Minimizing Energy Consumption of MPI Programs in Realistic Environment

Amina Guermouche, Nicolas Triquenaux, Benoit Pradelle, William Jalby

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Amina Guermouche, Nicolas Triquenaux, Benoît Pradelle and William Jalby
Université de Versailles Saint-Quentin-en-Yvelines

Abstract

Dynamic voltage and frequency scaling proves to be an efficient way of reducing energy consumption of servers. Energy savings are typically achieved by setting a well-chosen frequency during some program phases. However, determining suitable program phases and their associated optimal frequencies is a complex problem. Moreover, hardware is constrained by non negligible frequency transition latencies. Thus, various heuristics were proposed to determine and apply frequencies, but evaluating their efficiency remains an issue.

In this paper, we translate the energy minimization problem into a mixed integer program that specifically models realistic hardware limitations. The problem solution then estimates the minimal energy consumption and the associated frequency schedule. The paper provides two different formulations and a discussion on the feasibility of each of them on realistic applications.

1 Introduction

For a very long time, computing performance was the only metric considered when launching a program. Scientists and users only cared about the time it took for a program to finish. Though still often true, the priority of many hardware architects and system administrators has shifted to caring more and more about energy consumption. Solutions reducing the energy enveloppe have been put forth.

Among the different existing techniques, Dynamic Voltage and Frequency Scaling (DVFS) proved to be an efficient way to reduce processor energy consumption. The processor frequency is adapted according to its workload: When the frequency is lowered without increasing the execution time, the power consumption and energy are reduced.

With parallel applications in general, and more precisely with MPI applications, reducing frequency on one processor may have a dramatic impact on the execution time of the application: Reducing processor frequency may delay a message sending, and maybe its reception. This may lead to cascading delays increasing the execution time. To save energy with respect to application deadline, two main solutions exist: online tools and offline scheduling. The former try to provide the frequency schedule during the execution whereas the latter provide it after an offline study. They both require the application task graph (either through a previous execution or by focusing on iterative applications).

Many online tools [10, 20] identify the *critical path*: the longest path through the graph, and focus on processors that do not execute these tasks. Typically, when waiting for a message, the processor frequency is set to the minimal frequency until the message arrives [20]. Although online tools allow some energy savings, they provide suboptimal energy saving because of a lack of application knowledge.

On the other hand, offline scheduling algorithms [1, 19] provide the best frequency execution of each task. However, none of the existing algorithms consider most current multi-core architectures characteristics: (i) cores within the same processor share the same frequency [8] and (ii) switching frequency requires some time [16].

This paper presents two models based on linear programming which find the execution frequencies of each task while taking into account the mutlicore architecture constraints and characteristics (section 3) previously described. Moreover, we allow the execution time to be increased if this leads to more energy

savings. The user provides a maximum performance degradation that she can tolerate. The presented models provide optimal frequency schedule which minimizes the energy consumption. However, when considering large applications and large machines, no current solver can provide a result, even parallel ones. The reason behind this issue is discussed in section 3.

2 Context and execution model

We consider MPI applications running on a multi-node platform. The targeted architectures consider the following characteristics: (i) the latency of frequency switching is not negligible and (ii) cores within the same processor share the same frequency.

A process, running on every core, executes a set of tasks. A task, denoted T_i, is defined as the computations between two communications. The application execution is represented as task graph where tasks are vertices and edges are messages between the tasks. Figure 1 is an example of the task graph running on two processes. One process executes tasks T_1 and T_2 while the other one executes tasks T_3 and T_4.

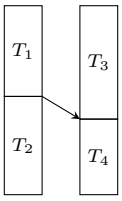


Figure 1: Task graph

Before going into more details on the execution model, let us provide an example of the problem we want to solve. Consider the example provided in Figure 2. The application is executed on 3 cores, 2 in the same processor and one in another processor. Tasks T_1, T_2, T_3 and T_4 are executed on processor 0 while tasks T_5 and T_6 are executed on processor 1. In order to minimize the energy consumption through DVFS, we make the same assumption as [19]: tasks may have several phases and each phase can be executed at a specific frequency. Typically on Figure 2, task T_1 is divided into 3 phases. The first one is executed at frequency f_1, the second one at frequency f_2 and the last one at frequency f_3.

As stressed out before, setting a frequency takes some time. In other words, when a frequency is requested, it is not set immediately. Thus, on Figure 2, when frequency f_2 is requested, it is set some time after. One needs to be careful of such situations since a frequency may be set after the task which it was requested from is over.

Moreover, cores within the same processor run at the same frequency. Hence, on Figure 2, when f_1 is first set on processor 0, all the tasks being executed at this time (T_1 and T_3) are executed at frequency f_1. T_5 is not affected since it is on another processor. To provide the best frequency to execute each task portion, we need to consider all parallel tasks which are executed at the same time on the processor.

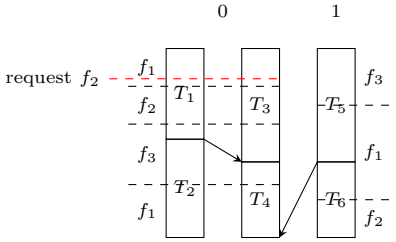


Figure 2: Frequency switch latency [1]

[1] Note that only the latency of the first request is represented

Our model requires the task graph to be provided (through profiling or a complete execution of the application). Thus, we consider deterministic applications: for the same parameters and the same input data, the same task graph is generated. In order to guarantee that edges are the same over all possible executions, one has to make sure that the communications between the processes are the same. Non deterministic communications in MPI are either receptions from an unknown source (by using *MPI_Any_Source* in the reception call), or non-deterministic completion events (*MPI_Waitany* for instance). Any application with such events is considered as non-deterministic, thus falls out of the scope of the proposed solution.

