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### ► To cite this version:

Virgile Lacharnay, Sylvain Lavernhe, Christophe Tournier, Claire Lartigue. A physically-based model for global collision avoidance in 5-axis point milling. *Computer-Aided Design*, 2015, 64, pp.1-8. 10.1016/j.cad.2015.02.003 . hal-01121686

HAL Id: hal-01121686

<https://hal.science/hal-01121686v1>

Submitted on 2 Mar 2015

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# A physically-based model for global collision avoidance in 5-axis point milling

Virgile Lacharnay<sup>a</sup>, Sylvain Lavernhe<sup>a</sup>, Christophe Tournier<sup>a,\*</sup>, Claire Lartigue<sup>a</sup>

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## Abstract

Although 5-axis free form surface machining is commonly proposed in CAD/CAM software, several issues still need to be addressed and especially collision avoidance between the tool and the part. Indeed, advanced user skills are often required to define smooth tool axis orientations along the tool path in high speed machining. In the literature, the problem of collision avoidance is mainly treated as an iterative process based on local and global collision tests with a geometrical method. In this paper, an innovative method based on physical modeling is used to generate 5-axis collision-free smooth tool paths. In the proposed approach, the ball-end tool is considered as a rigid body moving in the 3D space on which repulsive forces, deriving from a scalar potential field attached to the check surfaces, and attractive forces are acting. A study of the check surface tessellation is carried out to ensure smooth variations of the tool axis orientation. The proposed algorithm is applied to open pocket parts such as an impeller to emphasize the effectiveness of this method to avoid collision.

**Keywords:** 5-axis Machining, Collision-free, Potential field, Tool path generation, Ball-end milling

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## 1. Introduction

5-axis surface machining is an essential process in the field of aerospace, molds and dies industries. 5-axis milling is required for the realization of difficult parts such as blades and impellers and is also very convenient to improve quality for the machining of deep molds in plastic injection and casting by reducing tool length. Despite the evolution of CAM software, 5-axis tool path programming requires advanced skills and collision detection remains a challenge during tool path computation. One can distinguish two kinds of tool collision when addressing machining issues: local gouging, involving the active part of the tool and global collisions where the whole body of the tool, the tool holder and the spindle can be considered. In this paper, only global collisions are studied. In the literature numerous papers deal with global collision avoidance in 5-axis milling. Several approaches exist and are based on collision tests executed during the tool path computation or after during a post-processing of the tool path. The proposed methods often address the problem from point to point, without an entire view of the tool path,

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14 which leads to non-optimal tool paths and oscillations of the tool axis. Methods are usually based  
15 on models to represent the tool geometry and the environment (part surface, check surfaces, etc),  
16 a collision test between the obstacle and the tool and finally a correction or optimization of the  
17 tool axis orientation to avoid the obstacle. It is during this final stage of optimization that the  
18 smoothness of the trajectory may be corrected.

19 Two main approaches exist in the literature: geometric methods, which are the most used,  
20 and potential methods. In both approaches the modeling of the tool and the check surface is  
21 required. In most cases, the tool is divided into implicit surfaces (cylinders, cones) [1] leading  
22 to the description of the tool under the APT formalism [2]. The check surface, usually designed  
23 in the CAD system by a parametric surface, is modeled as a NURBS surface [3] by its convex  
24 envelope [4] or by a tessellated representation to simplify computations.

25 With the geometric approach, the problem is mainly treated in a local coordinate system attached  
26 to the tool using the C-Space approach [5]. Interferences between the tool and the check surface  
27 are detected using algorithms primarily based on surface intersections [6]. These tests lead to the  
28 definition of a collision-free area in the C-Space to orient the tool axis [7].

29 Another geometric method frequently used to evaluate the interferences is based on the cones  
30 and maps of visibility. This problem addressed by [8] and [9] enables, using a Gaussian sphere,  
31 to generate a local visibility map taking into account the part surface and then to integrate the  
32 machine constraints of accessibility (tool, tool holder, environment) to reduce the space available  
33 for the tool axis (global visibility). Other works increase the visibility relevance even further by  
34 taking into account the travel range of the machine tool which reduces the available area on the  
35 Gaussian sphere [10], [11].

36 The final step is the optimization of the tool path in the resulting C-Space collision-free do-  
37 main including constraints such as smoothness of the tool postures or tool length minimization  
38 [12],[13].

39 The other approach, based on potential fields, has been developed in the domain of mobile  
40 robotics for collision avoidance. This consists in using virtual potential fields that allow a robot  
41 to avoid the obstacle during an excessive approach [14]. Indeed, a repulsive force, calculated  
42 as the gradient of the scalar potential field, tends to infinity when the distance between the mo-  
43 bile robot and the obstacle tends to zero, thereby deflecting the initially programmed path. This  
44 method has been improved to handle special cases associated to the position of the obstacles and  
45 the "goal" point to reach [15] [16]. In addition, taking into account the dynamics [17] illustrates  
46 the presence of oscillations when the robot moves back towards the programmed position. How-  
47 ever, in the field of mobile robotics, this issue is less critical due to the large tolerances allowed  
48 on the trajectories.

