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# Impact of Virtual Bridging on Virtual Machine Placement in Data Center Networking

Dallal Belabed, Stefano Secci, Guy Pujolle, Deep Medhi\*

Sorbonne Universités, UPMC Univ Paris 06, UMR 7606, LIP6, F-75005, Paris, France. Email: firstname.lastname@upmc.fr

\* U. Missouri-Kansas City, 5100 Rockhill Road, Kansas City, MO 64110-2499 USA. Email: dmedhi@umkc.edu

**Abstract**—The increasing adoption of virtualization techniques has recently favored the emergence of useful switching functions at the hypervisor level, commonly referred to as virtual bridging. In the context of data center network (DCN) consolidations, for VMs colocated in the same virtualization server, virtual bridging becomes very useful to offload inter-VM traffic from access and aggregation switches, at the expense of an additional computing load. DCN consolidations typically chase traffic engineering (TE) and energy efficiency (EE) objectives, and both should be affected by virtual bridging, but it is not intuitive to assert whether virtual bridging acts positively or negatively with respect to TE and EE that should also depend on the DCN topology and forwarding techniques. In this paper, we bring additional understanding about the impact of virtual bridging on DCN consolidations. First, we present a repeated matching heuristic for the generic multi-objective DCN optimization problem, with possible multipath and virtual bridging capabilities, accounting for both TE and EE objectives. Second, we assess the impact of virtual bridging on TE and EE in DCN consolidations. Extensive simulations show us that enabling virtual bridging has a negative impact when EE is the goal and multipath forwarding is adopted, while it leads to important gains, halving the maximum link utilization, when TE is the DCN consolidation goal.

## I. INTRODUCTION

The recent achievement of x86 virtualization by advanced software techniques allows attaining virtualization of server and network functions at competitive performance-cost trade-offs with respect to legacy solutions. The increasing adoption of virtualization techniques has recently favored the emergence of useful switching functions at the hypervisor level, commonly referred to as virtual bridging. In the context of data center network (DCN) consolidations, for VMs colocated in the same virtualization server, virtual bridging becomes very useful to offload inter-VM traffic from access and aggregation switches, at the expense of an additional computing load on the physical server. DCN consolidations typically chase traffic engineering (TE) and energy efficiency (EE) objectives, and both should be affected by virtual bridging, but it is not intuitive to assert whether virtual bridging acts positively or negatively with respect to TE and EE that should also depend on the DCN topology and forwarding techniques. In this paper, we bring additional understanding about the impact of virtual bridging on DCN consolidations.

The literature on DCN consolidation problems is extensive. Often referred to as DCN optimization, VM placement or virtual network embedding, the various propositions at the state of the art often have a narrow scope, with a set of constraints so that it is not possible to jointly adopt EE

and TE objectives and model advanced multipath forwarding protocols [1] [2] [3], network link states optimization [4] and edge virtual bridging [5]. In particular, virtual bridging functionalities are rarely modeled, probably because they are marginally understood.

The impact of multipath forwarding protocols on DCN performance strictly depends on the DCN topology. While the most common legacy topology is the 3-tier [6] architecture, it is losing interest because with network virtualization inter-rack traffic in support of consolidation procedures is overcoming the amount of external traffic. Therefore, flats topologies become more interesting. Topologies such as fat-tree [7], DCell [8] and BCube [9] are gaining momentum. The flat nature of these topologies can also give virtual bridging a higher importance in the DCN interconnect. Our previous study investigated the impact of multipath forwarding on different topologies [10]; results showed that multipath forwarding is indeed effective with flattened topology, and that it can be counterproductive when the EE is the primary goal of DCN consolidations (without virtual bridging capabilities).

Furthmore, link states can be monitored when planning Virtual Machines (VMs) migrations. For instance, migrating a VM catalyst of significant traffic at a server (“VM container”) whose access link is close to saturation is not a wise decision. VM containers that are topologically attractive could therefore be favored when deciding where to host and migrate VMs. Commercial DCN consolidation tools, (e.g., VMware Capacity Planner [11], IBM CloudBurst [12]), typically are aware of CPU, memory, storage, and energy constraints of VM containers are not, however, aware of link states since the legacy hypothesis is to consider unlimited link capacity. With the emergence of network virtualization, related storage synchronization tools and pervasive virtual bridging, the hypothesis that DCN links have infinite capacity is today becoming inappropriate, especially for DCs facing capital expenditure limitations. Performing VM consolidation that is aware of both container and link states is, however, known to be NP-hard [4]. The complexity does naturally increase when considering multipath and virtual bridging capabilities.

In this context, the recent introduction and large deployment of virtual bridging in most hypervisor solutions, (e.g., XEN, KVM, VM Ware NSX), is introducing novel constraints as it becomes interesting to assign to a same container or nearby containers, VMs exchanging large traffic amounts. The impact of virtual bridging on DCN consolidations can be sensible,

depending on topology and forwarding situations as well as on DCN objectives. The contribution of this paper is twofold:

- We describe a virtual machine placement optimization problem in DCN, with multipath forwarding and virtual bridging capabilities meeting TE and EE objectives, in a novel, compact and versatile formulation. We design a repeated matching heuristic scaling with large DCN sizes. To the best of our knowledge, this is the first comprehensive formulation in this sense, i.e., the first allowing for the consideration of virtual bridging and multipath capabilities and of TE and EE objectives.
- We run our heuristic on realistic DCN consolidation instances, showing that enabling virtual bridging improves TE performance on one hand by roughly two times, and on the other hand it can be counterproductive when EE is the primary goal. With respect to the coexistence of virtual bridging and multipath forwarding, we determine under which circumstances the gain brought by both innovations can be positive.

In the following, Section II presents the background of our work. The DCN optimization model is formulated in Section III, our heuristic is in Section IV, and our simulation results are in Section V. Section VI concludes the paper.

## II. BACKGROUND

We briefly discuss state of the art work on Ethernet routing, DCN traffic, and consolidation models.

### A. Data Center topologies

The *3-tier* architecture [6] is the common legacy of DC topology. It has three layers: access, aggregation and core layers. At the access layer, servers or server units, (e.g., blades), are attached to the network, via access switches; at the aggregation layer, access switches connect to aggregation switches; at the core layer, each aggregation switch is connected to multiple core switches. Such an architecture typically relies on legacy VLAN and STP switching [13], which, while simple and fast, is known to underutilize the network resources. Even if TE mechanisms such as Multiple STP, root bridge priority and port cost optimization methods exist, and major problems still persist, namely in terms of convergence time upon failures, routing, and physical topology changes.

