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HAL Id: hal-01119358

https://hal.science/hal-01119358

Submitted on 23 Feb 2015

ANALYTICAL MODEL OF THE FLOW STAGE
OF A PNEUMATIC SERVO-DISTRIBUTOR
FOR SIMULATION AND NONLINEAR
CONTROL

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ABSTRACT

A typical electropneumatic control system consists of one or two servo-distributors and a simple or double effect jack. The nonlinearities of such systems are chiefly due to the mass flow rate of the fluid through the restrictions which vary with the control input. The modelling objective for simulation is to enable an accurate representation of the physical system, generally in an analytical form, to be obtained, this form enables easier analysis and control laws synthesis.

In this paper we have studied the Servotronic servo-distributor (Asco-Joucomatic Company). An approximation of the flow stage characteristic of this servo-distributor by several polynomial functions is described. We have developed analytical models for both simulation and control purposes. The choice of the polynomial functions and their degrees are discussed. Comparative results are presented and discussed.

KEYWORDS: Pneumatic, Servo-distributor, Modelling, Approximations, Simulation, Nonlinear Control.

1   NOMENCLATURE

A	Cross section (m^2)		Re	Reynolds number
b	Critical pressure ratio		T	Temperature (K)
Cq	Flow coefficient		u	Input control (V)
G	Static gain (Pa/V)		V	Volume (m^3)
$gst^{-1}(.)$	Inverse Function of static gain (V)		x	Spool position (m)
k	Polytropic coefficient		γ	Specific heat ratio
p	Pressure (Pa)		$\Gamma(.)$	Control polynomial function of (m^2)
$q_m(.,.)$	Mass flow rate (kg/s)		$\varphi(.)$	Leakage polynomial function (kg/s)
r	Perfect gas constant related to unit mass (J/kg/s)		$\psi(.)$	Polynomial function $(kg/s/m^2)$
R	Statistical criterion		τ	Time constant (s)

1.1   Subscript

dw	Downstream		S	Supply
E	Exhaust		up	Upstream
l, n, m	Polynomial degrees		v	Virtual

2   INTRODUCTION

An electropneumatic servo-drive is principally composed of an actuator (cylinder) and a power modulator (servo-distributor). Such systems are known to be highly nonlinear, which is mainly due to fluid compressibility [1]. For the synthesis of nonlinear control laws, a nonlinear electropneumatic servo-drive control model has to be established. This model must be linear in the input in order to use the tools of the nonlinear control theory of affine systems [2].

Furthermore the use of numerical simulation is more frequent in research teams as in industries for system conception, sizing (virtual prototypes) and behaviour analysis of new or existing systems. This requires a more accurate knowledge of the system in order to obtain as accurate as possible simulation models.

There are various types of models:

- knowledge models based on physical laws,
- representation models based on measurement,
- "grey box" models combining the two previous models.

Representation models can be used in the form of the table of measurement and/or in differential equations and/or algebraic equations obtained from an identification method [3]. In fact it is preferable to obtain an analytical model as this makes the analysis of the model and the control laws synthesis easier. Notice that from the analytical expression, the equilibrium points set and the linearised tangent model may be easily obtained.

Concerning electropneumatic servo-drives, the main difficulty of the modelling procedure is the determination of the analytical flow stage model of the servo-distributor. The other parts of the system (servo-distributor electromechanical part and

the cylinder) are now relatively well-modelled [4] [5] [6]. So in this paper several analytic nonlinear models of the flow stage of a 3/2-way servo-distributor Servotronic of the Joucomatic Company are presented. They correspond either to simulation models or to control models.

In this paper the structure of the Servotronic servo-distributor is described, it can be divided in two parts: the electromechanical part and the flow stage part. The global static characterisation of its flow stage part is shown later. Depending on the objective we use linear or nonlinear regression techniques with optimisation at the least square sense for obtaining several approximations of the servo-distributor flow stage characteristics by polynomial functions. A statistical criterion validates these approximations. A physical criterion is used too. Moreover static and dynamic results are presented with a simple application and are then discussed.

