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2D modelling of thin skeletonal elastic shallow shells

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The subject of this contribution is a certain thin skeletonal elastic shallow shell. The aim of analysis is to derive and apply a 2D-macroscopic model for shallow shells with the non-uniformly oscillating microstructure. The main feature of the proposed mathematical model is that the microstructure length parameter λ is similar compared to thickness δ of the shell ($\lambda \cong \delta$).The formulation of approximate mathematical model of these shells is based on a tolerance averaging approximation [5]. The general results of the contribution will be illustrated by the analysis of a specific problem.

1 Introduction

The considered skeletonal shell is made of two families of thin beams. The regions situated between the beams are fills a homogeneous matrix material (Fig.1). From a formal point of view, the structure under consideration can be described in the framework of the well-known theories for thin elastic shallow shells. However, due to the inhomogeneous microstructure of the shell, this direct description of the structure leads to shell equations with discontinuous and highly oscillating coefficients. The aim of the contribution is to formulate non-asymptotic averaged model describing composite shallow shell under consideration. The formulation of the macroscopic mathematical model for the analysis of dynamic behaviour of these shells will to be based on the tolerance averaging technique. The applications of this technique for the modelling of various dynamic problems of elastic periodic composites are given in a series papers [1], [2], [3], [4].

2 Direct description

The starting point of this contribution is the direct description of the structure in the framework of the well known theory of thin shallow shells. The displacement field of the arbitrary point of the shallow shell we write in form

$$w_3(x,z) = w_3(x) \qquad\qquad w_\alpha(x,z) = w_\alpha^0(x) - \partial_\alpha w_3(x)\, z \tag{1}$$

Denoting by $\mathbf{p}(\mathbf{x},t)$ the external forces, by ρ the mass density and setting $\partial_k = \partial/\partial x^k$, in the framework of the linear approximated theory for thin shallow shells, we obtain the following system of equations

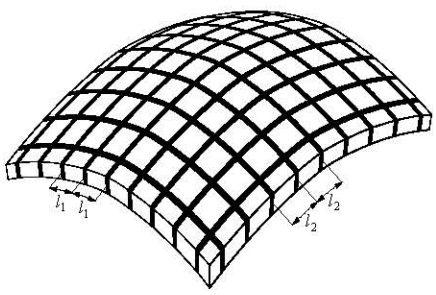

Fig. 1 Fragment of skeletonal shell.

(i) strain-displacement relations

$$\varepsilon_{\alpha\beta}(x,z) = \varepsilon_{\alpha\beta}^0(x) + \kappa_{\alpha\beta}(x)\; z, \qquad \varepsilon_{\alpha\beta}^0 = \partial_{(\beta} w_{\alpha)}^0 - b_{\alpha\beta}\, w_3, \qquad \kappa_{\alpha\beta} = -\partial_{\alpha\beta} w_3 \tag{2}$$

(ii) strain energy averaged over the shell thickness

$$E(x) = \tfrac{1}{2} B^{\alpha\beta\gamma\delta}\, \partial_{\alpha\beta} w_3\, \partial_{\gamma\delta} w_3 + \tfrac{1}{2} D^{\alpha\beta\gamma\delta} (\partial_{(\beta} w_{\alpha)}^0 - b_{\alpha\beta}\, w_3)(\partial_{(\delta} w_{\gamma)}^0 - b_{\gamma\delta}\, w_3) \tag{3}$$

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2 Averaged description

Introduce the orthogonal coordinate system $Ox^1x^2x^3$. Let the midsurface of the undeformed shell under consideration be given by $x^3 = z(x^1, x^2)$, $(x^1, x^2) \in \Omega$, where plane $\Omega = (-L_1/2,\ L_1/2) \times (-L_2/2,\ L_2/2)$ is the shell projection on $x_3 = 0$. In the case of a shallow shells we will be to consider the non-uniform cell distribution in Ω. Every $\Delta(x)$, where x belong to Ω, will be referred to the cell in Ω with center at x

$$\square(x) = \left(x_1 - \frac{\lambda_1}{2}, x_1 + \frac{\lambda_1}{2}\right) \times \left(x_2 - \frac{\lambda_2}{2}, x_2 + \frac{\lambda_2}{2}\right), \qquad x = (x_1, x_2) \in \Omega: \qquad \square \equiv \{\square(x),\ x \in \Omega\}. \tag{4}$$

Denoting by $\lambda_\alpha(x_\alpha)$ the diameter of $\Delta(x)$ and define $\lambda = \sup(\lambda_\alpha)$ as a microstructure length parameter, we assume that $\lambda_\alpha(x_\alpha)$ are small compared to the minimum characteristic length dimension of region Ω ($\lambda_\alpha << L_\alpha$). At the same time, the thickness h of the shell under consideration is supposed to be constant and of an order of the microstructure length parameter λ ($\lambda \approx h << \min\{L_1, L_2\}$).

The modeling technique will be based on the tolerance averaging approximation and on the restriction of the displacement field under consideration given by

$$w_3(x,z,t) = V_3(x,t), \qquad w_\alpha(x,z,t) = V_\alpha(x,t) + \varphi^{\mathrm{A}}(x) V_\alpha^A(x,t) + (-\partial_\alpha V_3(x,t) + \varphi^{\mathrm{A}}(x) u_\alpha^A(x,t))z \tag{5}$$

The governing equations derived from stationary action principle of the averaged lagragian $<\mathcal{L}> = <K> - <E> + <F>$ have the form

$$\begin{aligned}
&\partial_{\alpha\beta}\left(\left(\widetilde{B}^{\alpha\beta\gamma\delta} - \widetilde{B}^{\tau A\alpha\beta}\frac{\widetilde{B}^{\mu A\gamma\delta}}{\widetilde{B}^{\mu A\tau B}}\right)\partial_{\gamma\delta}V_3\right) - \widetilde{D}^{\alpha\beta}\,\partial_\beta V_\alpha - \widetilde{D}^{\alpha A}\,V_\alpha^A + \widetilde{D}^{33}\,V_3 + <\rho h>\ddot{V}_3 = <f^3> \\
&\partial_\delta(\widetilde{D}^{\alpha\beta\gamma\delta}\,\partial_\beta V_\alpha + \widetilde{D}^{\alpha A\gamma\delta}\,V_\alpha^A - \widetilde{D}^{\gamma\delta}\,V_3) - <\rho h>\ddot{V}_\gamma = -<f^\gamma> \\
&\widetilde{D}^{\alpha A\gamma\delta}\,\partial_\delta V_\gamma + \widetilde{D}^{\alpha A\gamma B}\,V_\gamma^B - \widetilde{D}^{\alpha A}\,V_3 + <\rho h\ \varphi^A\,\varphi^B>\ddot{V}_\alpha^B = <f^\alpha\,\varphi^A>
\end{aligned} \tag{1}$$

The above equations have the smooth and functional coefficients in contrast to equations in direct description with the discontinuous and highly oscillating coefficients.

3 Conclusion

The results obtained in this contribution justify to formulate the following conclusions:

1. The tolerance averaging approximation can be successfully applied to formulate averaging model of thin skeletonal shallow shells.
2. The obtained model leads to the governing equations with non-constant but the smooth coefficients in contrast to equations in direct description with the discontinuous and highly oscillating coefficients.
3. Since the proposed model equations have smooth and functional coefficients then in most cases solutions to specific problem for skeletonal shells under consideration have to be obtained using numerical methods.

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