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Optimal Control for a Self-Reacting Point Absorber: A One-Body Equivalent Model Approach

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Abstract—This paper deals with the optimal control of a self-reacting Wave Energy Converter (WEC) where the reaction force is obtained using a damping-plate. Model Predictive Control (MPC) is applied for unconstrained and constrained input control cases. Objective function attempting to optimise the power generation is directly formulated as an absorbed power maximisation problem and thus no optimal references, such as buoy and/or spar velocity, is required. Moreover, rather than using the full WEC model in the optimisation problem which can be time-consuming, and because of linear assumptions, we propose the use of a phenomenologically one-body equivalent model derived using the Thévenin's theorem.

Index Terms—wave energy converter, phenomenologically one-body equivalent model, optimal control, model predictive control.

I. INTRODUCTION

This paper is about the development of a new french wave energy converter referenced as the “EM Bilboquet” project (Fig. 1). The power take-off (PTO) extracts mechanical power due to incoming waves by a system made up of a cylindrical buoy sliding along a partially submerged structure. This structure is made up of a vertical cylinder, referenced in the following as spar, with a damping plate attached at its keel. Energy resulting from the relative motion between the two concentric bodies is harnessed by rack-and-pinion which drives a permanent magnet synchronous generator through a gearbox. Wave energy converters using a reaction source which is not the seabed i.e. such as a plate, are referenced in the wave energy literature as a self-reacting WEC and because horizontal dimensions of the buoy is small compared to the incident wave length, the term of self-reacting point absorber is used. The use of a submerged body acting as a reference for the floating body which can react against is not a new concept but have a promising future.

Regarding the control strategy, it is well established now, that it is an important WEC design aspect to make it more efficient, and lot of efforts have been put on it during the last decade. Several strategies have already been proposed and can be classified in two main categories, namely passive and, what we will call in the following, active loading. Model Predictive Control (MPC) in the wave energy context belongs to the latter, and has already received most attention from wave energy converter community. In [1], Cretel studies optimal control for a single-body point absorber case with objective function

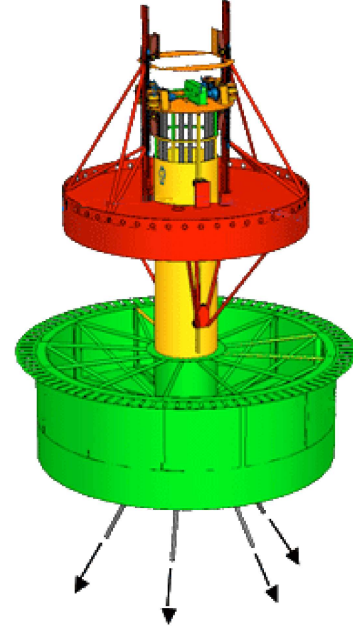


Fig. 1: Project “EM Bilboquet”.

formulated on the absorbed energy. In [2], author presented design details and simulation results for a Model Predictive Control (MPC) for a two-body WEC. Objective function in the optimisation problem is formulated as a tracking error problem where reference is the optimal buoy velocity. However the used hydrodynamic WEC model was too simple in the fact that it did not take into account the model frequency dependence and it restricted the two-body dynamics to a single one assuming a perfect stationary reference body (the central spar is perfectly motion-damped either via mooring or using a submerged damping-plate). In [3], author completed the previous work by considering a linear two-body dynamic. In [4] the same authors proposed a non-linear MPC (NMPC) formulation for a two-body WEC, in case where a non-linear mooring-line term is added to the model. In both papers, a full WEC model including a full radiation force description, was used but only for plant simulation purpose. Indeed, optimal control was formulated, based on a direct energy formulation, on a reduced model where authors assumed constant inertia and damping

terms, claiming that convolution terms in radiation forces are “fairly small compared to other external forces”. However, authors indicate that, if required, a full WEC model could be used in MPC formulation, based on a linear state-space approximation. The main counterpart in doing this, is that the WEC model order would increase considerably, which could be a real problem for real-time evaluation of the optimisation problem.

