A new concept for polychromatic laser guide stars: one photon excitation of 4P\(_{3/2}\) level of sodium atom

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Abstract

The correction of the atmospheric tip-tilt using a polychromatic laser guide star (PLGS) is the only solution currently suggested to run the adaptive optics of ground telescopes over 100% of the sky. The difficulty is to create a sufficiently intense UV chromatic component of the PLGS. We show that, if one uses a gravity centre technique, the returned flux of photons at 330nm must lie between 2\(\times10^5\) photons/s/m\(^2\) and 1.3\(\times10^6\) photons/s/m\(^2\). This paper describes a model which is validated on experimental results on the sky at the observatory of Keck, the LLNL and Pierrelatte. We give the results of this model for the solution already suggested which consists in exciting the 4D\(_{5/2}\) level of mesospheric sodium atoms with two photons, using two lasers operating at 589nm and 569nm (case 2). We present a new method which consists in exciting the level 4P\(_{3/2}\) with a single photon, using a laser operating at 330nm (case 1). Thanks to a modeless laser we show that 1W in case 1 produces the same flux as 30W in case 2. Moreover, to reach necessary flux at 330nm one needs 10W in case 1 whereas one would need more than
400W in case 2. This new method is very promising in terms of flux but also in terms of simplicity.

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1. Introduction

Near-diffraction-limited imagery on very large telescopes has become a major tool in astrophysics. Adaptive optics (AO) allows these observations if a source, within the isoplanetism domain of the observed object is bright enough, to be able to analyze the incident wave surface. The probability to find such bright sources is the main limitation of AO, especially in visible range. To increase the sky coverage, large astronomical facilities use or will implement a monochromatic laser guide star (LGS). An important progress was achieved in 2004 on a 10 meters family telescope at Keck Observatory. This result showed that AO works very well thanks to a 14th magnitude (~2×10^4 photons/s/m^2) natural guide star (that we call, in this paper, TTNGS from « tip-tilt natural guide star ») for tip-tilt correction and a 9.5th magnitude (~1.7×10^6 photons/s/m^2) LGS for higher order corrections. This important result has become a reference. The corresponding experimental data will be used often in this paper. At Keck Observatory, a 50cm diameter projector focuses in the mesosphere a 17W laser (on top of the telescope) at 589nm. LGS star provides the phase reference except for the slope θ, its position being indeterminate. The natural TTNGS star is used to measure θ, and, as a result, another limitation appears: the probability to find a natural star bright enough is in the best case ~10^-7 in visible at the galactic poles. The polychromatic laser guide star
(PLGS) resolves this major difficulty, by creating a multiple wavelengths radiating source which consist of the D$_2$ component corresponding to the LGS star and other UV-VIS-IR components that we call in this paper TTLGS from “tip-tilt laser guide star”.

The tip-tilt $\theta$ is measured via the differential tip-tilt $\Delta \theta$ between chromatic components. $\theta$ is proportional to its derivative $\Delta \theta$ as a function of $\lambda$. The R&D ELP-OA program is probably the only one in the world to ensure 100% sky coverage. This is necessary for very faint astrophysic objects, the priority of 10 meters optical telescopes (DOT). The initial proposal to produce a PLGS consists in exciting the mesospheric sodium atom 4D$_{5/2}$ level via the intermediate level 3P$_{3/2}$, with two dependents lasers at 589nm and 569nm. This excitation with classical lasers raised the first problem of intrinsic atomic saturation of $3S_{1/2} \rightarrow 3P_{3/2}$ and $3P_{3/2} \rightarrow 4D_{5/2}$ sodium transitions. We overcome this first limitation by developing a modeless laser that spread out the peak power over all velocity classes of the sodium atom vapour. Therefore we hope to gain at least a factor 10 on the returned flux. In spite of this gain, there is a risk that the returned flux is not sufficient for ELP-OA program. Indeed, the second limitation comes from the fact that the measurement of $\Delta \theta$, via the TTLGS chromatic components, has to be more precise than the direct measurement of $\theta$, via the TTNGS natural star, that is essential in the case of a monochromatic laser guide star LGS. The proportionality factor that links $\theta$ to $\Delta \theta$ is about 18. The TTNGS and TTLGS stars are measured in the full-field of view of telescope, thus the respective performances can be compared. As a result, photons flux of the TTLGS chromatic components should be $18 \times 18$ times more intense (ie of about 6 times fainter magnitude) than the TTNGS star photons flux. Keck’s and Gemini’s engineers estimate that the magnitude $m_v$ of natural star TTNGS cannot exceed 18$^{th}$
magnitude (~7×10^2 photons/s/m^2) and that the magnitude allowing an acceptable AO operation is about 16 (~4×10^3 photons/s/m^2). In paper 7, authors achieved a Strehl ratio of 36% in K’ band using a 14th (2.6×10^4 photons/s/m^2) magnitude TTNGS. Recent progress has shown that it is possible to achieve the same performance using a 16th magnitude. Thus we can see that the ELP-OA’s TTLGS chromatic components magnitude has to be under 12 (~2×10^5 photons/s/m^2) and equal to 10 (~1.3×10^6 photons/s/m^2) for a good correction. Thus the TTLGS chromatic components have to be almost as intense as the LGS star, measured at Keck Observatory. This is a challenge that requires a precise study of the laser-sodium interaction. The initial proposition to produce a PLGS star with two laser excitation at 589nm and 569nm does not allow this equalization. We demonstrate that the corresponding TTLGS chromatic components are 100 times fainter than the LGS D_2 component. Roughly, if the LGS star has a returned flux of about 10^6 photons/s/m^2 then, the TTLGS star components flux will be around 10^4 photons/s/m^2, that is extremely insufficient. If electronics and algorithms are still able to gain in sensitivity (a factor of 10 seems to be possible), it remains to find a method to increase at least one order of magnitude TTLGS star flux.

