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# Performance Analysis of IEEE 802.11e EDCA under Finite Load in an Error Prone Channel 

Yoram Haddad<br>Jerusalem College Of Technology<br>Email: haddad@jct.ac.il

Gwendal Le Grand<br>École Nationale Supérieure des Télécommunications<br>Email: gwendal.legrand@enst.fr


#### Abstract

Although the IEEE 802.11e standard including Quality of Service support in WiFi networks was recently approved, it does not increase wireless capacity. In order to optimize wireless resources, it is necessary to model the behavior of the system to assist network planning and deployment. In this article, we propose a Markov chain model for the EDCA access scheme described in IEEE 802.11e. Contrary to existing IEEE 802.11 models, our contribution studies more realistic assumptions since we consider a non-ideal channel moreover in unsaturated mode. The benefits of our model are manifold: we calculate the throughput more precisely, we can study various traffic loads, and finally we can examine various transmission environments.


## Keywords

Finite load, EDCA, Error prone channel.

## I. Introduction

Since the first IEEE 802.11 standard in 1997 [1], research laboratories kept on trying to model the behavior of access mechanisms to the medium.

Indeed, the difficulty to model DCF [1], which is the main access mechanism of IEEE 802.11, relies in the number of parameters that change during the transmission by a station (denoted STA in the following). A relevant and efficient model would constitute a key to assist the deployment of wireless networks, which is currently done in a quasi empirical way. As far as we know, there exists no software to evaluate the exact capacity of a cell where users have very different requirements in quality of service (QoS).

Thus, it is necessary to calculate the probability of collision, errors over the channel, and also the average time in contention period. These events being by nature random, the model will only approximate reality, but will not constitute an exact model. Many models have already been proposed in the literature, each one having its own specificities and its approximations. The model that we propose is closer to reality since it unveils two major approximations that are common to all former models, namely the assumptions that the channel is ideal and saturated. In section 2, we present existing models, their advantages, drawbacks, and limits. After having presented our model in section 3 , we will present the results provided by our model in section 4 and conclude by proposing improvements for our model.

## II. State of the art

We distinguish two main categories of models. Since 1996, the CSMA/CA mechanism used by the DCF and its performances were studied by Bianchi [2] . This model which is based on Markov chains was published in 1998 [3]-[4] . In parallel, Cali et al. [5] developed a model based on geometric distributions. The so-called Bianchi model is based on a two dimensional Markov chain. The first dimension $s(t)$ indicates the backoff stage which represents the number of transmission attempts which failed. The second dimension $b(t)$, indicates the value of the backoff timer, which corresponds to the number of time slots to wait before being able re-initialize a transmission after a failure. This model fits well to a saturated medium because it assumes that the STA have permanently data to transmit. This implies that the results represent the maximum throughput offered by a WiFi cell. However, this model uses several approximations. Firstly, the channel is supposed to be ideal, i.e., it does not introduce any error. Moreover, the limited number of retransmissions allowed in the standard is not taken into account in the model.
Cali's model also permits to compute the maximum flow offered in saturated mode for the DCF, but this time, the backoff time is evaluated as the average of a geometric distribution. Cali's model uses the same approximations as Bianchi's model. The major difference between those two models lies in the way the probability is computed for a station transmitting at a time $t$ (the computation being easier in Cali's model). In order to enrich these two models, several papers were published that tried to improve one of the approximations listed previously.

Using the model of Cali, [6] developed a model in unsaturated traffic context with an error-prone channel. Several improvements of Bianchi's model have also been proposed. [8] improves Bianchi's model by taking into account the error probability directly in the calculation of the flow, while keeping the approximation of the saturated mode only for the DCF. [9] examined unsaturated networks, by introducing an additional state into the Markov chain. This state takes into account the possibility of having an empty buffer after a transmission. The model deals with an additional problem, which is that of the multi-rate STA. However, this work was carried out within the framework of an ideal channel, and does not take into account several significant characteristics of the

DCF such as the frozen time when the channel is busy. The IEEE 802.11e standard [10] including mechanisms for QoS management, has been already designed, for use in saturated mode [11], [12] . These models are both based on Markov chains and assume that the system is in unsaturated mode and that the channel is ideal.

