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From depinning transition to plastic yielding of amorphous media: A soft modes perspective

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A mesoscopic model of amorphous plasticity is discussed in the wider context of depinning models. After embedding in a $d+1$ dimensional space, where the accumulated plastic strain lives along the additional dimension, the gradual plastic deformation of amorphous media can be regarded as the motion of an elastic manifold in a disordered landscape. While the associated depinning transition leads to scaling properties, the quadrupolar Eshelby interactions at play induce specific additional features like shear-banding and weak ergodicity break-down. The latters are shown to be controlled by the existence of soft modes of the quadrupolar interaction, the consequence of which is discussed in the context of depinning.

Most liquids flow as soon as they experience shear stress. In contrast many complex fluids (pastes, foams, colloidal suspensions, etc.) do not flow for shear stresses lower than some threshold yield limit. The rheological behavior of these yield-stress fluids parallels the plasticity of amorphous solids (oxide and metallic glasses, polymers, etc.). Both families of materials exhibit a rich phenomenology. Close to the yielding threshold, critical-like behaviors are observed: avalanches [1, 2], growth of a correlation length scale [3], Hershell-Bulkley law [4]... In parallel other properties are reminiscent of glassy phenomena: e.g. thermal [5] and mechanical [6–8] history dependence. In the same spirit, strain localization [9, 10], a phenomenon of crucial technological interest (since it controls the mechanical strength) can be analyzed as an ergodicity break-down process: plastic activity is trapped in a very limited sub-region of the phase space [11].

These two phenomenological archetypes (criticality and glass transition) have motivated parallel efforts of modeling. Building on trap models [12] designed to capture ergodicity breaking and aging at glass transition, Sollich et al. [13, 14] developed Soft Glassy Rheology (SGR) models and could associate different rheological behaviors of complex fluids to a parameter of their model, an effective temperature associated to mechanical noise (see a recent discussion in [15]). A different glassy approach has been pursued by Bouchbinder and Langer[16] who extended the Shear-Transformation-Zone theory[17] to explicitly account for an effective temperature related to the slow configurational degrees of freedom of the glassy material under shear.

The need to go beyond mean field description and understand the crucial effect of elastic interactions associated to the localized rearrangements (Eshelby events) [17–21] responsible of amorphous plasticity has early led to the development of mesoscopic models accounting for these interactions [22]. This effort of modeling at mesoscopic scale has, since then, been very active [23–35]. As early noticed in [23], the competition at play in mesoscopic models between microscopic disorder and elastic interaction strongly reminds the physics of the depinning transition [36] that naturally entails critical features. Recently summarized in Ref. [35], most features of the associated scaling phenomenology have been observed numerically [37–40] and experimentally [1, 2].

Noteworthily, some of the key non-ergodic features (e.g. aging and shear-banding[32, 41, 42]) can be also recovered within the framework of the mesoscopic elasto-plastic models. This has raised the question of the precise link with the depinning transition. In particular, the crucial effect of the non-positiveness of the quadrupolar elastic interaction induced by individual plastic events has been questioned. Recently Lin *et al.* [35] have shown the necessity of three independent exponents (instead of two for standard depinning) to account for the scaling properties of mesoscopic models of amorphous plasticity.

Here we show that the specific features observed in elasto-plastic models are controlled by the presence of multiple soft modes of the quadrupolar elastic interaction. The presence of such soft modes will be argued not to be an artifact of lattice discretization or of a specific numerical implementation [31], but rather a genuine characteristic of elasto-plastic models. In the present perspective, shear bands directly result from the Eshelby interaction symmetry i.e. extended modes of plastic deformation that satisfy compatibility and consequently induce no internal stress. This property, absent in classical depinning models, has dramatic effects on the stability, the dependence on initial conditions as well as the ergodicity properties of plastic yielding models.