3   DESCRIPTION OF THE SERVO-DISTRIBUTOR UNDER STUDY

Our research team, in collaboration with the Asco-Joucomatic Company, has developed the Servotronic servo-distributor [4]. So the main physical characteristics of the components are well-known and a good simulation model has been established by using many measurements and physical laws. It is assumed that at each instant during the spool displacement, the output mass flow rate of the servo-distributor for a spool position is the same as the one obtained in steady state for the same spool position and the same pressure conditions. The Servotronic servo-distributor model can be divided in two parts (see fig.1). The first part corresponds to the positioning dynamic model of the spool and depends mainly on the electromechanical system and the second part corresponds to the static flow stage model.
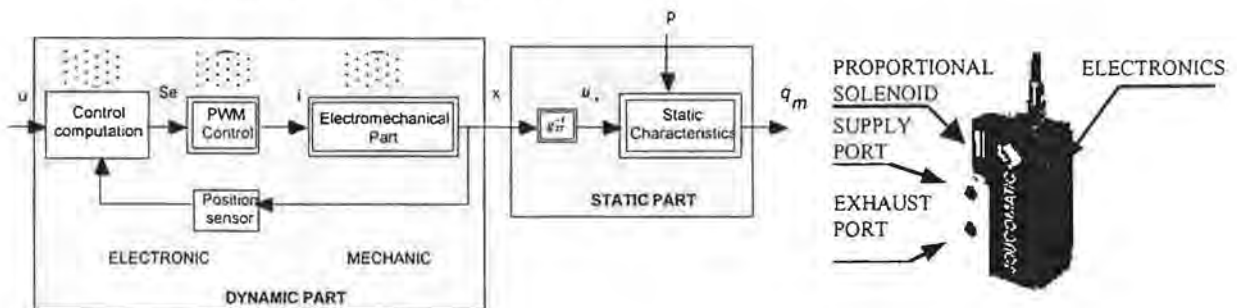

Fig. 1 Servotronic servo-distributor.

3.1 Dynamic part

Concerning the modelling of the servo-distributor dynamic part, many research papers consider that the servo-distributor dynamics can be represented by a third order transfer function model, which is a representation model and not a knowledge model [5][6].
In our case, a physical model has been established. It can be described by three state variables: the electromagnetic current, the velocity and the position of the spool. In the Servotronic servo-distributor, the spool position is controlled by a local state feedback.

One of the advantages of this position control is the negligible hysteresis between the spool position x and the input voltage u. So a static relation $x = g_{ST}(u)$ may be defined and experimentally obtained and the inverse relation $u = g_{ST}^{-1}(x)$ can be easily obtained.

Since the static flow stage models use the servo-distributor voltage input as input instead of the spool position, it is necessary to use the inverse of the static characteristic giving for each spool position the corresponding input control voltage. So in figure 1, 'u_V' is used for the virtual input corresponding to the spool position obtained from the dynamic model: $u_v = g_{ST}^{-1}(x)$.

Many research papers consider generally that the servo-distributor dynamic is very high. In fact, in our case, the bandwidth of the Servotronic servo-distributors is about *170 Hz* and the bandwidth of classic electropneumatic actuator systems is about *10 Hz*. This consideration generally justifies that the servo-distributor electromechanical part in electropneumatic actuator system modelling is not taken into account for a control model [7].

3.2 Static flow stage part

The figure 2 shows, in continuous lines, a symbolic representation of a 3/2-way servo-distributor. It is the classic half bridge Wheatstone representation with the two variable restrictions called A_S and A_E whose cross sections vary with spool position and therefore with the control.

In order to establish a mathematical model of the power modulator flow stage, many works in the literature present approximations based on physical laws [8][9][10] by the modelling of the geometrical variations of the restriction areas A_S and A_E as well as by the experimental local characterisation [11]. These methods are based on approximations of fluid flow through a convergent nozzle in turbulent regime, corrected by a Cq coefficient (1)[1]. The mass flow rate law in each restriction has the following form:

$$q_m(P_{up}, P_{dw}, u) = A(u) \frac{P_{up}}{\sqrt{T_{up}}} C_q \left(\frac{P_{up}}{P_{dw}} \, or \, (P_{up} - P_{dw}) \, or \, u \, or \, Re \right) C_m \left(\frac{P_{up}}{P_{dw}} \right) \qquad (1)$$

with:

- in subsonic regime, $\dfrac{P_{dw}}{P_{up}} > b$:

$$C_m\left(\frac{P_{dw}}{P_{up}}\right) = \sqrt{\frac{\gamma}{r}\frac{2}{\gamma-1}\left(\left(\frac{P_{dw}}{P_{up}}\right)^{\frac{2}{\gamma}} - \left(\frac{P_{dw}}{P_{up}}\right)^{\frac{\gamma+1}{\gamma}}\right)}$$

- in sonic regime, $\dfrac{P_{dw}}{P_{up}} \le b$:

$$C_m\left(\frac{P_{dw}}{P_{up}}\right) = \sqrt{\frac{\gamma}{r}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

The experimental ISO 6358 norm [12] method can also be used.

Sesmat [4] made a complete study of the Servotronic servo-distributor flow stage where all fluid flow circuits (dashed lines) and their influence on the flow stage characteristics were shown. The fluid does not flow through only two variable restrictions but through a more complex set of restrictions, the geometric forms of the restriction vary and the different flow regimes occur.

Therefore these previous modelling methods are not appropriate for fine modelling of the Servotronic servo-distributors.