## III. System model

Our article is placed in the context of IEEE 802.11e. We reuse the methodology presented in [11]. Our work is an improves IEEE 802.11 and IEEE 802.11e existing models Indeed, our objective is to provide a more realistic and extensive model. Thus, we propose the following improvements: we consider a non-ideal channel (i.e., which introduces errors into the packets, according to a fixed probability of error), we consider that stations do not always have a packet to transmit (i.e., the emission buffer of the network card can be empty). These two enhancements are motivated by the fact that the assumption of an ideal channel is (as in many quoted models) a rather coarse simplification in the field of wireless. Moreover, the saturated medium which is usually considered to evaluate the capacity of the network only indicates the maximum capacity of the link. To our knowledge, there is no EDCA model that takes into account a non-ideal channel, under finite load. We estimate that the introduction into the model of these two elements is necessary to obtain more precise models that are closer to reality.

## A. Four dimensional Markov Chain

The Markov chain, fig. 2 and more detailled on fig.4, models the behavior of an access category (AC) managed by EDCA, for a given station. In order to simplify the diagram, we did not represent all the transition probabilities from one state to another. They will all be described in section III-D. Our model comprises a great number of indexes and variables, which are summarized in the following table:

| Variable's <br> name | Explanation |
| :---: | :--- |
| $\mathrm{s}(\mathrm{t})$ | Number of retry at time t |
| $\mathrm{b}(\mathrm{t})$ | Backoff timer at time t |
| $v(\mathrm{t})$ | Timer in transmission, collision, error <br> or frozen period |
| $\mathrm{e}(\mathrm{t})$ | If error occurs e(t)=1 else 0 |
| j | Value of $\mathrm{s}(\mathrm{t})$ |
| k | Value of $\mathrm{b}(\mathrm{t}) \in$ <br> $[0, \mathrm{Wj}]$ |
| d | Value of $v(t)$ |
| e | Value of e(t) |
| i | Index of the Access Category ACi $\mathrm{i}=$ <br> $0,1,2$, or 3 |
| $A_{i}$ | Value of AIFSi decreased by 1 |
| N | Value of the initial frozen timer |
| $W_{j}$ | Maximal value of the backoff timer |
| m | Number of maximum retry with Wj <br> increasing |


| $\mathrm{m}+\mathrm{h}$ | Number of maximum retry before discarding <br> the packet |
| :---: | :--- |
| $P_{i}$ | Collision probability for ACi |
| $P_{b}$ | Probability that the channel is busy |
| q | Probability that the buffer isn't empty |
| $\gamma$ | Propagation time |
| $T_{e}$ | Average time to detect an error |
| $T_{s}$ | Average time for a successful transmission |
| $T_{c}$ | Average time to detect a collision |

Table 1: Variables and constants of the model
The first dimension has been explained in the state of art. The second dimension $b(t)$, is a stochastic process indicating the state of the backoff timer, for a given AC at time $t$. The initial value of the timer is drawn among an interval $[0, \mathrm{Wj}]$, where Wj depends on the backoff stage j with

$$
W_{j+1}=2 W_{j}+1
$$

The third dimension introduced by [11] is a variable which have various meanings according to the context. During the frozen period, it indicates the remaining time before being able to carry on decreasing the timer fig.3. In transmission or collision period, it indicates the remaining time before the end of the period, fig.1.