In this paper, we propose to use the “phenomenologically one-body equivalent model” concept, firstly introduced in [5]. Based on the well-known Thévenin’s theorem, we identify a one-body equivalent state-space model excited by an equivalent wave excitation force, that is useful for formulating the optimisation problem because of its low order.

II. THE SELF-REACTING POINT ABSORBER MODEL

A. Mathematical Background

In this section we present the mathematical formulation of the linearised model for a generic self-reacting WEC¹. For the sake of simplicity, the total structure dynamics is restricted to the heaving mode. Under the assumption of linear wave potential theory, the linearised motion equation for the two bodies is given by the Newton’s second law²

$$\begin{cases} m_1 \ddot{z}_1 = f_{\text{ex},1} + f_{\text{r},11} + f_{\text{r},21} + f_{\text{gen}} + f_{\text{s},1} \\ m_2 \ddot{z}_2 = f_{\text{ex},2} + f_{\text{r},22} + f_{\text{r},12} - f_{\text{gen}} + f_{\text{s},2} + f_{\text{drag}} \end{cases} \quad (1)$$

where m_1 and m_2 are respectively the mass of the buoy and the spar, z_i is the body i vertical displacement with respect to the equilibrium position. $f_{\text{ex},i}$ is the wave excitation force applied on body i . It can be expressed in the time-domain as

$$f_{\text{ex},i}(t) = \int_{-\infty}^{\infty} h_{\text{ex},i}(t - \tau) \eta_o(\tau) d\tau \quad (2)$$

where $\eta_o(t)$ is the wave elevation due to the incident wave at the origin O, located at the intersection of the undisturbed free surface level with cylinder axis and $h_{\text{ex},i}(t)$ is the impulse response of the wave excitation force related to the geometry of the body i [6]. $f_{\text{r},ij}$ is the force applied on the body j due to the motion of body i . This force is associated to the radiation problem. In linear potential theory, it is conventional to decompose this force in two parts which are frequency dependent. One is proportional to the body acceleration, the other is proportional to its velocity and, are respectively referenced as added mass and radiation damping

$$f_{\text{r},ij} = -m_{\text{a},ij} \ddot{z}_i(t) - b_{ij} \dot{z}_i(t) \quad (3)$$

Because of the hydrodynamic coefficient frequency dependence, it is convenient to replace (3) by an easiest computational formulation. Cummins [7] shown that it can be approximated by the following representation in the time-domain for the zero forward speed case

$$f_{\text{r},ij} = -m_{\text{a},ij}(\infty) \ddot{z}_i(t) - \int_{-\infty}^t k_{ij}(t - \tau) \dot{z}_i(\tau) d\tau \quad (4)$$

where $m_{\text{a},ij}(\infty)$ is defined as the infinite added mass and $k_{ij}(t)$ is the radiation convolution kernel. $f_{\text{s},i}$ is the net restoring force due to gravity and buoyancy. It is proportional to the displacement of the body structure from its equilibrium position. The coefficient of proportionality is denoted $\kappa_{\text{s},i}$ and is referenced as the buoyancy stiffness

$$f_{\text{s},i}(t) = -\kappa_{\text{s},i} z_i(t) \quad (5)$$

where the diagonal elements are respectively defined for the buoy and the platform by $\kappa_{\text{s},1}$ and $\kappa_{\text{s},2}$ such as