For our modelling, we propose to use the results of the global experimental method giving the static characteristics of the flow stage [13].
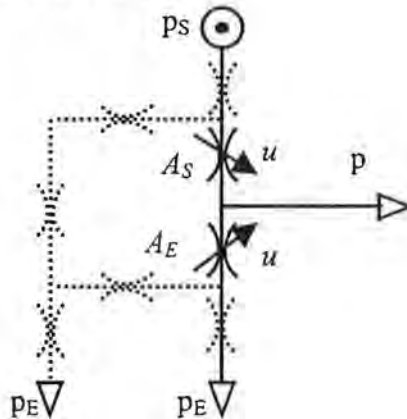

Fig. 2 The schematic flow circuit representation.

The global characterisation corresponds to the measurement of the output mass flow rate q_m depending on the input control u and the output pressure p. This characterises the component taken as a whole ("black box"). The global characterisation has the advantage of obtaining simply, by projection of the characteristics series $q_m(p, u)$ (see fig. 3a) on the different planes :

- the mass flow rate characteristics series ("pressure - mass flow rate" plane),
- the mass flow rate gain characteristics series ("input control - mass flow rate " plane),
- the pressure gain characteristics series("input control - pressure" plane).
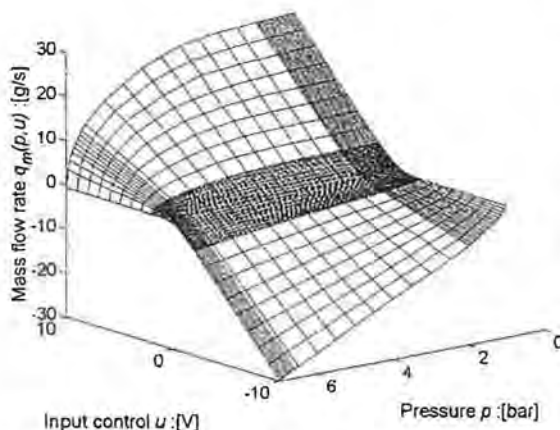

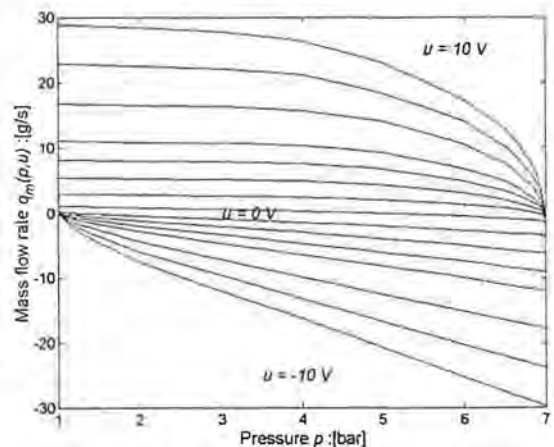**Fig. 3a** The global static characteristics.

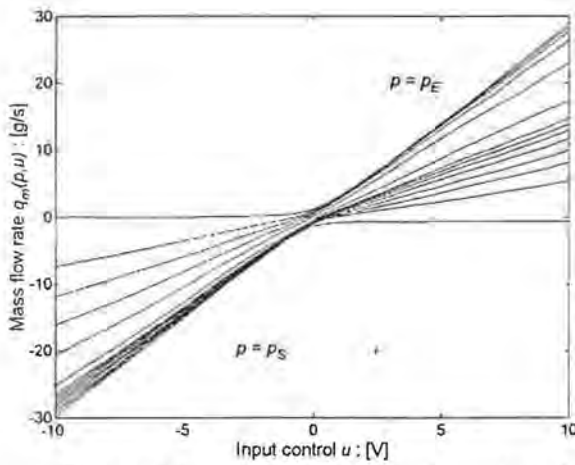**Fig. 3b** Mass flow rate characteristics series.

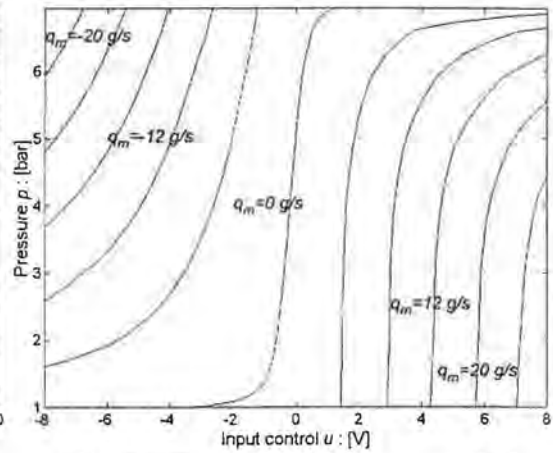**Fig. 3c** Mass flow rate gain characteristics series.

Fig. 3d Pressure gain characteristics series.

Figure 3b clearly shows the nonlinear character of the mass flow rate evolution according to pressure. The mass flow rate gain characteristics (see fig. 3c) give an idea of the variation of the restriction sections that occur during the fluid flow according to control input u and clearly show the nonlinear character of these for small values of u.