In our model, we introduced a fourth dimension, denoted $e(t)$. This variable may have two values:

1. $\mathrm{e}(\mathrm{t})=1$ the transmission is corrupted but did not undergo a collision
2. $e(t)=0$ in all the other cases

This variable was introduced in order to make the difference between, a transmission failed because of a collision and that which fails because of an error. This difference is fundamental since the objective of our model is to provide the collision probability in the most precise way, without using an any other factor such as the influence of the errors on channel. .There isn't this dimension in the previous models for the simple reason that the channel is generally considered as being ideal. Let $P_{i}$ be the collision probability and $P_{b}$ probability that the channel is busy. Beside, $P_{i}$, is made up of two parts: a probability of external collision, due to the transmissions of the other STA and a probability of internal collision, due to the virtual contention which takes place between ACi of the same STA. Thus:

$$
P_{i}=P_{i_{i n t}}+P_{i_{e x t}}
$$

At time $t$, a state of a given $A C$ is fully determined by the quadruplet ( $\mathrm{j}, \mathrm{k}, \mathrm{d}$, e) which corresponds to the values taken respectively by each dimension.

Let's describe the chain through some specific states fig. 2 .
The system is in the state:
$-\mathrm{j}=-1$, if a new packet, following a past transmission (either successful or not), is in the queue.
$-\mathrm{j}=-2$ indicates that the AC is in the transmission period, after having reached the channel successfully, without having meeting collision, nor errors.
The following states are specific to our model. Here lies one of the model's keys: the unsaturated buffer. Indeed, we consider the possibility of the arrival of a new packet, $j=-4$, as well as the possibility of not having a standby packet in the buffer $\mathrm{j}=-5$.
$-j=-4$ indicates that AC tries to transmit a packet lately arrived at the buffer and not just following a transmission.
$-j=-5$, indicates that the buffer of the AC is empty


Fig. 1. Collisions and Errors

## B. Markov Chain

We will now describe the chain starting from a given state and will observe the possible path through the chain. Let us suppose that $A C$ is in the state $(j, 1,0,0)$.

AC met thus j collisions and/or corrupted transmissions and undergoes its $j^{\text {th }}$ backoff. This is indicated by the first dimension of this state. Its backoff timer is equal to 1 (shown by the second dimension of the state) and is decreasing, as the value of the third dimension indicates it to 0 . Beside, knowing that the j -th transmission did not begin yet, the 4 th dimension is by default equal to 0 . From that state, two possibilities arise at ACi . If ACi observes that the channel is busy, the backoff timer is frozen, which involves the beginning of the frozen period, and brings ACi to the state $(\mathrm{j}, 1, \mathrm{~N}+\mathrm{A}, 0)$. If not, then go to $(\mathrm{j}, 0,0,0)$. This cycle is repeated until AC can reactivate its timer and be able to finally reach the channel. At this time, if no other ACj of higher priority within the same STA and no other STA tries to transmit at the same time, ACi will be able to reach the channel and transmit its packet. However, if a collision or an error occurs on the packet, a certain time respectively Tc or Te will run out before AC becomes aware of this collision or respectively of this error, and passes to state $\mathrm{j}+1$ for a new
attempt.


Fig. 2. Simplified Markov Chain

## C. Characteristic of our model: the unsaturated mode

After a successful transmission, if the buffer has already a new packet on standby (with a probability q), then ACi enters state $\mathrm{j}=-1$. In the case of an empty buffer (with a probability $1-q), A C i$ enters a waiting state noted ( $-5,0,0,0$ ). In each Time Slot, ACi, checks its buffer, if it still does not contain a new packet to be transmitted it loops on the same state $(-5,0$, $0,0)$. On the other hand, if a new packet arrives in the queue, the ACi moves to state $(-4,0, \mathrm{~A}, 0)$, which allows it to access the channel directly after having checked that it remained free for a certain time (AIFS).