49 This approach has already been applied within the context of 5-axis machining for collision  
50 avoidance in a static case. Indeed, the work of [18] uses a simplified version of the formulation  
51 of repulsive forces developed by [14] to treat local and global collisions. The distances be-  
52 tween the tool and the part and the check surfaces are reformulated into an energy minimization  
53 problem to iteratively determine a better tool posture. However, since the proposed approach  
54 is quasi-static, i.e. applied from point to point on the trajectory, the appearance of oscillations  
55 is a problem raised by the authors themselves. Finally, this type of static application was also  
56 developed as part of a haptic manipulation to guide the tool axis [19].

57  
58 Thus the aim of this paper is to show the benefit of a dynamic method using potential fields  
59 to compute the tool axis orientation along a given tool path ensuring collision avoidance and  
60 smooth trajectories in 5-axis ball-end milling. This new approach allows in particular to avoid

61 the optimization stage of the tool axis orientation in the collision-free C-Space domain required  
 62 to ensure the smoothness of the tool path. A particular attention is paid to the influence of the  
 63 check surface tessellation to compute the repulsive force. The computation of the cutter location  
 64 points according to a chordal deviation and a scallop height is out of the scope of this paper.  
 65 Cutter location points are modeled as continuous polynomial curves.

66 The rest of the paper is organized as follows: the mechanical model of the tool movement  
 67 computation is presented in section 2. Simulation parameter values are investigated in section  
 68 3. An application to the machining of a 5-axis open pocket is carried out in section 4 and  
 69 results demonstrate the efficiency of the proposed method in terms of collision avoidance and  
 70 smoothness. Finally, the conclusions are summarized in section 5.

## 71 2. The potential field approach

### 72 2.1. General framework

73 In the proposed approach, the tool is considered as a rigid body moving in the 3D space on  
 74 which repulsive and attractive forces are acting. 5-axis collision avoidance is managed thanks  
 75 to repulsive forces deriving from a potential field. Thus, the aim of this section is to set up the  
 76 equations of the tool movement along the tool path and between the obstacles.

77 In order to illustrate the effect of repulsive and attractive forces, the tool geometry is reduced to  
 78 a unique point such as its center of mass  $G$ , located on the tool axis. However, the tool could be  
 79 modeled as a set of points  $P$  which are distributed whether on the tool axis or on the tool surface.

80 In 5-axis ball-end milling, the tool axis orientation is defined in the local coordinate system  
 81  $(\underline{C}_L, \underline{f}, \underline{n}, \underline{t})$  where  $\underline{C}_L$  is the tool center,  $\underline{f}$  is the unit vector tangent to the tool path,  $\underline{n}$  is the unit  
 82 vector normal to the part surface and  $\underline{t}$  is given by  $\underline{t} = \underline{f} \wedge \underline{n}$  (Fig.1). In this coordinate system, the  
 83 tool axis can be rotated around each of the three unit vectors without generating local collision  
 84 on the active part. In the proposed method, roll angle  $(\theta_f, \underline{f})$  and pitch angle  $(\theta_t, \underline{t})$  are used to  
 85 control the tool axis orientation.

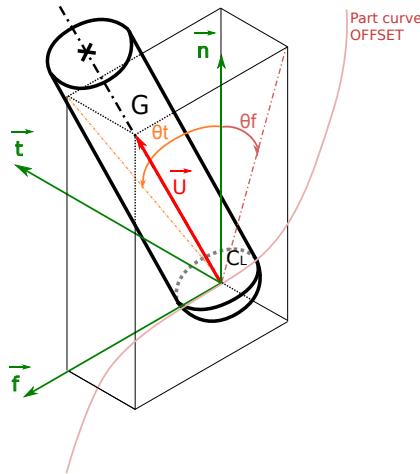


Figure 1: Tool position and tool axis orientation set-up

86 The tool center follows the programmed tool path whereas the tool axis orientation is com-  
 87 puted to avoid the obstacles by resolving the fundamental principle of dynamics. Furthermore,  
 88 the tool velocity along the tool path is supposed to be constant and equal to a value defined by the  
 89 end user. This also establishes a simple relationship between time  $t$  and the path displacement  $s$   
 90 (cumulative arc length) throughout the tool path. This principle applied to the center of mass  $G$   
 91 of the tool and expressed in the local frame  $(\underline{C}_L, \underline{f}, \underline{n}, t)$ , leads to the following Eq.(1):

$$\underline{J} \cdot \frac{d\underline{\Omega}(t)}{dt} = \underline{\mathcal{T}}(t) \quad (1)$$

92 where  $\underline{J}$  is the inertia tensor,  $\underline{\Omega}(t)$ , the angular velocity of the tool, which derives from the  
 93 angular position  $\theta_t$  and  $\theta_n$  within the local frame, and  $\underline{\mathcal{T}}(t)$  the total torque.  
 94 Since the tool axis can spin around the two vectors  $\underline{f}$  and  $\underline{t}$ , Eq.(1) can be split into two separated  
 95 scalar equations Eq.(2) and Eq.(3):

$$J_f \cdot \frac{d^2\theta_f(t)}{dt^2} = \mathcal{T}_f(t) \quad (2)$$

$$J_t \cdot \frac{d^2\theta_t(t)}{dt^2} = \mathcal{T}_t(t) \quad (3)$$

96 Thus the problem of tool axis orientation can be modeled as two independent pendulum  
 97 systems in two different planes and ordinary differential equation solver is used to compute the  
 98 tool motion.  
 99 Therefore, the behavior of the tool axis orientation is computed thanks to Eq.(1) whereas the tool  
 100 center follows the tool path at constant velocity  $v_0$  with:

$$s(t) = v_0 \cdot t + s(0) \quad (4)$$

101 Once the framework is established, it is necessary to define a model for the repulsive and  
 102 attractive forces acting on the tool, in order to compute the resulting torque.