Thanks to the remarkable properties of the modeless laser that we developed for ELP-OA, we report on, in this paper a solution that allows gaining a theoretical factor of 10. It consist in producing independent TTLGS and LGS, via a one photon direct excitation of 3S_{1/2} \rightarrow 4P_{3/2} Na transition at 330nm with a modeless laser. Only a modeless laser allows considering this possibility. From an instrumental point of view, this solution has an important advantage because it preserves the current LGS facilities, without any modification. Taking into account atmosphere transmission at 330 nm on the Keck’s site
in Hawaii, we show that a 1W modeless laser at 330nm at the mesosphere (~2-3W at the ground) produces a TTLGS components returned flux identical with the one obtained by an excitation using two lasers of 2×15W (at the mesosphere) at 589nm + 569nm. A 10 times higher flux can be obtained with an about 10W modeless laser at 330 nm. More than 400W at 589nm +569 nm would be necessary to get this gain. Such high power is problematic on astronomical facilities. The new solution that we propose is simpler and cheaper. It allows spatially, temporally and spectrally uncoupling for both TTLGS and LGS stars and corresponding lasers. This simplification brings the whole laser guide star system more robust and reliable.

In this paper we compare, from the theoretical point of view, the photons flux of the radiative cascade generated by one photon excitation of 4P\(_{3/2}\) level at 330 nm of Na atom, with the one generated by two photons excitation of 4D\(_{5/2}\) level at 589 nm and 569 nm for different types of lasers.

2. The model

The model that we have used is similar to the one described in paper 11 for the 3S\(_{1/2}\) \(\rightarrow\) 3P\(_{3/2}\) D\(_2\) transition. We consider the two cases quoted above.

2.1 Case 1: 4P\(_{3/2}\) level one photon direct excitation at 330nm.

4P\(_{3/2}\) level can be directly excited, from the ground state 3S\(_{1/2}\), with a laser centred at 30272.51cm\(^{-1}\) (~330nm). Figure 1 presents the energy levels involved in the radiative cascade of the process. The system of the rate equations that describes the atomic system is:
\[
\frac{\partial N_1(t, r, \nu)}{dt} = -N_1(t, r, \nu) \frac{g_1}{g_5} \int_{-\infty}^{+\infty} \sigma(\nu'-\nu) \Phi(t, r, \nu') d\nu' \\
+ \left( \frac{N_2(t, r, \nu)}{\tau_{12}} + \frac{N_2(t, r, \nu)}{\tau_{21}} + \frac{N_3(t, r, \nu)}{\tau_{31}} \right) \\
+ N_4(t, r, \nu) \int_{-\infty}^{+\infty} \sigma(\nu'-\nu) \Phi(t, r, \nu') d\nu' 
\]

(1)

\[
\frac{\partial N_2(t, r, \nu)}{dt} = \frac{N_4(t, r, \nu)}{\tau_{42}} - \frac{N_2(t, r, \nu)}{\tau_{21}} 
\]

(2)

\[
\frac{\partial N_3(t, r, \nu)}{dt} = \frac{N_4(t, r, \nu)}{\tau_{43}} - \frac{N_3(t, r, \nu)}{\tau_{31}} 
\]

(3)

\[
\frac{\partial N_4(t, r, \nu)}{dt} = -\frac{N_4(t, r, \nu)}{\tau_{42}} - \frac{N_4(t, r, \nu)}{\tau_{43}} - \frac{N_5(t, r, \nu)}{\tau_{54}} 
\]

(4)

\[
\frac{\partial N_5(t, r, \nu)}{dt} = N_1(t, r, \nu) \int_{-\infty}^{+\infty} \sigma(\nu'-\nu) \Phi(t, r, \nu') d\nu' \\
- \frac{N_5(t, r, \nu)}{\tau_{54}} - \frac{N_5(t, r, \nu)}{\tau_{51}} \\
- N_5(t, r, \nu) \frac{g_1}{g_5} \int_{-\infty}^{+\infty} \sigma(\nu'-\nu) \Phi(t, r, \nu') d\nu' 
\]

(5)

\[
\sum_{i=1}^{5} N_i(t, r, \nu) = N_D(\nu) = N_1(t = 0, r, \nu) 
\]

(6)

where

\[ t = \text{time (s)} \]

\[ r = \text{vector of the radial position in the laser beam (m)} \]

\[ \nu = \text{frequency Doppler shift (Hz)} \]

\[ g_1, g_5 = \text{degeneracy of the 3S}_{1/2} \text{ and } 4P_{3/2} \text{ states respectively} \]
\( I/\tau_{ij} = \text{probability of the transition } i \rightarrow j \ (s^{-1}) \)

\( \Phi = \text{laser photon flux density per unit area, unit time and unit frequency (m}^{2}\cdot s^{-1}\cdot \text{Hz}^{-1}) \)

\( \sigma = \text{homogeneous absorption cross section (m}^2 \)

\( N_i = \text{population of the level } i \)

\( N_D = \text{Doppler distribution population assuming a uniform temperature at the mesosphere across the laser beam} \)

The three terms of equation 1 correspond to absorption, spontaneous emission and stimulated emission. Using the same hypotheses as in paper 11, we define the spatial temporal saturation parameter as:

\[
S(t,r,\nu) = \tau_s \int_{-\infty}^{\infty} \sigma(\nu'-\nu)\Phi(t,r,\nu')d\nu' = \tau_s \phi(t)D(r)\sigma_{\text{eff}}(\nu) \tag{7}
\]

The effective cross-section \( \sigma_{\text{eff}}(\nu) \) is the convolution product of the homogenous absorption cross-section \( \sigma(\nu) \) of sodium atom and the normalize laser line shape \( g(\nu) \).

The homogenous absorption has a Lorentzian profile:

\[
\sigma(\nu) = \frac{\sigma_0(\Delta \nu / 2)^2}{(\nu_0 - \nu)^2 + (\Delta \nu / 2)^2} \tag{8}
\]

where \( \sigma_0 \) is the homogenous cross section centred at \( \nu_0 \), and \( \nu_0 \) is the frequency line center. \( \Delta \nu \) is the homogenous linewidth of the transition.