Alternative topologies have been proposed in recent years, as briefly mentioned in the introduction. Originally, the authors in [7] proposed a special instance of a Clos topology called “fat-tree” to interconnect commodity Ethernet switches as  $k$ -ary fat-tree. As depicted in Fig. 1a, all switches are identical and are organized on two layers: the core layer and the pod layer. Generally, at the pod layer there are  $k$  pods, each one containing two layers of  $\frac{k}{2}$  switches: edge switches and aggregation switches. Each  $k$ -port switch in the lower layer (edge layer) is directly connected to  $\frac{k}{2}$  hosts. Each of the remaining  $\frac{k}{2}$  ports is connected to  $\frac{k}{2}$  of the  $k$  ports in the aggregation layer. Concerning the core layer, there are  $(\frac{k}{2})^2$   $k$ -port core switches. Each core switch has one port

connected to each of the  $k$  pods. The  $i^{th}$  port of any core switch is connected to the  $i^{th}$  pod so that consecutive ports in the aggregation layer of each pod switch are connected to the core switches on  $(\frac{k}{2})$  strides. Fig. 1a shows a fat-tree example for  $k = 4$ . Another topology is BCube [9], a recursive architecture designed for shipping and container-based, modular data center. As depicted in Fig. 1b, the BCube solution has server devices with multiple ports (typically no more than four). Multiple layers of cheap commodity off-the-shelf mini-switches are used to connect those servers. A BCube<sub>0</sub> is composed of  $n$  servers connected to an  $n$ -port switch. A BCube<sub>1</sub> is constructed from  $n$  BCube<sub>0</sub>s and  $n$   $n$ -port switches. More generally, a BCube <sub>$k$</sub>  ( $k \geq 1$ ) is constructed from  $n$  BCube <sub>$k-1$</sub> s and  $n^k$   $n$ -port switches. For example, in a BCube <sub>$k$</sub>  with  $n$   $n$ -port switch, there are  $k + 1$  levels of switches. Each server has  $k + 1$  ports, numbered from level-0 to level- $k$ . Hence, BCube <sub>$k$</sub>  has  $N = n^{k+1}$  servers. Each level having  $n^k$   $n$ -port switches. The construction of a BCube <sub>$k$</sub>  is as follows. One numbers the  $n$  BCube <sub>$k-1$</sub> s from 0 to  $n - 1$  and the servers in each BCube <sub>$k-1$</sub>  from 0 to  $n^k - 1$ . Then one connects the level- $k$  port of the  $i^{th}$  server ( $i \in [0, n^k - 1]$ ) in the  $j^{th}$  BCube <sub>$k-1$</sub>  ( $j \in [0, n - 1]$ ) to the  $j^{th}$  port of the  $i^{th}$  level- $k$  switch. The BCube construction guarantees that switches only connect to servers and never connect directly to other switches, thus multipathing between switches is impossible. It is worth noting that this kind of architecture requires virtual bridging in containers to operate. Fig. 1b shows an example of a BCube<sub>1</sub>, with  $n = 4$ .

Similarly to BCube, DCell [8] uses servers equipped with multiple network ports and mini-switches to construct its recursive architecture. In DCell, a server is connected to several other servers and a mini-switch. Generally, a high-level DCell is constructed from low-level DCells. The connection between different DCell networks is typically done by using virtual bridging in containers. A DCell <sub>$k$</sub>  ( $k \geq 0$ ) is used to denote a level- $k$  DCell. DCell<sub>0</sub> is the building block to construct larger DCells. It has  $n$  servers and a mini-switch ( $n = 4$  for DCell<sub>0</sub> in Fig. 1c). All servers in DCell<sub>0</sub> are connected to the mini-switch.

In DCell<sub>1</sub>, each DCell<sub>0</sub> is connected to all the other DCell<sub>0</sub>s with one link; the Fig. 1c shows a DCell<sub>1</sub> example. DCell<sub>1</sub> has  $n + 1 = 5$  DCell<sub>0</sub>s. DCell connects the 5 DCell<sub>0</sub>s as follows. It assigns each server a 2-tuple  $[a_1, a_0]$ , where  $a_1$  and  $a_0$  are the level-1 and level-0 IDs, respectively. Thus  $a_1$  and  $a_0$  take values from  $[0, 5)$  and  $[0, 4)$ , respectively. Then two servers with 2-tuples  $[i, j - 1]$  and  $[j, i]$  are connected with a link for every  $i$  and every  $j > i$ . Each server has two links in DCell<sub>1</sub>. One connects to its mini-switch, and hence to other nodes within its own DCell<sub>0</sub>. The other connects to a server in another DCell<sub>0</sub>. In DCell<sub>1</sub>, each DCell<sub>0</sub>, if treated as a virtual node, is fully connected with every other virtual node to form a complete graph. Moreover, since each DCell<sub>0</sub> has  $n$  inter-DCell<sub>0</sub> links, a DCell<sub>1</sub> can only have  $n + 1$  DCell<sub>0</sub>s, as illustrated in Fig. 1c. A DCell <sub>$k$</sub> , is constructed in the same way to the above DCell<sub>1</sub> construction. The recursive DCell construction procedure [8] is more complex than the BCube

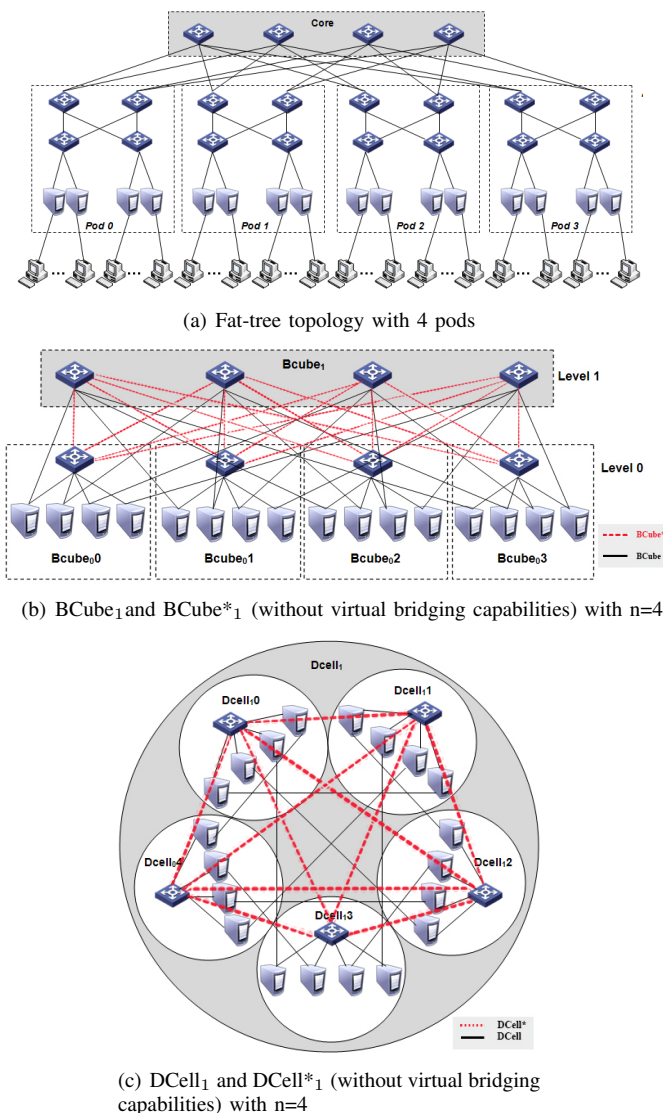


Fig. 1. Recently proposed DCN topologies

procedure.