### 103 2.2. Implementation of the repulsive and attractive forces

104 Repulsive forces acting on the tool are due to scalar potential attached to the check surfaces.  
 105 More precisely, each check surface is tessellated into a set of check points which are considered  
 106 as collision potential sources.  
 107 In order to ensure collision avoidance between the check surface and the tool, the expression of  
 108 the scalar potential generated by each check point  $O_i$  is the following Eq.(5):

$$\mathcal{U}_{rep_i} = \begin{cases} \frac{1}{2} \cdot \left( \frac{1}{(r_i - r_s)} - \frac{1}{r_0} \right)^2 & \text{if } (r_i - r_s) < r_0 \\ 0 & \text{else} \end{cases} \quad (5)$$

109 with:

- 110 •  $r_i$ : distance between the considered point  $P$  of the tool and the given check point  $(O_i)$
- 111 •  $r_0$ : check point neighborhood value (neighborhood sphere radius on Fig.3)
- 112 •  $r_s$ : security clearance

113 Given the scalar potential  $\mathcal{U}_{rep_i}$ , the repulsive vector field  $\underline{\mathcal{F}}_{rep_i}$  is defined by:

$$\underline{\mathcal{F}}_{rep_i} = -\nabla(\mathcal{U}_{rep_i}) = -\frac{\partial U_{rep_i}}{\partial r_i} \cdot \begin{pmatrix} \frac{\partial r_i}{\partial x} \\ \frac{\partial r_i}{\partial y} \\ \frac{\partial r_i}{\partial z} \end{pmatrix} \quad (6)$$

114 Assuming that  $r = r_i - r_s$  and  $\underline{u}_i = \frac{\underline{O}_i \cdot \underline{P}}{\|\underline{O}_i \cdot \underline{P}\|}$ , this leads to:

$$\underline{\mathcal{F}}_{rep_i} = \begin{cases} \left(\frac{1}{r} - \frac{1}{r_0}\right) \cdot \frac{1}{r^2} \cdot \underline{u}_i & \text{if } r < r_0 \\ 0 & \text{else} \end{cases} \quad (7)$$

115 The evolution of  $\|\underline{\mathcal{F}}_{rep_i}\|$  relative to  $r_i$ , the distance between the considered point of the tool  
 116 and the given check point  $O_i$  is plotted in Fig.2. The repulsive force tends to infinity when the tool  
 117 is entering the neighborhood of the check point and becomes closer to it. Collision avoidance is  
 118 ensured thanks to this behavior.

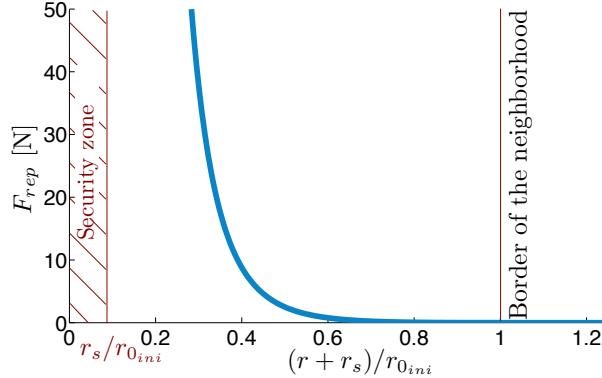


Figure 2: Repulsive force intensity evolution

119 Thus, the total repulsive force  $\underline{\mathcal{F}}_{rep}$  applied to the considered point  $P$  of the tool results from  
 120 the sum of the elementary repulsive forces produced by each of the  $n$  check points  $O_i$  with:

$$\underline{\mathcal{F}}_{rep} = \sum_{i=1}^n \underline{\mathcal{F}}_{rep_i} \quad (8)$$

121 The torque generated at the  $C_L$  point by the total repulsive force at each point of the tool  $P_j$   
 122 is given in Eq. (9):

$$\underline{\mathcal{T}}_{rep} = \sum_{j=1}^m \underline{C}_L \underline{P}_j \wedge \underline{\mathcal{F}}_{rep}(\underline{P}_j) \quad (9)$$

123 An attractive torque exerted by a spring is introduced to restore the tool axis orientation in  
 124 the programmed configuration as well as a viscous damper to allow the system to return to its  
 125 steady state without oscillating. Two attractive torques, for roll and pitch angles, are used. Thus  
 126 the additional repulsive and attractive torques lead to the two equations Eq.(10) and Eq.(11):

127  $J\ddot{\theta}_f + c\dot{\theta}_f + k(\theta_f - \theta_{f_{goal}}) = \mathcal{T}_{f_{rep}}$  (10)

128  $J\ddot{\theta}_t + c\dot{\theta}_t + k(\theta_t - \theta_{t_{goal}}) = \mathcal{T}_{t_{rep}}$  (11)

128 with:

129 •  $k$ : stiffness coefficient;

130 •  $c$ : damping coefficient;

131 •  $\theta_{f_{goal}}$ : programmed angle around  $f$ ;

132 •  $\theta_{t_{goal}}$ : programmed angle around  $t$ .