The pressure gain characteristic (see fig.3d) is used to determine the equilibrium set of the electropneumatic actuator system, in steady state for a given mass flow rate, generally equal to zero for example. This characteristic is therefore very important because it affects the steady state error of the global system.

4   APPROXIMATIONS OF THE SERVO-DISTRIBUTOR FLOW STAGE CHARACTERISTICS

In this section we present the approximations of the Servotronic servo-distributor flow stage characteristics by polynomial functions. Two cases have been studied:

- the first one is the approximation by a generalised polynomial function: approximation I,
- the second one is the approximation by several polynomial functions to obtain an input affine nonlinear model (nonlinear control objective): approximation II.

The quality of the approximation is given by a statistical criterion R called multiple correlation coefficient, which corresponds to a variance ratio. Moreover, in our case, a second criterion is fixed a posteriori, it corresponds to the error of the pressure gain characteristic obtained for a zero mass flow rate value.

4.1 The polynomial approximations

The mass flow rate is function of the pressure and the input control voltage so the polynomial approximations will be of multivariable type. The curve shapes of the global characteristics justify the choice of functions of a polynomial type and of their polynomial degrees. These functions lead then to easy calculus (matrix calculus).

In the case where a generalised polynomial function of the form (2) is chosen, the coefficients "c_{ij}" are linear according to the x and y variables and their combinations, so the linear regression technique may be used.

$$f(x,y) = \sum_{i=0}^{n}\sum_{j=0}^{m} c_{ij} x^i y^j = c_{00} + c_{01}y + .. + c_{0m}y^m + .. + c_{n0}x^n + .. + c_{nm}x^n y^m \qquad (2)$$

In the case where a polynomial function of the form (3) is chosen, the coefficients to be determined are nonlinear according to x and y variables and their combinations, so the nonlinear regression technique may be used.

$$f(x,y) = f(x) + g(x)h(y) \qquad (3)$$

with: $\quad f(x) = \sum_{i=0}^{n} a_i x^i$, $g(x) = \sum_{i=0}^{m} b_i x^i$ and $h(y) = \sum_{i=0}^{l} c_i y^i$

In the first case, the linear system of equations can be solved. It is not necessary to specify the initial conditions. In the second case a local linearisation of the nonlinear system is carried out and then the linearised system is solved. Then, new values of the unknown parameters are obtained and this process is repeated in order to minimise the least square criterion. In this case it is necessary to specify the initial conditions.

We note that the number of experimental points acquired during the flow stage characterisation is sufficient with a greater density in the nonlinear zones, therefore weighting coefficients are not used in the algorithm [14][15].

4.2 The analytical simulation model : approximation 1

The shape of the servo-distributor characteristics series curves indicates:

- in the plane (u, q_m), for high values of input control u the curves are linear but they are nonlinear for small values of u,

- in the plane (p, q_m), for the positive input control values u the curves are nonlinear for the high pressure values whereas for the negative values of u, the nonlinearities occur for the low pressure values.

It is difficult to take these different cases into account. This is why we consider a polynomial function of the following form:

$$q_m(p,u) = \sum_{i=0}^{n}\sum_{j=0}^{m} c_{ij} p^i u^j = c_{00} + c_{01}u + .. + c_{0m}u^m + .. + c_{n0}p^n + .. + c_{nm}p^n u^m \qquad (4)$$

Several approximations have been made with the following polynomial degrees:

$n = m = 5; n = 10, m = 5; n = 5, m = 10; n = m = 10$.

For all approximations the value of R is 0.99. The pressure gain characteristics of figure 4a show that the results are better for degrees greater than five. For the last two cases the results are similar. This shows that the degree of u has more influence than the degree of p. In the second case the results are worse than for the first case, this remark shows that an increase in the degree of p does not lead to a better solution. The third case, which seems to be a good compromise, requires 66 coefficients $((n+1)(m+1))$ whereas the table of experimental points contains 201 values. The space memory is also improved but the number of calculations is increased.
Figure 4b shows that the errors are less than $+/- 0.4$ g/s. The above method gives only one global model and may be used for the simulation models.

There are other methods that should give better results, such as the Tchebycheff polynomial approximation. Other solutions may be used with local models (piecewise approximation with spline technique for example). With this technique, the number of parameters computed for each domain is reduced but their implementation is difficult [15]
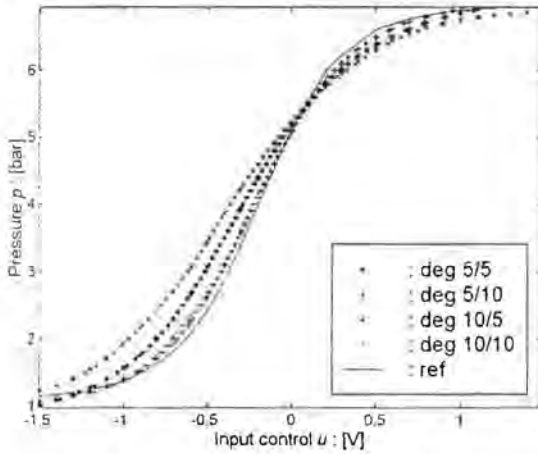

