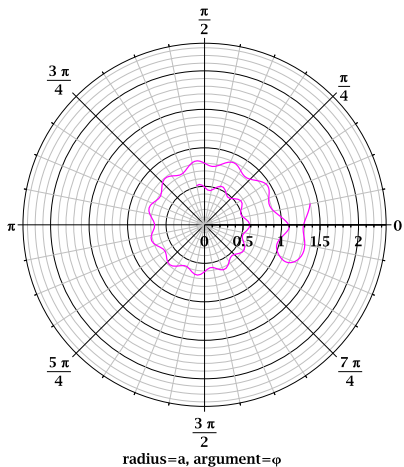


External force amplitude (A) as a function of the oscillation amplitude at the stationary solution (a_S).

Points J_n define a border between regions where n prevails.
 Multiple stable amplitudes for a given value of A .

No-capture case.

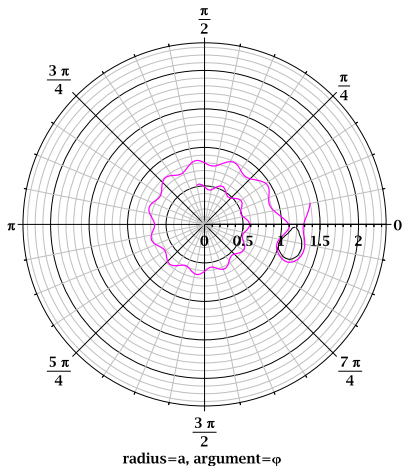
$\nu = 63.0, \lambda = 10.0357, \beta = 0.0050, \mu = -0.1667, A = 2.100, n = 11.$



- Integral curve winding up around the origin.
- Initial conditions: $a_0 = 1.4, \varphi_0 = 0.2$
- Parameters: $\lambda = \frac{\nu}{\omega_0} = 10.0357, \gamma = 10100.$

Capture region.

$v=63.0, \lambda=10.0357, \beta=0.0050, \mu=-0.1667, A=2.100, n=11.$

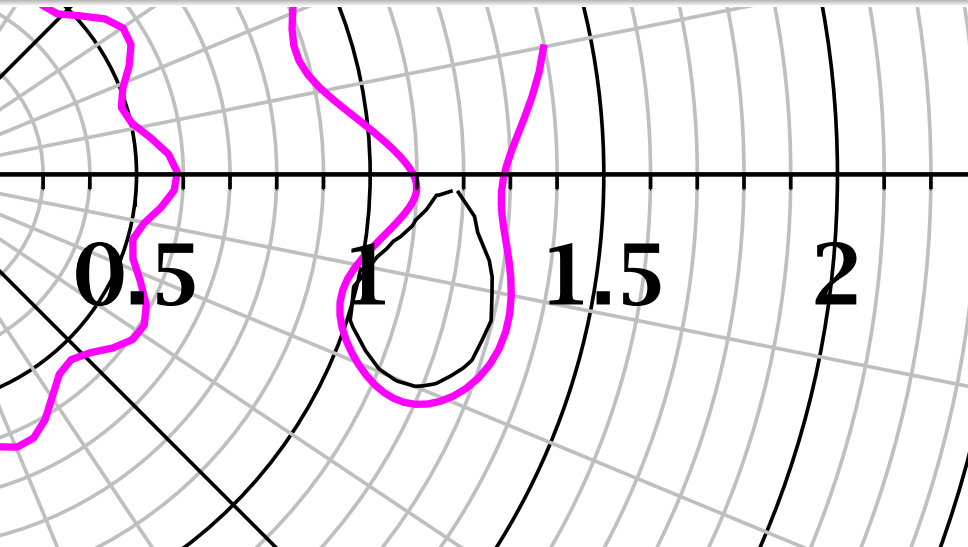


- Local study gives a capture region.
- The integral curve wraps around a capture region.

The argumentary Duffing oscillator.
Averaging.
Capture phenomenon.
Applications and future work.

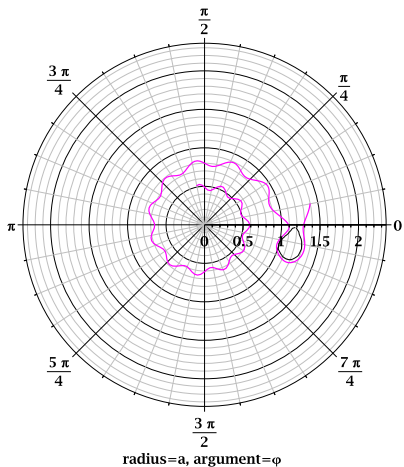
No-capture case.
Annulus structure.
Capture case. Diameters.
Capture probability.