Depinning-like models for amorphous plasticity Here we restrict ourselves to a simple scalar case [24]. Assuming, bi-axial loading conditions, we define respectively for stress and strain the scalar quantities $\sigma =$

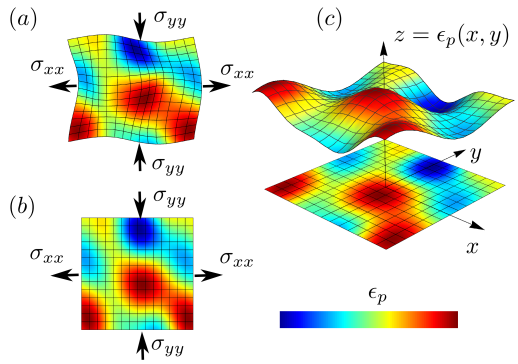

FIG. 1. Sketch of a 2d amorphous material upon bi-axial loading. (a) The mesh is deformed according to the displacement. The associated strain has a reversible elastic contribution and an irreversible plastic contribution. The latter is represented according to the color scale. (b) The plastic strain field (colors) is represented on the un-deformed reference frame. (c) The plastic strain field can be represented as a d-dimensional manifold moving in a $d+1$ space.

$\sigma_{yy} - \sigma_{xx}$, $\varepsilon = \varepsilon_{yy} - \varepsilon_{xx}$ from their tensorial counterparts. At mesoscopic scales, a simple plastic criterion can be defined from the comparison between the local values of the scalar equivalent stress field $\sigma(\mathbf{r}, \{\varepsilon^{\rm pl}\}) = \sigma^{\rm ext} + \sigma^{\rm int}[\mathbf{r}, \{\varepsilon^{\rm pl}\}]$ with a threshold stress $\sigma^c[\mathbf{r}, \varepsilon^{\rm pl}(\mathbf{r})]$. The local stress σ results from the addition of a (constant) external stress $\sigma^{\rm ext}$ and of a spatially fluctuating internal stress $\sigma^{\rm int}$ due to the successive plastic rearrangements mediated by the elastic interactions. Here the local stress threshold σ^c encodes the disordered nature of the structure, it depends both on space and on the local value of the plastic strain $\varepsilon^{\rm pl}$. From this local criterion a simple equation can be written to model the evolution of the plastic strain field:

$$\partial_t \varepsilon^{\rm pl}(\mathbf{r}, t) = \mathcal{P}\big(\sigma^{\rm ext} + G^{\rm el} * \varepsilon^{\rm pl}(\mathbf{r}, t) - \sigma^{\rm c}[(\mathbf{r}, \varepsilon^{\rm pl}(\mathbf{r}, t)]\big) \quad (1)$$

where $\mathcal{P}(\cdot)$ stands for the positive part function, i.e. $\mathcal{P}(x) = x$ if $x > 0$ and $\mathcal{P}(x) = 0$ if not. We recognize the usual formalism of depinning [36]. As sketched on Fig. 1, we can define an extra coordinate z, orthogonal to the space variable $\mathbf{r}$ after embedding in a $d+1$ dimensional space. The equation $z = \varepsilon^{\rm pl}(\mathbf{r})$ thus defines an elastic manifold whose propagation in the random landscape $\sigma^{\rm c}[\mathbf{r}, \varepsilon^{\rm pl}(\mathbf{r}, t)]$ is governed by Eq. (1).

The quenched noise $\sigma^{\rm c}$ is defined by its average $\langle \sigma^{\rm c} \rangle = \overline{\sigma^c}$ and its correlations $\langle \sigma^{\rm c}(\mathbf{r}, z)\sigma^{\rm c}(\mathbf{r} + \delta\mathbf{r}, z + \delta z)\rangle = \varsigma^2 f(\delta\mathbf{r})g(\delta z)$ where ς^2 gives the variance. Short-range correlations are considered, namely, $f(\delta\mathbf{r}) \to 0$ if $|\delta\mathbf{r}| \gg \ell$ and $g(\delta z) \to 0$ if $|\delta z| \gg b$. The length scale ℓ is given by the mesoscopic scale at which coarse-graining is performed. The length scale b corresponds to the typical plastic strain induced by elementary plastic events. Finally $G^{\rm el}$ is the elastic kernel associated to the reaction of the elastic matrix to a plastic inclusion[43]. This long-range interaction is characterized by a quadrupolar symmetry and can be written in an infinite medium as: $G_Q(\mathbf{r}) = A\cos(4\theta)/r^2$ where $r = |\mathbf{r}|$ and θ is the polar angle. Instead of directly integrating Eq. (1), an extremal dynamics algorithm of the model discretized on a lattice is implemented [23, 24]. Only one site experiences plastic deformation at each iteration step.