133 In these equations,  $\mathcal{T}_{f_{rep}}$  and  $\mathcal{T}_{t_{rep}}$  are the projections of the repulsive torque computed at the  $C_L$   
 134 point in the perpendicular plane to  $f$  and  $t$  respectively. The inertia  $J$ , the damping coefficient  
 135 and the mass are equal for both equations but they could be different as well, leading to a different  
 136 behavior in the two planes.

137 *2.3. Transient behavior set-up*

138 When the tool leaves the neighborhood area after avoiding obstacles, it is essential that the  
 139 tool axis returns in its steady state with a smooth response. In this area, the behavior of the tool  
 140 axis orientation can be modeled as a damped harmonic oscillator without external applied force.  
 141 The resulting differential equation is then:

$$J\ddot{\theta} + c\dot{\theta} + k\theta = 0 \quad (12)$$

142 which can be rewritten into the classical form:

$$\ddot{\theta} + 2\xi\omega_0\dot{\theta} + \omega_0^2\theta = 0 \quad (13)$$

143 with:

144 •  $\omega_0 = \sqrt{\frac{k}{J}}$ , the natural oscillating frequency;

145 •  $\xi = \frac{c}{2\sqrt{kJ}}$ , the damping ratio.

146 Thus, the damping parameter has to be determined according to the next equation relative to  
 147 a critically damped harmonic oscillator ( $\xi \geq 1$ ):

$$c \geq 2\sqrt{kJ} \quad (14)$$

148 where  $J$  has been set to 1.

149 Solving the differential equation without second member ensures a modification of the tool  
 150 axis orientation with an aperiodic behavior. The second order differential equations with second  
 151 member corresponding to equations 10 and 11 are not formally resolvable. These equations  
 152 are thus solved using a differential equation solver based on the Runge-Kutta RK4 numerical  
 153 method. The solver is the ODE 45 solver proposed in Matlab.

154 **2.4. First example of simple configuration**

155 The proposed example allows the validation of the transient behavior of the system when the  
 156 tool enters and exits an obstacle neighborhood. The tool, reduced to its center of mass  $G$  and  
 157 represented by its axis, is following a straight line. An obstacle, defined by a check point with a  
 158 given neighborhood, is located on the tool path.

159 The torque generated by the repulsive force is given in Eq.(9) with  $n = 1$  and  $P = G$ . As  
 160 equations (10) and (11) are independent, only the roll angle  $\theta_f$  around the feed vector  $f$  has been  
 161 used in this example. When the tool is entering the obstacle neighborhood, the repulsive force  
 162 is acting on the tool and modifies the tool axis orientation. The attractive torque generates the  
 163 return of the tool axis to its programmed orientation. Fig.3 illustrates the resulting smooth tool  
 164 axis motion. This simple example shows the benefit of the proposed method to avoid collisions.

165 **2.5. Parameters influence on collision avoidance**

166 The stiffness parameter  $k$ , the neighborhood value  $r_0$  and the inertia  $J$  of the tool model have  
 167 an influence on the way the tool axis orientation is modified when the tool comes closer to an  
 168 obstacle. To illustrate this behavior, the example of a single point obstacle is re-used. Several  
 169 values of  $r_0$ ,  $J$  and  $k$  are studied and the resulting behaviors of the tool axis orientation, especially  
 170 the location in the  $(f, t)$  plane of the center of mass  $G$  located on the tool axis, are plotted in Fig.4.  
 171 Initial values of  $r_0$ ,  $k$  and  $J$  produce the red curve.

172 The curve resulting from a multiplication by two of the neighborhood value  $r_0$  is the blue  
 173 one. Results show that the neighborhood parameter  $r_0$  can be seen as an anticipation parameter  
 174 to start modifying the tool axis orientation more or less further from the obstacle.

175 The influence of the inertia increase is illustrated by the black curve. As might be expected,  
 176 the result demonstrates that a heavier tool is pushed back later by the repulsive force. The behav-  
 177 ior after the obstacle is very different as the return of the tool axis to the programmed orientation  
 178 is very long.

179 As for the stiffness parameter  $k$ , the curve resulting from a multiplication of its value is the  
 180 green one. Its influence is on how fast the orientation of the tool axis is changed to avoid the

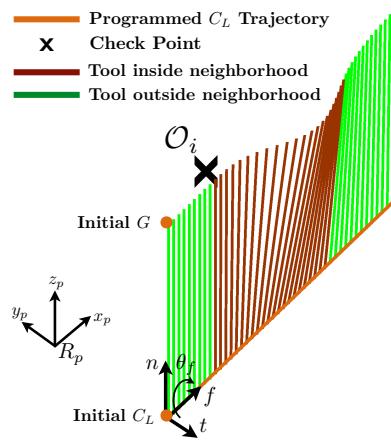


Figure 3: Tool axis orientation modified by a single point obstacle

181 obstacle. When the stiffness value increases, the modification of the tool axis orientation occurs  
 182 later and the spring force pulls back the tool quicker when leaving the neighborhood.