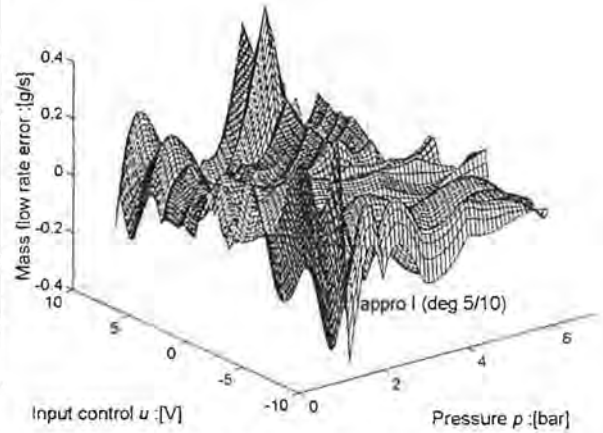Fig. 4a Pressure gains comparison (zoom).   Fig. 4b Mass flow rate errors.

4.3   The analytical control model: approximation II

The polynomial function form (4) is not appropriate for nonlinear control synthesis [2]. Nevertheless the equation (1) shows that the input control variable, which is the input control voltage, appears in a nonlinear manner. So it is necessary to use a new control input variable, which is a function of u.

The choice of the mass flow rate expression is essentially based on the expression (1)[1] with an additional function to take into account the mass flow rate leakage. So we propose the following expression for the mass flow rate:

$$q_m(p,u) = \varphi(p) + \psi(p, sgn(\Gamma(u))) \, \Gamma(u) \tag{5}$$

$\Gamma(u)$ is a polynomial function whose evolution is similar to the evolution of the equivalent section restriction that the fluid crosses as a function of spool position. $\Gamma(u)$ will therefore be deduced from the mass flow rate gain characteristics series.

$\varphi(p)$ is a polynomial function whose evolution corresponds to the mass flow rate leakage, it is identical for all input control values of u. $\psi(p, sgn(\Gamma(u)))$ is a polynomial function whose evolution is similar to the one described by the expression of mass flow rate laws (1). It is a function of the input control sign because the function is different for the inlet ($\Gamma(u)>0$) and for the exhaust ($\Gamma(u)<0$). The form of the mass flow rate characteristics (see fig. 3b) justifies this approximation by two different functions, one defined for $u>0$ and the other for $u<0$.

These functions must have bijectivity properties and $\Gamma(u)$ is defined as follows: $\Gamma(u) = -\Gamma(-u)$ and $\Gamma(0) = 0$. This last property is justified by the symmetrical mechanical structure of the Servotronic spool-sleeve assembly. Moreover, this property is required if the electropneumatic actuator system is a single input system, that is to say one input control for both servo-distributors [11].

The procedure for this approximation is more difficult than if there were no the constraints on the functions $\Gamma(u)$ and $\varphi(p)$.
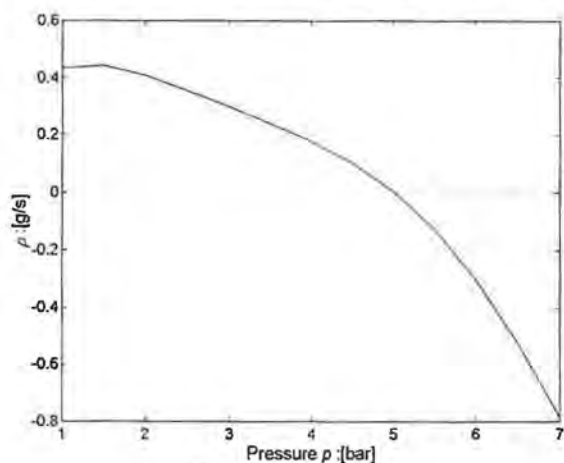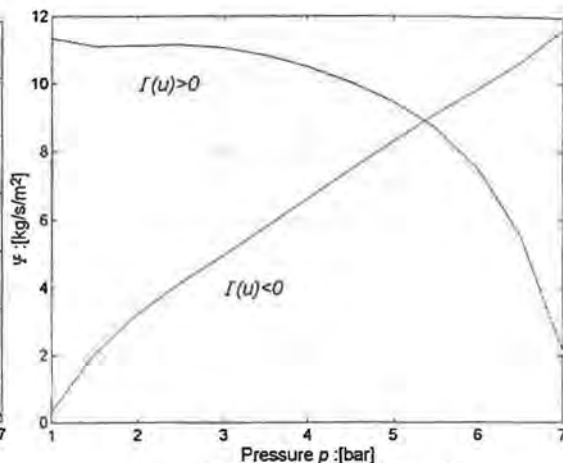