$$\kappa_{\text{s},i} = \rho g \iint_{S_{\text{F0},i}} dS \quad (6)$$

where ρ is the water density, g is the gravitational acceleration and $S_{\text{F0},i}$ is the water plane area at equilibrium condition. In the cylindrical shape case, we have $\kappa_{\text{s},1} = \rho g \frac{\pi}{4} (D_b^2 - D_s^2)$ for the buoy and $\kappa_{\text{s},2} = \rho g \frac{\pi}{4} D_s^2$ for the spar. In order to enhance the spar modelling during the resonant oscillation we use, as a first approximation, a constant linear damping term b_{drag} , proportional to the spar velocity such as $f_{\text{drag}} = -b_{\text{drag}} \dot{z}_2$. In this paper the additional damping is chosen in such a way that it corresponds to the maximum dissipation of the non-linear term [8] when optimal active control is applied³ which corresponds approximatively, after numerical investigation, to 14.5% of the critical damping defined as $b_{\text{crit},2} = 2\sqrt{(m_2 + m_{\text{a},22}(\infty))\kappa_2}$. Finally, f_{gen} denotes the force due to the generator which is also the control input. One could include additional restoring or non-linear forces due to mooring lines. However in the following, we would assume that the energy extraction in heaving mode is not or less-perturbed by this effort.

B. Approximated State-Space Model

Based on the above development and using a matrix notation, the equation system (1) can be rewrite as

$$\begin{aligned} (M + M_{\text{a}}(\infty)) \ddot{\xi}(t) + \int_{-\infty}^t K(t - \tau) \dot{\xi}(\tau) d\tau \\ + B_{\text{drag}} \dot{\xi}(t) + K_{\text{s}} \xi(t) = F_{\text{ex}}(t) + F_{\text{gen}}(t) \end{aligned} \quad (7)$$

where $\xi = [z_1 \ z_2]^T$. This integro-differential equation is referenced in the literature as the Cummins formulation. It is well established in the wave energy community that direct computation of (7), based on a discrete-time approximation, is not efficient and not appropriated for control purposes. The use of parametric models based on a state-space representation that approximate the convolution kernels (2) and (4) are more suitable. In [9], authors provide a MATLAB toolbox which approximate the convolution terms of (4) by a linear time-invariant system. Regarding the wave excitation forces, Falnes (1995), in [10], shown that the convolution kernel $h_{\text{ex},i}(t)$ of (2) is not necessary causal because of the mathematical assumptions made for the hydrodynamic parameter determination.

¹i.e. we do not make any assumption on the PTO principle.

²Indices 1 is used for what is refereed to the buoy and 2 for the spar

³Optimal active impedance is found in the frequency-domain by numerical exhaustive search based on the non-linear model.

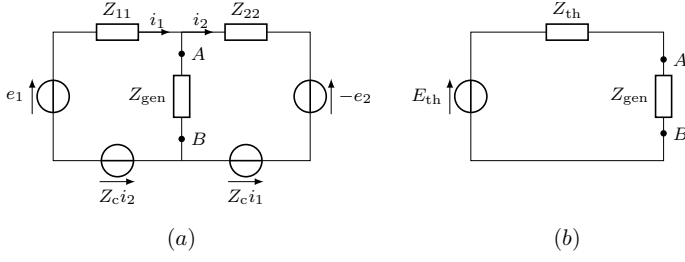


Fig. 2: Thévenin's equivalent model.

Hydrodynamic coefficients, i.e. added mass, radiation damping, and wave excitation force, that are required in the identification process, are computed in the frequency-domain by a semi-analytical method described in [11] and also in [12].

III. PHENOMENOLOGICALLY ONE-BODY EQUIVALENT MODEL

A. From a Frequency Domain Analysis...

Assuming a harmonic decomposition, i.e. $\xi(t) = \hat{\xi} \exp^{-i\omega t}$, the equation system (1) can be rewrite in the frequency-domain in a matrix form such as

$$[B(\omega) + B_{\text{drag}} + i\omega(M + M_a(\omega) - \frac{K_s}{\omega^2})]\hat{\xi} = \hat{F}_{\text{ex}} + \hat{F}_{\text{gen}} \quad (8)$$