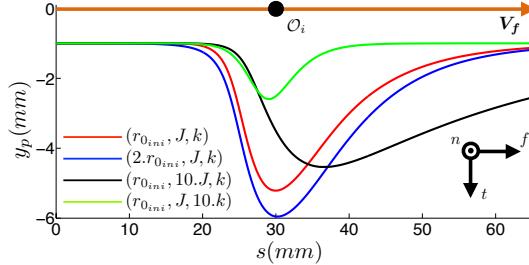


Figure 4: Influence of the stiffness, the inertia and the neighborhood on the tool orientation

183 This section has shown the relevance of the method to avoid a single point obstacle with a  
 184 critical damped behavior as well as the influence of the parameters on this behavior. The next  
 185 section is devoted to the general case of a surface obstacle as usually encountered in Computer-  
 186 Aided Manufacturing.

### 187 3. Continuous check surface discretization

#### 188 3.1. Problem definition

189 Usually, parts encountered in 5-axis milling exhibit continuous check surfaces modeled as  
 190 parametric surfaces. In order to apply the proposed approach, the check surfaces have to be  
 191 tessellated and each point of the mesh is considered as a repulsive point whether it belongs to  
 192 the neighborhood or not. By applying repulsive and attractive forces as described in the previous  
 193 section on a rough tessellation of a Bezier patch, the following behavior occurs (Fig.5).

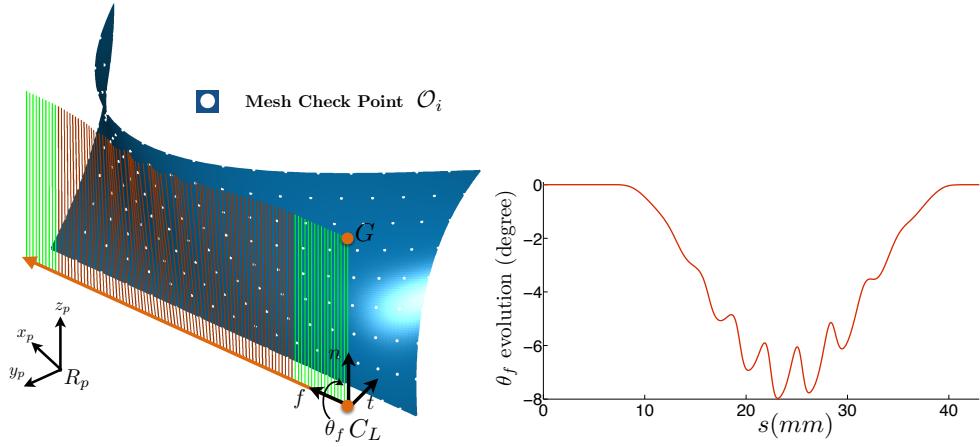


Figure 5: Low density of repulsive points ( $d=4\text{mm}$ )

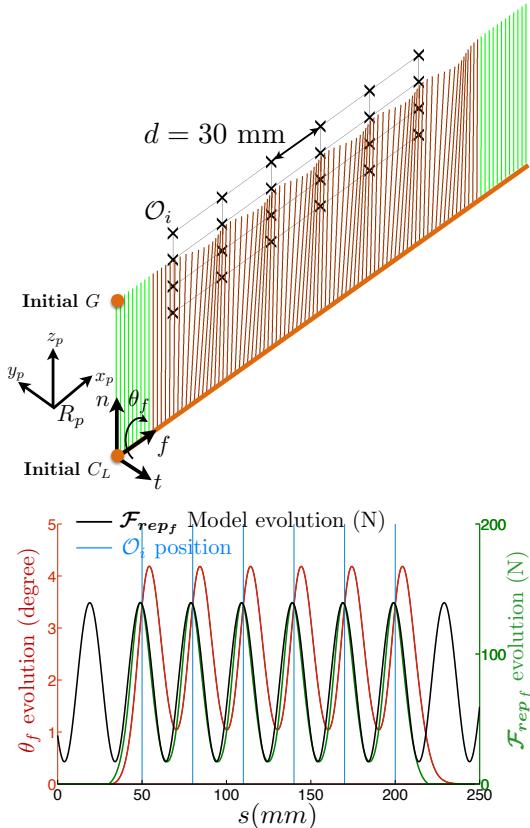


Figure 6: Repulsive force and tool axis orientation along a planar check surface with structured mesh

194 In this example, the mesh size is set to 4mm leading to 45 nodes in the mesh. The low  
 195 repulsive point density in the check surface mesh allows the tool to penetrate between the points,  
 196 thus generating collisions and oscillations. It is therefore important to study the density of the  
 197 check surface mesh to avoid this type of behavior.

198 *3.2. Steady-state solution analysis*

199 In order to determine the mesh size of the check surfaces to get a smooth response, the in-  
 200 fluence of points of the obstacle's mesh has to be investigated. The following case of study  
 201 illustrates the worst configuration in terms of steady-state solution. A tool moving along a planar  
 202 check surface enters successively different spherical potential fields created by a structured mesh  
 203 of the check surface which is aligned with the tool axis orientation. The tool geometry is reduced  
 204 to a unique point located at its center of mass. Before entering the first neighborhood, the orien-  
 205 tation of the tool axis is constant, equal to the programmed value. As the tool penetrates in the  
 206 different neighborhoods, large amplitude oscillations are generated due to the distance between  
 207 the aligned obstacles (Fig.6). Finally, the tool exits neighborhoods without oscillation respecting  
 208 the aperiodic response.