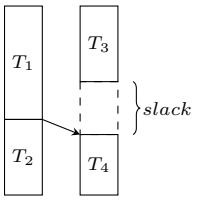


Figure 3: Slack time

Tasks within a core are totally ordered. If a task T_i ends with a send event, then the following task T_j starts exactly at the end of T_i. On Figure 1, task T_2 starts exactly after T_1 ends. On the other hand, when a task is created by a message reception (T_4 on Figure 1), it cannot start before all the tasks it depends on finish (T_1 and T_3) and it has to wait for the message to be received. If the message arrives after the end of the task which is supposed to receive it, the time between the end of the task and the reception is known as *slack* time. On Figure 3, tasks T_1 sends a message to T_3 but T_3 ends before receiving the messages creating the slack represented by dotted lines.

A task energy consumption E_i is defined as the product of its execution time $exec_i$ and its power consumption P_i. Since the application is composed of several tasks, its energy consumption can be expressed as the sum of the energy consumption of all the tasks. Thus, the goal translates into providing the set of frequency to execute each task. Hence, one can calculate the application energy consumption as:

$$E = \sum_i (E_i) = \sum_i (exec_i \times P_i) \tag{1}$$

Minimizing the energy consumption of the application is equivalent to minimizing E in equation (1).

For each task T_i, both $exec_i$ and P_i depend the frequency of the different phases of the task. In addition, tasks are not independent since when executed in parallel on the same processor, the tasks share the same frequency. Moreover, the overall execution time of the application depends on all the $exec_i$ and the slack time. To minimize the energy consumption while still controlling the overall execution time, we express the problem as a linear program.

3 Building the linear program

The following paragraphs describe how the energy minimizatoin problems translates into a linear programming. We first describe the precedence constraints between the tasks, then we describe two formulations which consider the architecture constraints. Finally, we discuss the feasibility of the described solutions.

3.1 Precedence constraints

Let T_i be a task defined by its start time bT_i and its end time eT_i. The beginning of tasks is bounded by the precedence relation between them. As already stressed out, a task cannot start before its direct predecessors complete their execution. As explained in section 2, if T_i sends a message, its child task T_j starts exactly when T_i ends since the end of the communication means the beginning of the next task. This translates to:

$$bT_j = eT_i$$

bT_i	Beginning of a task T_i
eT_i	End of a task T_i
bTs_i	Beginning of a slack task Ts_i
eTs_i	End of a slack task Ts_i
$exec_i^f$	The execution time of a task T_i if executed completely at frequency f
tT_i^f	The time during which the task T_i is executed at frequency f
δ_i^f	The fraction of time a task T_i spends at frequency f
M_j^i	Message transmission time from task T_j to task T_i

Table 1: Task variables

On the other hand, when T_i ends with a message reception from T_k, one has to make sure that its successor task T_j starts after both tasks end. Moreover, as pointed out in section 2, when a task receives a message, some slack may be introduced before the reception. Slack is handled the same way tasks are: it has a start and an end time and it can be executed at different frequencies depending on the tasks on the other cores. On Figure 3, the slack after T_3 may be executed at different frequencies whether it is executed in parallel with T_1 or T_2.

To ease the presentation, we assume that each task T_i receiving a message (from a task T_k) is followed by a slack task, denoted Ts_i. The beginning of Ts_i, denoted bTs_i is exactly equal to the end of T_i,

$$bTs_i = eT_i \tag{2}$$

whereas its end time, denoted eTs_i, is at least equal to the arrival time of the message from T_k. Let M_k^i denote the transmission time from T_k to T_i. Thus:

$$eTs_i \geq eT_k + M_k^i \tag{3}$$

Note that a task may receive messages from different processes (after a collective communication for example) and equation 3 has to be valid for all of them.

Finally, since T_j, the successor task of T_i has to start after T_i and T_k finish, one just needs to make sure that:

$$bT_j = eTs_i$$

In order to compute the end time of a task T_i (eT_i), one has to evaluate the execution time of T_i. As explained above, a task may be executed at different frequencies. Let $exec_i^f$ be the execution time of T_i if executed completely at frequency f. Every frequency can be used to run a fraction δ_i^f of the total execution of the task. Let tT_i^f be the fraction of time T_i spends at frequency f. It can be expressed as: $tT_i^f = \delta_i^f \times exec_i^f$. Thus, the end time of a task is:

$$eT_i = bT_i + \sum_f tT_i^f$$

Note that one has to make sure that a task is completely executed:

$$\sum_f \delta_i^f = 1 \tag{4}$$

Finally, since the power consumption depends on the frequency, let P_i^f be the power consumption of the task T_i when executed at frequency f. Using this formulation, the objective function of the linear program becomes:

$$min(\sum_{T_i}(\sum_f(tT_i^f \times P_i^f))) \tag{5}$$

One can just use tT_i^f in the objective function as it is expressed in equation (5), and the solver would provide the values of tT_i^f of all tasks at all frequencies. This solution was presented in [19]. The provided solution can be used on different architectures than the ones we target in this work. As a matter of fact, nothing constrains parallel tasks on one processor to run at the same frequency, and the threshold of switching frequency is not considered either. Moreover, no constraint on the execution time is expressed. The following paragraphs first describe how the performance is handled then they introduce additional constraints the handle the architecture constraints and execution time.