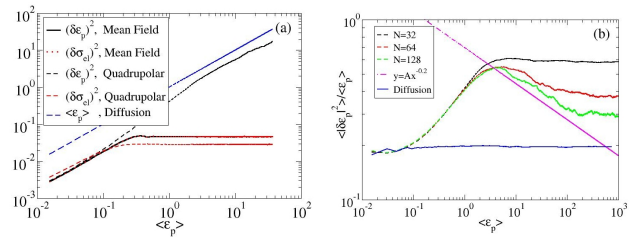

FIG. 2. (a) Strain and elastic stress variance vs cumulated plastic strain $\langle\varepsilon^{\rm pl}\rangle$ for a quadrupolar propagator G_Q and a Mean-Field G_{MF}. A linear behavior is represented for comparison. (b) Size-dependent diffusive-like behavior of the plastic strain field for the quadrupolar kernel G_Q for sizes $N = 32,\ 64,\ 128$ with $M = 1000,\ 100,\ 30$ realizations respectively. Results have been divided by the linear behavior expected for diffusion. The larger the system, the longer the anomalous sub-diffusive behavior.

Depinning vs plasticity In the following, G_Q denotes the quadrupolar interaction used in the plasticity model and G_{MF} a simple Mean-Field (MF) interaction, $G_{MF}(\boldsymbol{r}) = 1/(N^2 - 1)$ if $|\boldsymbol{r}| \neq 0$ and $G_{MF}(\boldsymbol{r}) = -1$ if not. The latter will be used (all other parameters being kept constant) to show the expected behavior of a standard reference depinning model. In order to to investigate the origin of the specific effects of the elastic kernel, we also define a weighted average of the two propagators: $G_a = (1-a)G_Q + aG_{MF}$ where the parameter a gives the relative weight of the mean field. For moderate values of a, the quadrupolar symmetry is mainly preserved in the sense that the Green function remains strictly negative in the 0 and $\pi/2$ directions.

We first show in Fig. 2 (a) the evolution of the variance of the plastic strain (the interface width in the depinning context) in the two cases of amorphous plasticity and mean field depinning. In the latter case, a classical Family-Vicsek scaling [44] is recovered: the interface width grows as a power law until it reaches a plateau. Saturation is obtained at a typical time scale $\tau \propto L^z$ such that the correlation length ξ has reached the system size $\xi(\tau) \approx L$. In the amorphous plasticity case, the first power-law growth regime is recovered but, past $\xi \approx L$, the interface width shows no saturation but rather a diffusive trend [24]. The evolution of the variance of the elastic stress is also shown in the two cases. Here saturation is recovered in plasticity as well as in MF depinning. Note that the elastic stress field can be directly obtained from the plastic strain field from a simple convolution with the propagator. The fact that the diffusive trend

at play with the strain field does not show in the stress fluctuations is a first indication that strain fluctuations are controlled by soft modes of the elastic interaction.

We show in more details the scaling properties of the variance of the plastic strain field in Fig. 2(b). In order to emphasize the specificity of the diffusion-like process, the variance is scaled by the mean plastic strain, a ratio expected to be constant for standard diffusion. A significant system size dependence is observed. In particular, a long sub-diffusive transient is obtained for large systems.