209 To study the amplitude of the observed oscillations when the tool enters the different neighborhoods, an analytical model is proposed based on a driven harmonic oscillator. The total  
210 repulsive force generated by the aligned obstacles is modeled as a sinusoidal driving force such  
211 as:  
212

$$\mathcal{F}_{rep} = \mathcal{F}_0 \cdot (1 + \cos(\omega \cdot t)) \quad (15)$$

213 The driving frequency  $\omega$  of these oscillations depends on the distance  $d$  between each obstacle  
214 and the velocity  $v_0$  of the tool along the trajectory:

$$\omega = 2\pi \cdot v_0 / d \quad (16)$$

215 The behavior of this model is depicted by the black curve in Fig.6 and the repulsive force  
216 created by the obstacles is the green one. Aligned mesh points are represented by the vertical  
217 blue lines. The amplitude of the model  $\mathcal{F}_0$  has been calibrated according to the real amplitude.  
218 Thus, the proposed model with the  $\cos$  function is relatively close to the actual behavior of the  
219 repulsive force.

220 Then, the considered differential equation is the following:

$$J \ddot{\theta} + c \dot{\theta} + k \theta = \mathcal{T}_0 \cdot (1 + \cos(\omega \cdot t)) \quad (17)$$

221 with:

$$\mathcal{T}_0 = C_L G \mathcal{F}_0$$

223 The response is the sum of the transient solution (without second member)  $\theta_1(t)$  and the  
224 steady-state solution  $\theta_2(t)$  with:

$$\theta(t) = \theta_1(t) + \theta_2(t) \quad (18)$$

225 with:

$$\begin{cases} \theta_1(t) = A_1 e^{-\frac{t}{\tau_1}} + A_2 e^{-\frac{t}{\tau_2}} \\ \theta_2(t) = B \cos(\omega \cdot t - \Phi) + \frac{\mathcal{T}_0}{k}. \end{cases} \quad (19)$$

226 Regarding the steady-state solution, the amplitude  $B$  and the phase  $\Phi$  are expressed as fol-  
227 lows:

$$B = \frac{\mathcal{T}_0 / k}{\sqrt{(1 - (\frac{\omega}{\omega_0})^2)^2 + 4 \cdot \xi^2 \cdot (\frac{\omega}{\omega_0})^2}} \quad (20)$$

$$\tan(\Phi) = \frac{2 \cdot \xi \cdot (\frac{\omega}{\omega_0})}{1 - (\frac{\omega}{\omega_0})^2} \quad (21)$$

228 The amplitude of the steady-state solution is illustrated in Fig.7 in the case of an aperiodic  
229 response ( $\xi = 1$ ) with the parameters of table 1. Thus, the choice of a driven frequency such as  
230  $\frac{\omega}{\omega_0} \rightarrow \infty$  corresponding to the reduction of the distance between the obstacles  $d$  along the tool  
231 path decreases the amplitude ratio.

232 Reducing the distance between the obstacles by a factor of 3,  $\omega_2 = 3\omega_1$ , leads to a greater  
233 attenuation of oscillations as illustrated in Fig.7 and Fig.8. However, oscillations are still visible  
234 in accordance with the model. Indeed, the case study is very unfavorable because the tool is  
235 reduced to a point so it can penetrate between two consecutive obstacles.

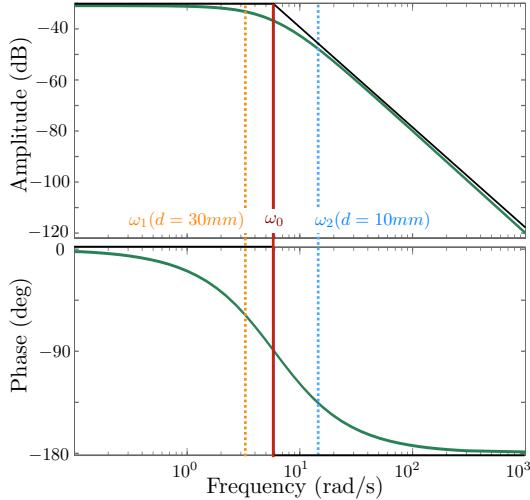


Figure 7: Gain pattern of damped system

236 3.3. Application to the Bezier patch

237 Based on this analysis, the mesh of the considered Bezier patch is densified with a mesh  
 238 size  $d$  equal to 1mm as illustrated in Fig.9. The total repulsive force  $\mathcal{F}_{rep}$  is plotted as well  
 239 as the resulting roll angle  $\theta_f$ . One can then observe that  $\mathcal{F}_{rep}$  presents small oscillations in  
 240 contrast to the smooth evolution of the angle  $\theta_f$ . Indeed, the mesh nodes of the check surface  
 241 are sufficiently close and are not aligned with the tool axis. The oscillations are attenuated on  
 242 one hand by the dynamic behavior of the second order system and on the other hand by the  
 243 numerical resolution done by the ODE 45 solver. Collision avoidance is respected and the return  
 244 to the initial programmed orientation is performed without oscillation. It should be noticed that  
 245 the mesh size must be adapted to the size of the check surface geometrical features in order  
 246 to avoid subsampling. Otherwise, collision may happen between the tool and these small local  
 247 features.