\( D(r) \) is the laser photons flux distribution per unit of area. In order to reduce the calculation time, a commonly used approximation consists in taking \( D(r) \) constant over the diameter \( 2w \) of the laser beam. In practice phase mirrors allow this spatial laser beam profile. We approximate the temporal laser linewidth \( \phi(t) \) as a rectangle function of linewidth \( \tau_L \). Consequently:
\[ S(t, \nu) = N_L \frac{\tau_{31}}{\tau_L} \frac{\sigma_{\text{eff}}(\nu)}{(\pi w^2)} \quad 0 < t < \tau_L \]

\[ S(t, \nu) = 0 \quad t < 0 \text{ et } t > \tau_L \]

where \( N_L = \frac{E}{h \nu} \) is the photons number in a laser pulse and \( E = P_\lambda / f_L \) the pulse energy (where \( P_\lambda \) is the mean laser power value at wavelength \( \lambda \)).

The Doppler distribution can be written as:

\[ N_D(\nu) = N_D^0 \frac{2\sqrt{\ln(2) / \pi}}{\Delta \nu_D} e^{-\frac{2\sqrt{\ln(2)} \nu^2}{\Delta \nu^2}} \]

where \( N_D^0 = \rho \Delta z^{-1} \) is the atomic density by unit of volume , \( \rho \) is the column density et \( \Delta z \) is the column height.

2.2 Case 2: 4D_{5/2} two photons excitation at 589nm and 569nm via 3P_{3/2} level.

Figure 2 shows the system of excitation and the radiative cascade induced by two lasers at 589nm and 569nm locked on the transitions 3S_{1/2} \rightarrow 3P_{3/2} and 3P_{3/2} \rightarrow 4D_{5/2}. The formalism used is the same as the one described above. Two effective cross sections are involved \( \sigma_{01} \) and \( \sigma_{02} \) corresponding to 3S_{1/2} \rightarrow 3P_{3/2} and 3P_{3/2} \rightarrow 4D_{5/2} \) transitions. We will not describe here the rate equations system that is obvious.

3. Returned fluorescence photons flux

3.1 Calculation of returned flux

Neglecting non radiative processes, the fluorescence flux of the TTLGS chromatic component corresponding to \( i \) towards \( j \) level transition is :

\[ \Phi_{ij} = T f_L \frac{A \Delta z}{4 \pi \nu^2} \frac{1}{\tau_{31}} \int d\nu \int d^2r \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} N_i(r, t, \nu) dt = T f_L \frac{A \Delta z}{4 \pi \nu^2} \frac{1}{\tau_{31}} \int d\nu \int d^2r \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} N_i(t, \nu) dt \]

(12)
Δz is the thickness and z is the height of the sodium layer, A is the receiver area at the
ground (fixed to 1m² in the following), T is the atmospheric transmission at the
wavelength corresponding to the transition i→j and fL is the lasers repetition rate.

The simulation was carried out for three types of laser lines g(ν):

1) **laser of type I**: a Gaussian inhomogeneous line shape of width \( \Delta \nu_L = 1 MHz \)
simulating longitudinal single-mode laser;

2) **laser of type II**: the same line as the previous one, followed by a double phase
   modulation at 180MHz and 325MHz in order to generate sidemodes;

3) **laser of type III**: a Gaussian inhomogeneous line shape of width \( \Delta \nu_L = 2.8 GHz \)
   (at 1/e²) for 3S₁/₂→3P₃/₂ and 3S₁/₂→4P₃/₂ transitions and \( \Delta \nu_L = 1 GHz \) for transition
   3P₃/₂→4D₅/₂; this spectral line shape simulates a modeless laser that excites all
   velocity classes of the sodium Doppler/Hyperfine profile\(^{11}\). It is well known that
   the hyperfine structure of 3S₁/₂ level is 1.77GHz.

The different atomic parameters used in this paper are summed up in table 1. In spite of
few divergences found in the literature, the transitions probabilities \( 1/\tau_{ij} \) and homogenous
absorption cross-section \( \sigma_0 \) and \( \sigma_{0i} \) have been taken equal to the one of reference \(^{10}\).

Sodium density \( N_D^0 \) is the most important source of dispersion of experimental results.
Indeed, Megie et al. showed\(^{16}\) that seasonal variations of the sodium column
density \( \rho = \Delta z N_D^0 \) is maximum in November-December, about \( 8 \times 10^9 \) atoms/cm², and
minimum in May-June, about \( 2 \times 10^9 \) atoms/cm², that is to say, a seasonal variation factor
of 4. Sodium concentration can also depend on the site. Moreover, Kwon and al.\(^{17}\)
obseved during the winter of 1987 at Mauna Kea Observatory (Hawaii), sodium
sporadic of about 40min duration that produced sharp sodium abundance peaks of around
6.7×10^9 atoms/cm^2, whereas the mean value was around 3.5×10^9 atoms/cm^2. A detailed study was carried out by Michaille et al.\textsuperscript{18} at La Palma from September 1999 to August 2000 and showed that the sporadic probability events is about 1 per night, on a time scale going from a few seconds to a few hours. The authors concluded that the origin of the sporadic is still uncertain and that adaptive optics operation will not be affected, in terms of defocusing. The mean value of sodium column density measured by the authors at La Palma is about 4×10^9 atoms/cm^2. Gardner et al\textsuperscript{19} also showed that tropospheric gravity waves propagate in the mesosphere creating spatial modulation of the sodium column density able to modify the local value by 100\% in a few hours. In conclusion, all the results found in the literature, show that the mean value of the sodium column density is \( \rho \approx 4 \times 10^9 \text{ atomes/cm}^2 \) \( (N_D^0 = 4 \times 10^9 \text{ atomes/m}^3 \text{ for a column height of } \Delta z = 10 \text{ km}) \).

As \( \Phi \) is proportional to the column density \( \rho = \Delta z N_D^0 \), the value of \( \Delta z \) does not have any effect in the calculation that we present in this paper.

We wrote a numerical program using MATLAB. Neglecting the coherence effects, the following calculations are taking into account each frequencies \( \nu \) with a step of \( \Delta \nu = 1 \text{ MHz} \). The convergence process has been approached very carefully. The numerical calculation of population state has been carried out with an adaptive step integrator. The initial population condition has been given for each frequencies \( \nu \), by the distribution \( N_D(\nu) \).

