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# ASYMPTOTIC MODELS OF THE HEAT TRANSFER IN LAMINATED CONDUCTORS

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## ABSTRACT

Laminated materials play an important role in civil engineering. The contribution is focused on the modelling of heat conduction in these materials. The analysis is carried out in the framework of the tolerance averaging technique, [1]. A new asymptotic procedure for finding solutions to the specific heat conduction problems is proposed. General results are illustrated by some numerical examples and compared with those derived from homogenization technique, [2].

**Key words:** heat transfer, laminated rigid conductors.

## INTRODUCTION

It is known that the direct approach to the analysis of heat conduction processes in composites with a dense periodic structure leads to ill-conditioned and complicated computational problems, [2]. That is why some averaged (macroscopic) mathematical models for finding solutions to special problems have been formulated. In most cases averaged models of periodic materials and structures are based on the concept of homogenization, [2-4], as well as on its modifications like the microlocal parameter approach, [5-7]. A certain drawback of the asymptotic homogenization is that it does not describe the effect of the period lengths on the overall behaviour of a periodically inhomogeneous solid, [8,9]. Hence some phenomena taking place near the solid boundaries and close to its initial state cannot be investigated in the framework of homogenization, [8,10,11]. That is why a number of alternative approaches to the modeling of periodic materials and structures have been formulated. The overview of these approaches can be found in [1], where foundations of what was called the tolerance averaging technique have been summarized. It has to be emphasized that in contrast to homogenization the tolerance averaging describes the effect of periodicity cell size on the overall behaviour of the periodically inhomogeneous solid.

In this contribution we apply tolerance averaging to the heat conduction problem in periodic two-component laminates. The main result is the formulation of a certain higher-order asymptotic technique for finding approximate solution to the known tolerance heat conduction equations. These equations are recalled in the subsequent section. Moreover, the proposed asymptotic approximation technique will be applied to the numerical analysis of some special problems. The obtained numerical results show that the proposed asymptotic approach can be effectively applied to the analysis of problems involving near-boundary and near-initial time effects. It is also shown that the homogenized model equations can be treated as the first step in the proposed higher-order asymptotic approach.

### Notations

Let the physical space be parameterized by the orthogonal Cartesian coordinate system  $Ox_1x_2x_3$ , and let  $t$  stand for the time coordinate. Subscripts  $i, j, k$  and  $\alpha, \beta$  run over sequences 1, 2, 3 and 1, 2, respectively. The partial derivatives with respect to arguments  $x_k$  are denoted by  $\partial_k$  and the time derivate by the overdot. We also define  $\mathbf{x} = (x_1, x_2, x_3)$ ,  $z = x_3$ .

For an arbitrary integrable function  $f(\cdot)$  of  $z \in \langle -L, L \rangle$  (which can also depend on  $(x_1, x_2)$  and  $t$ ) and a length parameter  $l$ ,  $l \ll L$ , we introduce mean value  $\langle f \rangle(z)$  of  $f(\cdot)$  setting

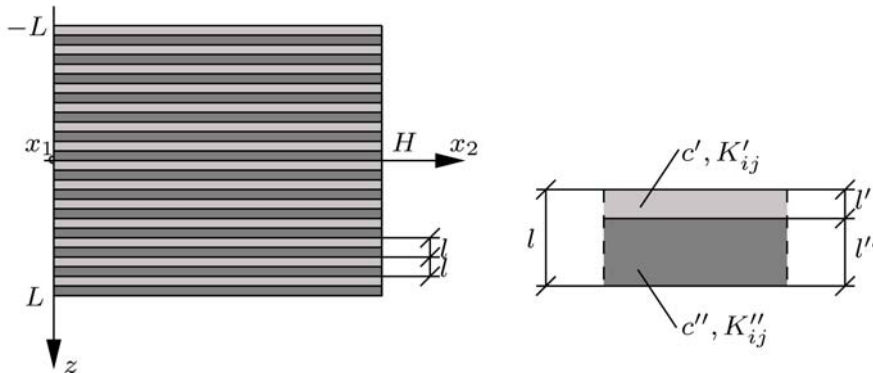
$$\langle f \rangle(z) = \frac{1}{l} \int_{z-l/2}^{z+l/2} f(y) dy, \quad z \in \left\langle -L + \frac{l}{2}, L - \frac{l}{2} \right\rangle \quad (1)$$

