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A distribution network design problem in the automotive
industry:
MIP formulation and heuristics

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Abstract

We consider a multi-product distribution network design problem arising from a case-study in the automotive industry. Based on realistic assumptions, we introduce minimum volume, maximum covering distance and single sourcing constraints, making the problem difficult to solve for large-size instances. We thus develop several heuristic procedures using various relaxations of the original MIP formulation of the problem. In our numerical experiments, we analyze the structure of the obtained network as well as the impact of varying the problem parameters on computation times. We also show that the implemented heuristic methods provide good quality solutions within short computation times on instances for which a state-of-the-art MIP solver does not produce any feasible solution.

Keywords: Supply chain network design, Location-routing, Minimum volume constraints, Linear relaxation

1. Introduction

A considerable part of a company costs is devoted to the supply chain management, in particular logistics operations such as transport and storage. For instance, according to the CSCMP (Council Of Supply Chain Management Professionals) annual reports [9], the logistics costs as a percentage of the GDP in the USA varied between 7.9% and 9.9% in the last decade. This is why more attention and consideration has to be given to optimizing the supply chain planning. One key question at the strategic planning level is designing the supply chain network and more precisely locating the major facilities of the company, namely plants and distribution centres. In the present work, we deal with a multi-product distribution network design problem arising from a case-study in the automotive industry. The planning horizon that we consider is not very long (one year in our case-study). The overall network structure consists of three levels: assembly plants in the first level, distribution centres (DCs) in the second one and customers (car dealers) in the third one. We assume that the number and location of the plants as well as the number and location of the customers are fixed. Given a deterministic demand of customers for each product and a list of potential DCs, our main concern is to locate DCs and to assign customers to them in such a way as to minimize the total distribution costs. In order to evaluate transport routes from DCs to customers, we construct groups of customers (clusters) using a clustering-based location-routing approach which allows dealing with large-size real-life instances. Moreover, we consider realistic hypotheses that have not been simultaneously taken into account in the literature. Namely, we introduce single sourcing restrictions and minimum volume constraints for both transport flows and facility throughputs. We also use maximum covering distance constraints, i.e. the length of the route between a DC and any cluster that it serves (computed as the length of the shortest route starting from the DC, visiting all the customers of the cluster then coming back to the DC) must not exceed a given distance. The combination of these operational features involves seeking a trade-off between conflicting objectives, thus the problem becomes difficult to solve for large-size instances (for some instances it is even

difficult to find feasible solutions). This is why we develop efficient heuristic methods based on various relaxations of the original mixed integer programming (MIP) formulation of the problem. To the best of our knowledge, it is the first time that this kind of approach is applied to determine both location and assignment variables in a supply chain network design problem subject to minimum volume and distance constraints.

The paper is organized as follows. In the next section, we propose an overview of the related literature. Section 3 is devoted to the explanation of the modeling considerations and to the description of the mathematical formulation of the problem. Then, efficient heuristic procedures are investigated in section 4 and computational results concerning our case-study are presented in section 5. Finally, some conclusions are provided in section 6.

2. Position in the literature

Our research is related to two main literature streams, the first one being *facility location* and the second one *supply chain network design*. This is why we first propose an overview on these expanding fields. We then discuss the integration of minimum volume and distance constraints in supply chain network design problems.

2.1. Facility location and supply chain network design

In a classical facility location problem, the objective is to locate new facilities and to determine the related product flows on a given network, where the locations of demand points are known. In the present work, we focus on a discrete location problem, i.e. facility locations have to be chosen among a list of eligible sites. A survey on discrete location science can be found, among other, in [25].

A typical approach in supply chain network design problems is the use of integrated models, i.e. simultaneously modeling and solving the strategic location problem and other operational issues. This body of works includes production-distribution problems [32], location-inventory problems [3] and location-routing ones (LRP) [24]. In the context of our study, we were particularly interested in the latter problem; see e.g. [22] for an overview on the classification of LRP. From the related literature, we can mainly identify three different approaches when dealing with vehicle routing within supply chain network design problems. The first one can be called the explicit method as routes are explicitly modeled as decisions in the optimization problem (see e.g. [33]) but this may result in the formulation of large-size MIP, most of which are hard to solve. The second method consists in using the continuous approximation, i.e. expressing continuous functions to characterize the customer demand distribution and their locations (for instance in [29] continuous approximation is used to approximate the optimal routing costs). The major disadvantage in this case is the need to resort to strong assumptions such as a uniform demand distribution or a uniform customer density. Finally, it is also possible to use a sequential clustering-based method, as proposed by Barreto et al. in [5]. We chose to apply this method in order to have a good approximation of the routing costs while keeping a manageable size for the optimization problem. This allows dealing with real-life instances involving many customers.

2.2. Integration of minimum volume constraints and maximum distance constraints

One of the constraints involved in our model is the maximum covering distance constraint. A customer is said "covered" if there exists an opened facility situated within a pre-specified distance of it. In order to formulate this

58 kind of distance constraints, we can either use covering objectives (mainly when locating public sites such as schools,
59 police stations, hospitals, parks, etc.) or impose covering distance constraints with any type of objective, usually
60 cost or distance minimization. The literature dealing with covering objectives is abundant (see [10] for a recent
61 literature review) whereas only a few papers, among which are [20], [27], [1] and [21], consider covering distance
62 constraints.

63 In our network design model, a major feature is imposing a minimum volume on each transport link and on
64 each opened DC. This is an approach used in facility location and supply chain network design to introduce flow
65 consolidation. The literature devoted to this stream appears to be relatively scarce. Table. 1 provides a summary
66 of the main characteristics of the seventeen papers that we found in this field. From a modeling point of view, we
67 can first notice that only four works consider minimum volume constraints for transport quantity on each link of
68 the network. Most of the papers deal with this kind of constraint for the throughput of facilities and none of them
69 propose a model using minimum volume constraints for both facility throughput and transport quantities. Multi-
70 product or single sourcing are restrictions that could make the problem more difficult to solve as the former increases
71 the variable number and the latter adds 0-1 constraints. Only two of the listed works ([11] and [26]) simultaneously
72 consider the multi-product and single sourcing features. From a solution method perspective, the computational
73 results provided in [26] concern only small instances (27 binary variables) and use a commercial solver. Other papers
74 implement heuristic solution procedures, mainly constant approximation [12, 13, 15, 19, 30] and linear relaxation
75 based heuristics [6, 16, 18, 31]. In the latter papers, the heuristic strategy is based on rounding fractional solutions
76 to 0 or 1, depending on their values. In [16], the idea consists in rounding to 1 the carrier selection variable having
77 the greatest fractional value. In the other works, the rounding of a location variable depends on the comparison of
78 its fractional value to an upper bound close to 1 and a lower bound close to 0. None of the four papers focuses on
79 rounding assignment variables either because they are not problematic in the solution procedure or because they
80 are continuous variables.

81 The present work features four main contributions as compared to the above-mentioned works. First, to fit real-
82 life requirements, we consider maximum distance and single sourcing restrictions simultaneously with minimum
83 volume constraints when modeling the assignment variables for multiple products. Second, in our study, we assign
84 minimum volume constraints to facility throughputs as well as to transport flows, which makes the problem more
85 realistic but increases the difficulty of solving it. Third, we propose MIP-based heuristic procedures using various
86 types of linear relaxations to determine location and assignment variables (both sets of variables being required to
87 be binary). Finally, we carry out numerical experiments using a case-study with large-size instances (16 assembly
88 plants, 51 potential DCs and 448 car dealers).

89 **3. Problem modeling**

90 *3.1. Problem description*

91 We study a three-level distribution network consisting of assembly plants, distribution centres (DCs) and cus-
92 tomers (see. Fig. 1). Cars are produced at assembly plants and are then transported by truck to intermediate
93 distribution centres (via "primary" transport links) before being sent by truck to the car dealers (via "secondary"

Paper	Min	volume	Complicating			Solution procedure and numerical					
	constraints		features			results					
	Facility location decisions	Throughput of facilities	Transport quantity	Multi-product	Single sourcing	Number of integer variables in the largest instances considered	Number of continuous variables in the largest instances considered	State-of-the-art MIP solver	Linear relaxation based heuristic	Other heuristic	Specialized exact method
Krumke and Thielen (2011) [15]			X			NM	NM			X	X
Lim et al. (2006) [16]			X			10890	0		X	X	X
Seedig (2011) [28]			X			50	50				X
Zhu et al. (2011) [34]			X			119643	119643	X			
Alumur et al. (2012) [2]	X	X		X		1200	58000	X			
Barros et al. (1998) [6]	X	X		X		NM	NM		X		
Correia et al. (2013) [8]	X	X		X		1728	83705	X			X
Geoffrion and Graves (1974) [11]	X	X		X	X	727	23513				X
Guha et al. (2000) [12]	X	X			X	NM	NM			X	
Karger and Minkoff (2000) [13]	X	X			X	NM	NM			X	
Melo et al. (2005) [17]	X	X		X		270	732810	X			
Melo et al. (2011) [18]	X	X		X		560	107057	X	X		
Meyerson (2001) [19]	X	X			X	NM	NM			X	
Ndiaye and Alfares (2008) [23]	X	X			X	370	1810	X			
Sabri and Beamon (2000) [26]	X	X		X	X	27	214	X			
Svitkina (2010) [30]	X	X			X	NM	NM			X	
Thanh et al. (2010) [31]	X	X		X		2185	30870	X	X		
Present work	X	X	X	X	X	62832	1632	X	X		

Table 1: Literature review on facility location and network problems featuring minimum volume constraints (NM=Not Mentioned).

or "last-mile" transport links). Products to be distributed may wait on manufacturing sites or on DCs until they are shipped but a maximum waiting time at each facility is imposed in order to reduce the total delivery time. In the present work, this maximum waiting time is considered as given for each plant and each DC; it typically amounts to a few days.