This is a major difference with the saturated models that assume a STA always has a packet to transmit, in other words, that its buffer is never empty. Therefore, when a new packet arrives, it has to wait and initiate a backoff, instead of being sent directly after an AIFS. In our model, we introduced $(-4,0, \mathrm{~d}, 0) \mathrm{d}=0$..A to correct this approximation. The introduction of this state $(-5,0,0,0)$ is fundamental, since a STA does not always have data to transmit. Thanks to our state, we can consider the possibility of having an empty buffer. We can in addition, via the parameter q, represent different load scenarios, which is a significant added value to existing models.

## D. Transitions probabilities

We will now describe the transition probabilities from a state to an other.

1) For states $(-2,0, d, 0), d=1,2, \ldots,\left[T_{s}\right]$,

$$
\begin{gathered}
\text { - } P\{(-2,0, d-1,0) /(-2,0, d, 0)\}=1 \\
2 \leq d \leq\left[T_{s}\right]
\end{gathered}
$$

Account for the fact that, during the transmission, time is decremented.

$$
\text { - } P\left\{\left(-1,0, A_{i} 0\right) /(-2,0,1,0)\right\}=q
$$

After a successful transmission, when a new packet is already waiting in the buffer

- $P\{(-5,0,0,0) /(-2,0,1,0)\}=1-q$

When the buffer is empty
2) For states $(j, 0,0,0), j=0,1, \ldots, m+h$,

- Successful transmission if no collision occurs nor error.

$$
\begin{gathered}
P\left\{\left(-2,0,\left[T_{s}\right], 0\right) /(j, 0,0,0)\right\}=\left(1-P_{i}\right)\left(1-P_{e}\right) \\
0 \leq j \leq m+h
\end{gathered}
$$

- If a collision occurs then AC enter the collision period

$$
\begin{gathered}
P\left\{\left(j, 0,\left[T_{c}\right], 0\right) /(j, 0,0,0)\right\}=P_{i} \\
0 \leq j \leq m+h
\end{gathered}
$$

- If no collision occurs but there is an error

$$
\begin{gathered}
P\left\{\left(j, 0,\left[T_{e}\right], 1\right) /(j, 0,0,0)\right\}=\left(1-P_{i}\right) \times P_{e} \\
0 \leq j \leq m+h
\end{gathered}
$$

3) For state ( $\mathrm{m}+\mathrm{h}, 0,0,0$ ),

- If no collision and no error occurs, then the transmission succeed


Fig. 3. A Frozen Period

$$
\begin{aligned}
& P\left\{\left(-2,0,\left[T_{s}\right], 0\right) /(m+h, 0,0,0)\right\}= \\
&\left(1-P_{i}\right)\left(1-P_{e}\right)
\end{aligned}
$$

- If a collision or an error occurs after the collision or error period, the packet is discarded because $m+h$ retries have been exhausted. Then the AC checks his buffer. If a new packet is already waiting:
- After an error:

$$
\begin{aligned}
P\left\{\left(-1,0, A_{i}, 0\right) /(m+h, 0,1,1)\right\} & = \\
& \left(1-P_{i}\right) P_{e} \times q
\end{aligned}
$$

- After a collision:

$$
P\left\{\left(-1,0, A_{i}, 0\right) /(m+h, 0,1,0)\right\}=P_{i} \times q
$$

Let: $P_{i}+\left(1-P_{i}\right) P_{e}=P_{f i}$
If the buffer is empty:

- After an error:

$$
\begin{aligned}
& P\{(-5,0,0,0) /(m+h, 0,1,1)\}= \\
& \quad\left(1-P_{i}\right) P_{e} \times(1-q)
\end{aligned}
$$

- After a collision:

$$
P\{(-5,0,0,0) /(m+h, 0,1,0)\}=P_{i} \times(1-q)
$$

4) For states $(\mathrm{j}, 0, \mathrm{~d}, 0), \mathrm{j}=0,1 \ldots \mathrm{~m}+\mathrm{h}$ and $d \geq 1$, when a collision occurs, time is decremented by 1 for each time slot elapsed, till the AC exit the collision period:

$$
\begin{aligned}
& P\{(j, 0, d-1,0) /(j, 0, d, 0)\}=1 \\
& 0 \leq j \leq m+h ; \quad 2 \leq d \leq\left[T_{c}\right]
\end{aligned}
$$