If  $f(\cdot)$  is a periodic function (with period  $l$ ) then  $\langle f \rangle$  is independent of  $z$ .

### MODEL EQUATIONS

The periodically laminated rigid heat conductor under consideration is assumed to be made of two kinds of laminae and occupies region  $\Omega = \Pi \times \langle -L, L \rangle$  in the physical space. The scheme of the conductor is shown in Fig. 1, where  $l$  is the period of inhomogeneity,  $l \ll L$  and  $l', l''$  are laminae thicknesses. The specific heat and the heat conduction tensor components in pertinent laminae are denoted by  $c', K'_{ij} = K'_{ji}$  and  $c'', K''_{ij} = K''_{ji}$ . We assume the material symmetry of the conductor setting  $K'_{\alpha 3} = K''_{\alpha 3} = 0$  and denote  $k' = K'_{33}$ ,  $k'' = K''_{33}$ .

Figure 1. The cross-section of the periodically laminated conductor



By  $\theta = \theta(\mathbf{x}, t)$ ,  $\mathbf{x} \in \Omega$ ,  $t \in R$ , we denote a temperature field at time  $t$ . We assume that in every lamina field  $\theta(\cdot)$  satisfies the well-known linearized Fourier heat transfer equation

$$c(x_3)\dot{\theta}(\mathbf{x}, t) - \partial_i [K_{ij}(x_3)\partial_j \theta(\mathbf{x}, t)] = 0. \quad (2)$$

On the interfaces between laminae we deal with the heat flux continuity conditions

$$K_{i3}^+ \partial_3^+ \theta(\mathbf{x}, t) = K_{i3}^- \partial_3^- \theta(\mathbf{x}, t) \quad (3)$$

where  $\partial_3^+$ ,  $\partial_3^-$  stand for the right-hand side and left-hand side derivatives, respectively, and  $K_{i3}^+$ ,  $K_{i3}^-$  are the boundary values of  $K_{i3}$  in the pertinent laminae. Equations (2), (3) have to be satisfied together with the appropriate initial and boundary conditions. The above equations are difficult to solve since  $l \ll L$  and coefficients  $c(\cdot)$ ,  $K_{ij}(\cdot)$  are uncontinuous highly oscillating functions. That is why we replace the above equations by the tolerance model equations which have constant coefficients.

Recalling results detailed in [1], we introduce the  $l$ -periodic function  $g(z)$  assumed to be a *saw-like shape function*; the diagram of  $g(\cdot)$  is shown in Fig. 2. The tolerance averaging technique is based on the assumption that the class of temperature fields  $\theta(\cdot, t)$  is restricted by condition

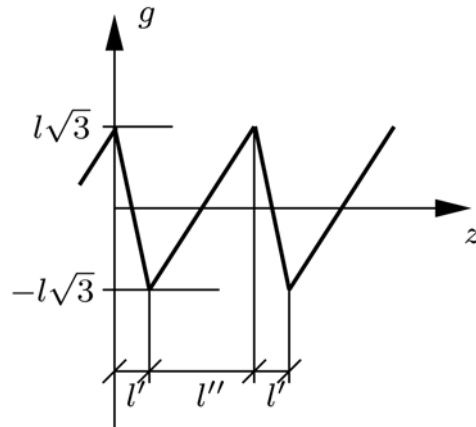
$$\theta(\mathbf{x}, t) = \vartheta(\mathbf{x}, t) + g(z)\psi(\mathbf{x}, t) \quad (4)$$