3.2 Execution time constraints

The performance of an application is a major concern; whether the energy consumption is considered or not. In this paragraph we provide constraints which consider the execution time of the application. In MPI, all programs end with *MPI_Finalize* which is similar to a global barrier. Let $last\_task^i$ be the last task on core i (the *MPI_Finalize* task). Since the application ends with a global communication, every task $last\_task^i$ is followed by a slack task $last\_slack\_task^i$. The difference between the global communication slack and the other slack tasks lies in the end time: the end time of all slack tasks of a global communication is the same (all processes leave the barrier at the same time). Thus, for every couple of cores (i, j):

$$elast\_slack\_task^i = elast\_slack\_task^j \tag{6}$$

Let $total\_Time$ be the application execution time: It is equal to the end time of the last slack task.

$$total\_Time = elast\_slack\_task^i \tag{7}$$

However, in some cases, increasing the execution time of an application could benefit to energy consumption. In order to allow this performance loss to a specified extent, the user limits the degradation to a factor x of the maximal performance. Let $exec\_Time$ be the execution time when all tasks run at the maximal frequency, and x the maximum performance loss percentage allowed by the user. The following constraint allows performance loss with respect to x:

$$total\_Time \leq exec\_Time + \frac{exec\_Time \times x}{100}$$

The next sections describe two different formulations. In the first formulation, the solver is provided with all possible task configurations and chooses the one minimizing energy consumption. In the second formulation, the solver provides the exact time of every frequency switch on each processor.

3.3 Architecture constraints: the workload approach

In order to provide the optimal frequency schedule, the linear program is provided with all possible task configurations, *i.e.*, all possible of parallel tasks, known as workloads. Then the solver provides the execution frequency of each workload.

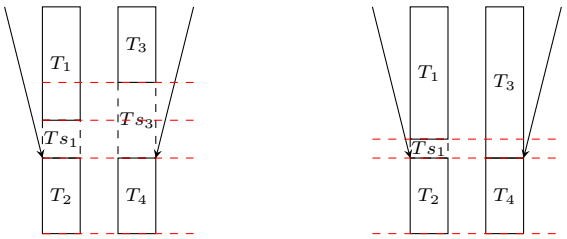
3.3.1 Shared frequency constraint

We need to express that tasks executed at the same time on the same processor run at the same frequency. Hence, we first need to identify tasks executed in parallel on the same processor. Depending on the frequency being used, the set of parallel tasks may change. Figure 4 is an example of two different executions running at the maximal and minimal frequency. Only processes that belong to the same processor are represented. In Figure 4a, when the processor runs at f_max, the set of couple of tasks which are parallel is: $\{(T_1, T_3), (T_1, Ts_3), (Ts_1, Ts_3), (T_2, T_4)\}$ (represented by red dotted lines). When the frequency is set to f_min (Figure 4b), the slack after T_3 is completely covered and the set of parallel tasks becomes: $\{(T_1, T_3), (Ts_1, T_3), (T_2, T_4)\}$.

bW_i	Beginning of a workload W_i
eW_i	End of a workload W_i
tW_i^f	The time a workload W_i is executed at frequency f
dW_i	The duration of a workload
$\overline{tW_i^f}$	A binary variable used to say if a workload is executed at a frequency f or not

Table 2: Workload formulation variables

In order to provide all possible configurations, we define the processor workloads. A workload, denoted W_i is tuple of potentially parallel tasks. In Figure 4, $W_1 = (T_1, T_3)$, $W_2 = (Ts_1, T_3)$, $W_3 = (T_1, Ts_3)$ represent a subset of the possible workloads. Note that there are no workloads with the same set of tasks. In other words, once a task in a workload is over, a new workload begins. On the other hand, a task can belong to several workloads (like T_1 in Figure 4a).



(a) f_max (b) f_min

Figure 4: Workloads

Recall that our goal in to calculate the fraction of time a tasks should spend at each frequency (tT_i^f) in order to minimize the energy consumption of the application according to the objective function (5). Since tasks may be executed at several frequencies, so does a workload. In Figure 5, the workload $W_1 = (T_1, T_3)$ is executed at frequency f_1 then at frequency f_2. Thus, since T_1 belongs to both $W_1 = (T_1, T_3)$ and $W_2 = (T_1, Ts_3)$, the execution time of T_1 at frequency f_1 ($tT_1^{f_1}$) can be calculated by using the fraction of time W_1 and W_2 spend at frequency f_1. In other words, the execution time of a task can be calculated according to the execution time of the workloads it belongs to. Let tW_i^f be the fraction of time the workload W_i spends at frequency f. Thus:

$$tT_i^f = \sum_{W_j, T_i \in W_j} tW_j^f \tag{8}$$

Using the execution time of a workload at a specific frequency (tW_i^f), one can calculate the duration of a workload, dW_i as:

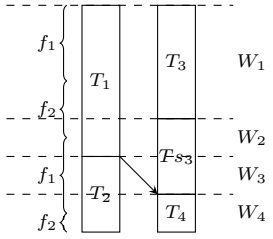


Figure 5: Workloads and tasks execution

$$dW_i = \sum_f tW_i^f$$

3.3.2 Handling frequency switch delay

Recall that one of the problems when considering DVFS is the time required to actually set a new frequency. Thus, before setting a frequency, one has to make sure that duration of the workload is long enough to tolerate the frequency change since changing frequency takes some time. In other words, if the frequency f is set in a W_i, tW_i^f is larger than a user-defined threshold, denoted Th.

$$\forall W_i, \forall f : tW_i^f \geq Th \times \overline{tW_i^f} \tag{9}$$

$\overline{tW_i^f}$ is a binary variable used to guarantee that definition (9) remains true when $tW_i^f = 0$.

$$\overline{tW_i^f} = \begin{cases} 0 & tW_i^f = 0 \\ 1 & otherwise \end{cases} \tag{10}$$

The expression of definition (10) as a mixed binary programming formulation is expressed in the appendix.