When the collision period finishes, the AC doubles the size of the Contention Window (CW), except when CW had already reached the maximum value CWmax, and chooses a random number from the uniformly distributed set $[0, \mathrm{Wj}+1]$ and then enters the next backoff stage

$$
\begin{aligned}
& P\{(j+1, k, 0,0) /(j, 0,1,0)\}=\frac{1}{\left(W_{j+1}+1\right)} \\
& 0 \leq k \leq W_{j+1} ; \quad 0 \leq j \leq m+h
\end{aligned}
$$

5) For states $(j, 0, d, 1), j=0,1 \ldots m+h$ and $d \geq 1$, it is similar to the collision period and time is still decremented by 1 :

$$
\begin{aligned}
& P\{(j, 0, d-1,1) /(j, 0, d, 1)\}=1 \\
& 0 \leq j \leq m+h ; \quad 2 \leq d \leq\left[T_{e}\right]
\end{aligned}
$$

$$
\begin{aligned}
& P\{(j+1, k, 0,0) /(j, 0,1,1)\}=\frac{1}{\left(W_{j+1}+1\right)} \\
& 0 \leq k \leq W_{j+1} ; \quad 0 \leq j \leq m+h
\end{aligned}
$$

6) For states $(\mathrm{j}, \mathrm{k}, 0,0), \mathrm{j}=0,1, \ldots, \mathrm{~m}+\mathrm{h}$ and $k \geq 1$, the backoff timer is decremented by 1 if the channel is idle,

$$
\begin{aligned}
& P\{(j, k-1,0,0) /(j, k, 0,0)\}=1-P_{b} \\
& \quad 1 \leq k \leq W j ; \quad 0 \leq j \leq m+h
\end{aligned}
$$

It is frozen if the channel is buzy and has to wait $\mathrm{N}+\mathrm{A}$ Time Slots

$$
\begin{aligned}
& P\{(j, k, N+A, 0) /(j, k, 0,0)\}=P_{b} \\
& 1 \leq k \leq W_{j} ; \quad 0 \leq j \leq m+h
\end{aligned}
$$

7) For states $(j, k, d, 0), j=0,1, \ldots, m+h, \quad k \geq 1$ and $d \geq 1$, when a time slot elapsed during the frozen period, the remaining frozen time is decremented by 1

$$
\begin{aligned}
& \quad P\{(j, k, d-1,0) /(j, k, d, 0)\}=1 \\
& 1 \leq k \leq W_{j} ; \quad 0 \leq j \leq m+h ; \quad A_{i}+1 \leq d \leq N+A_{i}
\end{aligned}
$$

After the frozen period, if the channel is idle, the backoff time is further decremented

$$
\begin{aligned}
& P\{(j, k, d-1,0) /(j, k, d, 0)\}=1-P_{b} \\
& 1 \leq k \leq W_{j} ; \quad 0 \leq j \leq m+h ; \quad 2 \leq d \leq A_{i} \\
& P\{(j, k-1,0,0) /(j, k, 1,0)\}=1-P_{b} \\
& \quad 1 \leq k \leq W_{j} ; \quad 0 \leq j \leq m+h
\end{aligned}
$$

But if the channel is still busy, then the frozen time returns to its initial value, i.e., $N+A_{i}$.

$$
\begin{aligned}
& P\{(j, k, N+A, 0) /(j, k, d, 0)\}=P_{b} \\
& \quad 1 \leq k \leq W_{j} ; \quad 0 \leq j \leq m+h ; \quad 1 \leq d \leq A_{i}
\end{aligned}
$$