### 3.2 Required flux

To estimate the required flux, we have used the results concerning LGS star and TTNGS star magnitude published by Keck\textsuperscript{6,7}. Until now, this is the only full experimental set-up
that is working on a 10m telescope facility. A LGS star of magnitude $m_{V}^{LGS} = 9.5$ has been obtained with 10 to 20W laser power (typically 17W) on top of the telescope. Taking into account atmospheric losses we estimate that 15W is projected in the mesosphere. As we said above good tip-tilt correction is ensured by a natural TTNGS star of magnitude $m_{V}^{TTNGS} \approx 16$. It is more convenient to work in terms of number of photons per second and unit area. Consequently, the following conversion\textsuperscript{20} is used:

$$m_{V} = -2.5 \log_{10} \frac{\Phi}{\Phi_{0}}$$ \hspace{1cm} (11)

With Vega as reference star, $\Phi_{0} = 2.54 \times 10^{-10}$ lumens$\cdot$cm$^{-2}$, i.e. in band V centred at 555nm $\Phi_{0} = 1.043 \times 10^{10}$ photons$ / s / m^{2}$.

Thus we calculate for the LGS and TTNGS stars:

$$m_{V}^{LGS} = 9.5 \Rightarrow \Phi_{LGS} = 1.7 \times 10^{6} \text{ photons} / s / m^{2}$$

$$m_{V}^{TTNGS} = 16 \Rightarrow \Phi_{TTNGS} = 4.2 \times 10^{3} \text{ photons} / s / m^{2}$$ \hspace{1cm} (12)

The tip-tilt correction requires a natural guide star TTNGS of a relatively weak intensity. But, the UV-VIS-IR chromatic components of TTLGS star needs much larger intensity. Indeed, the relation between the tip-tilt $\theta$ and the differential tip-tilt $\Delta \theta$ is:

$$\theta = \frac{n_{3} - 1}{n_{2} - n_{1}} \Delta \theta \approx 18 \times \Delta \theta$$ \hspace{1cm} (13)

where $n_{3}$ is the average value of the air refraction index at the observation wavelength and $n_{2}$ and $n_{1}$ are the indexes of the TTLGS chromatic components. The measurement of $\Delta \theta$ has to be 18 times more precise than the measurement $\theta$. Assuming photons noise limitation, the intensity of the TTLGS chromatic components has to be $18^{2} = 324$ times more intense than the intensity of the TTNGS star. The required photon flux is therefore:
\[ \Phi_{TTLGS} = 324 \times \Phi_{TNGS} \approx 1.4 \times 10^6 \text{ photons/s/m}^2 \approx \Phi_{LGS} \]

I.e. the intensity of the UV-VIS-IR chromatic components of the TTLGS must be the same order of magnitude as the LGS star intensity (D\(_2\) component). We show below that this is impossible using case 2 (two photons excitation at 589nm + 569nm).

We conclude that a TTLGS chromatic component flux of about \(10^6\) photons/s/m\(^2\) at 330nm is necessary for the ELP-OA project.

### 3.3 Validation of the model based on Keck, PASS-2 and PASS-1 experimental results

To reproduce the photons flux of the LGS star obtained at Keck, we have run our simulation program for case 2, considering the power of the second laser \(P_{569}=0\), \(f_L=11kHz\) and \(\tau_L=100ns\). The mean sodium column density has been taken equal to the one measured at Mauna Kea Observatory (\(\rho = 4 \times 10^9 \text{ atomes/cm}^2\)). Taking into account the atmospheric transmission in Hawaii, the laser power \(P_{589}\) at the mesosphere at 589 nm was equal to about 15W. The waist value \(w=0.4m\) (see below) has been taken in order to be compatible with the measurements of Keck’s LGS star shape.

Figure 3 displays the result for the three types of lasers presented above. We notice the favourable evolution of the intensity of the LGS star when using from a single mode laser to a modeless laser, via a phase modulated laser. This behaviour was already experimentally measured and theoretically calculated. Our calculation gives a LGS flux of \(1.6 \times 10^6\) photons/s/m\(^2\) for the experimental conditions of Keck Observatory. This value has an excellent agreement with the measurement of Keck \(m_V \approx 9.5\) (\(1.7 \times 10^6\) photons/s/m\(^2\)).
In the ELPOA project, the sky photometry experiment of PASS-2 Pierrelatte/France\textsuperscript{21,22,23} provided a photon flux at 330nm versus the laser power at 589nm and 569nm, for types I and II lasers. Table 2 and figures 4 and 5 compare the experimental results of PASS-2 with our theoretical calculation of case 2, with $P_{589}=P_{569}$. The sodium column density during PASS-2 experiment was not measured. Then, the calculation of the $\Phi_{51}$ flux at 330nm was carried out for a mean sodium column density of $\rho = 4 \times 10^9$ atoms / cm$^2$. Lasers power for PASS-2 experiment is taken at the ground. Thus for lasers beams attenuation, the atmospheric transmission equal to $T=0.8$ has been used. The experimental values considered are average values of the published results, corresponding to a total laser power at the ground $P=P_{589}+P_{569}$ of about 20W and 100W for type II laser and 14W for type I laser. At the ground, the near-diffraction lasers beams area is a rectangle of 28×35mm$^2$. To keep the same near diffraction area at the mesosphere and using the laser shape definition of our calculation, we have considered a waist $w$ of 1m. A factor better than 2 is obtained for PASS-2 experiment, between the experiment and our calculation (see table 2 and figures 4 and 5), that is a very good agreement, according to the hypothesis and experimental conditions.