3.3.3 Valid workload filtering

The linear program is provided with all possible workloads, then it provides the different tW_j^f for each workload. However, all workloads cannot be present in one execution. In Figure 4, $W_1 = (T_1, Ts_3)$ and $W_2 = (Ts_1, T_3)$ are both possible workloads, but they cannot be in the same execution, because if W_1 is being executed, it means that T_3 is over (since Ts_3 is after T_3) thus W_2 cannot appear later since Ts_1 and T_3 are never parallel. Thus, in order to prevent W_1 and W_2 from both existing in one execution, we just need to check whether the tasks of the workload can be parallel or not. Two tasks are not parallel if one ends before the beginning of the second. Since we consider workloads, we focus only on the beginning and end time of the workload itself. Let bW_i and eW_i be the start time and the end time of the workload $W_j = (T_1, \ldots, T_i, \ldots, T_n)$. They are such that:

$$bW_j >= bT_i \tag{11}$$

$$eW_j <= eT_i \tag{12}$$

Note that although the beginning and the end of the workload are not exactly defined, this definition makes sure that the beginning or the end of a task start a new workload. Moreover, the complete execution of a task are guaranteed thanks to equations (4) and (8).

Figure 6 is an example of a workload that cannot exist. Let us assume the execution represented in Figure 6, and let us focus on the workload $W_1 = (T_1, Ts_3)$. Let us also assume that with other frequencies, a possible workload is $W_2 = (T_3, Ts_1)$. As explained above, W_1 and W_2 cannot both exist in the same execution because of precedence constraints. It is obvious from the example that T_3 and Ts_1 are not parallel, let us see how it translates to workloads. Since W_2 has to start after both T_3 and Ts_1 begins, then it starts after Ts_1 (since $bTs_1 \geq bT_3$ Figure 6). The same way it ends before eT_3. But since $eT_3 \leq bTs_1$ (as shown in Figure 6) then the duration of W_2 should be negative which is not possible.

Thus, we identify workloads which cannot be in the execution as workloads which end before they begin. The duration of a workload is such that:

$$dW_i = \begin{cases} 0 & eW_i < bW_i \\ eW_i - bW_i & otherwise \end{cases} \tag{13}$$

In the appendix (section 6), we proove that if two workloads cannot be in the same execution (because of the precedence constraints), then the duration of at least one of them is 0 (paragraph 6.4.2).

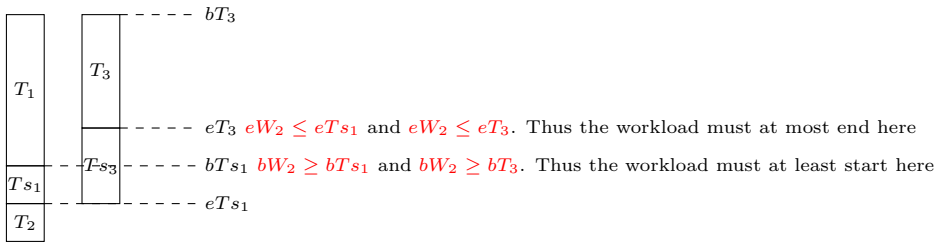


Figure 6: Negative workload duration for impossible workloads

3.3.4 Discussion

The appendix (section 6) provides a detailed formulation of the energy minimization problem using workloads. The formulation shows the use of two binary variables: one to express the threshold constraint and one to calculate the duration of the workload. With these two variables, the formulation is not linear anymore, which requires more time to solve (especially when the number of workloads is important).

Moreover, we tried providing all possible workloads of one of the NAS parallel benchmarks on class C on 16 processes (IS.C.16) on a machine equiped with 16 GB of memory. The application task graph is composed of 630 tasks. The generated data (*i.e.* the number of workloads) could not fit in the memory of the machine. Thus, even with no binary variables, providing all possible workloads is not possible when considering real applications.

In the following section, we provide another formulation which requires only the task graph.

3.4 Architecture constraints: the frequency switch approach

As explained earlier, our goal is to minimize the energy consumption of a parallel application using DVFS. In order to do so, we express the problem as a linear program. We consider that the program is represented as a task graph and each task can have several phases. The difficulty of the formulation is to provide, for each task, the frequency of each of its phases (tT_i^f) since one has to make sure that parallel tasks must run at the same frequency. In this section, we provide another formulation which considers the time to set a new frequency on the whole processor instead of considering tasks independently and then force parallel tasks to run at the same frequency.