In [5], Falnes presents a phenomenologically one-body equivalent model for a self-reacting wave energy converter working in heaving mode and using the relative motion between the two bodies to extract power. Using the complex intrinsic mechanical impedance concept defined as

$$Z_{ij} = [b_{ij}(\omega) + b_{\text{drag},ij}] + i\omega[m_{ij} + m_{a,ij}(\omega) - \frac{k_{s,ij}}{\omega^2}] \quad (9)$$

and assuming $\hat{f}_{\text{gen}} = Z_{\text{gen}}(\hat{z}_1 - \hat{z}_2)$, without loss of generality, and $Z_c = Z_{12} = Z_{21}$, we may rewrite (8) as

$$\begin{bmatrix} Z_{11} + Z_{\text{gen}} & Z_c - Z_{\text{gen}} \\ Z_c - Z_{\text{gen}} & Z_{22} + Z_{\text{gen}} \end{bmatrix} \begin{bmatrix} \hat{z}_1 \\ \hat{z}_2 \end{bmatrix} = \begin{bmatrix} \hat{f}_{\text{ex},1} \\ \hat{f}_{\text{ex},2} \end{bmatrix} \quad (10)$$

Using an electrical analogy i.e. $\hat{z}_i \rightarrow i_i$ and $\hat{f}_{\text{ex},i} \rightarrow e_i$, see Fig. 2, and applying the well-known Thévenin's theorem for dependent sources, we found that the complex relative velocity $\hat{z}_r = (\hat{z}_1 - \hat{z}_2) \rightarrow (i_1 - i_2)$ can be expressed as

$$\hat{z}_r = \frac{1}{Z_{i,\text{eq}}}(\hat{f}_{\text{ex},\text{eq}} + \hat{f}_{\text{gen}}) \quad (11)$$

where $Z_{i,\text{eq}} \rightarrow Z_{\text{th}}$ is the intrinsic mechanical impedance for the one-body equivalent system and $\hat{f}_{\text{ex},\text{eq}} \rightarrow E_{\text{th}}$ is the complex equivalent wave excitation force, respectively defined as

$$Z_{i,\text{eq}} = \frac{Z_{11}Z_{22} - Z_c^2}{Z_0} \quad (12)$$

$$\hat{f}_{\text{ex},\text{eq}} = \hat{f}_{\text{ex},1} \frac{(Z_2 + Z_c)}{Z_0} - \hat{f}_{\text{ex},2} \frac{(Z_1 + Z_c)}{Z_0} \quad (13)$$

with

$$Z_0 = Z_1 + Z_2 + 2Z_c \quad (14)$$

The obtained equivalent model is quite useful for discussing optimum performance in regular wave analysis. Indeed it can be shown that the maximum absorbed power, when reactive control strategy⁴ is applied and no constraints are considered, is given by (15) [5], [13]

$$Z_{\text{gen,opt}} = Z_{i,\text{eq}}^* \quad (15)$$

where * symbol denotes complex-conjugate.

B. ... to a Phenomenologically One-Body Equivalent State-Space Model

Equation (11) describes the equivalent wave excitation force to relative velocity relation. If we are able to identify a time-domain model based on data provided by this equivalent model, we could use it as it will be explained in the next section for formulating the control problem. In the time-domain, (11) can be expressed as a convolution product

$$\dot{z}_r(t) = \int_{-\infty}^{\infty} h(t - \tau)(u(\tau) + w(\tau))d\tau \quad (16)$$

where $h(t) = \mathcal{F}^{-1}\{H(i\omega) \equiv Z_{i,\text{eq}}^{-1}\}$, $u(t) \equiv f_{\text{gen}}(t)$ and $w(t)$ is the equivalent wave excitation force in the time-domain defined as

$$w(t) = \int_{-\infty}^{+\infty} g(t - \tau)\eta_o(\tau)d\tau \quad (17)$$

where $g(t)$ is the inverse Fourier transform of $G(i\omega) \equiv \hat{f}_{\text{ex},\text{eq}}$

$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(i\omega) \exp^{-i\omega t} d\omega \quad (18)$$