The company chose an organization using DCs to benefit from economies of scale for long-distance transport from manufacturing sites to DCs. In fact, starting from a given plant, shipments to various customers are consolidated and sent to a given DC that serves these customers. This allows achieving full truckload transport from plants to DCs and leads to meeting the targeted shipping frequencies. On the contrary, if direct transport from plants to customers were used, serving "small" customers would be problematic: either less-than-truckload transport would be used or waiting times would increase, which means raising transport costs or deteriorating customer service.

Around 35 distinct car types, without considering colors and options, have to be distributed through the network. A given plant can manufacture different car types and a given car type can be manufactured in several sites. Nevertheless, we assume that the demand of each car dealer for the different car types have already been assigned to assembly plants. This is indeed a strategic decision made by high management and not at the distribution level. Demands are thus expressed as quantities to be distributed from a given source (assembly plant) to a given

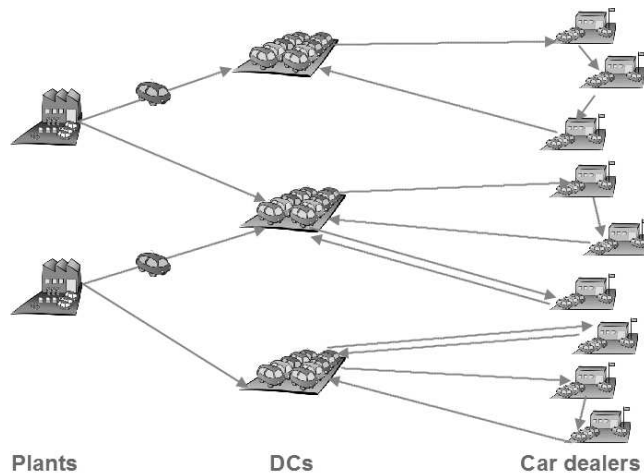


Figure 1: Car distribution network

109 destination (car dealer).

110 Distribution costs involve the costs of primary transport, secondary transport and the transit through DCs.
 111 These costs are obtained by multiplying unit costs by quantities. *No inventory costs are included* as only make-
 112 to-order products are considered in the study. Moreover, as the management of the distribution centres and the
 113 related activities are outsourced to logistics suppliers, there are no fixed opening costs to be charged to the car
 114 maker. There is only a unit transit cost (considered as given in our study) to be paid to the logistics supplier each
 115 time a car goes through a DC. The unit transport cost on each transport link is computed by dividing the cost of
 116 a truck by its load. The cost of a truck depends on the traveled distance, which consists for primary transport in
 117 the length of a direct trip from a plant to a DC. For secondary transport, it is equal to the length of the shortest
 118 route starting from a DC, visiting a cluster of customers then coming back to the DC.

119 To make the model as close to reality as possible, these are the operational constraints that we also consider:

- 120 • Maximum covering distance constraints: as mentioned in §1, a DC cannot be situated farther than a given
 121 distance from a cluster that it serves. In fact, deliveries are usually made by drivers that have to come back to
 122 the distribution centre at the end of the working day. The traveled distance per delivery trip thus should not
 123 be greater than a given limit, allowing the driver to comply with the legal daily driving time. In the reference
 124 dataset used in §5.1, the value of the maximum covering distance parameter is computed as the driving time
 125 multiplied by a constant average speed.
- 126 • Minimum volume constraints on transport links: maximizing the loading of trucks is one of the priorities
 127 in car distribution as cars are voluminous products transported by specific trucks with limited capacities,
 128 typically, a truck can carry up to 8-10 small cars. Thus, if it is not fully loaded, the unit cost per transported
 129 car could significantly increase. Products to be distributed to the same destination have to wait on the plant
 130 or the distribution centre until a full truckload is consolidated. If the total flow rate on a given transport link
 131 is low, the waiting time could be considerable and lead to excessive lead times. In this case, the best solution
 132 is to consolidate many flows on a same transport link. This is expressed in our model using minimum volume
 133 constraints that condition the use of primary and secondary transport links. The lower bound assigned to

134 each link has to be greater than the volume ensuring on average a full truckload within a fixed maximum
135 waiting time allowed at the sourcing point. Using this lower bound, we can assume that the transport of
136 products is made through full truckloads and thus the unit transport cost on the link is computed as the cost
137 of a truck divided by the truck capacity. Exact definitions of the parameters involved in the minimum volume
138 constraints can be found in §3.4.1.

- 139 • Minimum volume and maximum capacity constraints on DCs: The unit transit cost above mentioned results
140 from a commercial negotiation between the car maker and the logistics supplier managing the DC. It only
141 applies if the total throughput of the DC is between a minimum volume and a maximum capacity.
- 142 • Single sourcing restrictions: for a given cluster, all the products manufactured in a same plant should go
143 through a single DC. This is an operational feature that facilitates day-to-day operations. Thus, it is mean-
144 ingful to use an aggregate representation of the product types based on their sourcing plants. In the sequel,
145 we consider the different types produced in a same plant as a single product, which reduces the number of
146 products from 35 to 16. The model keeps however its multi-product feature as assignments of clusters to DCs
147 can differ according to the sourcing plant.

148 Aggregation of products per plant and the use of mixed product loads on primary (plant-DC) transport links
149 lead to considering average capacities for trucks from plants to DCs. We propose to use an average truck
150 capacity (W_i) specific to each plant (i) and to compute it as the weighted average load factor over the whole
151 demand of the various product types manufactured in the given plant with the demand of each product type
152 as weight. Notice that the load factor of a car type is the number of cars of this type that can be loaded on
153 a truck. It can differ according to the car size.

154 On secondary transport links, trucks of mixed product types are also shipped from DCs to clusters of cus-
155 tomers. As we do not know in advance what kind of product will be transported, we propose to use a common
156 weighted average capacity W over all the car types.

157 The simultaneous introduction of minimum volume, maximum covering distance and single sourcing constraints
158 makes the problem closer to real-life requirements but leads to several computational difficulties. Before providing
159 the detailed mathematical formulation of the network design problem and presenting a heuristic solution method
160 to solve it, we first explain the proposed clustering-based location-routing approach. Then, we analyze the main
161 trade-offs to be achieved regarding the distribution network structure.

162 3.2. The clustering-based location-routing approach

163 By introducing a minimum volume constraint for each transport link, we implicitly assume that the total demand
164 of the destination point (DC or customer) exceeds the transport minimum volume. For primary transport, this is
165 not problematic as a given DC is supposed to deal with the demand of many customers for many product types. For
166 secondary transport, this is not obvious. The demand of some car dealers could be below the threshold corresponding
167 to reaching a full truckload within the maximum waiting time allowed at a DC. This is why the company resorts
168 to grouping deliveries: a given truck starting from a distribution centre may have to visit several customers before
169 coming back to the distribution centre. We have then to answer the following question: which customers to serve

170 together in the same route? This leads to a Vehicle Routing Problem (VRP). In the present work, we propose to use
 171 a clustering-based location-routing approach to obtain a good approximation of secondary transport routes without
 172 increasing the problem size. In fact, using an integrated location-routing approach (LRP) would be expensive
 173 in terms of computational effort, in particular for real-life instances. To the best of our knowledge, no instances
 174 of uncapacitated LRP that are significantly larger than those in [4] (with 199 customers and 14 DCs) have been
 175 exactly solved in the literature, whereas for capacitated LRP, the largest instances involve only 100 customers and
 176 10 DCs (see the same work). Given that we deal in our case-study with about 500 customers, 15 products and
 177 50 capacitated DCs, considering an exact location-routing approach would be computationally intractable. This is
 178 why we develop a sequential procedure.

179 3.2.1. Main idea

180 The idea of the clustering-based method is to form groups of customers, which will be allocated to the same
 181 truck routes. Fig. 2 illustrates the clustering-based method that we can summarize in four steps:

- 182 1. Construct clusters of customers lying close to one another, while complying with a minimum volume limit
 183 and some other constraints.
- 184 2. Solve a Traveling Salesman Problem (TSP) for each distribution route from DCs to clusters.
- 185 3. Compute the unit cost per transported product for each route.
- 186 4. Solve a distribution network design problem using clusters as customers instead of the original car dealers.