3.4.1 Frequency switch overhead

Let c_{jp}^f be the time the frequency f is set on the processor p, j being the sequence number of the frequency switching. Figure 7 represents the execution of four tasks on two cores of the same processor p. In the example, we assume that there are only 3 possible frequencies. The different c_{jp}^f are numbered such that the minimum frequency f_1 corresponds to the switching time $c_{1p}^{f_1}, c_{4p}^{f_1}, \ldots$, the frequency f_2 corresponds to the frequency changes $c_{2p}^{f_2}, c_{5p}^{f_2}, \ldots$ and so on. A frequency f_1 is applied during a time which can be calculated as $c_{\{i+1\}p}^{f_2} - c_{ip}^{f_1}$. This can be translated to:

$$c_{\{i+1\}p}^{f_2} \geq c_{ip}^{f_1}$$

c_{ip}^f	Time of the i^{th} frequency switch on processor p. The frequency f is the one set
d_{ij}^f	The amount of time a frequency f is set for the task i for the frequency switch j

Table 3: Frequency switch formulation variables

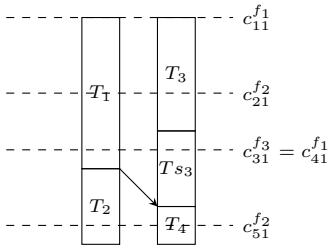


Figure 7: Frequency switches example

Note that some frequencies may not be set if the duration is zero. In figure 7, frequency f_3 is not set since $c_{31}^{f_3} = c_{41}^{f_1}$.

3.4.2 Handling frequency switch delay

As explained earlier, changing frequency takes some time. Thus, for a change to be applied, its duration has to be longer than the user-defined threshold Th. Let ζ_{ip}^{f} be a binary variable, such that:

$$\zeta_{ip}^{f} = \begin{cases} 0 & c_{\{i+1\}p}^{f'} - c_{ip}^{f} = 0 \\ 1 & otherwise \end{cases} \tag{14}$$

The threshold condition can be expressed as:

$$c_{\{i+1\}p}^{f'} - c_{ip}^{f} \geq Th \times \zeta_{ip}^{f}$$

We detail how equation (14) is translated into mixed binary programming constraints in the appendix.

3.4.3 Shared frequency constraints

Once the threshold condition is satisfied, one can calculate the time a task spends at each frequency, *i.e* tT_i^f, according to c_{jp}^{f}. On Figure 7, initially, tasks T_1 and T_3 run in parallel at frequency f_1. The time T_3 spends at frequency f_1 is $c_{21}^{f_2} - c_{11}^{f_1}$ whereas T_1 is executed twice at f_1. It spends $(c_{21}^{f_2} - c_{11}^{f_1}) + (eT_1 - c_{41}^{f_1})$ at frequency f_1. Let d_{ij}^{f} be the time the task T_i spends at frequency f after the frequency switch j. Back to Figure 7, $d_{11}^{f_1} = c_{21}^{f_2} - c_{11}^{f_1}$ and $d_{14}^{f_1} = eT_1 - c_{41}^{f_1}$. $tT_1^{f_1}$ becomes $tT_1^{f_1} = d_{11}^{f_1} + d_{14}^{f_1}$.

The above translates to:

$$tT_i^f = \sum_j d_{ji}^f$$

Note that a task is not impacted by a frequency change if it ends before the change or begins after the next change. In other words, $d_{ij}^{f_1} = 0$ if $eT_i \leq c_{jp}^{f_1}$ or $bT_i \geq c_{\{j+1\}p}^{f_2}$. Otherwise, $d_{ij}^{f_1}$ can be calculated as $min(eT_i, c_{\{j+1\}p}^{f_2}) - max(bT_i, c_{jp}^{f_1})$.

$$d_{ji}^{f} = \begin{cases} 0 & eT_i \leq c_{jp}^{f} \; or \; bT_i \geq c_{\{i+1\}p}^{f'} \\ min(eT_i, c_{\{j+1\}p}^{f'}) - max(bT_i, c_{jp}^{f}) & otherwise \end{cases} \tag{15}$$

3.5 Discussion

The appendix (section 6) provides the complete formulation of the problem using the frequency switch time variables. In addition to the binary variable used to satisfy the frequency switch overhead, for each task and for each frequency switch, five additionnal binary variables are used. Thus, for n tasks and m frequency

switch considered, $5 \times n \times m$ binary variables are required. Mixed integer programming is NP-hard [5], thus, with such a number of binary variables, no solution can be provided.

When comparing the workload approach and the frequency switch approach, one can notice that the former needs less binary variables and should be able to provide results. However, because all possible workloads have to be provided to the solver, it is as complex because of the memory required. Thus, if a very large memory is available, then the workload solution is the one to be used. And if new faster binary resolution techniques are provided, then the frequency switch solution should be used.

Several heuristics can be assumed in order to reduce the time to solve the problem. First, one can consider iterative applications, and solve the problems for only one iteration then apply it the remaining ones. However, this solution strongly depends on the number of tasks per iterations. We tried this solution on some kernels (NAS Parallel Benchmarks [2]) and the solver could not provide any result after several hours.

The most promising heuristic is to consider the tasks at the processor level instead of the core level. Thus, the only architecture constraint which needs to be considered is the frequency overhead one. This study is part of our current work and will be discussed in further studies.

4 Related Work

DVFS scheduling has been widely used to improve processor energy consumption during application execution. We focus on studies assuming a set of dependent tasks represented as a direct acyclic graph (DAG).

A lot of studies tackle task mapping problem while minimizing energy consumption either with respect to task deadlines [22] or by trying to minimize the deadline as well [13]. When considering an already mapped task graph, studies provide the execution speed of each task depending on the frequency model: continuous [3] or discrete [12]. Some studies also provide a set of frequencies to execute a task [6] (executing a task at multiple frequencies is known as VDD-Hopping). In [1], the authors present a complexity study of the energy minimization problem depending on the frequency model (continuous frequencies, discrete frequencies with and without VDD-Hopping). Finally studies like [18] and [17] consider frequency transition overhead. Although these studies should provide an optimal frequency schedule, they do not consider the constraints of most current architectures and more specifically the shared frequency among all cores of the same processor.