where  $\vartheta(\cdot)$  and  $\psi(\cdot)$  are differentiable functions *slowly varying* in argument  $z \in \langle -L, L \rangle$  with all their derivatives which appear in the problem under consideration. To underline this fact we write

$$\vartheta(x_1, x_2, \cdot, t) \in SV_\varepsilon(l), \quad \psi(x_1, x_2, \cdot, t) \in SV_\varepsilon(l) \quad (5)$$

where  $\varepsilon > 0$  was referred to in [1] as a tolerance parameter,  $\varepsilon \ll 1$ . It means that in every interval of the  $z$ -axis of the length  $l$ , functions  $\vartheta$ ,  $\psi$  together with their derivatives can be treated as constant with a certain approximation  $O(\varepsilon)$ . Conditions (5) are referred to as the physical reliability conditions imposed on the obtained solutions of specific problems. Function  $\vartheta(\cdot)$  is referred to as the macroscopic (averaged) temperature and  $\psi(\cdot)$  is called the amplitude of temperature fluctuations.

**Figure 2. Diagram of the periodic saw-like shape function**



Define

$$\langle f \rangle = v' f' + v'' f'', \quad v' = \frac{l'}{l}, \quad v'' = \frac{l''}{l}, \quad (6)$$

$$[f] = 2\sqrt{3}(f' - f''), \quad \{f\} = 12 \left( \frac{f'}{v'} + \frac{f''}{v''} \right), \quad (7)$$

where  $f$  stands for  $K_{\alpha\beta}$  and  $k = K_{33}$ .

After substituting the right-hand side of (4) to equation (2), and certain manipulations we obtain finally the following system of governing equations for  $\vartheta(\cdot)$ ,  $\psi(\cdot)$

$$\begin{aligned} \langle c \rangle \dot{\vartheta} - \langle K_{\alpha\beta} \rangle \partial_\alpha \partial_\beta \vartheta - \langle k \rangle \partial_3 \partial_3 \vartheta - [k] \partial_3 \psi &= 0, \\ l^2 \langle c \rangle \dot{\psi} - l^2 \langle K_{\alpha\beta} \rangle \partial_\alpha \partial_\beta \psi + \{k\} \psi + [k] \partial_3 \vartheta &= 0. \end{aligned} \quad (8)$$

Equations (8) with pertinent boundary and initial conditions, decomposition formula (4) and conditions (5) represent *the tolerance model* of a periodically laminated rigid conductor. The detailed discussion of equations (8) can be found in [1].

The homogenized equations can be treated as a asymptotic approximation of the tolerance model equations by the formal neglecting terms  $O(l^2)$  in (8). Under this approximation  $\psi$  can be eliminated from equation (8)<sub>1</sub> and hence

$$\begin{aligned} \langle c \rangle \dot{\vartheta} - \langle K_{\alpha\beta} \rangle \partial_\alpha \partial_\beta \vartheta - k^o \partial_3 \partial_3 \vartheta &= 0, \\ \psi &= -\frac{[k]}{\{k\}} \partial_3 \vartheta \end{aligned} \quad (9)$$

where we have denoted

$$k^o = \frac{k'k''}{v''k' + v'k''}.$$

Equations (9) with the boundary and initial conditions for  $\vartheta$  represent the homogenized model for the heat transfer in a periodically laminated rigid conductor, [5].