Fig. 5a Function φ.
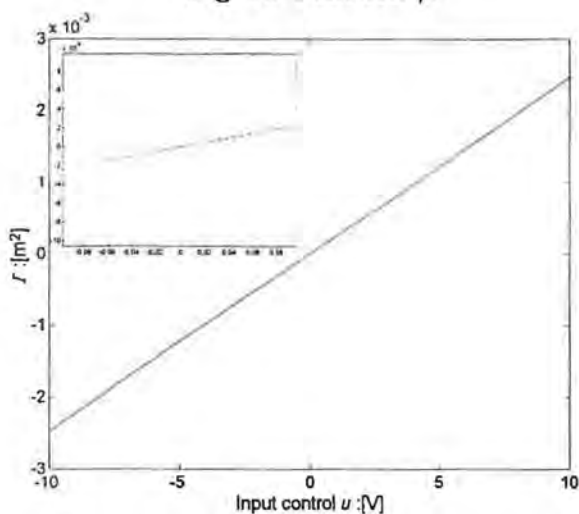
Fig. 5b Function Ψ.

Fig. 5c Function Γ.

Figure 5 shows the results where the polynomial functions $\varphi(p)$, $\psi(p, sgn(\Gamma(u)))$, $\Gamma(u)$ have degrees equal to five, five and two respectively. Figure 5b shows the curves $\psi(p, sgn(\Gamma(u)))$, which have a similar form to those, obtained by the expression (1).
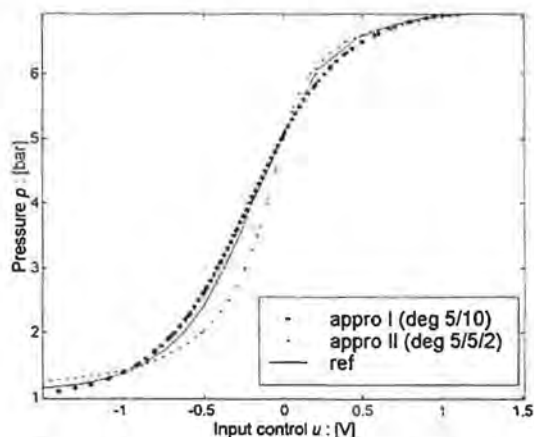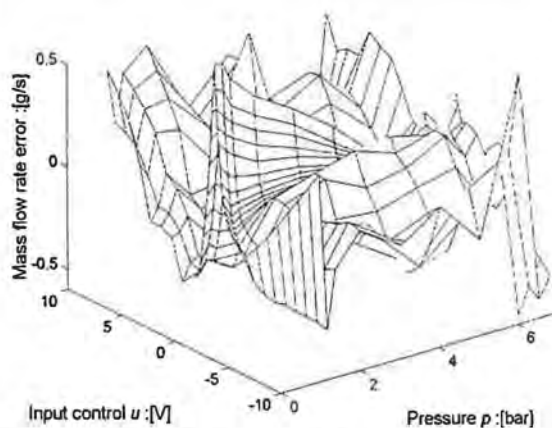

Fig. 6a Pressure gain comparison (zoom).

Fig. 6b Mass flow rate errors.

The value of the statistical criterion R is 0.99. Figure 6a shows that the pressure gain characteristic for approximation II is relatively correct, better than for approximation I with $n=5$ and $m=5$. Figure 6b shows that the mass flow rate errors are less than $+/- 0.5$ g/s. This technique gave a greater error near to the supply pressure and atmospheric pressure ("border effect"). The normal working pressures rarely reach these limit pressures, so these obtained results can be judged satisfactory. Other attempts have shown that increasing the polynomial degree of the different functions does not improve the results significantly. If there were no constraints on the functions the results would be better. An advantage of this type of model (approximation II) is the small number of coefficients $((n+1)+(m+1)+(l+1))$: 15 in our case. Moreover, a physical significance can be given to the polynomial functions.

5   STATIC AND DYNAMIC RESULTS : APPLICATION

We propose another method of validating the polynomial approximations. This consists of comparing the first partial derivatives of the polynomial approximations obtained in the two previous sections with the one deduced from the experimental characterisation.

For this, a system composed of a closed tank ($1l$) supplied by the Servotronic servo-distributor (see fig. 7) illustrates the influence of these first partial derivatives.
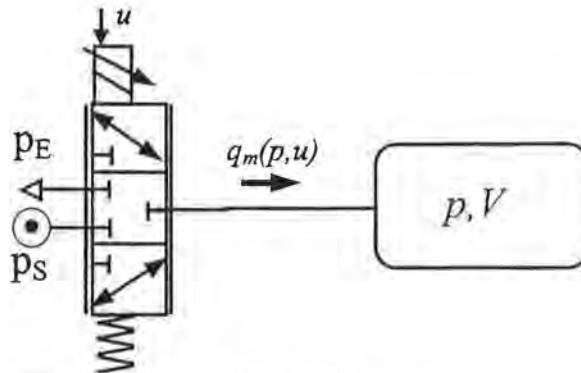

Fig. 7 Schema of the application.