Two transfer functions have to be determined for having a full description of the equivalent one-body model. One is related to the force to relative velocity behaviour whereas the second is related to the wave surface elevation to equivalent wave excitation force behaviour. In this paper we focus our talk only on the identification of $H(i\omega)$. A similar procedure than the one described below can be used for identifying $g(t)$. However, we have to consider a causal version of $g(t)$ in the identification process, and then the equivalent wave excitation force is provided by the following model

$$w(t) = \int_0^{t+t_c} g^{(c)}(t - \tau)\eta_o(\tau + t_c)d\tau \quad (19)$$

Because relative velocity is necessary a causal process, we used the Laplace formalism and then we look for an approximated transfer function such as

$$\tilde{H}(s) = \frac{P(s)}{Q(s)} = \frac{p_r s^r + p_{r-1} s^{r-1} + \dots + p_0}{s^n + q_{n-1} s^{n-1} + \dots + q_0} \quad (20)$$

Identification procedure is similar to the one presented in [14]. Convolution kernel properties of $h(t)$ are synthesised for both frequency- and time-domain in TABLE I. Once the model

⁴In case where optimal passive loading is applied, it can be shown that we have $R_{\text{gen,opt}} = |Z_{i,\text{eq}}|$

TABLE I: Convolution Kernel $h(t)$ Properties

Properties	Implication on parametric models
(1) $\lim_{\omega \rightarrow 0} H(i\omega) = 0$	There are a zero at $s = 0$
(2) $\lim_{\omega \rightarrow \infty} H(i\omega) = 0$	Strictly proper
(3) $\lim_{t \rightarrow 0^+} h(t) \neq 0$	Relative degree is 1
(4) $\lim_{t \rightarrow \infty} h(t) = 0$	BIBO stable

have been identified, an equivalent state-space model of (20) is directly given in a canonical form by

$$\begin{cases} \dot{x}(t) = A_c x(t) + B_c(u(t) + w(t)) \\ \dot{z}_r(t) \simeq C_c x(t) \end{cases} \quad (21)$$

where

$$A_c = \begin{bmatrix} -q_{n-1} & -q_{n-2} & \dots & -q_1 & -q_0 \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & \vdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix}, \quad B_c = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

and

$$C_c = [p_r \quad p_{r-1} \quad \dots \quad p_1 \quad p_0], \quad D_c = 0$$

Figure 3 presents frequency response for the equivalent wave excitation force to relative velocity transfer function and for the wave surface elevation to equivalent wave excitation force. A 6th order model have been found for the former and a 8th order, with a 5 [s] causalising time-shift t_c , for the latest seems to be good enough.

It is important to mention there, that one can see the most advantage of using the phenomenologically one-body equivalent model for control purpose. Indeed, a 6th order model is enough to fully describe the equivalent wave excitation force to relative velocity relation whereas at least a 24th order would be necessary if the two-body dynamic would be considered. This will considerably decrease the complexity and the time evaluation of the control strategy presented in the next section.

IV. OPTIMAL WEC CONTROL

A. Optimal Control Formulation

The control objective is to maximise the absorbed energy E_{abs} over a certain prediction horizon T_{hor}

$$E_{\text{abs}} = - \int_t^{t+T_{\text{hor}}} u(\tau) \dot{z}_r(\tau) d\tau \quad (22)$$

where $\dot{z}_r(t)$ is the relative velocity between the buoy and the spar such as $\dot{z}_r(t) = \dot{z}_1(t) - \dot{z}_2(t)$. Using a rectangle rule integration method, this can be rewrite as

$$E_{\text{abs}} \simeq -T_s \sum_{i=0}^{N_{\text{hor}}} u_{k+i-1} \dot{z}_{r,k+i} \quad (23)$$

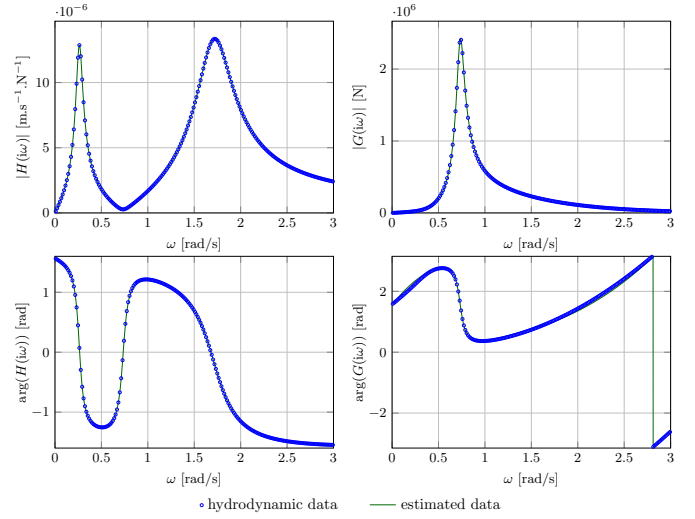


Fig. 3: Amplitude and phase of the frequency response for the equivalent one-body model.