Following [11] we shall transform equations (8) to a new form setting

$$\varphi \equiv \frac{[k]}{\{k\}} \frac{\partial \vartheta}{\partial z} + \psi. \quad (10)$$

Hence  $\varphi$  is the new unknown function slowly varying in  $z$ . Let us substitute the right-hand side of equation (10) into equation (8) and denote

$$D \equiv \langle c \rangle \frac{\partial}{\partial t} - \langle K_{\alpha\beta} \rangle \frac{\partial^2}{\partial x_\alpha \partial x_\beta}, \quad \partial \equiv \frac{\partial}{\partial z}, \quad \delta \equiv \left( \frac{l}{L_o} \right)^2, \quad \{k_L\} \equiv \frac{\{k\}}{L_o^2} \quad (11)$$

where  $L_o$  is a certain length dimension and then  $\delta \ll 1$  (we can assume  $L_o = L$  provided that  $L$  is finite). Under the above notations we obtain finally

$$\begin{aligned} D\vartheta - k^o \partial^2 \vartheta &= [k] \partial \varphi, \\ \delta D\varphi + \{k_L\} \varphi &= \delta \frac{[k]}{\{k\}} \partial D\vartheta. \end{aligned} \quad (12)$$

Equation (12) have to be considered together with the appropriate initial and boundary conditions for  $\vartheta$  and  $\varphi$  as well as with the decomposition formula derived from (4) and (10)

$$\theta(\mathbf{x}, t) = \vartheta(\mathbf{x}, t) - g(z) \left( \frac{[k]}{\{k\}} \partial \vartheta(\mathbf{x}, t) - \varphi(\mathbf{x}, t) \right) \quad (13)$$

and the physical reliability conditions

$$\vartheta(x_1, x_2, \cdot, t) \in SV_\varepsilon(l), \quad \varphi(x_1, x_2, \cdot, t) \in SV_\varepsilon(l)$$

constitute the basis for the subsequent analysis.

## ASYMPTOTIC MODELS

Due to the presence of the small parameter  $\delta$ ,  $0 < \delta \ll 1$ , in the resulting tolerance model equations (12), we shall apply the asymptotic approach to the analysis of initial/boundary value problems. To this end, independently of arguments  $x = (x_1, x_2)$  and  $t$ , we introduce arguments  $\bar{x} = (\bar{x}_1, \bar{x}_2)$  and  $\bar{t}$  setting

$$\bar{x}_\alpha \equiv \frac{x_\alpha}{\sqrt{\delta}}, \quad \bar{t} \equiv \frac{t}{\delta}, \quad \alpha = 1, 2$$

and define differential operator  $\bar{D}$  setting

$$\bar{D} \equiv \langle c \rangle \frac{\partial}{\partial \bar{t}} - \langle K_{\alpha\beta} \rangle \frac{\partial^2}{\partial \bar{x}_\alpha \partial \bar{x}_\beta}.$$

We shall seek an asymptotic approximation of solution to equations (12) in the form of asymptotic expansions

$$\begin{aligned} \varphi &= \varphi_0(\bar{x}, z, \bar{t}) + \delta \varphi_1(\bar{x}, z, \bar{t}) + \tilde{\varphi}_2(x, z, t) \\ \vartheta &= \vartheta_0(x, z, t) + \delta \vartheta_1(x, z, t) + \tilde{\vartheta}_2(x, z, t) \end{aligned} \quad (14)$$

where  $\tilde{\varphi}_2$ ,  $\tilde{\vartheta}_2$  are functions of an order  $\delta^2$ . Substituting the above expansion into tolerance model equation (14) and equating coefficients of  $\delta^0 = 1$  to zero we get

$$\begin{aligned} \bar{D} \varphi_0 + \{k_L\} \varphi_0 &= 0, \\ D \vartheta_0 + k^\circ \partial^2 \vartheta_0 &= [k] \partial \varphi_0. \end{aligned} \quad (15)$$

The above equations are assumed to be considered together with the initial/boundary conditions coinciding with those imposed on  $\varphi$  and  $\vartheta$ . Equations (15) represent what will be called the first order asymptotic approximation of the tolerance model equations (12).