The photometry experiment of PASS-1\textsuperscript{24} has been achieved with LLNL lasers (ALVIS project), in case 2. The laser experimental set-up is described in paper \textit{Erreur ! Signet non défini.}. Table 3 summaries the well-known experimental parameters. Taking into account the atmospheric transmission at LLNL, the laser average power at the mesosphere $P=P_{589}+P_{569}$, is estimated equal to 280W during the experiment. The variables parameters have been: the repetition rate (4.3-12.9 kHz), the laser power ratio $P_{569}/(P_{589}+P_{569})$ (30%, 50% and 70%), and the laser linewidth at 589nm (1-3GHz).
The set of results has shown a $\Phi_{51}$ returned flux at 330nm that presents little variations between $2.1 \times 10^5$ photons/s/m$^2$ and $5.1 \times 10^5$ photons/s/m$^2$ (figure 4 of paper 24). We lead our calculations for lasers type II and III, a sodium column density $\rho = 4.1 \times 10^9$ atoms/cm$^2$ and the laser power ratio 30%, 50% and 70%. In PASS-1 experiment the laser beam area at the mesosphere is supposed to be of about 1m$^2$ with a Fried parameter $r_0$ of 6cm; this corresponds for our model to a beam waist $w$ of 0.56m. But in paper 25, the measured area beam of the same experimental set-up was 3.14m$^2$ which gives a waist $w$ of 1m. The knowledge of the beam area is critical because with 280W of laser power the saturation of the sodium transitions involved is strong, even with modeless lasers. Table 3 shows the calculation with $w=0.56m$ for type II and III laser and with $w=1m$ for type III. This result displays clearly that the laser used at LLNL was equivalent to a type III laser, i.e. that the laser linewidth covered continuously the sodium Doppler/Hyperfine profile. The ALVIS lasers remained confidential, we do not know what kind of « modulator » and applied voltage was used, but we believe that the LLNL lasers, had the same efficiency as our modeless laser, i.e. all sodium velocities classes in the mesosphere were excited as it supposed in paper 24. The last column of table 3 shows a very good agreement between the experimental results and our theoretical approach. For waist $w=1m$ and type III lasers of 280W total mean power the returned flux is about $3 \times 10^5$ photons/s/m$^2$.

As a result, we can already conclude on two points: i) the experimental LGS and PLGS flux measured at Keck, PASS-1 and PASS-2 validates our model; ii) two 589nm and 569nm modeless lasers (type III) of total power $P=280W$ (at the mesosphere) will produce a returned flux at 330nm between $2 \times 10^5$ photons/s/m$^2$ and $5 \times 10^5$ photons/s/m$^2$. 
This flux is close to the one required (see section 3.2) but, with laser powers which can hardly be installed at an astronomical telescope.

**3.4 Returned flux comparison between case 1 and 2.**

In the following, the repetition rate and the laser pulse width were fixed at the current nominal values of ELP-OA project: $f_L = 17\text{kHz}$ and $\tau_L = 50\text{ns}$. Like we said before, the laser beam spatial shape $D(r)$ is approximated as a rectangular function, with the circular waist $w$. The different international LGS and PLGS projects recommended a 50cm diameter projector for a good astronomical site ($r_0 \sim 10\text{cm}$ at 500nm). Therefore, in this paper the waist value $w$ has been chosen in order to be compatible with Keck measurement of the full width half maximum LGS angular diameter $\alpha = 1.4\text{ arcsec}$ obtained with a 50cm diameter projector and a near-diffraction laser beam. According to the spatial shape definition considered in our calculation, the waist is:

$$w = \frac{tg(\alpha) z}{2 \sqrt{\ln(2)}} \approx 0.4m.$$  

Figure 6 displays the photon flux $\Phi_{51}$ at 330 nm in case 1 (curves 4, 5, 6) and 2 (curves 1, 2, 3). The flux for the other TTLGS wavelengths can be deduced as follows:

$$\Phi_{51} = \frac{1}{2} \Phi_{54} = \Phi_{43} = \Phi_{42} = \Phi_{21} \text{ in case 1 and 2, and } \Phi_{51} = \Phi_{51} \text{ in case 1.}$$

In case 2, $D_2$ transition flux $\Phi_{31}$ is essentially due to the one photon excitation-relaxation process at 589nm (LGS). In figure 6, $P$ corresponds to the total laser power on top of the telescope $P_{589} + P_{569}$ with $P_{589} = P_{569}$ in case 2, or to the UV laser power $P_{330}$ in case 1. Considering that the atmospheric transmission is strongly dependent on the astronomical site, the lasers powers used in section are the one at the mesosphere. The power 30W-VIS
corresponds to current ELP-OA project; i.e. two 20W lasers of type III at 589nm and 569nm at the ground. Table 4 summaries the different possibilities. In both cases 1 and 2, the modeless laser (type III) gives the best results, because the laser transitions considered are strongly saturable. The saturation intensities involved are:

\[ 3S_{1/2} \rightarrow 3P_{3/2} : I_{Sat}^{13} = \frac{h \nu_{589}}{\sigma_{01} \tau_{31}} \approx 185 W/m^2 \]

\[ 3P_{3/2} \rightarrow 4D_{5/2} : I_{Sat}^{36} = \frac{h \nu_{569}}{\sigma_{02} \tau_{63}} \approx 450 W/m^2 \]

\[ 3S_{1/2} \rightarrow 3P_{3/2} : I_{Sat}^{15} = \frac{h \nu_{530}}{\sigma_{05} \tau_{51}} \approx 45 W/m^2 \]

For these three transitions, if the whole laser power of respectively 15W, 15W and 1W excites only one velocity class, a saturation parameter of respectively: 190, 78 and 52 is obtained.