where T_s is the sampling period and $N_{\text{hor}} = T_{\text{hor}}/T_s$ denotes the prediction horizon length. Assuming a zero-hold order sampling, the one-body equivalent discrete state-space model (21) can be expressed as

$$\begin{cases} x_{k+1} = A_d x_k + B_d(u_k + w_k) \\ \dot{z}_{r,k} = C_d x_k \end{cases} \quad (24)$$

and the control input equation is given by

$$u_k = u_{k-1} + \Delta u_k \quad (25)$$

Defining $\bar{x}_k = [x_k \quad u_{k-1}]^T$ and considering $\bar{y}_k = [\dot{z}_{r,k} \quad u_{k-1}]^T$, we can define the augmented system such as

$$\begin{cases} \bar{x}_{k+1} = \begin{bmatrix} A_d & B_d \\ 0 & 1 \end{bmatrix} \bar{x}_k + \begin{bmatrix} B_d \\ 1 \end{bmatrix} \Delta u_k + \begin{bmatrix} B_d \\ 0 \end{bmatrix} w_k \\ \bar{y}_k = \begin{bmatrix} C_d & 0 \\ 0 & 1 \end{bmatrix} \bar{x}_k \end{cases} \quad (26)$$

or in a simple form

$$\begin{cases} \bar{x}_{k+1} = A \bar{x}_k + B_u \Delta u_k + B_w w_k \\ \bar{y}_k = C \bar{x}_k \end{cases} \quad (27)$$

The absorbed energy function (23) can then be reformulated as

$$E_{\text{abs}} \simeq -T_s \frac{1}{2} \sum_{i=0}^{N_{\text{hor}}} \bar{y}_{k+i|k}^T Q \bar{y}_{k+i|k} \quad (28)$$

with $Q = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and where notation $k+i|k$ means the prediction at the time $k+i$ based on the information available at the instant k .

B. Model Predictive Control Strategy

Maximising the absorbed energy (28) is equivalent to minimising

$$J' = \frac{1}{2} \sum_{i=0}^{N_{\text{hor}}} (\bar{x}_{k+i|k}^T \bar{Q} \bar{x}_{k+i|k} + \Delta u_{k+i|k}^T R \Delta u_{k+i|k}) \quad (29)$$

where $\bar{Q} = C^T Q C$ and where we have introduced a cost penalty $R > 0$ on the control input increment Δu_k . In (29), the first term $\bar{x}_{k|k}^T \bar{Q} \bar{x}_{k|k}$ does not contribute to the optimal solution as the last term $\Delta u_{k+N_{\text{hor}}|k}^T R \Delta u_{k+N_{\text{hor}}|k}$ can be neglected because it does not contribute to the energy absorption over the prediction horizon length. Then (29) can be re-written such as

$$J(\Delta u_k) = \frac{1}{2} \sum_{i=1}^{N_{\text{hor}}} (\bar{x}_{k+i|k}^T \bar{Q} \bar{x}_{k+i|k} + \Delta u_{k+i-1|k}^T R \Delta u_{k+i-1|k}) \quad (30)$$

or in a matrix notation form

$$J(\Delta u_k) = \frac{1}{2} \bar{X}_k^T \mathbf{Q} \bar{X}_k + \frac{1}{2} \Delta u_k^T \mathbf{R} \Delta u_k \quad (31)$$

where $\mathbf{Q} = \text{diag}(\overbrace{\bar{Q}, \dots, \bar{Q}}^{N_{\text{hor}}})$, $\mathbf{R} = \text{diag}(\overbrace{R, \dots, R}^{N_{\text{hor}}})$ and,

$$\bar{X}_k = \begin{bmatrix} \bar{x}_{k+1|k} \\ \bar{x}_{k+2|k} \\ \vdots \\ \bar{x}_{k+N_{\text{hor}}|k} \end{bmatrix}, \quad \Delta u_k = \begin{bmatrix} \Delta u_{k|k} \\ \Delta u_{k+1|k} \\ \vdots \\ \Delta u_{k+N_{\text{hor}}-1|k} \end{bmatrix}$$