Let  $\theta_0$  be a solution to a certain initial/boundary value problem in the framework of the homogenized model. It means that  $\theta_0$  has to satisfy equation

$$D \theta_0 - k^\circ \partial^2 \theta_0 = 0 \quad (16)$$

and the corresponding nonhomogeneous initial/boundary conditions. Hence

$$\theta'_0 = \vartheta_0 - \theta_0$$

is a solution to equation

$$D \theta'_0 + k^\circ \theta'_0 = [k] \partial \varphi_0 \quad (17)$$

which satisfies the pertinent homogeneous initial/boundary conditions. It follows that the first order approximation is governed by homogeneous system of equations

$$D \theta_0 + k^\circ \theta_0 = 0 \quad (18)$$

and

$$\bar{D} \varphi_0 + \{k_L\} \varphi_0 = 0 \quad (19)$$

with nonhomogeneous initial/boundary conditions and by the nonhomogeneous equation (17) with pertinent homogeneous initial/boundary conditions. Bearing in mind formula (13) for  $\theta$  we conclude that the initial/boundary conditions imposed on  $\theta_0$ ,  $\varphi_0$  can be coupled and the approximation formula has the form

$$\begin{aligned}\vartheta(x, z, t) &= \theta_0(x, z, t) + \theta'_0(x, z, t) + O(\delta), \\ \varphi(x, z, t) &= \varphi_0\left(\frac{x}{\sqrt{\delta}}, z, \frac{t}{\delta}\right) + O(\delta).\end{aligned}\quad (20)$$

It has to be remembered that the above formula has a physical sense only if functions  $\vartheta_0 = \theta_0 + \theta'_0$  and  $\varphi_0$  are slowly varying in argument  $z \in (-L, L)$  together with their first derivative.

Substituting formula (14) into tolerance model equations (12) and equating coefficients of  $\delta$  to zero we obtain

$$\begin{aligned}\bar{D}\varphi_1 + \{k_L\}\varphi_1 &= \frac{[k]}{\{k\}}\partial D\vartheta_0, \\ D\vartheta_1 + k^o\partial^2\vartheta_1 &= [k]\partial\varphi_1\end{aligned}\quad (21)$$

where

$$\vartheta_0 = \theta_0 + \theta'_0.$$

The above equations have to be considered together with the appropriate homogeneous initial/boundary conditions for  $\varphi_1$ ,  $\vartheta_1$ . Thus, the second order approximation is determined by formulae

$$\begin{aligned}\vartheta(x, z, t) &= \theta_0(x, z, t) + \theta'_0(x, z, t) + \delta\vartheta_1(x, z, t) + O(\delta^2), \\ \varphi(x, z, t) &= \varphi_0\left(\frac{x}{\sqrt{\delta}}, z, \frac{t}{\delta}\right) + \delta\varphi_1\left(\frac{x}{\sqrt{\delta}}, z, \frac{t}{\delta}\right) + O(\delta^2)\end{aligned}$$

where  $\theta_0$ ,  $\theta'_0$ ,  $\varphi_0$ ,  $\varphi_1$ ,  $\vartheta_1$  are found as solutions to the above mentioned initial/boundary value problems.

Following this procedure it is possible to formulate also higher order asymptotic approximations of the general tolerance model equations (12). Under the restriction of analysis to the expansion (14) we arrive at the residual equation for  $\tilde{\varphi}_2$ ,  $\tilde{\vartheta}_2$  in the form

$$\begin{aligned}\delta D\tilde{\varphi}_2 + \{k_L\}\tilde{\varphi}_2 &= \delta \frac{[k]}{\{k\}}\partial D(\delta\vartheta_1 + \tilde{\vartheta}_2), \\ D\tilde{\vartheta}_2 + k^o\partial^2\tilde{\vartheta}_2 &= [k]\partial\tilde{\varphi}_2.\end{aligned}\quad (22)$$

Obviously, functions  $\tilde{\varphi}_2$ ,  $\tilde{\vartheta}_2$  have to satisfy the pertinent homogeneous initial/boundary conditions. Let us observe that if term  $\delta\vartheta_1$  can be as negligibly small when compared to  $\vartheta_0$  then neglecting these terms we obtain only trivial solutions  $\tilde{\varphi}_2 = 0$ ,  $\tilde{\vartheta}_2 = 0$ . The above statement closes the proposed asymptotic procedure. A justification of this procedure is a separate mathematical problem and will be studied elsewhere.