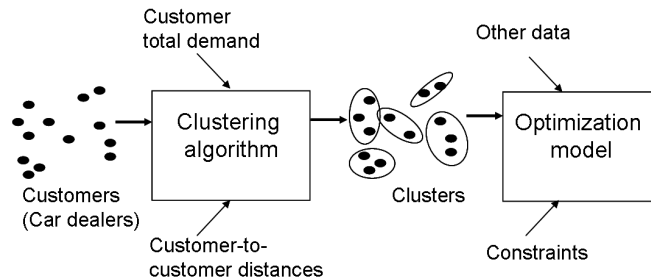


Figure 2: The clustering based approach

187 In step 1, we consider as input of the clustering algorithm the total demand of each customer, including all
 188 product types. This is due to the possible combination of different car types in a same secondary transport
 189 truckload. In step 2, we use a simple enumeration procedure to determine the shortest route starting from a DC,
 190 visiting all the customers of a cluster then coming back to the DC. In fact, a particular feature of car distribution is
 191 that a route consists of a small number of customers (usually between one and three). This makes solving the TSP
 192 problem through complete enumeration possible. In step 3, we use a classical formula with fixed and kilometric
 193 components to compute the cost of a truck for a given route. Then, we divide the truck cost by the average truck
 194 capacity to deduce the unit transport cost.

195 When using this sequential approach, we have to ensure consistency between the clustering step and the opti-
 196 mization step. We have then to make sure that the minimum volume required for each cluster is greater than the
 197 minimum volume required for secondary transport.

198 3.2.2. Algorithm

199 The proposed algorithm is a heuristic approach which consists in grouping the close customers into clusters (an
200 assessment of the quality of the solutions produced by the heuristic clustering is provided in §5.3.2).

201 The maximum number of customers per cluster is imposed by the decision maker and has to be strictly met by
202 the algorithm. The total demand of a cluster has to be greater than a given lower bound. For instance, we can
203 consider as a lower bound, the volume ensuring on average one or two full truckloads every week (if the maximum
204 waiting time allowed at DCs is one week). The clustering produced by the algorithm has also to meet, as far as
205 possible, two other constraints: a maximum total demand per cluster and a maximum distance between any two
206 customers of a cluster. The first constraint is added in the first phase of the clustering to balance the total demand
207 over the clusters. This could prevent for instance from grouping two customers already having a demand greater
208 than the minimum required quantity. The second constraint is used in the first phase of the clustering to impose a
209 proximity between the customers of a same cluster. However, these two constraints are not mandatory and may be
210 relaxed in order to find feasible solutions.

211 Input distances between customers are calculated using a Geographic Information System (GIS) to form a
212 point-to-point distance matrix. Algorithm 1 illustrates the pseudo-code of the first phase of the implemented
213 algorithm. At the beginning of the procedure, the cluster list is initialized to single-element clusters (i.e. each
214 customer corresponds to a cluster). We chose the following distance definition between two clusters p and q :
215 $Distance(p, q) = Min \{Distance(i, j), i \in C_p, j \in C_q\}$. C_p is the set of customers of the cluster p . This proximity
216 measure is called the "single linkage" measure; see [5] for a discussion of other possible measures.

217 Once the first phase of the clustering algorithm has been completed, a second phase has to be applied in order
218 to consider the clusters discarded in the first phase due to the different constraints. In the second phase, only the
219 minimum volume and the maximum number of customers constraints are imposed. We check that each cluster
220 has a total demand greater than the minimum demand required. If it is not the case for some cluster q then we
221 attach each of its customers to the nearest cluster having fewer customers than the maximum allowed number of
222 customers.

223 3.3. Main trade-off analysis

224 Before detailing the mathematical formulation of the optimization problem under study, we first analyze the
225 main trade-offs to be achieved. These trade-offs which are mainly driven by minimum volume and maximum
226 distance constraints not only influence the number of opened DCs but also the existence of a feasible solution.

227 In classical facility location problems, one of the main purposes of optimization is to decide about the number
228 of DCs to open so as to achieve the best possible trade-off between fixed-charge opening costs and transport costs.
229 In our problem, in addition to minimizing transport costs, we have also to cope with maximum covering distance
230 constraints. This could lead to opening many DCs in order to be close to customers. As previously mentioned, no
231 fixed-charge opening costs are included in the proposed model but minimum quantities are required. Thus, opening
232 a DC at each potential location may not be feasible. It would result in splitting the global volume to be distributed
233 from the plants in many DCs so that both the constraints of minimum throughputs and the constraints of minimum
234 volume on primary transport links are likely to be violated in the corresponding solution. One of the purposes of the

235 proposed optimization model is thus to build a network so as to meet the maximum covering distance constraints
 236 and to minimize the total transport costs while ensuring that minimum volume constraints are satisfied.

237 Furthermore, experiences with typical parameters and data of our case-study show that it is not always possible
 238 to comply with minimum volume constraints for primary transport links in the presence of maximum covering
 239 distance constraints. This means that shipping fully loaded trucks from plants to distribution centres within tight
 240 waiting times is not always feasible. Therefore, in the model, we have considered the possibility of violating the
 241 strict minimum volume constraints on primary transport links but keeping total constraint violation as low as
 242 possible via penalties (see §5.3.1 for details about the choice of the penalty coefficients).

Algorithm 1: The pseudo-code of the first phase of the clustering procedure

Data:

n : Total number of customers;
 m : Number of clusters remaining in the clustering procedure;
 V : Minimum volume per cluster;
 V' : Maximum volume per cluster;
 D_p : Total demand of cluster p ;
 maxDistance: Maximum distance between two customers of a same cluster;
 maxNumber: Maximum number of customers per cluster;
 $|C_p|$: Number of customers in cluster p ;

Algorithm:

Initialize the n customers to n individual clusters;

Initialize $m=n$;

while $m > 1$ **do**

 Find the pair of authorized clusters (p,q) such as $\text{distance}(p,q)$ is the minimum of the distance matrix;

if $\text{distance}(p,q) \leq \text{maxDistance}$ **then**

if $|C_p| + |C_q| \leq \text{maxNumber}$ **then**

if $D_p + D_q \leq V'$ **then**

 Merge clusters p and q ;

$m=m-1$;

if $D_p + D_q \geq V$ **then**

 Remove p and q from the clustering procedure;

$m=m-1$;

else

 In the distance matrix, update the distances between the cluster (p,q) and the other remaining clusters;

end

else

 Forbid merging p and q ;

end

else

 Forbid merging p and q ;

end

else

 Break;

end

end

243

244 *3.4. Mathematical formulation*

245 *3.4.1. Model parameters*

246	I	Set of plant indices ($i = 1, \dots, I$)
247	J	Set of DC indices ($j = 1, \dots, J$)
248	Q	Set of customer cluster indices ($q = 1, \dots, Q$)
249	$G = (V, A)$	A graph where V is the set of nodes and A is the set of arcs. $V = V_{PL} \cup V_{DC} \cup V_{CL}$ where V_{PL} is the
250		set of plant nodes, V_{DC} is the set of DC nodes and V_{CL} is the set of cluster nodes. $A = A_{PR} \cup A_{SE}$
251		where $A_{PR} = \{(i, j) \in A \mid i \in V_{PL}, j \in V_{DC}\}$ and $A_{SE} = \{(j, q) \in A \mid j \in V_{DC}, q \in V_{CL}\}$
252	D_{qi}	Total demand of cluster q for the product manufactured in plant i during the whole planning horizon.
253	$totProd_i$	Total volume of cars produced by plant i .
254	$minVol_j$	Minimum volume of cars that has to go through DC j if it is chosen.
255	$maxVol_j$	Maximum volume of cars that can go through DC j if it is chosen.
256	PTC_{ij}	Cost of a truck going from plant i to DC j (Primary Transport Cost).
257	STC_{jq}	Cost of a truck starting its route at DC j and visiting all the customers of cluster q before going
258		back to j (Secondary Transport Cost).
259	TC_j	Unit transit cost for a car going through DC j .
260	W_i	Average truck capacity for the cars manufactured in plant i .
261	W	Average truck capacity for the whole volume of cars.
262	M	Big value, $M = \min(maxVol_j, totProd_i)$
263	NWD	Number of working days in the planning horizon.
264	$T_{max}(i)$	Maximum waiting time allowed at plant i before shipping to distribution centres.
265	T	Maximum waiting time allowed at a distribution centre before shipping to car dealers.
266	CD	Maximum covering distance parameter (i.e. the maximum length of a route starting at a DC, visiting
267		the customers of a given cluster then coming back to the DC).
268	PI_{ij}	Low volume penalty amount for primary transport from plant i to DC j .
269	$R(j, q)$	Length of the optimal route starting at DC j and visiting the customers of cluster q before coming
270		back to j .
271	$V_{min}(i)$	Minimum volume of cars that has to go through any opened primary transport link starting at
272		plant i . It has to be equal at least to the minimum volume ensuring on average a full truckload within
273		$T_{max}(i)$ from plant i to a given DC hence $V_{min}(i) \geq \frac{W_i}{T_{max}(i)} NWD$
274	V_{min}	Minimum volume of cars that has to go through any opened secondary transport link. It has to be
275		equal at least to the minimum volume ensuring on average a full truckload within T from a DC to
276		any cluster of customers hence $V_{min} \geq \frac{W}{T} NWD$.

