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► **To cite this version:**

Monika Wragowska, Czeslaw Wozniak, Olga Szlachetka. A model of the heat conduction for functionally graded laminate with uniformly distributed micro inclusions. PAMM, 2012, 12, pp.413-414. hal-01117991

**HAL Id: hal-01117991**

**<https://hal.science/hal-01117991>**

Submitted on 18 Feb 2015

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# A model of the heat conduction for functionally graded laminate with uniformly distributed micro inclusions

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The aim of the contribution is to formulate an asymptotic model of heat conduction for functionally graded two component laminate reinforced by periodically spaced micro inclusions.

## 1 Introduction

Laminate occupies region  $(0, L) \times \Xi$  when  $\Xi$  is a regular region on  $Ox^2x^3$ - plane. The composite is made of a large number  $p$ ,  $\frac{1}{p} \ll 1$ , of layers with the same thickness  $l$ ,  $l = \frac{L}{p}$  and every layer has a symmetry plane  $x_n = \frac{l}{2} + (n-1)l$ ,  $n = 1, \dots, p$ . The scheme of this composite is shown in Figure 1.

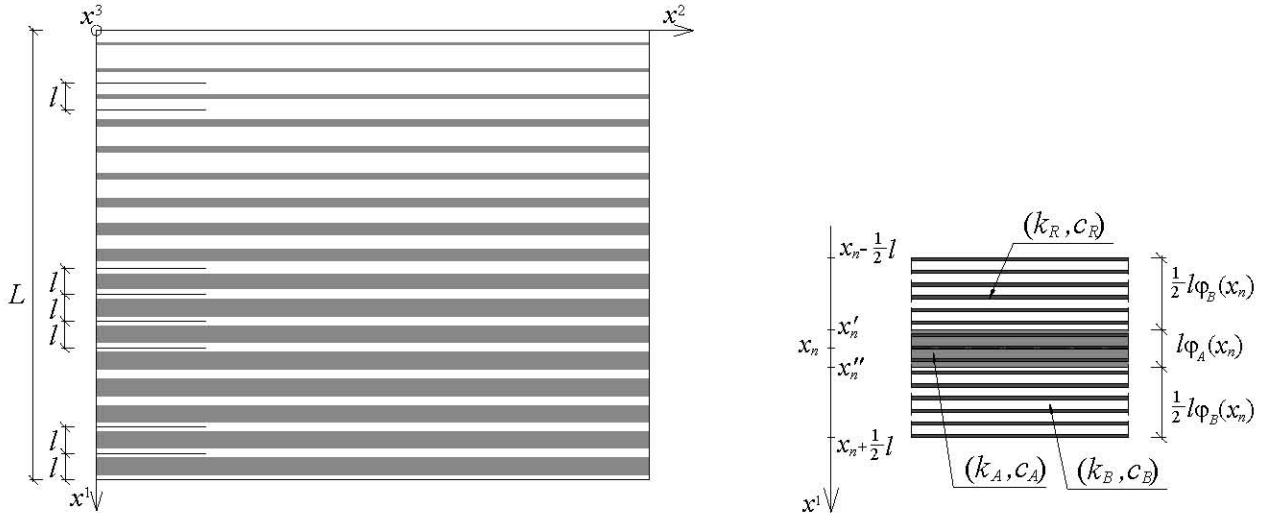


Fig. 1: A cross section of a functionally graded laminate and its fragment with periodically spaced micro inclusions.

## 2 Modelling Procedure

In order to obtain the model of this laminate we introduce material fractions  $\varphi_A$ ,  $\varphi_B$  such that  $\varphi_A + \varphi_B = 1$ ,  $\varphi_A \in C^1([0, L])$ , and  $\forall x^1 \in (0, L)$   $[\varphi_A(x^1) \in (0, 1)$  and  $l |\partial_1 \varphi_A(x^1)| \ll 1]$ . It has to be remembered that the lamina materials  $A$  and  $B$  have an extra reinforcement micro periodic spaced micro inclusions. Every lamina is made of the isotropic material having heat conductivity  $k_A$  and  $k_B$  and specific heat  $c_A$  and  $c_B$ , respectively. The uniform distribution of micro inclusions in every lamina is characterized by constant heat conductivity  $k_R$  specific heat  $c_R$  and by constant fraction coefficient  $\nu_R$ ,  $\nu_R \ll 1$ .