### B. Ethernet fabric evolution

In the last decade, several evolutions to the legacy Ethernet switching architecture in terms of TE features have occurred. Under the perspective of incremental upgrade of the Ethernet switching architecture to meet TE requirements, we can consider that the Multiple Spanning Tree Protocol [14] has been the first attempt to actively perform TE in a legacy Ethernet switched network running STP and hence suffering from unused links in normal situations. The multiplexing of multiple clients or VLANs into one among several spanning trees can also be optimized as presented in [14]. Then, other protocols trying to solve bottleneck issues along the spanning tree(s) have been standardized, as notably the Link Aggregation Group or the multi-chassis EtherChannel protocols [15], allowing a switch to use multiple links as a single one with respect to the STP control-plane. Eventually, the real bottle-

neck in performing TE efficiently in an Ethernet switching context being the spanning tree bridging of Ethernet traffic, the STP control-plane has been removed from more recent carrier Ethernet solutions implementable in DCNs, namely: the Provider Backbone Bridges with Traffic Engineering (PBB-TE) [16], where centralized control servers push MAC tables to backbone switches (in a similar philosophy OpenFlow [3] does too); the L2LSP [17] effort suggesting to use the VLAN fields as MPLS label fields; the already mentioned SPB [2] and TRILL [1] protocols where the control-plane is distributed adapting a layer-3 link state routing protocol (ISIS) to work with the Ethernet data-plane. Nodes in this context are no longer simple bridges since they perform a routing function. Hence, in TRILL, as well as in the following, we refer to them as router-bridges (referred to as RBridges, or RBs).

While differing in terms of scalability and deployability, the latter three solutions have proven to be viable ones and have been adopted by many vendors. Notably, these protocols enabled multipath routing of Ethernet frames, and hence opened the way to active load-balancing over multiple paths across virtual and physical switches. In this paper, we assume DCN multipath capabilities are enabled by one of these protocols.

### C. DCN traffic models

At present, little is known about DCN traffic characteristics. This is likely because of the important heterogeneity context and because of non-disclosure confidentiality reasons. There are, however, a few studies worth being mentioned.

Supposing a legacy 3-tier architecture, the authors in [18], [19] collected information from edge, aggregation, and core devices, finding that the traffic originating from a rack showing an ON/OFF behavior following heavy-tailed distributions. It is also important to mention that, as presented in [19], most of the DC server-originated traffic, 80%, stayed within the rack, while for the university and private enterprise DCs, between 40% and 90% left the rack.

In terms of transferred volume, the authors in [20] showed that more than 90% of transfers had a volume from 100 MB to 1 GB. They also showed that 50% of the time, a VM had approximately 10 concurrent flows, and that at least 5% of the time, a machine had more than 80 concurrent flows.

In [21], the authors studied the incoming and outgoing traffic rates for seventeen thousand VMs. Their results showed that 80% of the VMs had an average rate less than 800 KBytes/min. However, 4% of them had a rate ten times higher. Moreover, the traffic rate's standard deviation of 82% of the VMs was lower than or equal to two times of the mean.

### D. Virtual Machine Placement in Data Center Networks

We review thereafter relevant works that take network constraints in the VM placement problem into consideration modeled as an optimization problem.

In [21], the authors proposed a VM placement solution considering network resource consumption. They assumed that a VM container could be divided into CPU-memory slots, where each slot could be allocated to any VM. They considered the

number of VMs equal to the number of slots; if the number of slots was higher than the number of VMs, they added dummy VMs (with no traffic), and did not affect the algorithm. Due to a communication cost between slots, defined as the number of forwarded frames among them, the objective was set as the minimization of the average forwarding latency. They also assumed static single-path routing and focused on two traffic models. A dense one where each VM sent traffic to every VMs at an equal and constant rate, and a sparse Infrastructure as a Service (IaaS)-like one with isolated clusters so that only VMs in the same IaaS could communicate.

In [22], the authors consolidated VM placement considering a non-deterministic estimation of bandwidth demands. The bandwidth demand of VMs was set to follow normal distributions. They formulated the consolidation in a Stochastic Bin Packing problem and introduced a new heuristic approach to resolve it. In [23], the authors considered network constraints in addition to CPU and memory constraints in the VM placement problem. They defined a network-aware VM placement optimization approach to allocate VM placement while satisfying predicted traffic patterns and reducing the worst case cut load ratio in order to support time-varying traffic. Interested by network cuts, they partitioned the set of hosts into non-empty connected subsets, which are bottlenecks for the traffic demand between VMs placed in different sides of the cut.

In [24], the authors revisit the virtual embedding problem by distinguishing between server and bridge nodes with respect to the common formulation. They proposed an iterative 3-step heuristic: during the first step an arbitrary VM mapping was done; the second step mapped virtual bridges to bridges nodes, and the third one mapped virtual links accordingly. If one of these steps failed, the heuristic would come back to the previous one until a solution was found. The quality of the solution seemed dependent on the first step, the other steps just minimized the impact of the previous step. Further, there may have been a scaling problem due to the uncontrollable backtracking. More generally, virtual embedding approaches in the literature often discarded specificities of the network control-plane such as the routing protocol and TE capabilities.

In [25], the authors optimized jobs placement where each job required a number of VMs; the objective function minimized the network and the node costs. The authors did not handle the link capacity constraints, and did not consider multipath forwarding capabilities instead multipath routing with one single egress path. In [26], the authors minimize the power energy consumption of activated servers, bridges and links, to maximize the global energy saving. The authors converted the VM placement problems into a routing problem, and so they addressed the network and server optimization problem as a single one. So, there was no trade-off between the network-side and server-side optimization objective.

Some of these studies ignored link capacity constraints, others excluded dynamic routing as in [21], or just considered the traffic volume to reduce the number of containers as in [22], or just the network resources as in [21] and [23],