When considering linear programming formulation to minimize application energy consumption, many formulations have been proposed in the past. When considering single processor, [9] provides an integer linear programming formulation with negligible frequency switching overhead. The same problem but considering frequency transition overhead was addressed in [21]. The author also provide a linear-time heuristic algorithm which provides near-optimal solution.

The work presented in [19] is the closest to the work presented in this paper. In [19], the authors present a linear programming formulation of the minimization energy problem where tasks can be executed at several frequencies. Both slack energy and processor energy consumption are considered in the minimization and a loose deadline is considered. In a similar way, [15] provides a scheduling algorithm and an integer linear programming formulation of the energy minimization problem on heterogeneous systems with a fixed deadline. The formulation is very close to the one described in [19], but the authors also considered communication energy consumption. However, they do not consider slack time and its power consumption when solving the problem. In [14] the authors use an integer linear programming formulation of the problem where only task with slack time are slowed down, whereas other tasks are run at maximal frequency. The program is used to compute the best frequency execution of a task.

Although previous studies provide different solutions and formulations for DVFS scheduling, few of them consider current architecture constraints. While some previous studies consider frequency transition overhead [21, 11], none of them consider the fact that cores within the same processor run at the same frequency. This paper describes a mixed linear programming formulation that guarantees that parallel tasks on the same processor run at the same frequency. Moreover, it shows that it is possible to relax the deadline if it leads to energy saving.

5 Conclusion

The goal of this paper was to provide a study on how energy minimization problem of a parallel execution of an MPI-like program can be addressed and formulated when considering most current architecture constraints. In order to do so, we used linear programming formulation. Two different formulations were described. Their goal is to minimize the energy consumption with respect to a user-defined deadline by providing the optimal frequency schedule. Both solutions use a number of binary variables which is proportional to the number of tasks. Used as they are, these formulations should provide an optimal solution but are costly in terms of memory and resolution time, despite the use of fast parallel solvers like gurobi [7].

We are currently working on introducing heuristics to relax the architecture constraints by building tasks on the processor level instead of the core level. Using such heuristics seems to drastically reduce the time needed to solve the problem.

6 Appendix

This appendix summarizes the set of constraints of both formulations described in paragraphs 3.3 and 3.4. We start by describing how each non linear constraint which appears in sections 3.3 and 3.4 is expressed. For a more complete description and explanation, the reader can refer to [4].

6.1 Expressing non linear constraints

Section 3 presents different non continuous variables (definitions 10, (13) and (14), (15)). In this section, we briefly explain how this kind of expressions translates to inequalities using binary variables.

1. If-then statement with 0-1 variables: Expressing conditions like:

$$\overline{x} = \begin{cases} 0 & x = 0 \\ 1 & otherwise \end{cases}$$

(for instance, definition 10) requires the use of a large constant M such that:

$$x \leq M \times \overline{x} \tag{16}$$
$$x \geq \overline{x} \times \epsilon \tag{17}$$

Thus, when $x = 0$, (17) forces $\overline{x}$ to be equal to 0 and when $x \neq 0$, (16) is used to set the value of $\overline{x}$ to 1.

Note that, equation (9), which guarantees that $tW_i^f \geq Th \times \overline{tW_i^f}$ makes (17) useless (since $Th > \epsilon$). Thus, (17) is never used in the set of constraints.

2. If-then statement with real variables: Expressing formulas like:

$$z = \begin{cases} 0 & y < x \\ y - x & otherwise \end{cases}$$

(definition (13) for instance) is similar to the previous formulation in the sens that it requires the use of a big constant M. A binary variable bin is used such that when $y - x \leq 0$, $bin = 0$.

$$y - x \leq M \times bin \tag{18}$$
$$x - y \leq M \times (1 - bin) \tag{19}$$

Thus, when $y \leq x$, (18) is always valid regardless the value of bin. Hence, (19) forces bin to be equal to 0. Similarly, when $y \geq x$, equation (18) forces bin to 1.

Once bin is defined, z can be expressed as:

$$\begin{aligned} y - x &\leq z \leq M \times bin & (20)\\ y - x + z &\leq 2 \times (y - x) + M \times (1 - bin) & (21) \end{aligned}$$

Thus, when $y \leq x$, $bin = 0$ (from (18)) and (20) forces z to be 0 (since all variable are positive) and (21) is always valid. Similarly, when $y \geq x$, $bin = 1$ (from (19)) and (20) and (21) become:

$$\begin{aligned} y - x &\leq z \leq M \\ z &\leq y - x \end{aligned}$$

Thus $y - x \leq z \leq y - x$ which makes $z = y - x$.

3. Maximums: Maximums can be expressed by reformulating the definition as:

$$z = max(x, y) = x + \begin{cases} 0 & x \geq y \\ y - x & otherwise \end{cases}$$

Let w be such that:

$$w = \begin{cases} 0 & x \geq y \\ y - x & otherwise \end{cases}$$

We can express w by using (20) and (21).

4. Minimums: Expressing minimums is based on the same idea than expressing maximums:

$$z = min(x, y) = x - (x - y) \begin{cases} 0 & x \leq y \\ x - y & otherwise \end{cases}$$

We do not detail how minimums are expressed, since it is done the same way as maximums.