248 Fig.9 also emphasizes the positive effect of the dynamic approach compared to a static ap-  
 249 proach. As the differential equation of the tool motion is of second order, the variation of the  
 250 tool axis orientation always starts with a tangency continuity even for the worst case which is  
 251 the step-response. The resulting tool axis motion is smooth and the delay time is visible at the  
 252 beginning and at the end of the tool path. Finally, various parameters are at our disposal to pre-  
 253 vent oscillations, including the damping parameter and the mesh size of the check surface. If  
 254 necessary, it also remains the possibility of further improving the method using an unstructured  
 255 mesh (Fig.10). Indeed, when the mesh is unstructured, the check points are randomly distributed  
 256 and the repulsive force cannot be regular as a sinusoidal signal. Thus it helps to reduce the am-  
 257 plitude of the repulsive force oscillations and to remove oscillations of the tool axis orientation.  
 258 It would, for the same efficiency, reduce the number of nodes while ensuring a non-sinusoidal  
 259 repulsive force.

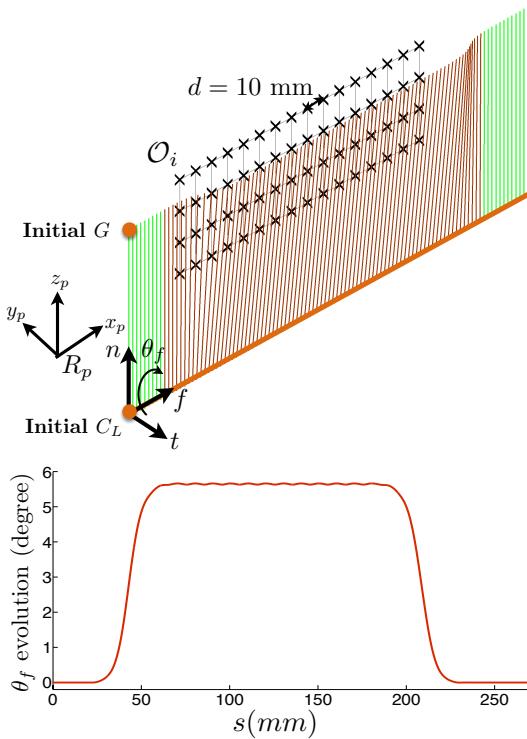


Figure 8: Tool axis behavior with an intermediate distance of 10mm with aligned obstacles

260 **3.4. Mesh size and repulsive force intensity**

261 As the total repulsive force  $\mathcal{F}_{rep}$  is computed as the sum of the elementary forces produced by  
 262 each check point (Eq.8), the densification of the mesh generates a greater total repulsive torque  
 263 on the tool. Consequently, the steady state solution is modified according to Eq.10 and Eq.11  
 264 and leads to:

$$\theta - \theta_{goal} = \frac{\mathcal{T}_{f_{rep}}}{k} \quad (22)$$

265 Thus, the tool axis orientation is more tilted when the density of the mesh increases. The stiffness  
 266 parameter  $k$  has to be modified according to the mesh size in order to maintain the same tool axis  
 267 behavior whatever be the mesh density.

268 **4. Application to an impeller**

269 To show the benefit of the proposed approach, the following example deals with the ma-  
 270 chining of an impeller, CAD model of which is displayed in Fig.11. The study focuses on the  
 271 5-axis sweeping of the vanes along the isoparametric curves of the surface with a ball-end mill,  
 272 diameter of which is equal to 5mm.

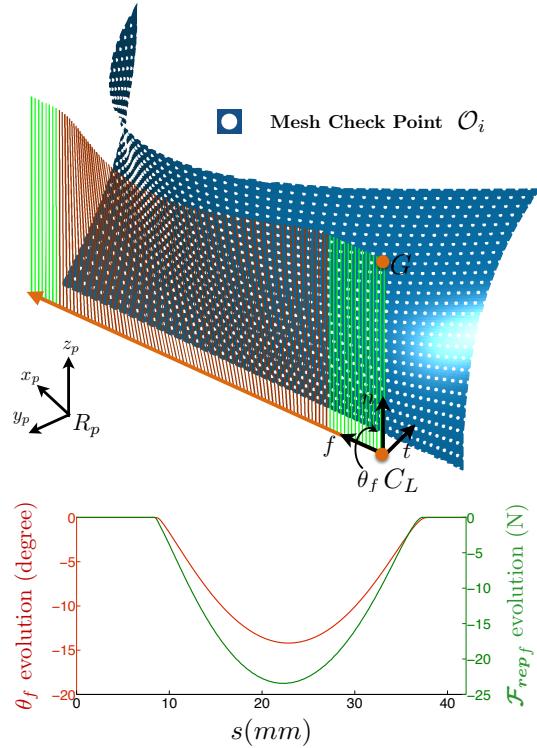


Figure 9: Simulation with a densified mesh ( $d=1\text{mm}$ )