8) For states $(-1,0, \mathrm{~d}, 0), \mathrm{d}=0,1 \ldots N+A_{i}$, before transmitting the packet, the channel has to be idle during an AIFSi time. If it is still idle, then the backoff process is started, and if not the frozen period is initiated.

$$
\begin{aligned}
& P\{(-1,0, d-1,0) /(-1,0, d, 0)\}=1 \\
& \qquad A_{i}+1 \leq d \leq N+A_{i} \\
& P\{(-1,0, d-1,0) /(-1,0, d, 0)\}=1-P_{b} \\
& \quad 1 \leq d \leq A_{i}
\end{aligned}
$$

$$
\begin{aligned}
& P\{(-1,0, N+A, 0) /(-1,0, d, 0)\}=P_{b} \\
& 0 \leq d \leq A_{i} \\
& P\{(0, k, 0,0) /(-1,0,0,0)\}=\frac{1-P_{b}}{W_{0}+1} \\
& \quad 0 \leq k \leq W_{0}
\end{aligned}
$$

9) For states $(-4,0, d, 0), \mathrm{d}=0, \ldots, A_{i}$, the new packet is already available. If the channel is idle, the packet is directly transmitted without going through the backoff process, since it is a new packet which is not following a last transmission. In case of a busy channel, the backoff process is initiated

$$
\begin{aligned}
P\{(-4,0, d-1,0) /(-4,0, d, 0)\}= & 1-P_{b} \\
& 1 \leq d \leq A i \\
P\{(-2,0, T s, 0) /(-4,0,0,0)\}= & 1-P_{f i} \\
P\{(0,0, T c, 0) /(-4,0,0,0)\}= & P_{i} \\
P\{(0,0, T e, 0) /(-4,0,0,0)\}= & \left(1-P_{i}\right) P_{e}
\end{aligned}
$$

10) For states $(-5,0,0,0)$, if the buffer is empty, then the AC waits until a packet arrives, and then initiates the regular contention process through state $\left(-4,0, A_{i}, 0\right)$.

$$
P\{(-5,0,0,0) /(-5,0,0,0)\}=1-q
$$

$$
P\{(-4,0, A, 0) /(-5,0,0,0)\}=q
$$

## E. Probability in steady state and equation systems

Let $b_{j, k, d, e}$ be the probability to be in state ( $\mathrm{j}, \mathrm{k}, \mathrm{d}, \mathrm{e}$ ), when the system is steady (in other words when $t \rightarrow+\infty$ ). As mentioned previously, $P_{f i}$ stands for the probability of a failed transmission, because of collision or error.

We calculated those probabilities using the same methodology as [4] and [10], but it was naturally necessary to adapt to the requirements and states of our model.

We obtained:

$$
\begin{gather*}
b_{j, 0,0,0}=\left(P_{f i}\right)^{j} \times b_{0,0,0,0}  \tag{1}\\
0 \leq j \leq m+h \\
b_{0, k, 0,0}=\frac{W_{0}-k+1}{W_{0}+1} \\
\times\left(1-(1-q)\left(1-P_{b}\right)^{A+1}\right)  \tag{2}\\
\times b_{0,0,0,0} \\
1 \leq k \leq W_{j}
\end{gather*}
$$

$$
\begin{align*}
b_{j, k, 0,0}= & \frac{W_{j}+1-k}{W_{j}+1} \cdot b_{j, 0,0,0}  \tag{3}\\
& 1 \leq j \leq m+h ; \quad 0 \leq k \leq W_{j}
\end{align*}
$$