277 *3.4.2. Decision variables*

- 278 • Location variables y : $y_j = 1$ if DC j is selected, $y_j = 0$ otherwise
- 279 • Assignment variables x : $x_{ijq} = 1$ if cluster q is assigned to DC j for the products of plant i , $x_{ijq} = 0$ otherwise

280 x_{ijq} is defined only if $R(j, q) \leq CD$.

281 • Variables z : $z_{ij} = 1$ if the route from plant i to DC j is selected, $z_{ij} = 0$ otherwise

282 • Variables a : $a_{jq} = 1$ if cluster q is assigned to DC j for at least one product, $a_{jq} = 0$ otherwise

283 a_{jq} is defined only if $R(j, q) \leq CD$.

284 • v'_{ij}, v''_{ij} : Continuous variables used to write the minimum volume constraints for primary transport links. v'_{ij} is
 285 a positive variable that has to be greater than $V_{min}(i)$. v''_{ij} is a positive variable used to compute the amount
 286 of violation of the minimum transport quantity on a given primary transport link $(i, j) \in A_{PR}$. It has to be
 287 less than $V_{min}(i)$ and will be minimized, set to zero if possible, as it is penalized in the objective function.

288 3.4.3. MIP formulation of the optimization problem

289 The multi-product distribution network design problem:

290

Minimize:

$$\begin{aligned}
 \text{Total distribution cost} &= \sum_{i \in \mathbf{I}} \sum_{j \in \mathbf{J}} \frac{PTC_{ij}}{W_i} \sum_{\substack{q \in \mathbf{Q} \\ R(j, q) \leq CD}} D_{qi} x_{ijq} && (\text{Primary transport cost} = \text{unit cost} \times \text{volume}) \\
 &+ \sum_{j \in \mathbf{J}} \sum_{\substack{q \in \mathbf{Q} \\ R(j, q) \leq CD}} \frac{STC_{jq}}{W} \sum_{i \in \mathbf{I}} D_{qi} x_{ijq} && (\text{Secondary transport cost} = \text{unit cost} \times \text{volume}) \\
 &+ \sum_{j \in \mathbf{J}} TC_j \sum_{i \in \mathbf{I}} \sum_{\substack{q \in \mathbf{Q} \\ R(j, q) \leq CD}} D_{qi} x_{ijq} && (\text{Transit cost} = \text{unit cost} \times \text{volume}) \\
 &+ \sum_{i \in \mathbf{I}} \sum_{j \in \mathbf{J}} PI_{ij} v''_{ij} && (\text{Low volume penalties for primary transport})
 \end{aligned}$$

Subject to:

$$\sum_{\substack{j \in \mathbf{J} \\ R(j,q) \leq CD}} x_{ijq} = 1 \quad \forall i \in \mathbf{I}, q \in \mathbf{Q}; D_{qi} \geq 0 \quad (1)$$

$$\sum_{q \in \mathbf{Q}} D_{qi} x_{ijq} = v'_{ij} - v''_{ij} \quad \forall i \in \mathbf{I}, j \in \mathbf{J} \quad (2)$$

$$v'_{ij} \geq V_{min}(i) z_{ij} \quad \forall i \in \mathbf{I}, j \in \mathbf{J} \quad (3)$$

$$v'_{ij} \leq M z_{ij} \quad \forall i \in \mathbf{I}, j \in \mathbf{J} \quad (4)$$

$$v''_{ij} \leq V_{min}(i) z_{ij} \quad \forall i \in \mathbf{I}, j \in \mathbf{J} \quad (5)$$

$$\sum_{i \in \mathbf{I}} \sum_{\substack{q \in \mathbf{Q} \\ R(j,q) \leq CD}} D_{qi} x_{ijq} \geq \min Vol_j y_j \quad \forall j \in \mathbf{J} \quad (6)$$

$$\sum_{i \in \mathbf{I}} \sum_{\substack{q \in \mathbf{Q} \\ R(j,q) \leq CD}} D_{qi} x_{ijq} \leq \max Vol_j y_j \quad \forall j \in \mathbf{J} \quad (7)$$

$$\sum_{i \in \mathbf{I}} D_{qi} x_{ijq} \geq V_{min} a_{jq} \quad \forall j \in \mathbf{J}, q \in \mathbf{Q}; R(j,q) \leq CD \quad (8)$$

$$\sum_{i \in \mathbf{I}} x_{ijq} \leq I a_{jq} \quad \forall j \in \mathbf{J}, q \in \mathbf{Q}; R(j,q) \leq CD \quad (9)$$

$$z_{ij} \leq y_j \quad \forall i \in \mathbf{I}, j \in \mathbf{J} \quad (10)$$

$$y_j, x_{ijq}, a_{jq}, z_{ij} \in \{0, 1\} \quad \forall i \in \mathbf{I}, j \in \mathbf{J}, q \in \mathbf{Q}; R(j,q) \leq CD \quad (11)$$

$$v'_{ij}, v''_{ij} \geq 0 \quad \forall i \in \mathbf{I}, j \in \mathbf{J} \quad (12)$$

291 The objective function consists in the total distribution cost, i.e. primary transport cost, secondary transport cost,
 292 transit cost and penalties for violating the minimum volume constraints on primary transport links. Each of the
 293 transport and transit cost components are computed as the unit cost per car multiplied by the volume of cars on
 294 each link/DC. Notice that binary variables y , z and a are not used in the objective function as they are created
 295 only to express minimum volume constraints for DCs and transport links. Constraints (1) state that the demand
 296 of cluster q for each product of plant i is satisfied and is routed through a single DC (as x are binary variables).
 297 Constraints (2)-(5) enable us to compute the violation of minimum volume constraints and to penalize it in the
 298 objective function. Constraints (6) state that if DC j is selected ($y_j = 1$), then the flows going through j have to
 299 be greater than the corresponding minimum volume limit. Constraints (7) stipulate that:

- 300 • If DC j is selected ($y_j = 1$) then the flows going through j must not exceed its maximum capacity.
- 301 • If DC j is not selected ($y_j = 0$) then there is no flow transiting by it (all x_{ijq} have to be set equal to 0).

302 Constraints (8) ensure that if the link between j and q is selected ($a_{jq} = 1$) then the corresponding total volume
 303 has to be greater than the minimum volume V_{min} . Constraints (9) stipulate that if the link between j and q is
 304 not selected ($a_{jq} = 0$) then all of the variables x_{ijq} have to be set equal to 0. Constraints (10) stipulate that if DC
 305 j is not opened ($y_j = 0$) then all of the variables z_{ij} have to be set equal to 0. Constraints (11) and (12) are the
 306 integrality and non negativity constraints.

307 Notice that maximum covering distance constraints are implicitly modeled when defining assignment variables x_{ijq} .
308 In fact, assignment of cluster q to DC j for the products of plant i is not allowed if the route distance between j
309 and q exceeds the maximum covering distance CD .

310 4. Heuristic solution approach

311 In this section, we propose a heuristic solution approach for the MIP problem presented in the previous modeling
312 section. For small-size instances of the problem, we can use a MIP commercial solver such as CPLEX to obtain
313 an optimal solution. However, computation times become prohibitively high for large-size real-life instances not
314 only for getting optimal solutions but even for getting the first feasible solutions. Therefore, we had to consider the
315 development of a specific and possibly more efficient heuristic solution method.

316 4.1. The main idea of the heuristic approach

317 The main idea of the proposed heuristic method is to exploit as much as possible the information provided by
318 the optimal solution of the linear relaxation of the problem. An argument supporting this approach is the tightness
319 of the lower bounds provided by the linear relaxation solution of the earlier defined formulation (typical deviations
320 are less than 1.3%).

321 In the problem under study, we have two main types of binary variables: location variables and assignment
322 variables. We propose a two-stage heuristic method: the first stage (S1) focuses on location decisions whereas
323 the second one (S2) deals with assignment decisions. The location variables are fixed in the first stage and the
324 corresponding values are used in the second stage. Three different heuristic methods are implemented for the second
325 stage.

326 4.2. First stage (S1): location decisions

327 In the first stage, we focus on deciding which DCs should be opened, i.e. we aim at determining the value of
328 each location variable y in such a way as to be as close as possible to the optimal solution. To this end, we propose
329 to solve a partial linear relaxation of the original MIP where the integrality constraints on variables x , z and a have
330 been removed whereas they have been kept for y variables. Different tests showed that this is an efficient way to
331 quickly identify the relevant main structure of the distribution network, i.e. to determine the DCs to open.