A dramatic effect on the diffusion regime is obtained when running a model with $a > 0$. In Fig. 3 (top) the time evolution of the interface width is shown for different values of a. It turns out that even the lowest MF contribution is enough to recover saturation at long times. A transient diffusive regime appears when a tends to zero, and the level of the final plateau increases accordingly. But when the interface gets too distorted, if $a > 0$ the (low) MF restoring force eventually stops the divergence of the strain fluctuations. A negative MF contribution $a < 0$ has the opposite effect: after a transient, the plastic strain gets unstable and its variance diverges very fast. The strong effect of the MF contribution is manifest in the plastic strain field for a cumulated plastic strain $\langle \varepsilon^{\rm pl} \rangle \approx 5$ for $a = -0.01,\ 0,\ 0.01$ using the same color scale in Fig. 3 (bottom). The plastic case ($a = 0.0$) shows a superposition of patterns localized at $\pm\pi/4$ following the symmetry of the quadrupolar kernel. Similar patterns survive with a positive MF contribution ($a = 0.01$) but get very attenuated (the interface width is much lower). A negative MF contribution induces conversely a strong localization behavior: plastic activity is restricted along a unique very thin shear band.

As above mentioned, shear-banding can be analyzed as an ergodicity breakdown: plastic deformation only visits a sub-part of the phase space. It is thus tempting to analyze the present model along these lines. In Fig. 4 we show two-point correlation functions computed after various "waiting times" (here the cumulated plastic strains). For the bare plasticity model, a striking mechanical history effect is observed: the larger the waiting time, the larger the decorrelation time. Again, the addition of a very small MF contribution is enough to destroy this mechanical history behavior. Such results are reminiscent of recent studies of depinning lines [45] that revealed aging properties but only in the roughness growing stage. Here the saturation of the interface roughness is postponed at infinity and aging can persist forever. This regime is thus naturally associated to the divergence of the interface width. This behavior could even be observed in a simple diffusion process. As above mentioned, the diffusion regime here at play is actually non trivial. In particular, as shown in Fig. 2 (b), for large systems, a very long sub-diffusive transient regime is obtained. This observation again supports weak ergodicity breakdown. The latter behavior is indeed associated to sub-diffusion [46].

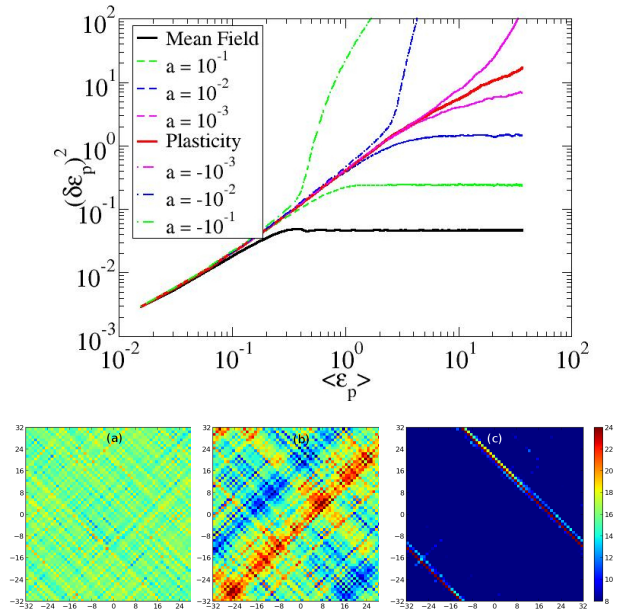

FIG. 3. Top: Strain variance (equivalently interface width) vs cumulated plastic strain $\langle \varepsilon^{\rm pl} \rangle$ for 5 different propagators: G_Q, G_a ($a = 10^{-1}, 10^{-2}, 10^{-3}$), G_{MF}. Bottom: Maps of plastic strain field obtained for a mere quadrupolar elastic interaction (b), and with a positive (a) and a negative (c) MF contribution $a = \pm 10^{-2}$ for $\langle \varepsilon^{\rm pl} \rangle \approx 5$, past the transient regime. The same color scale has been used in the three cases.

Fourier space and soft modes The introduction of yet a tiny MF component has thus dramatic consequences on the localization behavior In the following, a rewriting in Fourier space allows one to emphasize the crucial role of the soft modes of the propagator in this phenomenon. In the model presented above, the "Eshelby" quadrupolar interaction was defined through its Fourier transform in order to handle periodic boundary conditions [24]:

$$\widetilde{G^Q_{pq}} = A\left(-\cos(4\theta_{pq}) + 1\right) = -8A\left(\frac{p^2 - q^2}{p^2 + q^2}\right)^2 , \quad (2)$$

where θ_{pq} is the polar angle and (p, q) the wavevector in Fourier space. A is a constant chosen so that $G(0, 0) = -1$. The Fourier transform of the plastic strain field is defined as:

$$\varepsilon^{\rm pl}_{mn} = \frac{1}{N^2} \sum_{p=-N/2}^{N/2-1} \sum_{q=-N/2}^{N/2-1} \widetilde{\varepsilon^{\rm pl}_{pq}}\, e^{-i\frac{2\pi mp}{N}} e^{-i\frac{2\pi nq}{N}} . \quad (3)$$