TABLE I  
MATHEMATICAL NOTATIONS

$N$	set of VM containers and RBridges (RB); $n \in N$ .
$C$	container set; $C \subset N$ .
$V$	VM set; $V \subset N$ .
$R$	RB set; $R \subset N$ . $R_a \subset R$ is the access RB set.
$T^V$	set of VM pairs; $T^V \subset V \times V$ .
$T^C$	set of container pairs; $T^C \subset C \times C$ .
$T^R$	set of RB pairs; $T^R \subset R \times R$ .
Variables	
$e_{v,c}$	1 when $v$ is at $c$ ; 0 otherwise, $v \in V, c \in C$ .
$b_c$	1 if $c$ is enabled; 0 otherwise, $c \in C$ .
$a_{c,r}$	1 when $c$ traffic transits by $r$ if unipath; $\in [0, 1]$ if multipath, $c \in C, r \in R$ .
$q_{s,d}^k$	1 if traffic from $r_s$ to $r_d$ transits by the $k^{th}$ path if unipath. $\in [0, 1]$ if multipath, $(r_s, r_d) \in T^R$ .
$t_{c_i,c_j}$	traffic from $c_i$ to $c_j$ , $(c_i, c_j) \in T^C$ .
$t_{r_i,r_j}$	traffic from $r_i$ to $r_j$ ; $(r_i, r_j) \in T^R$ .
$t_{c,r}$	traffic from $c \in C$ to $r \in R$ .
$U$	maximum network link utilization.
Parameters	
$K_c^P$	power capacity of container $c \in C$ .
$K_c^M$	memory capacity of container $c \in C$ .
$d_v^P$	computing power demand of VM $v \in V$ .
$d_v^M$	memory demand of VM $v \in V$ .
$t_{v_i,v_j}$	traffic from $v_i$ to $v_j$ , $(v_i, v_j) \in T^V$ ; $t_{v_i,v_i} = 0$ .
$K_{i,j}$	$(i, j)$ link capacity, null if no link; $(i, j) \in N \times N$ .
$P_{i,j}^{k,s,d}$	1 when $k^{th}$ path from $r_s$ to $r_d$ uses link $(r_i, r_j)$ .
$\alpha$	trade-off coefficient between TE and EE objective, $\alpha \in [0, 1]$ .

only [26] considered multipath forwarding capabilities. Commonly, because of the relatively recent employment of virtual bridging for transiting traffic at the server level, virtual bridging capabilities for external traffic forwarding were ignored. To the best of our knowledge, our study is the first one to analyze virtual bridging impact on VM placement optimization and TE objective considering multipath forwarding.

### III. OPTIMIZATION PROBLEM

In the following, we present the mathematical notations of our reference optimization problem first, then we describe constraints in the case where multipath and virtual bridging are not enabled, we then show how they can be easily extended to enable multipath and virtual bridging. The notations are provided in Table I. First, we present integrity constraints, then capacity constraints and the objective function, and finally, we position the formulation with respect to the state of the art.

The objective of our problem is to balance between maximum link utilization and the number of containers to be activated.

$$\text{minimize } \alpha U + (1 - \alpha) \sum_{c \in C} b_c \quad (1)$$

Subject to the following constraints. A VM can be assigned to only one container:

$$\sum_{c \in C} e_{v,c} = 1; \quad \forall v \in V \quad (2)$$

A container is enabled only if it hosts at least one VM:

$$b_c \leq \sum_{v \in V} e_{v,c}; \quad b_c \geq e_{v,c}; \quad \forall c \in C, \forall v \in V \quad (3)$$

Each container is assigned to one RB:

$$\sum_{r \in R} a_{c,r} = 1; \quad \forall c \in C \quad (4)$$

Traffic between two access RBs is sent over a single path:

$$\sum_k q_{r_s, r_d}^k = 1 \quad \forall (r_s, r_d) \in R_a \times R_a \quad (5)$$

A VM is assigned to a container only if there are available residual computing resources:

$$\sum_{v \in V} d_v^P e_{v,c} \leq K_c^P; \quad \sum_{v \in V} d_v^M e_{v,c} \leq K_c^M; \quad \forall c \in C \quad (6)$$

Container-RB traffic is less than the access link capacity:

$$t_{c,r} \leq K_{c,r}; \quad \forall c \in C \quad \forall r \in R \quad (7)$$

Inter-RB traffic is less than the aggregation-core link capacity:

$$\sum_{r_s, r_d} \sum_k t_{r_s, r_d}^k q_{r_s, r_d}^k p_{r_i, r_j}^{k, r_s, r_d} < U K_{r_i, r_j} \quad \forall (r_i, r_j) \in T^R \quad (8)$$

Where:

$$t_{c,r} = \sum_{(v_i, v_j) \in T^V} (t_{v_i, v_j} + t_{v_j, v_i}) e_{v_i, c} a_{c, r}; \quad \forall r \in R, \forall c \in C$$

$$t_{r_s, r_d} = \sum_{\substack{i \neq j \\ (c_i, c_j) \in T^C}} t_{c_i, c_j} a_{c_i, r_s} a_{c_j, r_d}; \quad \forall (r_s, r_d) \in T^R$$

$$t_{c_i, c_j} = \sum_{(v_x, v_y) \in T^V} (t_{v_x, v_y} + t_{v_y, v_x}) e_{v_x, c_i} e_{v_y, c_j}; \quad \forall c_i, c_j \in C$$

We have a bi-criteria objective function that consists of the minimization of  $U$ , the maximum link utilization (TE goal), and the number of enabled containers (EE goal), weighted by the  $\alpha$  factor to assess the trade-off between the two goals and its impact on VM placement and DCN performance.

1) *Enabling multipath capabilities*: Multipath forwarding between containers and RBs (in the place of LAG, link bonding, or similar approaches) can simply be enabled by declaring  $a_{c,r}$  as a non-negative real variable instead of a binary variable. Hence, (4) becomes an integrity constraint on the sum of traffic ratios for each active container to its RBs. Similarly, a multipath between RBs can simply be enabled by declaring  $q_{s,d}^k$  as a non-negative real variable instead of a binary variable. Hence, (5) becomes an integrity constraint on the sum of traffic ratios for each pair of RBs to its used paths.

2) *Enabling virtual bridging*: Enabling virtual bridging means that the container absorbs the function of a bridge (typically at the hypervisor level). This feature can be easily included by transforming the variable  $a_{c,r}$  in a parameter and extending the RB set including the container nodes. Given that virtual bridging consumes additional power and memory, (6) should be slightly changed so that such an additional component, as a function of the traffic load, is included.

The provided optimization model is an extension of the baseline multi-commodity flow (MCF) problem for network routing with link capacity constraints [27], taking into account: peculiar data center networking constraints due to VM mobility, VM container switching on and off, virtual bridging, and multipath forwarding. In order to control the MCF complexity when handling TE and multipath parameters and variables, we adopted above the link-path formulation [27].

Given the elasticity related to VM migrations and multipathing, requiring double mapping between VMs and VM containers, and between VM containers and usable paths, our optimization problem defined by (2)-(1) even if comprehensive and versatile (considering both unipath and multipath modes, with and without virtual bridging, and VM attachment constraints) is a non-linear problem and cannot be linearized. Single mapping could be linearized but not double mapping.

#### IV. HEURISTIC APPROACH

Classically, mapping problems can be revisited as facility location problems, and when capacity constraints need to be verified as a function of the type of mapping, there are similarities with the capacitated facility location problem [28] and, in particular, with the single source facility location problem (SSFLP) [29], [30]. It is easy to derive that our DCN optimization problem can be reduced to the SSFLP and hence is NP-hard. Recently, modeling an optical network dimensioning problem as a facility location problem, the authors in [31] extended a primitive repeated matching heuristic described in [29], [30] to solve the SSFLP and proved it can reach optimality gaps below 5% also for many instances of the problem. A similar approach was later adopted for an optical network dimensioning approach, also providing outstanding performance for very large instances as described in [31].