332 4.3. Second stage (S2): assignment decisions

333 In the second stage, we focus on deciding which DCs should serve which clusters, i.e. we aim at determining
334 the value of each assignment variable x in such a way as to meet the minimum volume constraints for secondary
335 transport. After fixing the location variables in the first stage of the heuristic approach, the resulting assignment
336 problem is easier to solve than the original MIP. A rather large fraction (on average 44%) of the variables x are
337 indeed set to 0 due to the fact that the corresponding DCs are not opened. However, this second-stage problem
338 is not a standard assignment problem: since it includes knapsack constraints (due to capacity restrictions when
339 assigning customers to DCs), it can be shown to be NP-Hard (NP-Hardness of the whole distribution network
340 design problem was already settled in [14]).

341 In order to solve the second-stage assignment problem, three distinct methods were studied.

342 *4.3.1. Reinroduction of all the integrality constraints (S2M1)*

343 The method S2M1 reintroduces the integrality constraints for all the free assignment variables x just after the
344 location stage then solves the resulting MIP using a commercial solver. This is much quicker than directly solving
345 the original MIP as the number of binary variables is reduced after fixing the location variables.

346 *4.3.2. Gradual reintroduction of the integrality constraints (S2M2)*

347 The method S2M2 consists in gradually reintroducing the relaxed integrality constraints. As previously men-
348 tioned, in the first stage, we fix the location variables y . Then, we add the integrality constraints to all the secondary
349 transport related variables, namely, x and a . We solve the resulting MIP using a commercial solver and we fix x and
350 a at their values in the obtained solution. Finally, we add the integrality constraints to all the primary transport
351 related variables (i.e. z) and we solve the resulting problem. Here, it is worth pointing out that the last step does
352 not change any assignment decision as the x and a variables are fixed in the previous step. It only re-evaluates
353 the objective function taking into account the penalty resulting from violating the minimum volume constraints on
354 primary transport links.

355 *4.3.3. Fixing strategy (S2M3)*

356 We now discuss a fixing strategy exploiting the information provided by the solution of the first stage of the
357 heuristic. The idea is to try to fix as many assignment variables as possible before reintroducing the integrality
358 constraints for the currently free variables and solving the resulting MIP with a commercial solver. The fixing
359 strategy should enable us to decrease the number of binary variables before solving the final MIP and thus to
360 reduce computation times in the second stage of the heuristic procedure. However, a key issue is to ensure that the
361 fixing decisions thus made do not lead to infeasibility.

362 In order to implement the fixing strategy, we started at the optimal solution of the first location stage that
363 we examined to identify the main issues. We noticed two problems: firstly, fractional values are obtained for only
364 a small proportion of the assignment variables (of about 2%); secondly, there are of about 50% of the secondary
365 transport links where the minimum volume constraint is violated. These figures clearly show that the violation
366 of the minimum volume constraints for secondary transport links is the main issue in the optimal solution of the
367 location stage. This is why the fixing strategy detailed in Algorithm 2 aims at setting to 0 any secondary transport
368 flow (j, q) violating the minimum volume constraint, provided that the problem remains feasible. Thus, we make
369 sure that after closing (j, q) , the throughput of DC j will be kept above its minimum volume $minVol_j$ and that there
370 is another DC k able to deliver the demand of q without exceeding its maximum volume of throughput $maxVol_k$.

371 We repeat this procedure for all the secondary transport links then we solve the resulting linear problem. In
372 case the problem becomes infeasible, we remove the lately added fixing constraints and stop the loop. Otherwise,
373 we iterate the procedure until there are no further flows that can be fixed at 0 or the problem becomes infeasible.
374 The second step of the fixing strategy is to reintroduce the 0-1 constraints for all the free assignment variables and
375 to solve the resulting MIP with a commercial solver.

376 As detailed in the literature review, we found only four papers dealing with linear relaxation based heuristics for
377 facility location or network problems with minimum volume constraints. The management of feasibility is among
378 the main differences between our fixing algorithm and the solution methods implemented in these papers. In the

379 present work, through the tests introduced before setting flows to 0, we try to keep feasibility while fixing assignment
 380 variables. In [31], an infeasible solution is corrected after running the fixing algorithm using a feasibility pump.
 381 Melo et al. used in [18] a search in the neighborhood of the infeasible solution. In [6] and [16], the authors do not
 382 address the feasibility issues at the end of the heuristic procedure.

Algorithm 2: Fixing strategy cancelling inconsistent flows (S2M3)

Data:

V_{jq} : Volume going through the secondary transport link $(j, q) \in A_{SE}$ during the planning horizon;

$\forall j \in \mathbf{J} \forall q \in \mathbf{Q}$ such as $R(j, q) \leq CD$ $V_{jq} = \sum_{i \in \mathbf{I}} D_{qi} x_{ijq}$;

V_j : Volume going through the DC j during the planning horizon;

$\forall j \in \mathbf{J}$ $V_j = \sum_{\substack{q \in \mathbf{Q} \\ R(j, q) \leq CD}} \sum_{i \in \mathbf{I}} D_{qi} x_{ijq}$;

$nbFlows_0$: The number of flows that can be set to 0 according to volume tests (Initialization: $nbFlows_0 = 1$);

Algorithm:

Step 1

while $nbFlows_0 > 0$ **do**

$nbFlows_0 = 0$;

for each opened transport link (j, q) from DC j to cluster q **do**

if $V_{jq} < V_{min}$ **then**

if $V_j - V_{jq} \geq minVol_j$ **then**

if \exists DC k within the covering distance CD of q such as $(V_k + V_{jq}) \leq maxVol_k$ **then**

 Add the constraint $V_{jq} = 0$ (i.e. $x_{ijq} = 0 \forall i$);

$nbFlows_0 = nbFlows_0 + 1$;

end

end

end

end

 Solve the resulting linear problem;

if infeasible problem **then**

 | Cancel the lately added fixing constraints, break while

end

end

Step 2

Reintroduce the integrality constraints for all the free variables then solve the resulting MIP;

384 **5. Numerical results**

385 In this section, we aim at discussing the numerical experiments of our model based on real-life data from our
 386 case-study. We employed the C++ language to implement the MIP model and the commercial solver ILOG CPLEX
 387 version 12.1 to solve it. We carried out all the tests on a Pentium Core 2 Duo (2.53 GHz) with 1.92Go of RAM,
 388 running under Windows XP. After describing the case-study, we analyze the optimal solution given by CPLEX in
 389 terms of network configuration (location decisions and different trade-offs) and of computation times. Then, we
 390 study the computational performance of the two-stage heuristic approach as compared to applying CPLEX to the
 391 original MIP.

392 5.1. Case-study description

393 The case-study that motivated this research deals with a car distribution network in France. The network
394 consists of 16 assembly plants (or ports), 51 potential distribution centres and 448 car dealers. We employed a free
395 online tool [7] to obtain the geospatial coordinates of the different sites and the Geographic Information System
396 (GIS) Microsoft MapPoint to evaluate the route distances.

397 Based on the preferences of the decision maker, we defined a reference dataset where the planning horizon is
398 fixed at one year, the maximum covering distance is set to 460 kilometres. The minimum and maximum volumes per
399 DC are set according to some hypothesis on the flexibility of each DC. The minimum volume per primary transport
400 link is fixed at 1 full truckload within a maximal waiting time of 1 week. The minimum volume per secondary
401 transport link is also fixed at 1 full truckload within a maximal waiting time of 1 week. The clustering parameters
402 are the following: maximum number of customers per cluster = 3, maximum volume per cluster = 3 full truckloads
403 within 1 week, maximum distance between any two customers in a cluster = 50 kilometers and minimum volume
404 per cluster = 2 full truckloads within 1 week. Here, it is worth pointing out that a difference of 1 truckload between
405 the minimum quantity per cluster and the minimum quantity per secondary transport link gives more flexibility to
406 the optimization algorithm with respect to the possibility of assigning a cluster to several DCs.

407 5.2. Test instances

408 In order to test the performance of exact and heuristic solutions, we constructed 60 test instances by varying
409 the main parameters of the problem in the reference dataset.

- 410 • Instances A.1 to A.12: we vary the value of the maximum covering distance from 460 kilometres to 680
411 kilometres while fixing the other parameters of the reference dataset. This leads to increasing the number of
412 binary variables (see fig. 4).
- 413 • Instances B.1 to B.20: we vary the demand of customers. In each instance, we assign to each customer a
414 different pseudo-random coefficient lying between 0.5 and 1.5. The new customer demand is computed as the
415 reference demand (used in the reference dataset) multiplied by the pseudo-random coefficient.
- 416 • Instances C.1 to C.8: we vary the minimum volume parameters for primary transport, secondary transport
417 and DC throughputs as well as the maximum capacities for DCs (see Table. 2). In two instances (C.5 and
418 C.6), we also change the maximum covering distance value because tight minimum volume constraints lead
419 to an infeasible problem.
- 420 • Instances D.1 to D.20: each of these instances contains 46 potential DCs instead of 51 as we randomly select
421 5 DCs to remove from the list of potential DCs.