Capture region.



Capture region.

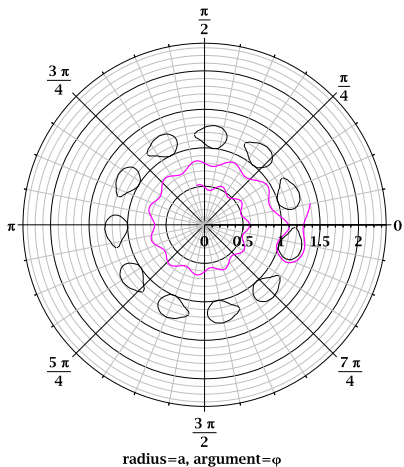
$\nu = 63.0, \lambda = 10.0357, \beta = 0.0050, \mu = -0.1667, A = 2.100, n = 11.$



- Local study gives a capture region.
- The integral curve wraps around a capture region.

One annulus.

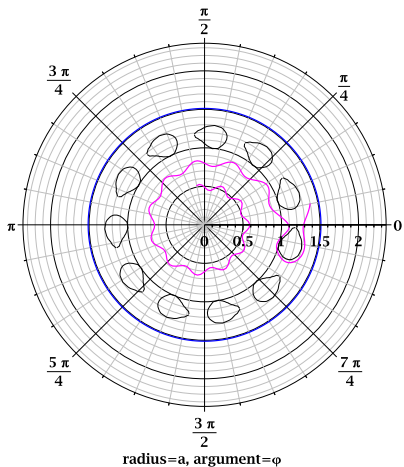
$v=63.0, \lambda=10.0357, \beta=0.0050, \mu=-0.1667, A=2.100, n=11.$



- The capture regions relative to a given value of n .
- Invariance vs $\frac{2\pi}{n}$ rotation.

Annulus border.

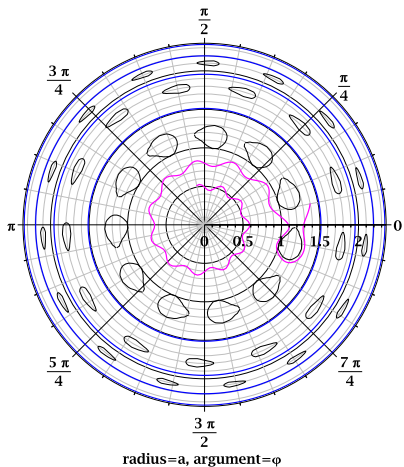
$v=63.0, \lambda=10.0357, \beta=0.0050, \mu=-0.1667, A=2.100, n=11.$



- Annulus border, linked to points J.

Complete annuli structure.

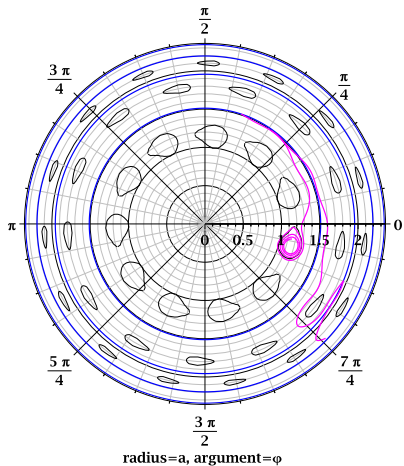
$\nu = 63.0$, $\lambda = 10.0357$, $\beta = 0.0050$, $\mu = -0.1667$, $A = 2.100$, n from 11 to 17.



- One annulus for each n .

Capture case. Integral curve through multiple annuli.

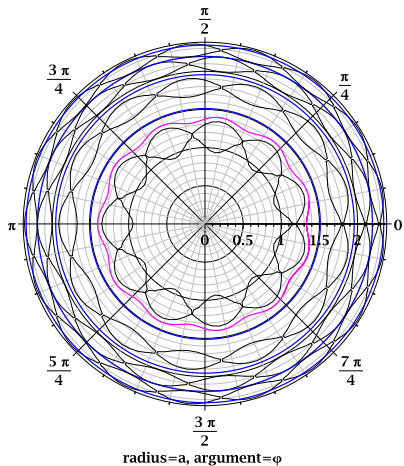
$\nu = 63.0, \lambda = 10.0357, \beta = 0.0050, \mu = -0.1667, \Lambda = 2.100, n$ from 11 to 17.