In the first step of modelling we homogenize reinforcement material independently with isotropic component materials. The effective heat conductivities and specific heat in every lamina after homogenization take the form:

$$\begin{aligned} K_A^1 &\equiv \frac{k_A k_R}{\nu_R k_A + (1 - \nu_R) k_R}, & K_A^2 = K_A^3 &\equiv (1 - \nu_R) k_A + \nu_R k_R, & C_A &\equiv (1 - \nu_R) c_A + \nu_R c_R, \\ K_B^1 &\equiv \frac{k_B k_R}{\nu_R k_B + (1 - \nu_R) k_R}, & K_B^2 = K_B^3 &\equiv (1 - \nu_R) k_B + \nu_R k_R, & C_B &\equiv (1 - \nu_R) c_B + \nu_R c_R. \end{aligned} \quad (1)$$

In the second step of modelling [1, 2] we shall deal with functionally graded laminate made of two orthotropic materials having material properties given by formulas (1).

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The idea of the modelling is based on the concept of the local layer [1] which has a form similar to that given above but has a symmetry plane at an arbitrary but fixed  $x \in (0, L)$ . We also introduce a local periodic shape function  $g_x(\cdot)$  defined on  $[x - \frac{l}{2}, x + \frac{l}{2}]$  as  $g_x(x - \frac{l}{2}) = g_x(0) = g_x(x + \frac{l}{2}) = 0$ ,  $g_x(x - \frac{l}{2}\varphi_A(x)) = \frac{l}{2}$ ,  $g_x(x + \frac{l}{2}\varphi_A(x)) = -\frac{l}{2}$  and is linear in  $(x - \frac{l}{2}, x - \frac{l}{2}\varphi_A(x))$ ,  $(x - \frac{l}{2}\varphi_A(x), x + \frac{l}{2}\varphi_A(x))$ ,  $(x + \frac{l}{2}\varphi_A(x), x + \frac{l}{2})$ ,  $x \in (0, L)$ .

The modelling procedure is based on two assumptions. The first assumption is strictly related to the asymptotic passage  $l \rightarrow 0$ . Let for every  $x \in (0, L)$   $f_x$  stand for  $K_A^1, K_A^2, K_A^3, C_A, K_B^1, K_B^2, K_B^3, C_B$ , then for  $x \in (0, L)$   $f_x \in L_{loc}^2(R)$ .

If  $l \rightarrow 0$  then  $f_x \rightarrow \langle f_x \rangle \equiv \frac{1}{l} \int_{x-\frac{l}{2}}^{x+\frac{l}{2}} f_x(y) dy$  for every  $x \in (0, L)$ .

The second assumption is that the temperature field  $\theta_x(\cdot)$  in every local layer is assumed in the form:  $\theta_x(y, t) = \vartheta(y, t) + g_x(y^1) \psi(y, t)$ ,  $y^1 \in (-l/2, l/2)$ ,  $y = (y^1, y^2, y^3) \in (-l/2, l/2) \times \Xi$ , where  $\vartheta(\cdot)$ ,  $\psi(\cdot)$  are new unknown functions. From the above assumptions applying limit passage  $l \rightarrow 0$  and taking into account the heat flux continuity at the interfaces we obtain the averaged heat fluxes in the form:

$$Q^1(\mathbf{x}, t) \equiv -\tilde{K}^1(x^1) \partial_1 \vartheta(\mathbf{x}, t), Q^2(\mathbf{x}, t) \equiv -\tilde{K}^2(x^1) \partial_2 \vartheta(\mathbf{x}, t), Q^3(\mathbf{x}, t) \equiv -\tilde{K}^3(x^1) \partial_3 \vartheta(\mathbf{x}, t), \quad (2)$$

where

$$\begin{aligned} \tilde{K}^1(x^1) &\equiv \frac{K_A^1 K_B^1}{K_A^1 \varphi_B(x^1) + K_B^1 \varphi_A(x^1)}, \\ \tilde{K}^2(x^1) &= \tilde{K}^3(x^1) \equiv K_A^2 \varphi_A(x^1) + K_B^2 \varphi_B(x^1), \\ \tilde{C}(x^1) &\equiv C_A \varphi_A(x^1) + C_B \varphi_B(x^1), \quad x^1 \in (0, L). \end{aligned} \quad (3)$$

and the balance equation:

$$\partial_1 Q^1(\mathbf{x}, t) + \partial_2 Q^2(\mathbf{x}, t) + \partial_3 Q^3(\mathbf{x}, t) + C(x^1) \partial_t \vartheta(\mathbf{x}, t) = 0. \quad (4)$$

Formulas (3), (4) represent the proposed model equations of heat conduction for a functionally graded media with micro-periodic inclusions under consideration.

### 3 Final Remark

The obtained system of equations for functionally graded laminate with uniformly distributed micro inclusions is characterized by continuous and nonperiodic coefficients.

### References

- [1] C.Woźniak, B.Michalak, and J.Jędrzyński, (eds.), Thermomechanics of heterogeneous solids and structures, tolerance averaging approach, Wydawnictwo Politechniki Łódzkiej, Łódź (2008).
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