422 5.3. Optimal solution using a commercial solver

423 In the present subsection, we study the solution given by the commercial solver CPLEX for the proposed MIP
424 problem. Different tests showed that proving the optimality, in some cases, is very time-consuming. Consequently,
425 we limited the optimality gap of CPLEX to 0.2% in order to reduce the computational effort (the optimality gap is

Test instance	Min volume for primary transport (nb truckload/week)	Min volume for secondary transport (nb truckload/week)	Min throughput per DC	Max throughput per DC	Max covering distance (km)
C.1	1	1	RefData	RefData	460
C.2	2	1	RefData	RefData	460
C.3	1	2	RefData	RefData	460
C.4	2	2	RefData	RefData	460
C.5	1	1	1.5×RefData	RefData	620
C.6	1	1	2×RefData	RefData	660
C.7	1	1	RefData	0.8×RefData	460
C.8	1	1	RefData	2×RefData	460

Table 2: Test instances C: varying the minimum and maximum volume parameters. RefData denotes the data of the reference dataset.

426 defined as the relative difference between the solution obtained and the best lower bound found by the Branch &
427 Bound procedure).

428 We first explain the setting of the penalty parameter related to primary transport links. Then, we analyze the
429 configuration of the distribution network obtained in the final solution given by CPLEX. Finally, we study the
430 impact of varying the main parameters of the problem on computation times.

431 5.3.1. Setting the penalty parameter

432 As already suggested in the modeling section, we handle the minimum volume constraints on primary transport
433 links using penalties. We penalize each unit below the targeted minimum volume by a given amount in the objective
434 function. In our context, we can study two alternatives to evaluate this amount. The first one is to set the penalty
435 to a huge value (10^9 for example), which means that the constraint will be violated only when there is no other
436 solution to reach feasibility. The second alternative consists in using a suitable penalty that makes violating the
437 constraint possible only if there is some opportunity to save costs. This is indeed the case if we choose to set the
438 penalty to the unit transport cost. Fig. 3 illustrates the total cost as a function of the transported volume in this
case. This situation means that if the volume on a given primary transport link $(i, j) \in A_{PR}$ is greater than the

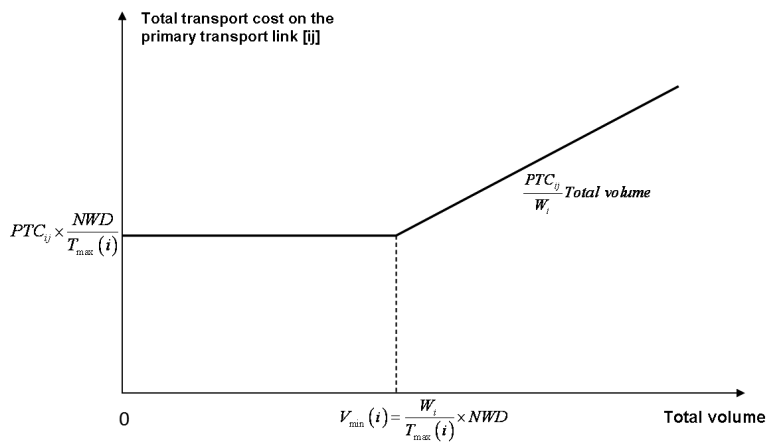


Figure 3: Primary transport cost as a function of the volume

439

440 minimum required volume then all the products can be shipped in full truckloads. We pay a unit transport cost
441 equal to $\frac{PTC_{ij}}{W_i}$, PTC_{ij} is the cost of a truck going from plant i to DC j and W_i is the average truck capacity for
442 the trips starting at plant i . On the contrary, if the volume on the link is less than the minimum required volume,
443 then regardless to the quantity, we pay a total cost equal to $PTC_{ij} \cdot \frac{NWD}{T_{max}(i)}$. Observe that this amounts to keep
444 shipping trucks with the same frequency $T_{max}(i)$ even if they are not fully loaded.

445 The computational experiments that we carried out using test instances A showed that when the unit penalty
446 is fixed at 10^9 , the computation time is prohibitively high regardless of the value of the maximum covering distance
447 (for instance, it is about 14 hours when the maximum distance is set to 460 kilometers and 20 hours when it is
448 equal to 480 kilometers). When the unit penalty is set to the unit transport cost, we found optimal solutions for
449 all the test examples except the one with a maximum distance set to 700 kilometers, that we aborted after 4 days
450 of running time without finding any feasible solution. The computation times remain acceptable until a distance
451 equal to 560 kilometers, after which the running time exceeds 4 hours (see fig. 4)

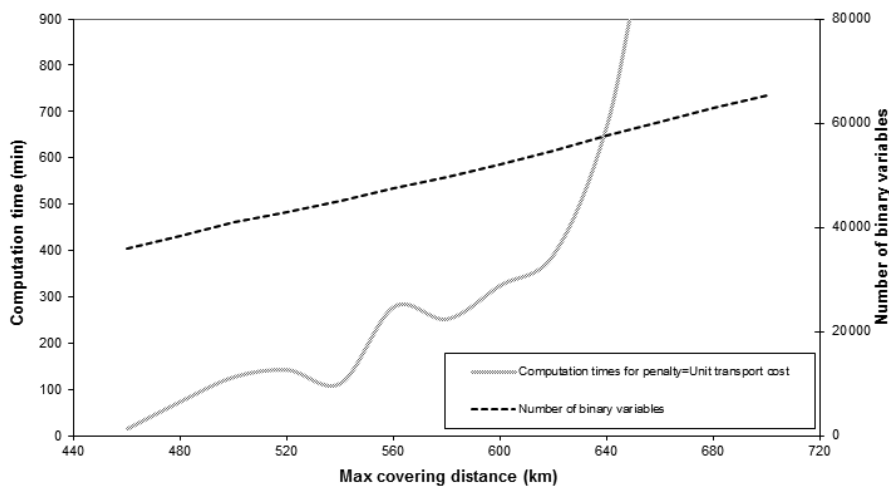


Figure 4: Computation time as a function of the maximum covering distance value for the second penalization alternative

452 In the subsequent experiments reported in this paper, we fix the penalty value for each primary transport link
453 at the unit transport cost on this link.

454 5.3.2. Assessment of the heuristic clustering

455 We propose to assess the heuristic clustering presented in §3.2.2 by comparing it with an exact clustering based
456 on a set-partitioning formulation (detailed description of this method can be found in [14]). To this aim, we
457 compute on instances B the relative difference in the total distribution cost between a) the optimal solution to the
458 MIP problem (1)-(12) using the heuristic clustering; b) the optimal solution to the MIP problem (1)-(12) using
459 the exact clustering. The values obtained for these instances are displayed in Table. 3. The figures show that the
460 increase in total cost due to the use of heuristic clustering rather than the exact clustering is only about 0.63% on
461 average, which confirms the good quality of the solutions produced by the heuristic.

462 We note here that one of the limitations of the exact approach is that the set-partitioning model possibly
463 requires the enumeration of a potentially huge number of candidate clusters (there are more than 60000 of these

464 in the instances shown in Table. 3, where no more than 3 customers per cluster are allowed). This may lead
 465 to numerically intractable problems when the number of customers or the maximum number of customers per
 466 cluster increase. By contrast, an advantage of the heuristic clustering is that it is far less dependent on these two
 467 parameters.

Test instance	B.1	B.2	B.3	B.4	B.5	B.6	B.7	B.8	B.9	B.10	B.11	B.12	B.13	B.14	B.15	B.16	B.17	B.18	B.19	B.20	Avg
Heuristic cost increase (%)	0.82	0.64	0.50	0.42	0.62	0.52	0.53	0.85	0.49	0.83	0.64	0.19	0.66	0.44	0.66	0.66	0.92	0.98	0.46	0.81	0.63

Table 3: Distribution cost increase when using a heuristic clustering rather than an exact one on instances (B) varying customer demand.

468 5.3.3. Distribution network

469 The first step of our location-routing procedure is the application of the heuristic clustering algorithm over the
 470 448 customers of the country. This results in 302 clusters meeting the various grouping constraints. Then, we run
 471 the network optimization on the reference dataset in order to obtain the best locations for DCs and the assignment
 472 of clusters to them. The first conclusion that can be made from the obtained figures is that the number of opened
 473 DCs is relatively high: 28 among 51 potential sites. This means that secondary transport considerably influences
 474 the network configuration, first because of the maximum covering distance constraint and second as the cost per
 475 car per kilometer for secondary transport is higher than the cost for primary transport, due to the difference of
 476 truck speed (secondary transport usually concerns last-mile deliveries in urban areas where vehicles are slower). As
 477 customers are scattered all over the country, we try to get close to them by opening many DCs. Fig. 5 shows that
 478 DCs are specialized by region and that transport flows assigned to them are related to close customers.