Table 4 shows the flux gain $\gamma$ for different situations versus the normalised situation C$_1$ (case 2, two single-mode type I lasers of 2×15W). A gain $\gamma$ of 11 is obtained in case 2 with two modeless lasers of 2×15W (see C$_3$ of table 4). It is a significant result. But remarkable result is that the same gain $\gamma$ is obtained with only 1W at 330 nm of a modeless laser in case 1 (see C$_6$). Figure 7 emphasizes this important result versus laser power. Consequently, the modeless laser allows considering with great interest the 330nm sodium excitation which was rejected in paper 10 probably because of the strong saturation induces with a single-mode excitation (see C$_4$ of table 4). Figure 6 displays the evolution of the photon flux at 330nm $\Phi_{330}$. A flux $\Phi_{330}$ of $4.2 \times 10^4$ photons/s/m$^2$ can be obtained in case 2 with two type III lasers of 2×15W at 589nm and 569nm, and in case 1 with one type III laser of only 1W at 330nm (see C$_3$ and C$_6$ of table 4). This flux is insufficient. But, we can see on figure 6 that, in case 1, the average slope is $\eta_1=3 \times 10^4$
This very encouraging result shows that 10W at 330nm (case 1) is enough to produce a flux of $3 \times 10^5 \text{photons/s/m}^2$ whereas 400W at 589nm and 569nm (case 2) produces only $1.8 \times 10^5 \text{photons/s/m}^2$ (see C, and C, of table 4). This last power cannot be implemented on an astronomical site, whereas 10W at 330nm is much easier. The UV star being absolutely necessary for PLGS, because is situated in a strong atmospheric dispersion wavelength region, case 2, initially considered, would need a large effort in order to reach the required specifications of section 3.2. Fundamentally, the reason comes from the fact that, in case 2, the flux $\Phi_{31}$ at 330nm is at least two orders of magnitude smaller than the flux $\Phi_{31}$ at 589nm ($D_2$). Figure 8 displays the ratio evolution $\Phi_{31}/\Phi_{51}$ ($=\Phi_{D2}/\Phi_{330}$) versus $P$, in case 2 and for three lasers types’ I-II-III. For the value 30-VIS in figure 6, that corresponds to the current ELP-OA program, using two modeless lasers of 2x20W (2x15W in the mesosphere), the theoretical flux $\Phi_{31}$ at 330nm and $\Phi_{31}$ at 589nm are respectively $4.2 \times 10^4 \text{photons/s/m}^2$ and $4.4 \times 10^6 \text{photons/s/m}^2$. The flux ratio $\Phi_{D2}/\Phi_{330}$ of ~105 is an intrinsic property of sodium atom.

4. Discussion

In the following, we consider the most favourable situation, corresponding to type III laser with ELP-OA characteristics.

4.1 Case 2: two photons excitation

The pulsed delay and the power balance between the two lasers can be optimized. Figure 9 shows an optimum delay of 3.7ns. A 10% gain is expected, but lasers temporal jitter of a few ns implies gain losses of same order of magnitude. Likewise, figure 10 shows an
optimum gain of 10%, when the power is distributed as a ratio of 35% on 589nm laser and 65% on 569nm laser (case $P=30W$). These gains are not sufficient to justify an extra cost of development.

It was shown during PASS-2 experiment that the radial superposition of the two lasers beams was not better than 10%. This implies a flux decrease of 20%. Moreover, this superposition will be difficult to realise, when the two lasers will have to follow the observation star. Indeed, we will have to consider the prism effect of the atmosphere that must be compensated in real time when the laser beam is launch out of zenith. This is a major complication.

Without AO on laser emission the two lasers spots at the mesosphere have a speckle structure when the beam size in the low atmosphere is larger than $r_0$. The speckle structure that is a function of wavelength is different for each wavelength. As a result, the efficiency of the two photons excitation could decreases dramatically.

As seen before, case 2 would need lasers powers of hundreds of watts. Although the wavelength at 589nm can be produced using solid state lasers (frequency addition of two YAG lasers or two fibre lasers, Raman laser…), no solid state lasers can generate 569nm. For the moment only dye lasers allow these wavelengths with high powers and high efficiency. The isotopic separation programs have demonstrated that is possible to obtain 1000W around 589nm and 569nm. Moreover, around 100W, the modeless laser starts again to raise the problem of the saturation of sodium atom transitions.

Therefore, we see that in case 2, they are several difficult technical limitations and there are little possibilities to increase significantly the flux of the chromatic components of the TTLGS star. An incoherent excitation in case 2, with type III lasers allows a maximum
population transfer of 7% for 2×15W and 17% for 2×200W in 4D\textsubscript{5/2} state. Only a coherent excitation, with a theoretical transfer of population of 100% in 4D\textsubscript{5/2} state could allow a significant gain\textsuperscript{30}. This solution is definitely very difficult to implement on the sky and on an astronomical site, because it requires ultra-fast lasers, with energies per pulse of about few µJ/cm\textsuperscript{2} that imply, at the mesosphere, mean power approaching 100W with required spot size and lasers repetition rate. Many technical problems will probably imply a transfer significantly weaker than 100%.

4.2 Case 1: one photon excitation

The direct excitation of 4P\textsubscript{3/2} at 330nm (case 1) is more promising for several reasons. First, this solution is fundamentally more efficient, because only one laser of type III of 1W allows a transfer of 3% in the sodium state 4P\textsubscript{3/2} and 20% for 10W. It is well known, that assuming a two levels absorption transition, the maximum population transfer can be 50% in the case of an incoherent excitation. Let us recall that a flux $\Phi_{51}$ of $3\times10^5$ photons/s/m\textsuperscript{2} is achieved in the conditions defined in section 3.2: laser of type III of 10W (in the mesosphere), $f_c=17$kHz, $\Delta t=50$ns, $w=0.4m$, $\rho=4\times10^9$atoms/cm\textsuperscript{2}. Many parameters can allow the optimization of the returned flux. For example, a cw laser of 10W at 330nm would give a photons flux $\Phi_{51}$ of $1.1\times10^6$ photons/s/m\textsuperscript{2} (see C\textsubscript{9} of table 4). This value is much larger than the flux value $\Phi_{51}$ of $4.2\times10^4$ photons/s/m\textsuperscript{2} obtained from two lasers of 2×15W (in the mesosphere) of type III for current ELP-OA project.

Despite the weaker atmospheric transmission at 330nm on a good astronomical site (40%-50% instead of 80%), a modeless laser 2-2.5 W (at the ground) is enough to equal the current solution considered for ELP-OA project, and a modeless laser of 20-25W for a flux 10 times more intense. Contrary to 589nm and 569 nm, for 330nm it exist efficient
solution of solid states lasers. Quasi-cw laser of 10W at 355 nm with $M^2<1.2$ is already in sale. There are solid state systems such as: Nd:YAG pumped by laser diodes and frequency tripling. Nd doped YAG or YLF matrix has intense laser line at 1321nm. Nd :YLF has its maximum gain at 1321nm. Using frequency quadrupling we can obtain in the previous case the laser line at 330nm. There are also high power optical fibers lasers at 1321nm. We can also realise frequency addition of frequency doubling of a 532nm Nd :YAG with a 870nm Sa :Ti laser or a high power laser diodes. UV laser diode progress is promising. Laser diode working directly at 375 nm is in sale. Other solutions already exist. While waiting for this kind of solid sources development, we can easily use a frequency doubling high yield DCM dye laser at 660nm, to obtain power higher than 10 watts at 330 nm.