## NUMERICAL ANALYSIS

In order to verify efficiency of the proposed asymptotic approximation technique we shall restrict ourselves to the solution of two benchmark problems. Comparing derived approximations of the averaged temperature and temperature fluctuation allows to determine the influence of the right-hand sides of equations (12) to a solution.

For the sake of simplicity we assume that every lamina is homogeneous and isotropic. Hence  $K'_{\alpha\beta} = \delta_{\alpha\beta} k'$ ,  $K''_{\alpha\beta} = \delta_{\alpha\beta} k''$ .

Let us consider firstly the one-dimensional stationary problem with boundary conditions

$$\begin{aligned}\theta(x_2, -L) = \theta(x_2, L) = \theta(0, z) = 0, \\ \theta(H, z) = \theta_A \cos \frac{\pi}{2L} z.\end{aligned}$$

We look for solution to (12) in the form

$$\begin{aligned}\vartheta(x_2, z) = \bar{\vartheta}(x_2) \cos \frac{\pi}{2L} z, \\ \varphi(x_2, z) = \bar{\varphi}(x_2) \frac{\pi}{2L} \sin \frac{\pi}{2L} z.\end{aligned}\tag{23}$$

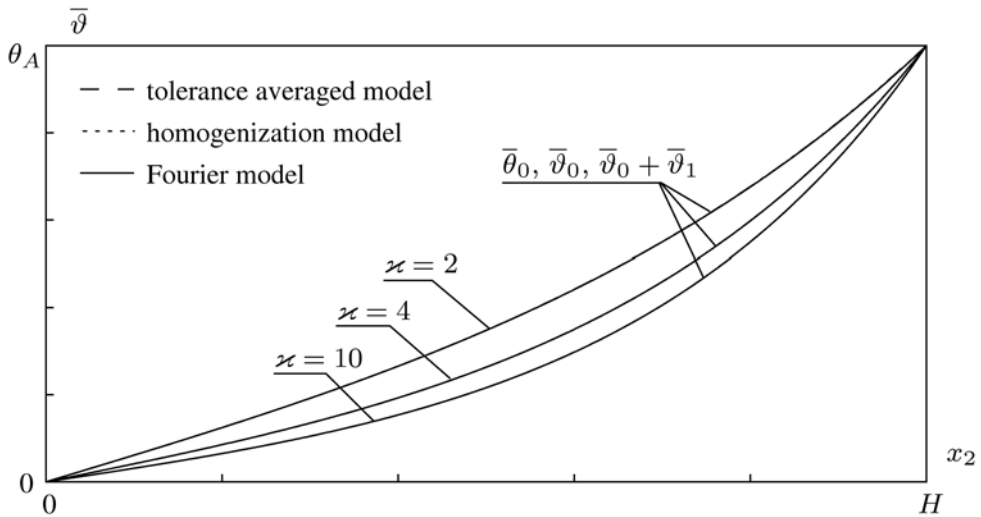
The computations have been carried out for different values of the inhomogeneity coefficient  $\kappa$  defined by

$$\kappa = \frac{k'}{k''}$$

and under assumption  $k'' = const$ .