Δu_k is the control sequence of Δu_k over the prediction horizon length N_{hor} . When the state and/or the control input are subject to constraints, the optimisation problem can be formulated as

$$\begin{aligned} & \underset{\Delta u_k}{\text{minimise}} && J(\Delta u_k) \\ & \text{subject to} && u_k \in \mathbb{U}, k = 1, \dots, N_{\text{hor}} \\ & && \bar{x}_k \in \mathbb{X}, k = 1, \dots, N_{\text{hor}} \end{aligned} \quad (32)$$

Model predictive control is based on the receding horizon concept. In a receding horizon framework, optimal control sequence Δu_k^* is obtained when solving (32) at the current time k but only the first optimal computed increment $\Delta u_{k|k}^*$ is applied.

$$\Delta u_{k|k}^* = [1, 0, \dots, 0] \Delta u_k^* \quad (33)$$

At the next sampling period the process is repeated with new measurements and estimations from the plant. From (27) and using notations previously introduced, we can define the state prediction equation over the horizon N_{hor} as

$$\bar{X}_k = J_x \bar{x}_{k|k} + J_u \Delta u_k + J_w W_k \quad (34)$$

where

$$J_x = \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^{N_{\text{hor}}} \end{bmatrix}, \quad W_k = \begin{bmatrix} w_{k|k} \\ w_{k+1|k} \\ \vdots \\ w_{k+N_{\text{hor}}-1|k} \end{bmatrix},$$

$$J_j = \begin{bmatrix} B_j & 0 & \dots & 0 \\ AB_j & B_j & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{N_{\text{hor}}-1} B_j & A^{N_{\text{hor}}-2} B_j & \dots & B_j \end{bmatrix}$$

with $j \in \{u, w\}$. It is important to mention that (34) is only dependent on the current state $\bar{x}_{k|k}$ and on the equivalent wave excitation force prediction. Substituting (34) in (31) and omitting terms that do not depend of the control input increment sequence Δu_k , optimisation problem cost function (31) can be rewritten in a quadratic form such as

$$J(\Delta u_k) = \frac{1}{2} \Delta u_k^T H \Delta u_k + f^T \Delta u_k \quad (35)$$

with

$$\begin{cases} H &= J_u^T \mathbf{Q} J_u + \mathbf{R} \\ f &= J_u^T \mathbf{Q} (J_x \bar{x}_{k|k} + J_w W_k) \end{cases} \quad (36)$$

By inspection, we find that the Hessian matrix H is positive definite hence optimisation problem (32) has a unique solution and can be efficiently solved using quadratic programming.

V. NUMERICAL RESULTS FOR OPTIMAL CONTROL

In this section, we present numerical results for both, regular and irregular wave conditions. All time-domain simulations are performed with MATLAB software based on a Runge-Kutta algorithm with a sampling period of $\Delta t = .01$ s. A sampling period of $T_s = .1$ s is used for the control strategy. Future wave excitation force information is supposed to be perfectly known. Influence of how good wave excitation force is predicted, is out of topic in this paper. Unconstrained input control is presented in order to prove the proposed technique feasibility. After numerical investigation in regular wave analysis case, it have been found that a 14 s horizon time prediction seems to be required to provide accurate results. Constrained analysis is then performed for power production prediction at a given see-state. In this paper, we consider a 2.4 MW generator power rating and a 3.75 m.s^{-1} maximum linear relative velocity which impose a constraint of 640 kN on the linear force that the power take-off could provide. In order to be more exhaustive, we have also considered a more restrictive control input of 530 kN.