The Fourier components of the elastic interaction is thus:

$$\widetilde{\sigma^{\rm el}_{pq}} = \widetilde{G^Q_{pq}}\widetilde{\varepsilon^{\rm pl}}_{pq} = -8A\left(\frac{p^2-q^2}{p^2+q^2}\right)^2 \widetilde{\varepsilon^{\rm pl}}_{pq} . \quad (4)$$

Denoting $\mathbf{e}_{(p,q)} = e^{-i\frac{2\pi mp}{N}} e^{-i\frac{2\pi nq}{N}}$ the (p, q) Fourier mode, we get $G^Q * \mathbf{e}_{(p,q)} = \lambda_{pq}\mathbf{e}_{(p,q)}$ with $\lambda_{pq} = -8A[(p^2 -$

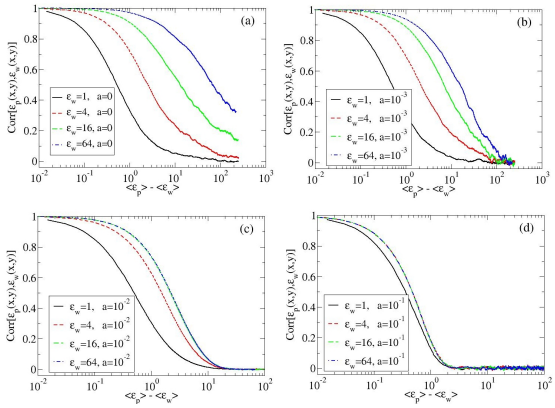

FIG. 4. Two-point correlation of the plastic strain field for 4 "waiting times" $\langle \varepsilon^{\rm pl} \rangle = 1,\ 4,\ 16,\ 64$ and for 4 different propagators: G_Q (a), G_a with a Mean-Field weight $a = 10^{-3}$ (b), $a = 10^{-2}$ (c) and $a = 10^{-3}$ (d). A clear aging effect show for the quadrupolar propagator G_Q: the longer the waiting time, the slower the decorrelation. The small MF contributions in propagators G_a gradually kills the aging behavior.

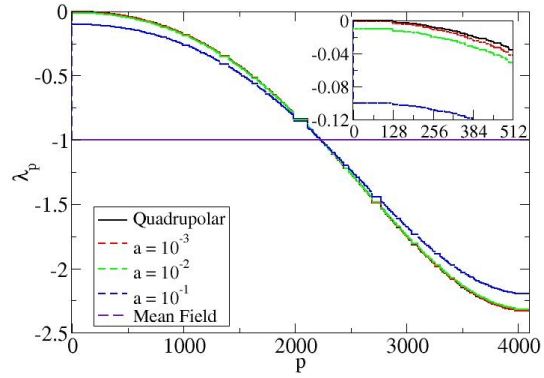

FIG. 5. Spectrum of eigenvalues of elastic propagators: Mean-Field (MF), quadrupolar interaction and MF-weighted quadrupolar interactions. The introduction of a fraction a of MF opens a gap between the translational mode having a null eigenvalue and the other modes $\lambda < 0$. The evolution of the gap is zoomed in the inset.