Another important advantage is the simplification on an astronomical site. The solution that we propose does not modify monochromatic laser guide star facility (LGS) that is working on very large telescopes. For tip-tilt correction it is sufficient to replace the TTNGS star by TTLGS star produced by an independent laser at 330nm. The TTLGS star can be positioned anywhere in the isoplanetism field. It is not necessary to extract the superimposed component $D_2$ and at least two UV-VIS-IR components of the PLGS star. This is a gain in simplicity and yield. For example, no grating is needed to extract the $D_1$ and the $D_2$ components, as in the case of PASS-1 and PASS-2. Furthermore in case 2, $D_2$ line has two components: one comes from the one photon excitation at 589nm (LGS) and the other one from the radiative cascade (TTLGS) (see figures 1 and 2). The last one would be useful to measure $\Delta \theta$. However they cannot be separated. Thus, to measure $\Delta \theta$
in case 1, D\(_2\) component can be used together with D\(_1\) component, that doubles the returned flux at 589 nm and eliminates the use of a grating.

The technical problems of spatial superposing and time synchronisation evoked above disappear because the two lasers that produce TTLGS and LGS are independent. This means a better reliability. An additional flexibility is the choice of lasers repetition rate. Indeed, the atmospheric coherence times of the wave front and of the tip-tilt are:

\[
\tau_{0,\text{wavefront}} = \frac{r_0}{\Delta \nu} \tag{14}
\]

\[
\tau_{0,\text{tilt}} = 12.33 \left( \frac{D}{r_0} \right)^{1/6} \left( \frac{r_0}{v} \right) \tag{15}
\]

\(r_0\) is the Fried parameter, \(D\) the telescope diameter and \(v\) the wind velocity. In visible range (~0.5\(\mu\)m) and for telescopes of 8-10m range, \(\tau_{0,\text{wavefront}}\) and \(\tau_{0,\text{tilt}}\) are about to 10 ms and 100ms. As a result, the laser at 330nm that produces TTLGS star can have a repetition rate of 10 times weaker than the one at 589 nm used for high orders correction. This possibility can be interesting. For example, in order to increase the returned flux, \(4P_{3/2}\) direct excitation could also be produced by an ultra fast laser for 100% transfer of the ground state population to the excited state \(4P_{3/2}\) using laser coherent transfer or chirped laser adiabatically transfer. The essential point for a 100% transfer is the required energy per pulse which must be of the order of few \(\mu\)J/cm\(^2\). At 17 kHz repetition rate this implies huge mean laser power (850W for a 1m\(^2\) spot, 5\(\mu\)J/cm\(^2\) and repetition rate of 17 kHz). Then, we could think that the laser repetition rate can be decreased until it becomes inversely proportional to the time of coherence of the tip-tilt, in order to reach the required energy per laser pulse for the same average power. However, in terms of photon flux per second it is clear that to keep a 100% population in the excited state for each
laser pulse during time $dt$ at 1kHz is equivalent to maintain a 10% population during time $dt$ at 10kHz. The cost and technical difficulties determine the choice.

Another question is the angular extension of the laser beam versus wavelength. Atmospheric turbulence limits the beam propagation. The Fried parameter $r_0$ varies with wavelength: $r_0 \propto \lambda^{6/5}$. An average value of the Fried parameter $r_0$ at a good astronomical site is about 10cm at 589nm and 5cm at 330nm. If the laser beam size is close or larger then the angular spot size is limited by the seeing $\lambda/r_0$. The seeing varies slowly with $\lambda$ as $\lambda^{-1/5}$. This means, that a spot size at the mesosphere needs a projector of approximately the same diameter at 589-569nm and 330nm.

If the noise in measuring $\Delta\theta$ was dominated by the Rayleigh residual scattering of atoms and molecules of the higher atmosphere observed in the telescope field of view, we could think that the solution presented in case 1 would be worse from this point of view. However, this is not the case. Indeed, the Rayleigh scattering increases like: $\eta_R = \frac{P_{330}}{P_{589}} \left( \frac{\lambda_{589}}{\lambda_{330}} \right)^4$. Case 1, involving less power laser, is in fact more convenient. For 1W at 330nm and 30W at 589nm+569nm $\eta_R$ is 0.3 and for 10W at 330nm and 400W at 589nm+569nm $\eta_R$ is 0.25. For an identical returned flux, the Rayleigh scattering is thus 3 or 4 times weaker in case 1. Moreover, we will demonstrate in a future paper that thanks to the modeless laser, the Rayleigh scattering can be eliminated.

Finally, one may expect that, on the new generation telescopes, the telescope vibration will be negligible and the optical axes will be stable. As a result the UV beam could be projected at the mesosphere by the telescope itself. Thus, the measurement of $\Delta\theta$ will not require the measurement of the gravity centre of the UV component of TTLGS because,
thanks to the light principle of reverse return light, the gravity centre of the UV component will not move. The measurement of only one VIS or IR chromatic component will thus be enough. This will represent a gain factor of 2 in the calculation time and an additional simplification in the optical system of observation.

5. Conclusion

The calculations using the model presented in this paper are in very good agreement with the experimental results on the sky of Keck, the LLNL and Pierrelatte. Our model clearly shows that the new solution suggested (case 1) which consists in exciting the sodium atom with one 330nm photon needs only 1W of a modeless laser to produce a returned flux of $4.2 \times 10^4$ photons/s/m$^2$ at 330nm. This flux is equivalent to that obtained by an excitation with two photons using two modeless lasers operating at 589nm and 569nm of 15W each (case 2). The flux which is required for the differential measurement of the tip-tilt $\theta$ is the main issue addressed in this paper. Taking the coherence time of the atmospheric tip-tilt into account, $\theta$ must be corrected at a frequency close to 100Hz. Under these conditions only a measurement of the centre of gravity of the chromatic components of TTLGS allows such a frequency. Preceding flux is then largely insufficient because one would have only a few hundreds of photons for the measurement of the centre of gravity which must be of an extreme precision. We showed that a returned flux at 330nm ranging between $2 \times 10^5$ photons/s/m$^2$ and $1.3 \times 10^6$ photons/s/m$^2$ is necessary. Our calculations clearly show that a modeless laser or quasi-continuous laser of 10W operating at 330nm makes it possible to fill the schedule of conditions whereas
one would need more than $2 \times 200W$ at $589nm + 569nm$. In order to validate this proposal, experiments in laboratory are in progress.