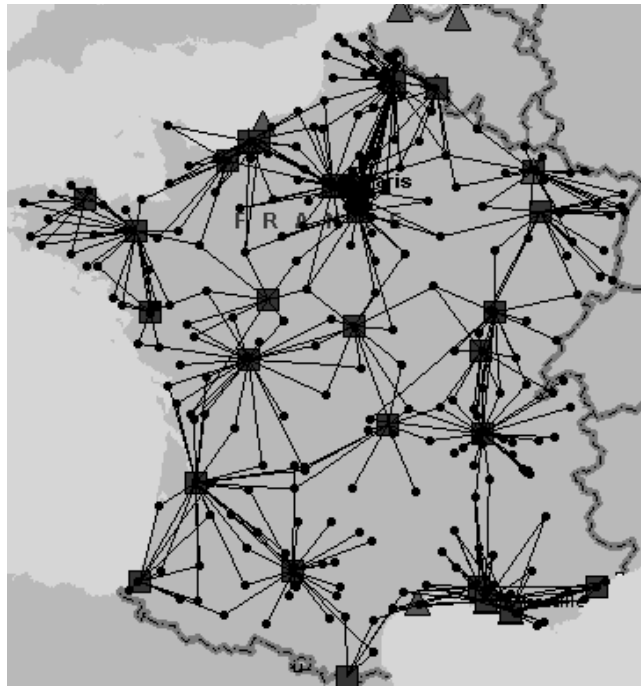


Figure 5: Secondary transport in the optimal solution given by CPLEX. Plant locations are represented using triangles, opened DCs using squares, and cluster barycentres using dark circles. As the figure is focused on secondary transport, some plants do not appear.

479 However, DCs were not opened at each potential location as this may violate minimum throughput constraints
480 for DCs. The throughput of many opened DCs is indeed close to the minimum required quantity. This reports on
481 the impact of minimum throughput constraints on the network configuration. Furthermore, the number of DCs
482 assigned to each cluster varies between 1 and 5. Obviously, clusters are more likely to be served by their closest
483 DCs. This is why the number of single assignments is important (34% of clusters are assigned to only one DC).
484 Other clusters are assigned to several DCs either because the problem is highly constrained or because sometimes
485 it is better to use a farther facility which costs less in terms of primary transport.

486 5.3.4. *Impact of varying parameters on computation times*

487 Different tests showed that the following parameters could have a considerable impact on computation times:

- 488 • Clustering parameters: Depending on these parameters, the number of clusters resulting from the algorithm
489 could vary and then influences the problem size. The most influential parameter is the minimum volume per
490 cluster V . Decreasing V leads to an increased number of clusters and consequently to an increased number
491 of assignment variables x .
- 492 • Minimum volume parameters: Increasing the minimum volume required for transport links or for the through-
493 put of DCs makes the problem more difficult to solve as it is not easy to find a trade-off between all the con-
494 straints. For instance, computation time went from 15 minutes to 35 hours when we doubled the minimum
495 volume required per secondary transport link (2 full truckloads instead of 1).
- 496 • Maximum covering distance parameter: Increasing the value of the maximum distance parameter leads to
497 increasing the number of possible assignments, which results in raising the number of the assignment variables
498 and of the related constraints. For instance, the size of the problem goes from 35853 binary variables and
499 13946 constraints when the maximum covering distance is fixed at 460 kilometers to 62832 binary variables
500 and 17120 constraints when the maximum covering distance is set to 680 kilometers. Fig. 4 shows how the
501 computation time significantly increases when the distance increases. If the distance is set to 700 kilometers
502 then the computation time to find a feasible solution can become prohibitive (more than 24 hours in some
503 cases).

504 Thus, even if the computation time needed by CPLEX MIP solver to solve the problem on the reference dataset is
505 acceptable (15 minutes), the analysis carried out in this paragraph showed that the variation of the main parameters
506 of the problem, in particular the maximum covering distance, can result in extensive running times as the number
507 of integer variables and constraints significantly increases. Moreover, this study has an interactive character: the
508 decision maker wants to run many what-if scenarios and requires to quickly view the impact of certain decisions.
509 This is why implementing a heuristic method appeared to be the best way to find a near-optimal solution in a
510 reasonable runtime, so that the decision making process is facilitated without deteriorating the solution quality.

511 5.4. *Experiments with the heuristic approach of section 4*

512 In the present subsection, we study the performance of the heuristic approach investigated in section 4 as
513 compared to the reference solution given by CPLEX for the original MIP. The optimality gap was limited to 0.2%

514 for the reference solution and for all the mixed integer programs solved within the heuristic methods (using CPLEX
515 also). The parameter setting was based on the reference dataset defined in §5.1.

516 As previous tests with CPLEX (based on default settings) show some difficulties in finding feasible solutions, we
517 tried to emphasize feasibility over optimality in the branching procedure using the "MIPEMPHASIS" parameter
518 of CPLEX. Nonetheless, this approach did not lead to significant improvements in computation times. We then
519 examined another alternative using the "setPriorities" function, which drives CPLEX to give priority to location
520 variables over assignment variables during branching. Location decisions have indeed a structural role in our network
521 design problem. This change in the settings of CPLEX resulted in a noticeable decrease in the computational effort.
522 The first part of Table. 4 shows computation times in minutes for CPLEX applied to the original MIP both with
523 default settings (6th column) and with priorities assigned to location variables (7th column) on instances A varying
524 the value of the maximum covering distance.

525 It is seen that CPLEX behaves much better when using priorities, on average CPU is half the initial value. This
526 is why in the subsequent tests, we will keep using priorities. The resulting figures are compared with those given
527 by heuristic methods (S2M1, S2M2 and S2M3). The second part of Table. 4 shows the solution quality of the three
528 heuristic methods and of the solver output for the original MIP within 10 minutes and 60 minutes respectively. In
529 fact, this information is useful to analyze the performance of the two-stage heuristic approach as compared to the
530 exact one.

531 Overall, Table. 4 shows that the gradual reintroduction of the integrality constraints in the second stage (S2M2)
532 leads to running times shorter than S2M1 and S2M3 but to higher deviations from the original MIP solutions (up to
533 1.71%). In fact, in the second step of S2M2, we ignore the minimum volume constraints for primary transport links
534 as the corresponding variables could have fractional values. Consequently, some expensive decisions are made for
535 the transport from plants to distribution centres. On the other hand, the computational results clearly show that
536 S2M1 yields excellent quality solutions and sometimes outperforms CPLEX applied to the original MIP (negative
537 values in the second part of Table. 4). This could be explained by the fact that the MIP optimality gap was limited
538 to 0.2%. In return, the computation times obtained with S2M1 could increase up to 133 minutes. Thus, to achieve
539 a trade-off between time and value, we should apply the fixing strategy S2M3, which is indeed better than S2M1
540 in terms of running time and which solution is less expensive than the one provided by S2M2. According to the
541 choice of the decision maker, it is possible to prioritize the solution speed (S2M2), the solution quality (S2M1) or
542 a trade-off between these two features (S2M3).

543 In summary, using the two-stage heuristic approach appears to be significantly more competitive than running
544 the solver on the whole MIP model of section 3.4.3. We can indeed notice according to the last two columns of
545 Table. 4 that within 10 minutes CPU, CPLEX cannot find any feasible integer solution. Within 60 minutes CPU,
546 it yields a solution better than the heuristic one only in the first two instances. This conclusion was also validated
547 through additional extensive numerical tests using test instances B, C and D defined in §5.2. The computational
548 results are summarized in Tables. 5, 6 and 7; the obtained figures again show the good performance of the 2-stage
549 heuristic procedures.

CPU(min)		Heuristic solution			CPLEX applied to the original MIP	
Instance	Max covering distance (km)	S2M1	S2M2	S2M3	Optimality gap set to 0.2 %, default settings	Optimality gap set to 0.2 %, higher priorities for location variables
A.1	460	0.8	0.3	0.5	15.3	8.4
A.2	480	0.6	0.3	0.7	74.4	22.8
A.3	500	2.1	0.4	0.7	127.3	28.8
A.4	520	1.9	0.4	0.4	142.3	76.4
A.5	540	5.0	0.3	0.4	113.8	41.7
A.6	560	6.2	0.5	0.5	279.1	255.2
A.7	580	6.6	0.6	0.7	252.4	202.1
A.8	600	19.8	0.4	0.9	325.5	209.4
A.9	620	20.2	0.4	0.5	392.7	159.2
A.10	640	14.7	0.5	0.6	> 10 h	> 10 h
A.11	660	15.9	0.5	0.9	> 10 h	> 10 h
A.12	680	91.2	0.6	2.3	> 10 h	> 10 h

Solution quality(%)		Heuristic solution			CPLEX applied to the original MIP	
Instance	Max covering distance (km)	S2M1	S2M2	S2M3	Time limit set to 10min	Time limit set to 60min
A.1	460	-0.06	1.49	0.73	NFS	-0.12
A.2	480	0.07	1.61	0.75	NFS	0.03
A.3	500	0.06	1.54	0.70	NFS	NFS
A.4	520	0.06	1.71	0.90	NFS	NFS
A.5	540	0.05	1.64	0.97	NFS	NFS
A.6	560	-0.01	1.70	0.97	NFS	NFS
A.7	580	0.08	1.69	0.99	NFS	NFS
A.8	600	-0.04	1.40	0.86	NFS	NFS
A.9	620	-0.06	1.36	0.87	NFS	NFS
A.10	640	0.02	1.40	0.88	NFS	NFS
A.11	660	0.01	1.44	0.89	NFS	NFS
A.12	680	0.03	1.42	0.92	NFS	NFS

Table 4: CPU (min) and solution quality (%) as a function of the maximum covering distance (instances A). CPU of CPLEX solutions is mentioned in 2 cases: when using default settings and when higher priorities are assigned to location variables during the branching procedure. Solution quality is measured as the relative difference between the heuristic solution and the solution produced by CPLEX applied to the original MIP within 0.2% of optimality. The heuristic uses one of the methods S2M i , $i = 1..3$ in the 2^{nd} stage. Optimality gap for any MIP used in the heuristics was limited to 0.2%. NFS means that no feasible solution was found within the allowed time limit.