For the simulation model, we assume that:

- air is a perfect gas,
- pressure and the temperature are homogeneous in the chamber,
- kinetic energy is negligible,
- evolution law is polytropic with coefficient k ($k=1.2$),
- $T_S = T_{up} = T_{dw}$, the temperature variation is negligible,
- dynamic servo-distributor part is negligible (see sect. 2.1).

Then the model is as follows:

$$\dot{p} = \frac{krT_S}{V} q_m(p,u) \tag{6}$$

At steady state ($\dot{p} = 0$, p^e, u^e), the mass flow rate is equal to zero ($q_m(p^e,u^e) = 0$). This equation corresponds to the pressure gain characteristic for a zero mass flow rate value.

To study the behaviour of this system near an equilibrium point, a linearised tangent model is calculated and we obtain a first order system of the following form:

$$\dot{p}^* = -\frac{1}{\tau}p^* + \frac{G}{\tau}\dot{u} \qquad \Rightarrow \qquad p^*(s) = \frac{G}{1+\tau s}\dot{u}^*(s) \qquad (7)$$

with:

$$p^* = p - p^e, \quad Cp = -\frac{\partial q_m(p,u^e)}{\partial p}\bigg|_e, \quad Gu = \frac{\partial q_m(p^e,u)}{\partial u}\bigg|_e, \quad \tau = \frac{V}{krT_S Cp}, \quad G = \frac{Gu}{Cp} \qquad (8)$$
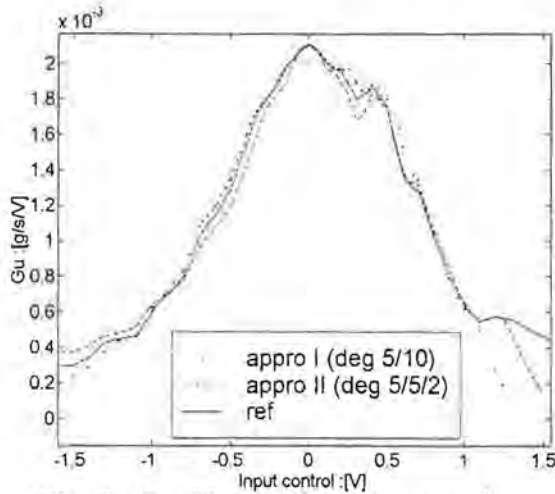

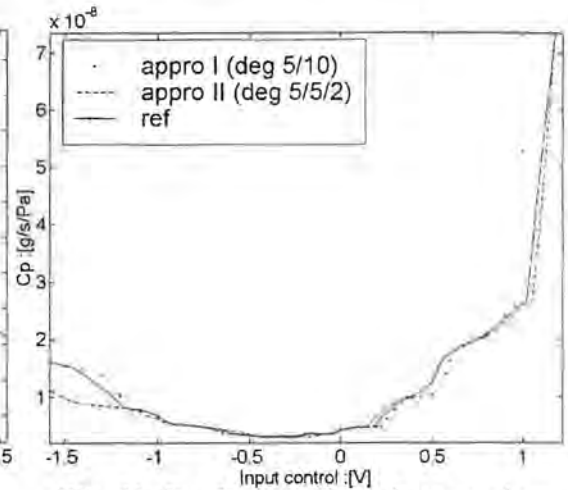**Fig. 8a** Coefficient Gu and comparison. **Fig. 8b** Coefficient Cp and comparison.

This linearised tangent model uses the two first partial derivatives of the mass flow rate function. Figure 8 shows the variations and the comparisons of the parameters Cp and Gu obtained from the experimental characteristics, approximation I ($n=5$, $m=10$) and approximation II ($n=5$, $m=5$, $l=2$). It can be noted that the variations are parabolic near zero volts for u, which shows the nonlinear behaviour of the electropneumatic system. The comparison is difficult to do but we note that the control approximation seems to be better. The error between the different approximations and the reference seems to be acceptable but a further analysis is required to see the real influence of these errors on the dynamic behaviour around an equilibrium point.

For that, we have made simulations, which consist of comparing the tank pressure evolutions due to input control steps obtained in the three previous cases.