- Integral curve captured.
- Initial conditions:
 $a_0 = 2.171791,$
 $\varphi_0 = -0.758700.$

No-capture case with $\beta = 0$.

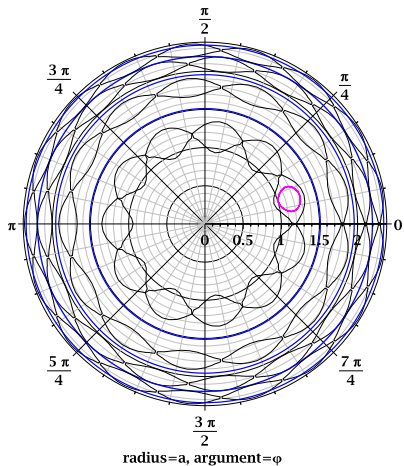
$\nu = 63.0, \lambda = 10.0357, \beta = 0.0000, \mu = -0.1667, A = 2.100, n$ from 11 to 17.



- Integral curve not captured.
- Closed curve.
- Amplitude stays above non-null minimum.
- Initial conditions:
 $a_0 = 1.4, \varphi_0 = 0.22$.

Capture case with $\beta = 0$.

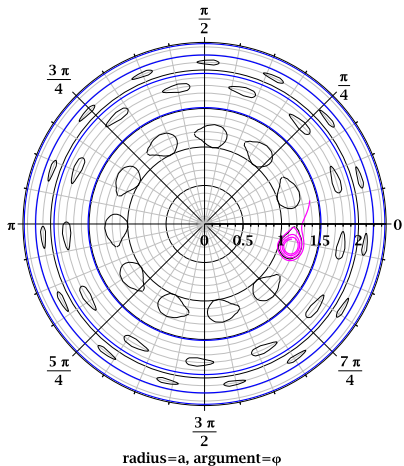
$\nu = 63.0, \lambda = 10.0357, \beta = 0.0000, \mu = -0.1667, \Lambda = 2.100, n$ from 11 to 17.



- Integral curve captured.
- Closed curve, not a spiral.
- Initial conditions:
 $a_0 = 1.2, \varphi_0 = 0.15$.

Capture case. Angular and radial diameters.

$\nu = 63.0, \lambda = 10.0357, \beta = 0.0050, \mu = -0.1667, A = 2.100, n$ from 11 to 17.

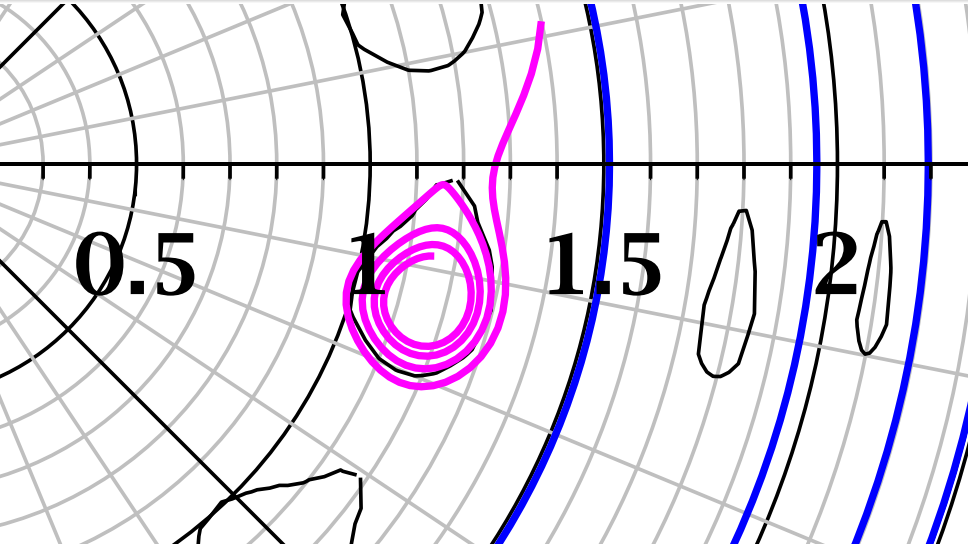


- Integral curve captured.
- Initial conditions:
 $a_0 = 1.4, \varphi_0 = 0.22$.
- Size and shape of the capture region:
 D_{ang} and D_{rad} .