6. Conclusion and future research

In this paper, we studied a multi-product distribution network design problem arising in the context of car distribution. We proposed a modeling approach considering several realistic assumptions. This led to the formulation of a large-size MIP with minimum volume, maximum covering distance and single sourcing constraints. We analyzed the main trade-offs related to the network structure and provided numerical results using real-life data. These results showed that it is possible to handle problems featuring about 500 customers, 15 products, 50

CPU(min)	Heuristic solution			CPLEX applied to the original MIP
Test instance	S2M1	S2M2	S2M3	Optimality gap set to 0.2%
B.1	1.6	0.2	0.4	29.9
B.2	1.0	0.3	0.2	13.0
B.3	2.4	0.4	0.2	243.1
B.4	0.9	0.3	0.2	41.6
B.5	0.4	0.2	0.2	8.3
B.6	0.3	0.2	0.2	11.8
B.7	1.0	0.2	0.2	11.7
B.8	0.9	0.2	0.2	17.7
B.9	1.1	0.2	0.2	22.1
B.10	2.3	0.5	0.3	31.5
B.11	0.8	0.4	0.2	31.6
B.12	1.2	0.8	0.2	19.8
B.13	0.3	0.2	0.2	23.3
B.14	0.4	0.3	0.3	40.7
B.15	0.9	0.5	0.3	25.0
B.16	2.0	0.2	0.2	67.2
B.17	1.1	0.2	0.3	35.0
B.18	2.2	0.2	0.2	19.2
B.19	0.9	0.2	0.2	20.3
B.20	0.7	0.2	0.3	16.3
Avg	1.1	0.3	0.2	36.5

Solution quality(%)	Heuristic solution		
Test instance	S2M1	S2M2	S2M3
B.1	0.10	1.03	0.61
B.2	-0.03	1.51	0.82
B.3	0.09	1.75	0.63
B.4	-0.03	1.19	0.72
B.5	0.06	1.50	0.93
B.6	-0.07	1.25	0.63
B.7	0.07	1.40	0.70
B.8	0.01	1.93	0.70
B.9	-0.08	1.37	0.49
B.10	-0.07	1.55	0.69
B.11	0.08	1.40	0.51
B.12	0.04	1.57	0.86
B.13	0.08	1.41	0.74
B.14	0.04	1.43	0.72
B.15	0.01	1.51	0.34
B.16	0.06	1.37	0.64
B.17	0.02	1.43	0.83
B.18	0.06	1.68	0.54
B.19	-0.03	1.66	0.53
B.20	0.02	1.46	0.64
Avg	0.02	1.47	0.66

Table 5: CPU (min) and solution quality (%) when varying customer demand (test instances B).

CPU(min)	Heuristic solution			CPLEX applied to the original MIP
Test instance	S2M1	S2M2	S2M3	Optimality gap set to 0.2%
C.1	0.8	0.3	0.5	8.4
C.2	7.4	0.2	1.0	56.4
C.3	8.7	0.6	1.0	551.9
C.4	1431.9	0.9	8.8	OOM
C.5	6.6	3.0	1.2	377.7
C.6	OOM	3.9	8.7	OOM
C.7	1.7	0.2	0.2	19.3
C.8	1.2	0.1	0.2	25.9

Solution quality(%)	Heuristic solution		
Test instance	S2M1	S2M2	S2M3
C.1	-0.06	1.49	0.73
C.2	0.06	6.06	1.88
C.3	0.01	1.17	1.24
C.4	-	-	-
C.5	0.04	1.00	0.43
C.6	-	-	-
C.7	-0.03	1.27	0.67
C.8	0.01	1.25	0.89
Avg	0.01	2.04	0.97

Table 6: CPU (min) and solution quality (%) when varying the minimum and maximum volume parameters (test instances C). OOM denotes an out of memory status without finding any feasible solution and "-" means that the solution quality cannot be evaluated as no reference solution was found.

CPU(min)	Heuristic solution			CPLEX applied to the original MIP
Test instance	S2M1	S2M2	S2M3	Optimality gap set to 0.2%
D.1	1.1	0.7	0.7	26.4
D.2	2.4	0.2	0.2	103.6
D.3	0.4	0.3	0.3	32.7
D.4	10.9	0.2	0.5	47.2
D.5	0.2	0.2	0.2	7.3
D.6	1.1	0.2	0.2	6.8
D.7	0.2	0.2	0.2	6.2
D.8	0.6	0.2	0.2	5.7
D.9	0.5	0.3	0.2	8.0
D.10	1.0	0.2	0.2	5.4
D.11	0.6	0.2	0.1	5.7
D.12	0.5	0.3	0.3	11.4
D.13	0.6	0.2	0.2	1.6
D.14	0.8	0.2	0.5	12.4
D.15	0.9	0.2	0.2	4.9
D.16	0.2	0.2	0.2	2.2
D.17	0.4	0.1	0.1	6.5
D.18	0.9	0.2	0.2	22.3
D.19	0.9	0.4	0.2	3.6
D.20	0.6	0.2	0.2	23.6
Avg	1.2	0.2	0.2	17.2

Solution quality(%)	Heuristic solution		
Test instance	S2M1	S2M2	S2M3
D.1	0.08	1.56	0.56
D.2	-0.01	1.20	0.70
D.3	0.09	0.87	0.47
D.4	0.08	1.65	0.57
D.5	-0.001	0.89	0.44
D.6	0.09	1.32	0.75
D.7	0.002	0.85	0.61
D.8	-0.05	1.01	0.64
D.9	0.04	1.15	0.81
D.10	0.06	1.29	0.82
D.11	-0.05	0.99	0.69
D.12	0.02	0.69	0.42
D.13	0.04	0.88	0.75
D.14	-0.07	1.47	0.71
D.15	-0.02	1.16	0.41
D.16	-0.04	1.02	0.65
D.17	-0.04	0.69	0.62
D.18	0.003	1.14	0.74
D.19	-0.07	1.10	0.68
D.20	0.05	1.12	0.78
Avg	0.01	1.10	0.64

Table 7: CPU(min) and solution quality(%) when varying the list of potential DCs (test instances D)

556 potential locations for DCs and, for each product, an average of 6 possible assignments of customers to DCs. The
557 corresponding MIP models, which can be solved within 0.2% accuracy in less than 2 hours CPU, typically involve
558 45100 0-1 variables and 15000 constraints.

559 In order to improve the computational performance of the solution procedure, several heuristic methods using
560 various relaxations of the original MIP formulation of the problem have been described. They were validated through
561 extensive computational experiments where the produced solutions have been compared with those obtained using
562 an efficient state-of-the-art exact MIP solver. To the best of our knowledge, it is the first time that this kind
563 of approach is applied to determine both optimal location and assignment decisions in a supply chain network
564 design problem subject to minimum volume and distance constraints. All the related papers that we found in the
565 literature worked only on rounding the location decisions. The results of our computational experiments confirm
566 the good performance of the proposed heuristic approaches even for the hardest case (62832 0-1 variables and 17120
567 constraints). Good quality feasible solutions are consistently obtained within short computation times on instances
568 for which a state-of-the-art MIP solver does not produce any feasible solution.

569 In terms of future research, it would be interesting to study other possible approaches to constructing heuristic
570 solutions. For instance, within the fixing strategy, one such possibility would be to accept infeasible solutions and to
571 repair them instead of building only feasible solutions. Another challenging option that is worth being considered
572 for future research is to look at a dynamic or multi-scenario version of the current model in order to deal with
573 demand seasonality. Various scenarios of demand could indeed have different impact on the network structure and
574 in particular on the major decision variables related to distribution centre location. Moreover, this extension of
575 the model is likely to take full benefit from the good performance of our two-stage heuristic approach especially
576 designed for tackling problems of larger sizes.

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