Figures 9 and 10 show the pressure and absolute pressure error evolutions of the system (6) for great ($1V$) and small ($0.1V$) step amplitudes of input control near the equilibrium pressure ($5.110^5 Pa$) and the equilibrium input control ($0V$).
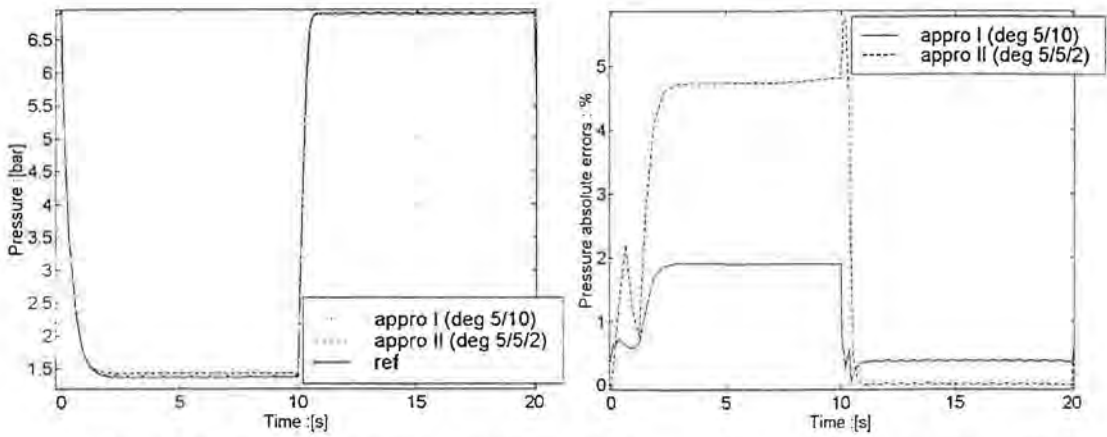
Fig. 9a,b Pressures and absolute pressure errors for 1 volt input control steps.
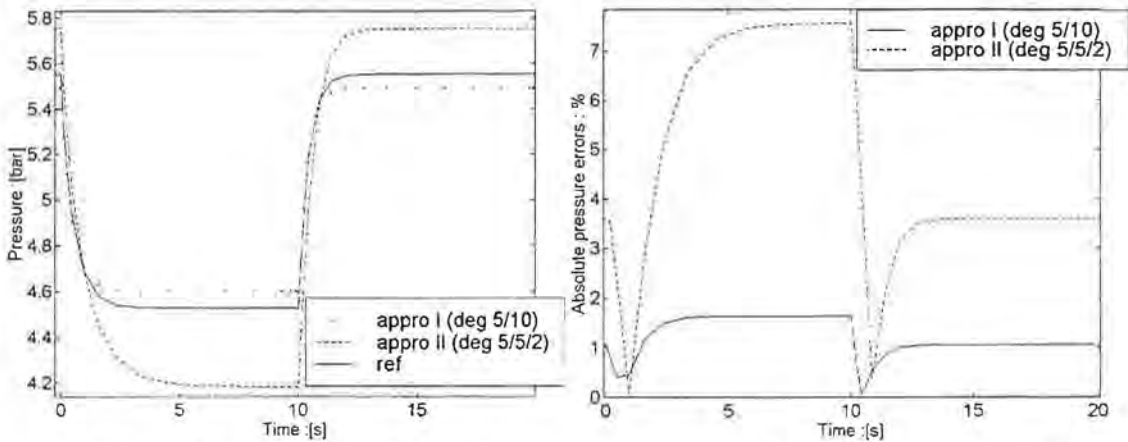

Fig. 10a,b Pressures and absolute pressure errors for 0.1 volt input control steps.

The figure 9 shows that the behaviour is correctly reproducible with the different approximations and the errors are smaller for $u>0$. This is verified too for the small input control steps (see fig.10) even though the errors are greater. The time constant does not reproduce well for the small step amplitude. The approximation I is better for all input control step amplitudes, the pressure error is less than *2%*.

The results show that the coefficients Cp and Gu can be other good criteria to validate the analytic approximations of the flow stage model. It is interesting to note that the errors of the steady state pressure values originate from the errors of the pressure gain characteristics shown in figure 6a.

6   CONCLUSION

In this paper we have shown that it is possible to obtain an analytical model of the servo-distributor flow stage. Two approximation techniques are used, the first method leads to a simulation model defined by a generalised polynomial function and the second leads to a control model defined by several polynomial functions and this last model can also be used for simulation.

The results obtained with our techniques are relatively correct, the first approximation is simpler and give the better results than the second approximation that has the advantage on a physical significance of polynomial functions. To obtain good results it is not

always necessary to increase the polynomial degrees. In fact the use of other different techniques could give better results.

It can be noticed that the errors introduced by the control analytical models are usually well compensated by the feedback used in the control algorithms. This remark explains why the simulation model needs to be more accurate than the control model.

We have defined a physical criterion which corresponds to the pressure gain characteristic but another criterion is necessary to take into account the dynamic aspect: the reproducibility of the first partial derivatives at the equilibrium of the mass flow rate characteristics can be an additional criterion.

These analytical models obtained enable the analysis and synthesis of the nonlinear control to be carried out. This approximation methods proposed may be easily used for other types servo-distributor and servo-valve.

From the control model, an inversion of the servo-distributor characteristic model can be carried out so it can be used, for example for the determination of desired pressure trajectories in applications of the force tracking control [16].

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