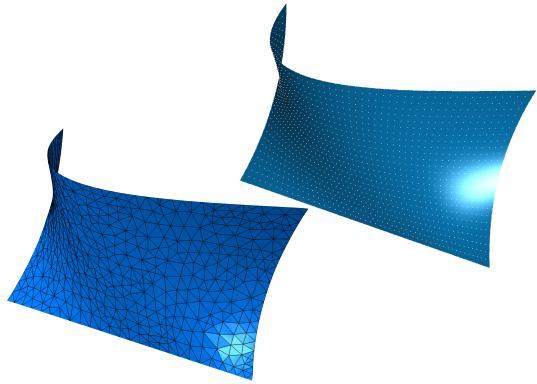


Figure 10: Structured and unstructured mesh

273 In this computation, the total repulsive force  $\mathcal{F}_{rep}$  is calculated by summing the forces generated by the mesh points of the left and right blades considered as check surfaces. The mesh of the  
 274 check surfaces leads to 4800 nodes. The programmed orientation of the tool axis is set normal to  
 275

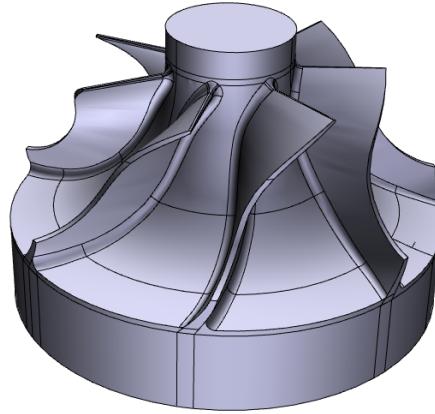


Figure 11: Impeller

276 the part surface which means  $\theta_f = \theta_n = 0$ . With such a tool axis orientation strategy, the tool  
 277 should inevitably collide with the blades. The constant parameters of the computation are listed  
 278 in table 1. In this table, one can notice that only three parameters are driving the simulation, the  
 279 stiffness parameter  $k$ , the neighborhood  $r_0$  and the mesh size  $d$ .

Table 1: Parameters table

Parameters	Symbol	value
Tool radius [m]	$R$	$2.5.10^{-3}$
Tool cylinder height [m]	$H$	$20.10^{-3}$
Programmed initial angle [rad]	$\theta_{goal}$	0
Tool inertia [ $kg.m^2$ ]	$J$	1
Damping ratio (no unit)	$\xi$	1
$C_L$ Curvilinear speed [ $m.mn^{-1}$ ]	$v_0$	1
Neighborhood value [m]	$r_0$	$15.10^{-3}$
Stiffness coefficient [ $N.m.rad^{-1}$ ]	$k$	32
Mesh size [m]	$d$	$2.10^{-3}$

280 Results are illustrated in Fig.12 where only the isoparametric path for the value  $v^* = 0.5$  of  
 281 the part surface is plotted for better readability. At the beginning of the tool path, the tool is  
 282 outside the check surfaces neighborhood and the tool axis orientation is not modified. Then, the  
 283 tool axis orientation is continuously modified and the tool passes between the two check surfaces  
 284 without collision. At the end of the tool path, the tool axis orientation is only modified by the  
 285 right hand check surface which pushes the tool on the left. This behavior is visible for  $s = 80mm$   
 286 on Fig.12. Thus the computation generates a smooth trajectory and the potential collisions with  
 287 the two check surfaces are avoided. In this example, the chosen tool has enough space to pass  
 288 between the two check surfaces. It is possible to encounter cases where the tool axis orientation  
 289 can not be modified without collisions due to the spatial arrangement of the check surfaces. In  
 290 this case the ODE 45 solver would not find a solution and the algorithm would stop according to  
 291 a termination criterion.

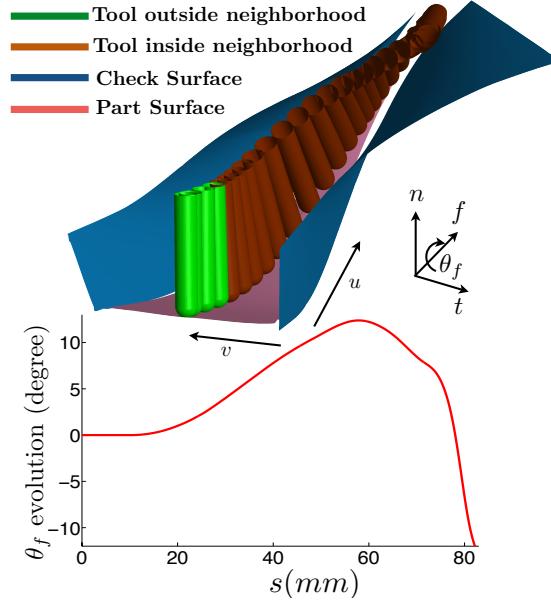


Figure 12: Computed tool axis orientation along the isoparametric curve  $v^* = 0.5$

292 Therefore, the proposed approach provides a good alternative to conventional approaches in  
 293 the case of 5-axis open pocket parts.

294 **5. Conclusions**

295 This paper presents an original method for 5-axis collision avoidance between the tool and  
 296 the check surfaces. Newton's laws are used to compute a continuous tool motion along the tool  
 297 path. The use of potential fields allows that no collision will occur with the obstacles as the  
 298 neighborhood generates a repulsive force growing to infinity when the tool gets closer to the ob-  
 299 stacle. As the tool axis orientation behaves like a damped harmonic oscillator, investigations are  
 300 carried out to prevent the tool axis orientation from oscillating and to ensure a smooth behavior  
 301 along the check surfaces. Numerical investigations show that the proposed approach is efficient  
 302 and does not require advanced CAM programming skills to compute the collision-free tool path.

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