Motivated by those results, we redesigned the repeated matching heuristic to our DCN optimization problem. Nevertheless, the double mapping we have handle in our problem and the multiple capacity constraints to care about (at both link and server sides) made this problem is much more difficult to solve and comparison to the optimum is not possible differently than in previous applications [29], [30], [31].

##### A. Reformulation of the optimization problem

Recall that DCN communications are between VMs that can be hosted behind the same VM container or behind distant containers interconnected by a DCN path. Certainly, external communications can be modeled introducing fictitious VMs and VM containers acting as egress point, from a functional standpoint. When multipath is enabled, multiple paths can be used, and when virtual bridging is enabled, a VM container can transit external traffic if the topology supports it. When communicating VMs are not colocated, inter-VM communication should involve a pair of containers and at least a DCN path between them.

Let a virtual node be designated by  $v$ ,  $v \in V$ , and a VM container node pair be designated by  $cp$ ,  $cp \in T^C$ , so that  $cp = (c^i, c^j)$ , i.e., a container pair is composed of two containers  $c^i$  and  $c^j$ . When  $c^i = c^j$  the container pair  $cp$  is said to be *recursive*. A subset of container node pairs is designated by  $D^C$  so,  $D^C \subseteq T^C$ . Let the  $k^{th}$  path from RB  $r^1$  to RB  $r^2$  be designated by  $rp = (r^1, r^2, k)$ . A set of RB paths is designated by  $D^R$  so that  $D^R \subset T^R$ .

**Definition IV.1.** Kit  $\phi$

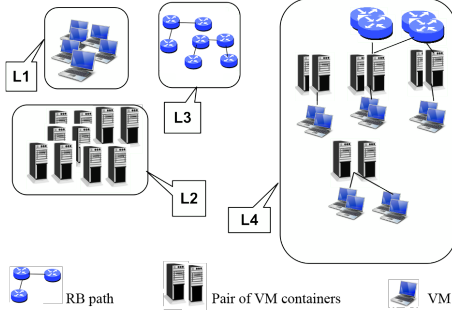


Fig. 2. Representation of heuristic sets:  $L_1$ ,  $L_2$ ,  $L_3$ , and  $L_4$

A **Kit**  $\phi$  is composed of a subset of VMs  $D^V$ , a VM container pair  $cp \in T^C$  and a subset of RB paths  $D^R$ . In a Kit  $\phi$ , each VM  $v \in D^V$  is assigned to one of the containers in a pair  $cp$  ( $c^1, c^2$ ). A container pair  $cp$  ( $c^1, c^2$ ) is connected by each RB path  $rp$  ( $r^1, r^2, k$ )  $\in D^R$ , so that  $c^1$  and  $c^2$  are respectively mapped to  $r^1$  and  $r^2$ . The Kit is recursive when its  $cp$  is recursive, and in such a case,  $D^R$  must be empty. When the multipath is not enabled,  $|D^R| = 1$ . The Kit is denoted by  $\phi(cp, D^V, D^R)$ .

#### Definition IV.2. Feasible Kit

A Kit  $\phi(cp, D^V, D^R)$  is said to be feasible if:

- $D^V$  is not empty, i.e.,  $D^V \neq \emptyset$ .
- Memory and power demands of each VM are satisfied, i.e., (6) restricted to  $D^V$  and  $cp$ .
- In case of non-recursive Kit, the link capacity constraints between VM containers are satisfied, i.e., (7) restricted to  $D^V$ ,  $D^R$  and  $cp$ .

#### Definition IV.3. $L_1, L_2, L_3$ , and $L_4$

$L_1$  is the set of VMs not matched with a container pair.  $L_2$  is the set of VM container pairs not matched with an RB path.  $L_3$  is the set of RB paths not matched with a container pair.  $L_4$  is the set of Kits.

#### Definition IV.4. Packing II

A Packing is a union of Kits in  $L_4$ . A Packing is said to be feasible if its Kits are feasible and  $L_1$  is empty.

#### B. Matching Problem

Given the DCN optimization problem's elements using the above described sets, it can be reformulated as a matching problem between them. The classical matching problem can be described as follows. Let  $A$  be a set of  $q$  elements  $h_1, h_2, \dots, h_q$ . A matching over  $A$  is such that each  $h_i \in A$  can be matched with only one  $h_j \in A$ . An element can be matched with itself, which means that it remains unmatched. Let  $s_{i,j}$  be the cost of matching  $h_i$  with  $h_j$ . We have  $s_{i,j} = s_{j,i}$ . We introduce the binary variable  $z_{i,j}$  that is equal to 1 if  $h_i$  is matched with  $h_j$  and zero otherwise. The matching problem consists in finding the matching over  $A$  that minimizes the total cost of the matched pairs.

$$\min \sum_{i=1}^q \sum_{j=1}^q s_{i,j} z_{i,j} \quad (9)$$

$$\text{s.t.} \quad \sum_{j=1}^q z_{i,j} = 1, \quad i = 1, \dots, q \quad (10)$$

$$\sum_{i=1}^q z_{i,j} = 1, \quad j = 1, \dots, q \quad (11)$$

$$z_{i,j} = z_{j,i}, \quad i, j = 1, \dots, q \quad (12)$$

$$z_{i,j} \in \{0, 1\}, \quad i, j = 1, \dots, q \quad (13)$$

(10) and (11) ensure that each element is exactly matched with another one. (12) ensures that if  $h_i$  is matched with  $h_j$ , then  $h_j$  is matched with  $h_i$ . (13) sets  $z_{i,j}$  as binary.

In our heuristic, one matching problem is solved at each iteration between the elements of  $L_1$ ,  $L_2$ ,  $L_3$ , and  $L_4$ . At each iteration, the number of matchable elements is  $n_1 + n_2 + n_3 + n_4$  where  $n_1, n_2, n_3$ , and  $n_4$  are the current cardinalities of the four sets, respectively. For each matching iteration, the costs  $s_{i,j}$  have to be evaluated. The cost  $s_{i,j}$  is the cost of the resulting element after having matched element  $h_i$  of  $L_1, L_2, L_3$ , or  $L_4$  with element  $h_j$ . The costs  $z_{i,j}$  are stored in a matrix  $Z$ . The dimension of the cost matrix  $Z$  is  $(n_1 + n_2 + n_3 + n_4) \times (n_1 + n_2 + n_3 + n_4)$ . Note that this dimension changes at each iteration.  $Z$  is a symmetric matrix. Given the symmetry, only ten blocks have to be considered. The notation  $[L_i - L_j]$  is used hereafter to indicate the matching between the elements of  $L_i$  and the elements of  $L_j$  as:

$$Z = \begin{pmatrix} [L_1 - L_1] & [-] & [-] & [-] \\ [L_2 - L_1] & [L_2 - L_2] & [-] & [-] \\ [L_3 - L_1] & [L_3 - L_2] & [L_3 - L_3] & [-] \\ [L_4 - L_1] & [L_4 - L_2] & [L_4 - L_3] & [L_4 - L_4] \end{pmatrix}$$

Selecting the least cost matching vector enables solution improvements via set transformations in next iterations. Obviously,  $L_1 - L_1$ ,  $L_2 - L_2$  and  $L_3 - L_3$  matchings are ineffective. To avoid a matching, e.g., because infeasible, its cost is set to infinity (a large number in practice). Matching corresponding to other blocks without  $L_4$  lead to the formation of Kits. Other matchings involving elements of  $L_4$  shall lead to the improvement of the current Kits, also generating local improvements due to the selection of better VM containers or RB routes; note that for these block local exchange linear optimization problems are to be solved for determining an exchange of VMs, VM containers and Kits between the heuristic sets while satisfying computing capacity constraints.