5. Expressing several conditions: In definitions like (15), several conditions can force the value of a variable.

$$w = \begin{cases} 0 & x \leq y \; or \; z \geq u \\ 0 & otherwise \end{cases}$$

Translating such definitions into inequalities requires the use of one binary variable for each condition and one binary variable to express the "or".

Let $bin1$, $bin2$ be such that: $bin1 = \left\{ \begin{array}{lll} 1 & if & z - u \geq 0 \\ 0 & & otherwise \end{array} \right.$ and $bin2 = \left\{ \begin{array}{lll} 1 & if & x - y \leq 0 \\ 0 & & otherwise \end{array} \right.$

These two definitions can be expressed using (16) and (17).

Finally $bin3$ is a binary variable which is equal to 1 if $bin1$ or $bin2$ are equal to 1 and 0 otherwise:

$$bin3 = \begin{cases} 1 & bin1 + bin2 \geq 1 \\ 0 & otherwise \end{cases} \quad (22)$$

Since $bin1$, $bin2$ and $bin3$ are binary variables, (22) can be easily expressed as:

$$\begin{aligned} bin1 &\leq bin3 & (23)\\ bin2 &\leq bin3 & (24)\\ bin3 &\leq bin1 + bin2 & (25) \end{aligned}$$

Thus, when $bin1$ and $bin2$ are 0, (25) forces $bin3$ to be 0 whereas when $bin1$ or $bin2$ are equal to 1, (23) and 24 forces $bin3$ to be equal to 1.

6.2 Objective function

Minimizing the energy consumption of a program described as a set of tasks is the objective function of the linear programming formulations described above. For a task T_i with a power consumption at a frequency f, P_i^f and executed at frequency f during tT_i^f, the energy consumption of the whole program for its whole execution time is:

$$min(\sum_{T_i}(\sum_{f}(tT_i^f \times P_i^f)))$$

6.3 Task constraints

Let $T_i, T_{i+1}, T_{i+2}, T_j$ be four tasks such that: T_i, T_{i+1}, T_{i+2} are consecutive and on the same processor. T_i ends with a message sending creating T_{i+1} which ends with a reception from T_j which generates T_{i+2} as shown in Figure 8.

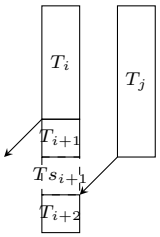


Figure 8: Task configuration

$$\begin{aligned}
eT_i &= bT_i + \sum_f tT_i^f \\
\sum_f \delta_i^f &= 1 \\
bT_{i+1} &= eT_i \\
bTs_{i+1} &= eT_{i+1} \\
eTs_{i+1} &\geq eT_j + M_j^{i+1} \\
eTs_{i+1} &\geq bTs_{i+1} \\
bT_{i+2} &= eTs_{i+1} \\
tT_i^f &= \delta_i^f \times execT_i^f
\end{aligned}$$

6.4 Workload approach

6.4.1 Additional variable

γ_i : A binary variable used to say if a workload duration is 0 or not
M : A large constant

$$\begin{aligned}
bW_i &\geq bT_j \\
eW_i &\leq eT_j \\
tT_i^f &= \sum_{\substack{W_j \\ T_i \in W_j}} tW_j^f \\
dW_i &= \sum_f tW_i^f
\end{aligned}$$

Using (16), (17) and (9), we express definition (10) as:

$$\begin{array}{rcl} tW_i^f & \geq & Th \times \overline{tW_i^f} \\ tW_i^f & \leq & M \times \overline{tW_i^f} \end{array}$$

Using (16), (17), (20) and (21) and γ_i as the binary variable, we express definition (13) as:

$$\begin{array}{lcccl} eW_i - bW_i & \leq & M \times \gamma_i & & \\ bW_i - eW_i & \leq & M \times (1-\gamma_i) & , \gamma_i \in \{0,1\} & \\ eW_i - bW_i & \leq & dW_i & \leq & M \times \gamma_i \\ eW_i - bW_i + dW_i & \leq & 2 \times (eW_i - bW_i) + M \times (1-\gamma_i) & & \end{array}$$

6.4.2 Proof of workload duration

We want to proove that if two workloads W and W' are possible, but they violate the precedence constraint between the tasks, then the duration of at least one of them is zero. We provide the proof for workloads with a cardinality equals to 2 since the proof remains the same for larger workloads.

Let $W = (T_i, T_j)$ and $W' = (T'_i, T'_j)$ such that T_i preceeds T'_i and T'_j preceeds T_j. We want to prove that $dW = 0$ or $dW' = 0$.

Lemma 1. *Let $W = (T_i, T_j)$ and $W' = (T'_i, T'_j)$. If $bT'_i \geq eT_i$ and $bT_j \geq eT'_i$, then $dW = 0$ or $dW' = 0$.*

Proof. Let us proove lemma 6.4.2 by contradiction. Let us assume that $dW \neq 0$ and $dW' \neq 0$.