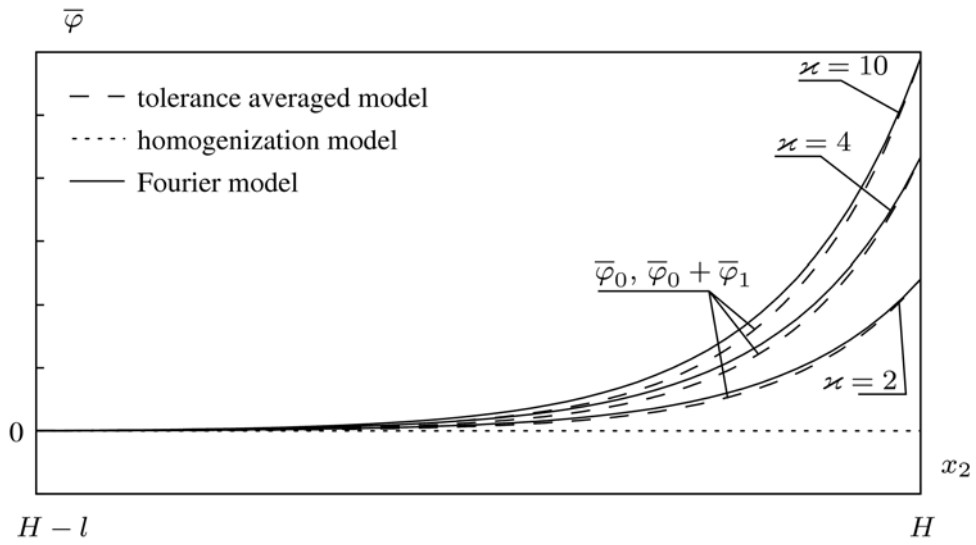
Solutions obtained both in the framework of the tolerance model using the proposed asymptotic approximation technique and homogenization are presented in Fig. 3 and Fig. 4 for  $H = 20l$ . The first approximation introduces an insignificantly small correction to the zeroth approximation.

**Figure 3. Averaged temperature for the stationary problem versus spatial coordinate parallel to layering. Difference between the first and second approximation as well as between the different models for a given  $\kappa$  are insignificant**





**Figure 4. Amplitude of intrinsic temperature fluctuation for the stationary problem**



Let us consider in turn a non-stationary problem. We introduce initial conditions