The Kit cost computation has to maintain the same rationale as in the reference optimization problem when setting individual matching costs. The cost needs to be computed to de-motivate under-loading VM containers in terms of CPU and RAM utilization, while avoiding over-loading RB paths in terms of link utilization and respecting computing capacity constraints. The Kit cost function has to appropriately model two opposite forces due to the dual aspects stressing DCNs: computing and network resources. On the one hand, the Kit

feasibility, in terms of link capacity constraints as described above does not need to be enforced during the repeated matching iterations, but to be motivated via the classical TE costs inducing the minimization of the maximum link utilization, and hence maximizing the minimum residual link capacity. On the other hand, residual computing capacities at the VM container level should be considered as costs; it is not suitable to have idle memory and CPU capacities when reducing the VM container's fixed energy consumptions is one of the goals of the DC provider. The overall Kit cost is not meant to represent a direct monetary cost, but it is such that the repeated matching promotes less expensive and more efficient Kits. Therefore, to align with the objective function (1), and remembering that the cost of a Packing corresponds to the cost of its Kits, we set the cost of a Kit  $\phi(cp, D^V, D^R)$  as:

$$\mu(\phi) = (1 - \alpha)\mu^E(\phi) + \alpha\mu^{TE}(\phi) \quad (14)$$

Where  $\alpha$  is the trade-off scaling factor between the EE and the TE components, that are, respectively:

$$\mu^E(\phi) = \sum_{c^i \in cp} \left( \frac{K_{c^i}^P}{\sum_{v \in D_i^V} d_v^P} + \frac{K_{c^i}^M}{\sum_{v \in D_i^V} d_v^M} + \Gamma T_v \right) \quad (15)$$

$$\mu^{TE}(\phi) = \max_{(n_i, n_j) \in rp, rp \in \phi} U_{n_i, n_j}(\Pi) \quad (16)$$

Where  $T_v$  represents the global traffic  $v$  sends and receives, i.e.,  $T_v = \sum_{v' \in V, v' \neq v} t_{v, v'}$ ;  $\Gamma$  is the additional power and memory, to take into account the impact of traffic to VM container's CPU and memory consumption when virtual bridging is enabled (zero otherwise). Note that the computing capacity constraints are indirectly enforced within the  $L_4L_4$  matching cost computation.  $U_{n_i, n_j}(\Pi)$  is the link utilization of each link used by the current Packing  $\Pi$  solution, so that the maximum link utilization experienced by the Kit's RB paths can be minimized. In our heuristic, in order to linearly compute the RB paths' link utilization, the aggregation and core links of RB paths are considered as congestion free, while access container-RB links are considered as prone to congestion. Then generally adheres to the reality of most DCNs today as access links are typically 1 G Ethernet links while aggregation/core links reach the 10 Gbps and 40 Gbps rates. This is a realistic approximation acceptable in a heuristic approach, especially because it allows a significant decrease in the heuristic's time complexity.

### C. Steps of the repeated matching heuristic

Due to the advantage of repeated matching between the different sets as described above, we can get rid of the non-linearities of the reference optimization problem with a heuristic approach that, based on the state of the art, is geared to achieve low optimality gaps. Its steps are as follows.

- **Step 0:** The algorithm starts with a degenerate Packing with no Kits and all other sets full.
- **Step 1:** A series of Packings is formed.
- **Step 1.1:** The cost matrix  $Z$  is calculated for every block.

- **Step 1.2:** The least cost matching vector is selected.
- **Step 1.3:** Go back to 1.1 for a new iteration unless the Packing cost has not changed in the last three iterations.
- **Step 2:** The heuristic stops, and in the case  $L_1$  is not empty a local incremental solution is created assigning VMs in  $L_1$  to enabled and available VM containers or, if none, to new containers.

The least cost matching computation (Step 1.2) can be hard to solve optimally because of the symmetry constraint (12). In our heuristic, we decided to solve it in a suboptimal way to lower the time complexity. We have implemented the algorithm in [32], based on the method of Engquist [33]. Its starting point is the solution vector of the matching problem without the symmetry constraint (12) obtained with the algorithm described in [34] that was chosen for its speed performance; its output is a symmetric solution matching vector.

Designing the matching costs in an efficient and rational way, the Packing cost across iterations should be decreasing, monotonically starting by the moment when  $L_1$  gets empty; moreover, Step 2 should be reached and the heuristic converges, and  $L_1$  at the last step should be empty.

## V. SIMULATION RESULTS

We implemented our heuristic using Matlab, we used CPLEX for the computation of matching costs of some blocks. The adopted VM containers correspond to an Intel Xeon 5100 server with 2 cores of 2.33GHz and 20GB RAM, able to host 16 VMs. We study the virtual bridging impact on our virtual machine placement model under two scenarios: when TE is the primary goal and when EE is the primary goal. We also analyze what happens when multipath forwarding is enabled at bridge level.

We executed our heuristic with the following DCN topologies: 3-tier, fat-tree, BCube and DCell. We note that only BCube, and DCell had a server centric architecture so that their servers had to employ virtual bridging. However, for the sake of comparison with 3-tier and fat-tree topologies, we included also variations of the conventional BCube and DCell topologies in the analysis, while maintaining their flat nature. Instead of connecting BCube<sub>0</sub> or DCell<sub>0</sub> containers with the higher level bridges, we connected BCube<sub>0</sub> or DCell<sub>0</sub> bridge, so they could work without virtual bridging; these variations were marked as BCube\* (Fig. 1b) and DCell\* (Fig. 1c).