From definition (10): $\begin{array}{rcl} dW \neq 0 & \Leftrightarrow & eW \geq bW \\ dW' \neq 0 & \Leftrightarrow & eW' \geq bW' \end{array}$

From constraints (11) and (12):

$$\begin{array}{lcl} bW \geq bT_i & & bW' \geq bT'_i \\ bW \geq bT_j & \text{and} & bW' \geq bT'_j \\ eW \leq eT_i & (26) & eW' \leq eT'_i \\ eW \leq eT_j & & eW' \leq eT'_j \end{array}$$

But $bT'_i \geq eT_i$ and $bT_j \geq eT'_i$, thus:

$$bW \geq bT_j \geq eT'_j \geq eW' \quad (27)$$

$$bW' \geq bT'_i \geq eT_i \geq eW \quad (28)$$

If we consider (27), (28) and (26):

$$bW' \geq bT'_i \geq eT_i \geq bW \geq eW'$$

Thus $bW' \geq eW'$ which by definition (10) implies that $dW' = 0$ which leads to a contradiction. □

6.5 Frequency switch approach

Note that we do not detail how the threshold condition is handled since it is done the same as for the workloads.

6.5.1 Additional variables

ζ_{ip}^f : A binary variable used to say if a workload is executed at a frequency f or not
y_{ij}^f : The maximum between bT_i and c_{jp}^f
w_{ij}^f : A variable used to express y_{ij}^f. It is equal to 0 if bT_i is the maximum, and $c_{jp}^f - bT_i$ otherwise
α_{ij}^f : A binary variable used to verify whether $bT_i \geq c_{jp}^f$
z_{ij}^f : The minimum between eT_i and $c_{\{j+1\}p}^f$
g_{ij}^f : A variable used to express z_{ij}^f. It is equal to 0 if eT_i is the minimum, and $eT_i - c_{\{j+1\}p}^f$ otherwise
β_{ij}^f : A binary variable used to verify whether $eT_i \leq c_{\{j+1\}p}^f$
ψ_{ij}^f : A binary variable used to check if $bT_i - c_{\{i+1\}p}^f \geq 0$
ϕ_{ij}^f : A binary variable used to check if $eT_i - c_{ip}^f \leq 0$
ρ_{ij}^f : A binary variable used to check if ψ_{ij}^f or ϕ_{ij}^f are true
M : A large constant

6.5.2 Constraints

$$\begin{array}{lcl} c_{\{i+1\}p}^{f'} & \geq & c_{ip}^f \\ c_{\{i+1\}p}^{f'} - c_{ip}^f & \geq & Th \times \zeta_{ip}^f \\ c_{\{i+1\}p}^{f'} - c_{ip}^f & \leq & M \times \zeta_{ip}^f \\ tT_i^f & = & \sum_j d_{ij}^f \end{array}$$

Expressing definition (15) as inequalities requires the use of (20) and (21) for the maximum and the minimum such that:

$$\begin{array}{rcl} y_{ij}^f & = & max(bT_i, c_{jp}^f) \\ & = & bT_i + w_{ij}^f \end{array} \quad \text{such that: } w_{ij}^f = \begin{cases} 0 & if \quad bT_i \text{ is the maximum} \\ c_{jp}^f - bT_i & otherwise \end{cases}$$

$$\begin{array}{rcl} z_{ij}^f & = & min(eT_i, c_{\{j+1\}p}^{f'}) \\ & = & eT_i - g_{ij}^f \end{array} \quad \text{such that: } g_{ij}^f = \begin{cases} 0 & if \quad eT_i \text{ is the minimum} \\ c_{\{j+1\}p}^f - eT_i & otherwise \end{cases}$$

Let α_{ij}^f be the binary variable used for the maximum and β_{ij}^f the one used for the minimum. By replacing the corresponding variables in (20) and (21), we obtain the following inequalities for the maximum:

$$\begin{array}{lcl} c_{jp}^f - bT_i & \leq & M \times \alpha_{ij}^f \\ bT_i - c_{jp}^f & \leq & M \times (1 - \alpha_{ij}^f) \\ c_{jp}^f - bT_i & \leq & w_{ij}^f \\ c_{jp}^f - bT_i + w_{ij}^f & \leq & 2 \times (c_{jp}^f - bT_i) + M \times (1 - \alpha_{ij}^f) \end{array} \quad , \alpha_{ij}^f \in \{0,1\} \quad \leq \quad M \times \alpha_{ij}^f$$

and the following for the minimum:

$$\begin{array}{lcl} eT_i - c_{\{j+1\}p}^{f'} & \leq & M \times \beta_{ij}^f \\ c_{\{j+1\}p}^{f'} - eT_i & \leq & M \times (1 - \beta_{ij}^f) \\ eT_i - c_{\{j+1\}p}^{f'} & \leq & g_{ij}^f \\ eT_i - c_{\{j+1\}p}^{f'} + g_{ij}^f & \leq & 2 \times (eT_i - c_{\{j+1\}p}^{f'}) + M \times (1 - \beta_{ij}^f) \end{array} \quad , \beta_{ij}^f \in \{0,1\} \quad \leq \quad M \times \beta_{ij}^f$$

Finally, using (23), (24) and (25) and the binary variables ψ_{ij}^f, ϕ_{ij}^f and ρ_{ij}^f as $bin1$, $bin2$ and $bin3$ respectively and using (20) and (21), d_{ij} can be expressed as:

$$
\begin{array}{lcl}
\phi_{ij}^f & \leq & \rho_{ij}^f \\
\psi_{ij}^f & \leq & \rho_{ij}^f \\
\rho_{ij}^f & \leq & \phi_{ij}^f + \psi_{ij}^f \\
z_{ij}^f - y_{ij}^f & \leq & d_{ij}^f \qquad \leq \quad M \times (1 - \rho_{ij}^f) \\
z_{ij}^f - y_{ij}^f + d_{ij}^f & \leq & 2 \times (z_{ij}^f - y_{ij}^f) + M \times \rho_{ij}^f
\end{array}
$$

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