$$\vartheta(z,0) = 0,$$

$$\psi(z,0) = \psi_A \sin \frac{\pi}{2L} z$$

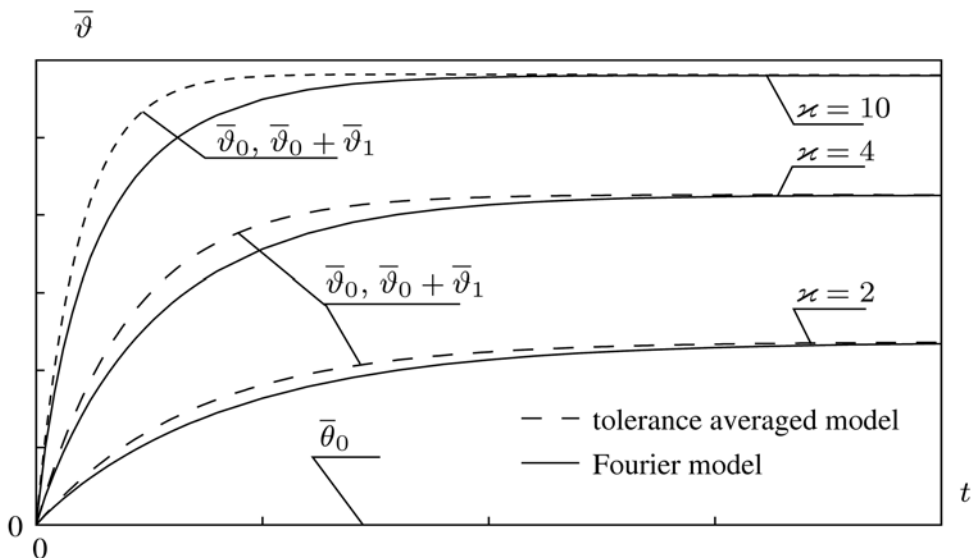
and we seek a solution in the form

$$\vartheta(z,t) = \bar{\vartheta}(t) \cos \frac{\pi}{2L} z,$$

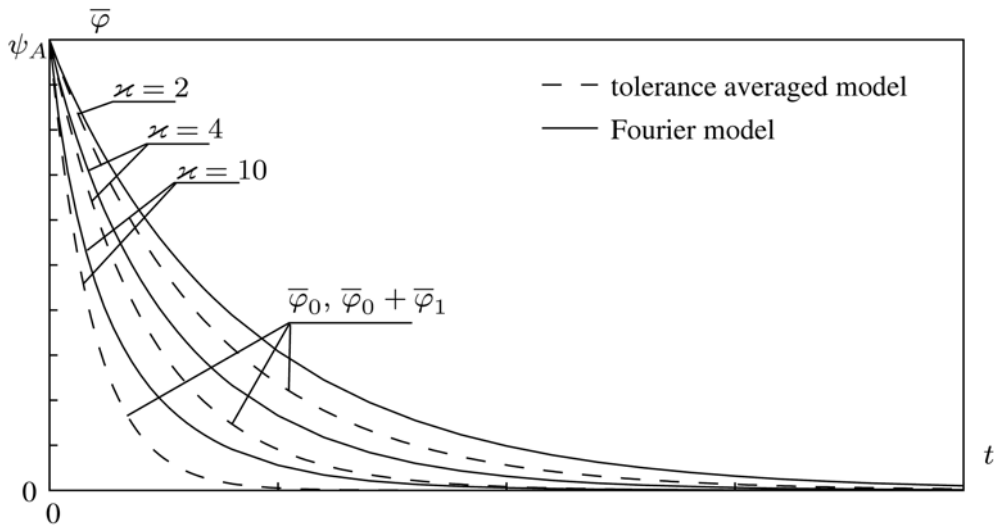
$$\varphi(z,t) = \bar{\varphi}(t) \sin \frac{\pi}{2L} z. \tag{24}$$

The obtained results are presented in Fig. 5 and Fig. 6.

**Figure 5. Averaged temperature for the non-stationary problem versus time coordinate**  
**Difference between the first and second approximation is not noticeable. The solution  $\bar{\theta}_0$  obtained in the framework of the homogenization model is inadequate**



**Figure 6. Amplitude of intrinsic temperature fluctuation for the non-stationary problem**



In order to verify these results we shall solve the problems in the framework of the Fourier model. The above problems will be solved numerically using the finite difference method (FDM). We approximate the temperature field  $\theta_{FDM}$  using formulae (4) and (23) or (24), where  $\bar{\vartheta}$  and  $\bar{\varphi}$  are treated as unknown approximation parameters. The approximation will be realized by the least square method. To this end we shall look for the values of functions  $\bar{\vartheta}$ ,  $\bar{\varphi}$  minimizing the least square error

$$\int_{-L}^L (\theta_{FDM} - \theta)^2 dz,$$

where  $\theta$  is a temperature field in the form (4).

The results of computations are shown in Fig. 3-6.

### CONCLUDING REMARKS

Let us summarize new results and information on the heat conduction in a laminated rigid conductor which have been obtained in this contribution.

1° It was shown that the proposed asymptotic technique allows to find solutions to tolerance model with sufficient accuracy. Thereby the considered model in general form without a simplification by omitting the right-hand sides of equation (12) can be used.

2° An influence of right-hand sides of the tolerance model equations to solution is of an order  $O(\delta)$ . Two first approximations seem to be sufficient in most practical cases.

3° Solutions to the selected problems obtained in this contribution are compared with those derived from homogenization. The main conclusion is that the tolerance model (in contrast to the known homogenization model) describes the boundary layer effect on the heat conduction in a laminated rigid conductor. Moreover, differences between solutions obtained in the framework of the Fourier equation and those related to the tolerance model are negligible in the boundary layer. Outside this layer solutions obtained by using the tolerance model, those derived from the homogenization model, as well as results calculated in the framework of the Fourier theory nearly coincide.

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