In the simulations, all DCNs are loaded at 85% in terms of computing and network capacity. Note that with all topologies, we allowed for a certain level of overbooking in the resource allocation for the sake of algorithm fluidity especially at starting and intermediate iterations. The capacity of the access link was set to 1Gbps. As not all VMs communicate to each other in today's DCNs adopting network virtualization, but instead traffic is segmented by IaaS management, we built an IaaS-like traffic matrix as in [21], with clusters of up to 30 VMs communicating with each-other and not communicating with other IaaS's VMs. Within each IaaS, the traffic matrix was built accordingly to the traffic distribution of [20]. We ran 30 different instances with different traffic matrices for each



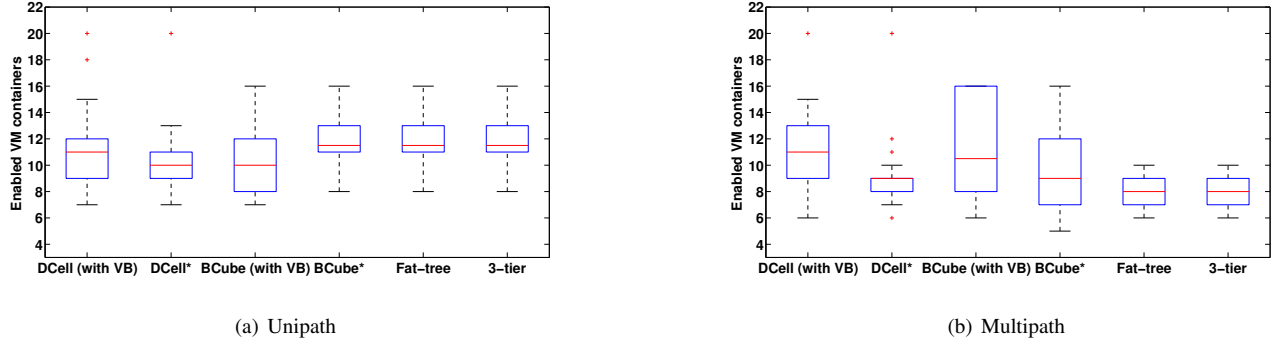


Fig. 3. Number of enabled VM containers (VB=Virtual Bridging)

case, and the results reported in the following were shown with a confidence interval of 95%. Our heuristic was fast (reached convergence roughly within a dozen of minutes per execution) and successfully reached a steady state, (i.e., three iterations led to the same solution, characterized by a feasible Packing as previously described).

#### A. Virtual bridging impact under EE-oriented consolidations

Fig. 3 illustrates the results in terms of enabled VM containers for different topologies when EE was the goal, (i.e.,  $\alpha = 0$  in the problem formulations). We report results for both the cases when multipath forwarding was not enabled, (i.e.,  $|D^R| = 1$  for all Kits) and the case where it is enabled. Observing the results we can assess the following:

- the impact of virtual bridging in DCN consolidations when the EE was the goal leads to negligible differences in EE performance;
- with multipath forwarding, the use of virtual bridging was counterproductive;
- the DCell topology showed better EE performance than the BCube, especially when multipath forwarding was enabled. This can be explained by the higher path diversity at the DCell container;
- hierarchical topologies, fat-tree, and 3-layer, did show the overall worst EE performance for single-path forwarding and better EE performance for multipath forwarding, with negligible difference to each other.

All in all, the main outcome of this analysis is that enabling virtual bridging does not bring any useful EE gain, and can even worsen the EE performance, when the consolidation EE objective is minimizing the number of enabled VM containers.

#### B. Virtual bridging impact under TE-oriented consolidations

As already mentioned, EE goals can be considered the opposite of TE goals. Chasing EE tends to minimize the number of enabled VM containers, yet no care is given to network link utilization. We rerun the experimentations setting the traffic engineering goal as the DCN consolidation objective, i.e.,  $\alpha = 1$ , considering singlepath and multipath forwarding, for the different topologies. Results are reported in Fig. 4. Observing the results we can assess that:

- virtual bridging always leads to sensible TE performance gains;
- with singlepath forwarding, the DCell gets the largest TE gain, from a median of roughly 65% of the maximum link utilization to roughly 45%. This is due to the fact that virtual bridging in the DCell allows indirectly minimizing the number of links used to interconnect servers.
- with multipath forwarding, the BCube gets the largest TE gain, with maximum link utilization being halved from about 80% to 40%.
- BCube and DCell do have similar TE performances, with slighter better performance with BCube probably because the gain in path diversity brought by virtual bridging is higher with BCube, (which keeps a core layer unlike the DCell);
- the TE gain with respect to hierarchical topologies (Fat-tree, 3-tier) is always positive and slightly higher with enabled multipath forwarding.

These TE performance results are not intuitive and relevant. It is definitely interesting to adopt virtual bridging when the primary goal of DCN consolidations is traffic engineering. Flat topologies show a sensible gain with respect to more hierarchical topologies, which once more motivate the migration to such new topologies for IaaS-based DCNs.

## VI. CONCLUSIONS

Data Center Networking is a challenging field of applications of old and new technologies and concepts. In this paper, we investigate how traffic engineering and EE goals in virtual machine consolidations can coexist with the emergence of virtual bridging, i.e., the capability to switch Ethernet traffic at the hypervisor level in virtualization servers. We also study such an impact when multipath forwarding is enabled.

We provide a versatile formulation of the virtual machine placement problem supporting virtual bridging capabilities and multipath forwarding, and describe a repeated matching heuristic.

Moreover, through extensive simulation of realistic instances with legacy and novel flat DC topologies, we discovered that when EE is the primary goal of DNC optimization, virtual bridging can be counterproductive and should not

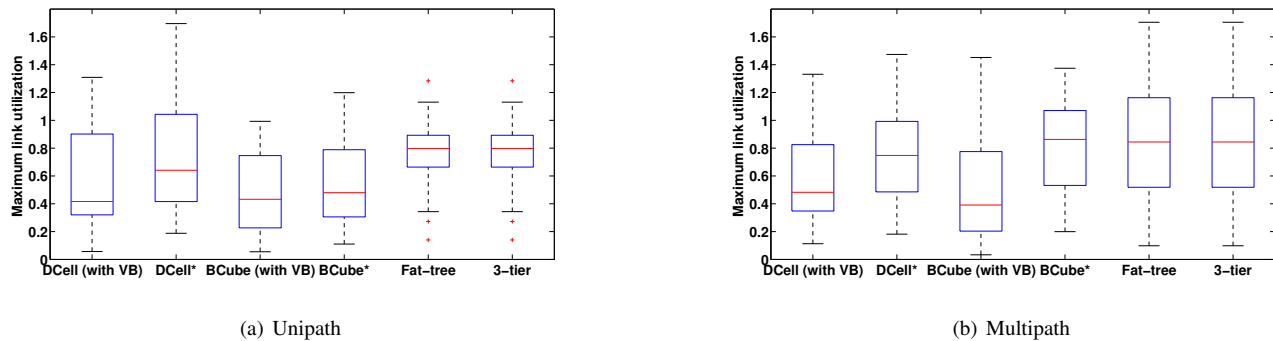


Fig. 4. Maximum link utilization (VB=Virtual Bridging)

be enabled. When TE is the primary goal instead, the TE performance gain can be very important and improved up to two times, with a maximum link utilization that can be halved for the BCube DCN topology, while remaining important also for the DCell topology. The gain with respect to more hierarchical topology (Fat-tree, 3-tier) is also important.