Figure 4 shows numerical results in case of regular wave analysis. Considered wave amplitude and wave frequency are respectively $A = 1$ m and $\omega_p = .85 \text{ rad.s}^{-1}$. As expected from theoretical background [13], it appears that the relative

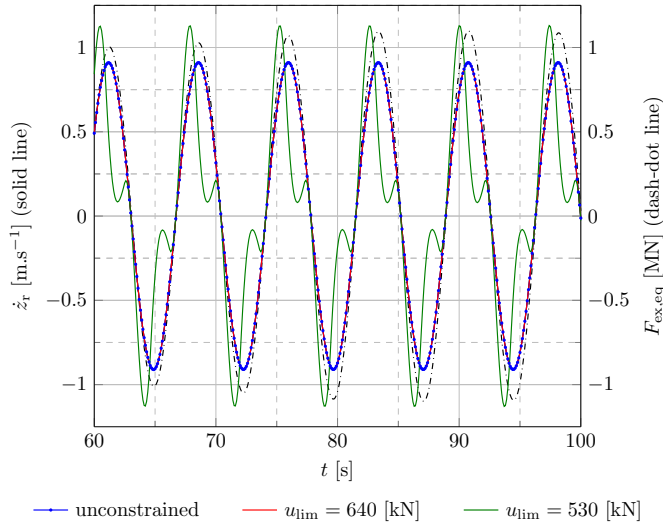


Fig. 4: Comparison between relative velocity and wave excitation force in case of regular wave analysis ($A = 1$ m, $\omega_p = .85$ rad.s $^{-1}$) for both unconstrained and constrained control strategy.

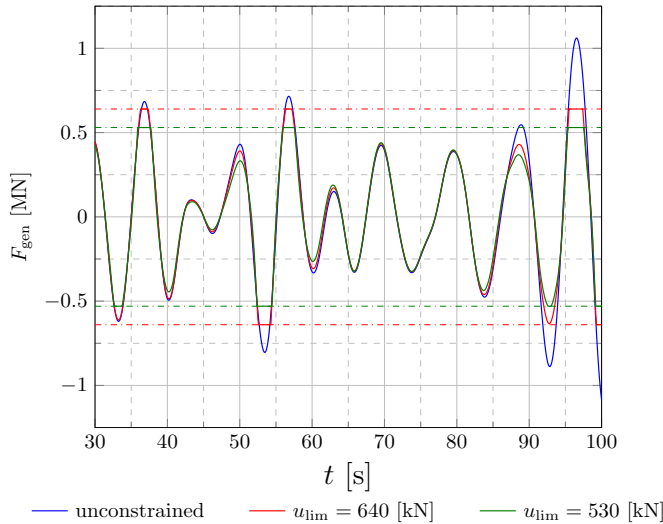


Fig. 5: Linear force due to the generator in case of irregular wave analysis.

velocity is in phase with the wave excitation force even when constrained control strategy is applied.

Figure 5 shows a more realistic example when the WEC is subjected to irregular wave conditions. Here, a JONSWAP's spectrum have been used which is characterised by a 2 m significant wave height, a 9 s peak period and, a 3.3 peak enhancement factor. Obviously it is clear that the control strategy is able to keep the PTO force within the desired control input constraints.

VI. CONCLUSION

This paper dealt with the special case modelling and control of a self-reacting wave energy converter where reaction force

is obtained using a damping plate. After presenting the full WEC model used for plant simulation, optimal WEC energy absorption in the time-domain was presented and the optimal control was formulated in a receding horizon fashion. In this paper, we proposed to use the “phenomenologically one-body equivalent model” description in the control problem formulation which reduces the model order and then is more appropriated for real-time application. Numerical results were presented for both regular and irregular wave analysis and also in unconstrained and constrained case in order to prove the proposed control strategy efficiency. Numerical results are encouraging and future works will investigate the use of this “phenomenologically one-body equivalent model” in case of a non-linear WEC model description.

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