The solution suggested has also other advantages:

1. Two completely independent lasers at $330nm$ and $589nm$ leave a richer technological choice. All solid state lasers are then possible. One of the major disadvantages of case 2 is the wavelength at $569nm$ which does not have, to our knowledge, any solid state laser solution. It thus imposes dyes lasers for the two wavelengths because they must have rigorously identical characteristics.

2. The independence of LGS and TTLGS simplifies the system largely and allows an up-grade without deep modification of the existing LGS. The problems of tip-tilt and higher orders corrections can then be optimized more easily and separately. To obtain a sufficient intensity at $330nm$, case 2 would require a power of more than $200W$ for the $589nm$ laser. But, such a power is useless for the production of LGS. $20W$ are enough.

3. The $330nm$ laser power to be implemented is relatively low and thus one can already consider very compact solid state systems which would be fixed directly on the telescope. It would not be then necessary to transport the beam any longer.

Acknowledgments
Table caption

Tab. 1.

Mesospheric sodium atoms parameters.

Tab. 2.

Parameters of the PASS-2 experiment and simulation. The lasers powers indicated for PASS-2 are ground powers. T is the atmospheric transmission at 589nm and 569nm which is supposed to be 0.8 at Pierrelatte.

Tab. 3.

Parameters of the PASS-1 experiment and simulation. See text.
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Table 4
Figure captions

Fig. 1.
Energy diagram and relaxation pathways of the one photon excitation of the 4P\textsubscript{3/2} sodium level at 330nm (case 1).

Fig. 2.
Energy diagram and relaxation pathways of the two photons excitation of the 4D\textsubscript{5/2} sodium level at 589nm+569nm (case 2).

Fig. 3.
Returned fluorescence flux at 589nm (Φ\textsubscript{31}, LGS) versus average laser power (at the mesosphere) with Keck’s laser characteristics for three types of lasers: i) 1 MHz single longitudinal mode (dashed), ii) 1 MHz single longitudinal mode followed by a double phase modulation at 180MHz and 325MHz (dashed point), iii) modeless (solid). The arrow corresponds to the typical laser power and returned flux of Keck Observatory. This calculation corresponds to case 2. It shows a gain of 6 between single mode and modeless excitation.

Fig. 4.
Returned flux at 330nm of PASS-2 experiment at CEA Pierrelatte (see paper 23) versus the peak power at 589nm (\(\frac{P_{589}}{f_L\tau_L}\)) and total mean power (\(P_{589}+P_{569}\)). The dots correspond to the experimental results using type II lasers and the solid curve corresponds to the numerical simulation in case 2 with a column sodium density of \(4\times10^9\) atoms/cm\(^2\).
Fig. 5.

Experimental and simulation of PASS-2 with type I lasers.

Fig. 6.

Return fluorescence flux at 330nm versus average laser power for the three types of lasers: a) 1 MHz single mode laser (curves 1 and 4), b) 1 MHz single mode laser followed by a double phase modulation at 180MHz and 325 MHz (curves 2 and 5) and c) modeless laser (curves 3 and 6). Solid curves correspond to the one photon excitation at 330nm (case 1) and dashed curves to the two photons excitation at 589nm + 569nm (case 2). The cross at 30W-VIS on the curve 3 corresponds to the ELP-OA project. The cross at 1W-UV on the curve 6 corresponds to the UV laser power at 330nm that equalize the flux of photons of the ELP-OA project. The cross at 10W-UV on the curve 6 corresponds to the flux that is closed to the required flux. 10W in case 1 gives 12 times more flux than 30W in case 2.

Fig. 7.

Ration between the returned flux at 330nm for a modeless laser excitation and for a single mode laser in the two cases as a function of laser power. Case 1: one photon excitation (solid curve). Case 2: two photons excitation (dashed curve).

Fig. 8.

Ratio evolution between the returned flux at 589nm and 330nm as a function of total laser power in case 2 and for the three types of lasers: i) type I - 1 MHz single mode laser (dashed), ii) type II - 1 MHz single mode laser followed by a double phase modulation at 180MHz and 325 MHz (dashed point) and iii) type III - modeless laser (solid). The cross
indicates that at 30W total laser power (ELPOA) the returned flux at 330nm is at least two orders of magnitude smaller than the one at 589nm.

**Fig. 9.**

Returned flux at 330nm as a function of the delay between the 589nm and 569nm pulses in case 2. A gain of 10% is expected for a 3.7ns delay. Total lasers power is 30W.

**Fig. 10.**

Returned flux at 330nm as a function of the ratio between 589nm laser power and total lasers power (589nm + 569nm) in case 2. A gain of 10% is achieved when the power is distributed as a ration of 35% at 589nm and 65% at 569nm. Total lasers power is 30W.
Figure 1
Figure 2
Figure 3
Total mean power \( P = P_{589} + P_{569} \) (W)

Figure 4
Flux at 330nm (photons/s/m²)

Total mean power (W)

Figure 5

PASS2 simulation:
Single mode
f_L = 5kHz
τ_L = 35ns
w = 1m
ρ = 4x10⁹ (atoms/cm²)
\[ \Phi_{330\text{nm}} = \Phi_{51} \text{ (photons/s/m}^2) \]

1: 589nm+569nm: Laser type I
2: 589nm+569nm: Laser type II
3: 589nm+569nm: Laser type III
4: 330nm: Laser type I
5: 330nm: Laser type II
6: 330nm: Laser type III

\[ P = P_{589} + P_{569} \text{ or } P_{330} \text{ (W)} \]

Figure 6
Figure 7
$\Phi_{D_2}/\Phi_{D_1} = \Phi_{31}/\Phi_{51}$

Case 2: 589nm+569nm

- Lasers type I
- Lasers type II
- Lasers type III

$P = P_{589} + P_{569}$ (W)

Figure 8
Figure 9
$\Phi_{330}$ (photons/s/m$^2$)

Figure 10
References


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