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#### REFERENCES

- [1] J. Touch and R. Perlman, "Transparent interconnection of lots of links (TRILL): Problem and applicability statement," *RFC 5556*, 2009.
- [2] M. Seaman, "IEEE 802.1aq shortest path bridging," *IEEE std*, 2006.
- [3] N. McKeown and et al., "Openflow: enabling innovation in campus networks," *ACM SIGCOMM Computer Communication Review*, vol. 38, no. 2, pp. 69–74, 2008.
- [4] Q. Zhang, L. Cheng, and R. Boutaba, "Cloud computing: state-of-the-art and research challenges," *Journal of Internet Services and Applications*, vol. 1, no. 1, pp. 7–18, 2010.
- [5] "IEEE Standard for Local and Metropolitan Area Networks—Virtual Bridged Local Area Networks - Amendment: Edge Virtual Bridging," in *IEEE 802.1Qbg, 802.1Q-2012 Edition*. IEEE, 2012.
- [6] "Data center architecture overview," in *Cisco Data Center Infrastructure 2.5 Design Guide*. Cisco, 2011, pp. 7–16.
- [7] M. Al-Fares, A. Loukissas, and A. Vahdat, "A scalable, commodity data center network architecture," in *ACM SIGCOMM Computer Communication Review*, vol. 38, no. 4. ACM, 2008, pp. 63–74.
- [8] C. Guo and et al., "Dcell: a scalable and fault-tolerant network structure for data centers," in *ACM SIGCOMM Computer Communication Review*, vol. 38, no. 4. ACM, 2008, pp. 75–86.
- [9] C. Guo, G. Lu, D. Li, H. Wu, X. Zhang, Y. Shi, C. Tian, Y. Zhang, and S. Lu, "Bcube: a high performance, server-centric network architecture for modular data centers," *ACM SIGCOMM Computer Communication Review*, vol. 39, no. 4, pp. 63–74, 2009.
- [10] D. Belabed, S. Secci, G. Pujolle, and D. Medhi, "Impact of ethernet multipath routing on data center network consolidations," *Proc. of the 4th Int. Workshop on Data Center Performance (DCPerf 2014)*, ICDCS, 2014.
- [11] "VMware Capacity Planner," VMware. [Online]. Available: <http://www.vmware.com/products/capacity-planner/overview.html>
- [12] "IBM Workload Deployer," IBM. [Online]. Available: <http://www-01.ibm.com/software/webservers/workload-deployer/>
- [13] R. Perlman, "An algorithm for distributed computation of a spanningtree in an extended lan," in *ACM SIGCOMM Computer Communication Review*, vol. 15, no. 4. ACM, 1985, pp. 44–53.
- [14] D. Santos, A. de Sousa, F. Alvelos, M. Dzida, and M. Pióro, "Optimization of link load balancing in multiple spanning tree routing networks," *Telecommunication Systems*, vol. 48, no. 1-2, pp. 109–124, 2011.
- [15] "Multi-chassis etherchannel (mec)," in *Cisco Data Center Infrastructure 2.5 Design Guide*. Cisco, 2011, pp. 7–16.
- [16] "Provider backbone bridges with traffic engineering," *IEEE Ratifies Computer Society-Sponsored 802.1Qay*, 2009.
- [17] D. Papadimitriou, E. Dotaro, and M. Vigoureux, "Ethernet layer 2 label switched paths," in *Next Generation Internet Networks, 2005*. IEEE, 2005, pp. 188–194.
- [18] T. Benson, A. Anand, A. Akella, and M. Zhang, "Understanding data center traffic characteristics," *ACM SIGCOMM Computer Comm. Review*, vol. 40, no. 1, pp. 92–99, 2010.
- [19] T. Benson, A. Akella, and D. Maltz, "Network traffic characteristics of data centers in the wild," in *Proceedings of the 10th annual conference on Internet measurement*. ACM, 2010, pp. 267–280.
- [20] A. Greenberg and et al., "V12: a scalable and flexible data center network," *ACM SIGCOMM Computer Communication Review*, vol. 39, no. 4, pp. 51–62, 2009.
- [21] X. Meng, V. Pappas, and L. Zhang, "Improving the scalability of data center networks with traffic-aware virtual machine placement," in *in Proc. of IEEE INFOCOM 2010*, 2010, pp. 1–9.
- [22] M. Wang, X. Meng, and L. Zhang, "Consolidating virtual machines with dynamic bandwidth demand in data centers," in *in Proc. of IEEE INFOCOM 2011*, 2011, pp. 71–75.
- [23] O. Biran and et al., "A Stable Network-Aware VM Placement for Cloud Systems," in *in Proc. of IEEE/ACM CCGrid 2012*, 2012, pp. 498–506.
- [24] M. G. Rabbani and et al., "On tackling virtual data center embedding problem," in *IM*, F. D. Turck, Y. Diao, C. S. Hong, D. Medhi, and R. Sadre, Eds. IEEE, pp. 177–184.
- [25] J. W. Jiang, T. Lan, S. Ha, M. Chen, and M. Chiang, "Joint vm placement and routing for data center traffic engineering," in *INFOCOM, 2012 Proceedings IEEE*. IEEE, 2012, pp. 2876–2880.
- [26] H. Jin, T. Cheocherngngarn, D. Levy, A. Smith, D. Pan, J. Liu, and N. Pissinou, "Joint host-network optimization for energy-efficient data center networking," *IEEE IPDPS, Boston, MA*, 2013.
- [27] M. Pióro and D. Medhi, *Routing, flow, and capacity design in communication and computer networks*. Elsevier/Morgan Kaufmann, 2004.
- [28] M. Balinski, "On finding integer solutions to linear programs," in *Mathematica*, May 1964.
- [29] M. Rönnqvist, S. Tragantalerngsak, and J. Holt, "A repeated matching heuristic for the single-source capacitated facility location problem," *European Journal of Operational Research*, vol. 116, pp. 51–68, 1999.
- [30] M. Rönnqvist, K. Holmberg, and D. Yuan, "An exact algorithm for the capacitated facility location problems with single sourcing," *European Journal of Operational Research*, vol. 113, pp. 544–559, 1999.
- [31] A. Reinert, B. Sansò, and S. Secci, "Design optimization of the petaweb architecture," *IEEE/ACM Trans. on Netw.*, vol. 17, no. 1, pp. 332–345.
- [32] M. Forbes, J. Holt, P. Kilby, and A. Watts, "A matching algorithm with application to bus operations," *Australian Journal of Combinatorics*, vol. 4, pp. 71–85, 1991.
- [33] M. Engquist, "A successive shortest path algorithm for the assignment problem," in *INFOR*, 1982, vol. 20, pp. 370–384.
- [34] R. Jonker and A. Volgenant, "A shortest augmenting path algorithm for dense and sparse linear assignment problems," *Computing*, vol. 38, pp. 325–340, 1986.