The argumentary Duffing oscillator.
Averaging.
Capture phenomenon.
Applications and future work.

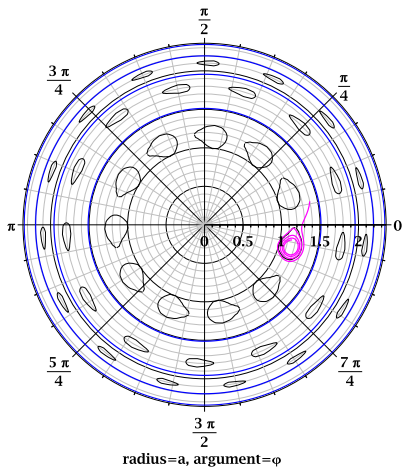
No-capture case.
Annulus structure.
Capture case. Diameters.
Capture probability.

Capture case. Angular and radial diameters.



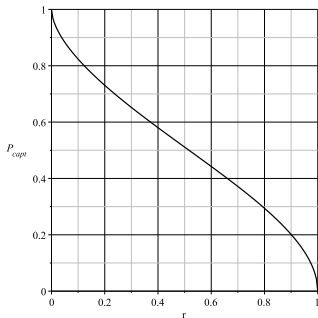
Capture case. Angular and radial diameters.

$\nu = 63.0, \lambda = 10.0357, \beta = 0.0050, \mu = -0.1667, A = 2.100, n$ from 11 to 17.



- Integral curve captured.
- Initial conditions:
 $a_0 = 1.4, \varphi_0 = 0.22$.
- Size and shape of the capture region:
 D_{ang} and D_{rad} .

Capture probability - Initial condition on circle $a = a_n$.



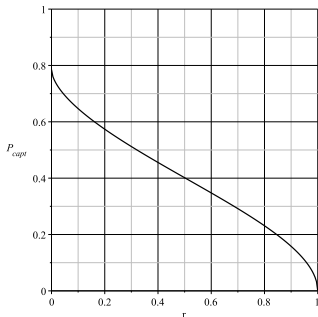
Capture Probability:

$$P \approx \frac{\text{Length inside capture region}}{\text{Length of circle crossing capture region}}$$

$$P \approx 1 - \frac{1}{2\pi} \left(\arcsin(r) + \frac{3}{2} \pi \left(\frac{2}{\pi} |\arcsin(r)| \right)^{0.56} \right).$$

$$\text{with } r = \frac{\frac{\beta}{A}}{\left(\frac{\beta}{A}\right)_{crit}} \quad \text{and} \quad \left(\frac{\beta}{A}\right)_{crit} = \frac{S(a_n)}{4\rho a_n}.$$

Capture probability - Initial condition inside annulus.



Capture Probability:

$$P \approx n \frac{\text{Area of capture region}}{\text{Area of annulus}}.$$

$$D_{ang}(n) \approx a_n \left(\varphi_T - \frac{\pi}{n} \left(\frac{3}{2} \left(\frac{2}{\pi} |n \varphi_T| \right)^{0.56} - 2 \right) \right),$$

with a_n =minimum of $A(a)$ for n , a_S =stationary amplitude for n and A ,

$$\text{and } \varphi_T \approx -\frac{1}{n} \arcsin \left(\frac{4 \beta \rho a_S}{A S(a_n)} \right).$$

$$D_{rad}(n) = a_n \frac{\sqrt{\eta^2 + 4(\zeta - \eta) \left(\beta \left(\frac{\pi}{n} + 2\varphi_T \right) + 2\eta \right)}}{\zeta - \eta},$$

$$\text{with } \eta = \frac{\beta}{n} \cot(n\varphi_T) \text{ and } \zeta = \frac{3}{8} \frac{\mu}{\rho} a_n^2.$$

$$P = \frac{3|\mu|}{64\lambda^2} \frac{n^3(n+2)^2}{n+1} D_{ang}(n) D_{rad}(n),$$

$$\text{with } \lambda = \frac{\nu}{\omega_0}.$$

Applications

The symbolic assessment of the capture probability of an argumentary oscillator should help to:

- Avoid argumentary resonance when resonance is unwanted (e.g. structure vibration, rotating machines).
- Induce argumentary resonance when resonance is desired (industrial processes, e.g. fluid vaporization).

Future work

- Establishment of a symbolic assessment of the error due to the averaging method.
- Development of other models (1 DOF).
- More than 1 DOF.

Thank you for your attention.