$q^2)/(p^2+q^2)]^2$. In other terms, the eigenmodes of the Green operator are precisely Fourier modes, and the associated eigenvalues are the above written λ_{pq}. This property stems from the translation invariance of the elastic propagator. The same property also holds for the MF propagator: $G^{MF}_{mn} = -\delta_m\delta_n + (1-\delta_m\delta_n)/(N^2-1)$ and $\widetilde{G^{MF}_{pq}} = -\delta_p\delta_q + (1-\delta_p\delta_q)/(N^2-1)$ where N is the linear size of the system. Let us now discuss the eigenvalue spectrum of the quadrupolar interaction. One first recognizes the translation mode of zero eigenvalue $\lambda_{00} = 0$. In the classical depinning case (say MF, Laplacian or power-law in distance) this mode is the only one characterized by a zero eigenvalue. It is the signature of the invariance of the model with respect to a uniform translation of the manifold along its propagation direction.

In the present quadrupolar case, a set of non-trivial eigenmodes are also characterized by a null eigenvalue. Namely $\mathbf{e}_{(p,p)}$ and $\mathbf{e}_{(p,-p)}$ with $p \in [-N/2, N/2-1]\backslash\{0\}$. Thus there is one trivial zero translation eigenmode and $2(N-1)$ non-trivial ones. These non-trivial modes corresponds in real space to arbitrary amplitudes for any combination of shear bands oriented at $\pm\pi/4$. Shear bands thus appear as soft modes of the quadrupolar elastic interaction. The diffusive-like behavior observed at long times for the plastic strain thus results from a competition between the different soft modes controlled by the quenched disorder. It is of interest to rewrite the argument of the $\mathcal{P}(\cdot)$ function in Fourier space

$$\mathcal{F}\left[\sigma^{\rm ext} + G^{\rm el} * \varepsilon^{\rm pl}(\mathbf{r},t) - \sigma^{\rm c}(\mathbf{r},\varepsilon^{\rm pl})\right] = \sigma^{\rm ext}\delta_p\delta_q + \widetilde{G^Q_{pq}}\widetilde{\varepsilon^{\rm pl}_{pq}} + \widetilde{\eta_{pq}}\ . \tag{5}$$

In real space, the spatial coupling is induced by the elastic interaction kernel, $G^{\rm el}$, while the noise term is local. In the space of eigenmodes, the opposite character is observed, namely the restoring force is local, but noise is not. Since all eigenvalues are null or negatives (otherwise the dynamics would be unstable) a competition emerges between the relaxation of the eigenmodes induced by the elastic contribution and a random forcing due to the quenched noise contribution. However, this interpretation only holds if we ignore the $\mathcal{P}(\cdot)$ function that intervenes in Eq. 1. At long time scales, the loading contributes to a positive average that allows for such an interpretation, however, at short time scales, the positive part function unfortunately cannot be simply expressed in Fourier space. Note that a similar situation appears in classical depinning models. The point is that for the latter ones, a long time integration gives a finite restoring force to any wavelength of manifold fluctuations.

When soft modes are present in the propagator, the relaxation contribution cancels out and one expects the strain field to be asymptotically dominated by the sole superimposition of soft modes, that is shear bands at $\pm\pi/4$, consistently with the diffusive behavior observed in the quadrupolar case. The strong effect of a small MF contribution to the quadrupolar propagator can thus be re-read as the consequence of the opening of a gap in the spectrum of eigenvalues, in other words to the vanishing of the soft modes. In Fig. 5, the spectra of eigenvalues of the stress redistribution kernel show the gradual gap opening due to the introduction of a MF contribution to the elastic quadrupolar interaction.

Conclusion The interpretation of the shear-bands as soft modes of the Eshelby elastic interaction may clarify the long debate about the relative importance of localized rearrangements and large scale shear-band like events [21],[47] as microscopic mechanisms of amorphous plasticity and complex rheology. It appears in particular that localized plastic events is the rule at short time

scales but on a larger time horizon, mostly shear bands account for the kinematics, as these are the only displacement fields that prevent large shear stress build-up.

While the present study has been concerned with the modeling of amorphous media plasticity, a similar phenomenology is expected for any depinning model as soon as the elastic propagator exhibits soft modes. As discussed in [35] in the case of plastic yielding, this new sub-class of depinning model is expected to exhibit non-trivial scaling properties. More generally it is tempting to study in more details the ergodic behavior of such models at finite temperature in relation with the Soft Glassy Rheology models [13] and with the recent observation of the strong effect of Eshelby events on relaxation processes in the liquid state [48].

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