For the third dimension, due to the regularity of the Markov chain, we get:

$$
\begin{gather*}
b_{j, k, d, 0}=\frac{P_{b}}{\left(1-P_{b}\right)^{A}} \times b_{j, k, 0,0}  \tag{4}\\
A_{i} \leq d \leq N+A_{i} ; \quad 0 \leq j \leq m+h ; \quad 1 \leq k \leq W_{j} \\
b_{j, k, d, 0}=\frac{P_{b}}{\left(1-P_{b}\right)^{d}} \times b_{j, k, 0,0}  \tag{5}\\
1 \leq d \leq A_{i}-1 ; \quad 0 \leq j \leq m+h ; \quad 1 \leq k \leq W_{j} \\
b_{j, 0, d, 0}=P_{i} \times b_{j, 0,0,0}  \tag{6}\\
1 \leq d \leq[T c] ; \quad 0 \leq j \leq m+h \\
b_{j, 0, d, 1}=\left(1-P_{i}\right) \times P_{e} \times b_{j, 0,0,0}  \tag{7}\\
1 \leq d \leq[T e] ; \quad 0 \leq j \leq m+h \\
b_{-2,0, d, 0}=\left(1-P_{f i}\right)^{(m+h+1)} \times b_{0,0,0,0}  \tag{8}\\
1 \leq d \leq[T s] \\
b_{-1,0, d, 0}=\frac{q \times b_{0,0,0,0}}{\left(1-P_{b}\right)^{d+1}}  \tag{9}\\
0 \leq d \leq A_{i} \\
b_{-1,0, d, 0}=q \times b_{0,0,0,0} \times \frac{1-\left(1-P_{b}\right)^{A_{i+1}}}{\left(1-P_{b}\right)^{A_{i+1}}}  \tag{10}\\
A_{i}+1 \leq d \leq N+A_{i}
\end{gather*}
$$

And for states $(-5,0,0,0)$

$$
\begin{align*}
& b_{-5,0,0,0}=\frac{(1-q) \times b_{0,0,0,0}}{q}  \tag{11}\\
& b_{-4,0, d, 0}=\left(1-P_{b}\right)^{A_{i-d}} \times(1-q) \times b_{0,0,0,0}  \tag{12}\\
& \quad 0 \leq d \leq A_{i}
\end{align*}
$$

By substituting (1) into (3), and (3) into (4)-(7), all the probabilities $b_{j, k, d, e}$ can be derived from $P_{b i}, P_{i}, P_{e}, \mathrm{q}$, which respectively stand for the probability that the channel is busy, the probability of collision for ACi , the packet error rate, and the probability that the buffer is not empty.

Finally we derive $b_{0,0,0,0_{i}}$ from the normalization condition:

$$
b_{0,0,0,0_{i}}=
$$

$$
\left[\begin{array}{l}
\frac{1-q}{q}+[T s]\left(1-\left(P_{f i}\right)^{m+h+1}\right) \\
+(1-q)\left(\frac{1-\left(1-P_{b}\right)^{A+1}}{P_{b}}\right) \\
+q \frac{\left(1+N P_{b}\right)}{P_{b}} \frac{1-\left(1-P_{b}\right)^{A+1}}{\left(1-P_{b}\right)^{A+1}+\left([T c] P_{i}+1\right)\left(\frac{1-P_{f i}^{m+h}}{1-P_{f i}}\right)} \\
+\left([T e]\left(1-P_{i}\right) P_{e}\right)\left(\frac{1-P_{f i}^{m+h}}{1-P_{f i}}\right) \\
+\frac{1+N P_{b}}{2\left(1-P_{b}\right)^{A}}\left(\left[1-(1-q)\left(1-P_{b}\right)^{A+1}\right] \times W_{0}\right) \\
+\sum_{j=1}^{m+h} W_{j} P_{f i}^{j} \\
+P_{f i}^{m+h}
\end{array}\right]^{-1}
$$

Thus, to derive $b_{0,0,0,0_{i}}$ we have must get the values of $T_{s}, T_{c}, T_{e}, P_{b}, P_{i}, P_{e}, m, h, W_{j}, \mathrm{~A}, \mathrm{~N}$, and q.
The derivation of $T_{s}, T_{c}$, and Te will be explained further. They depend on the considered ACi but also on the packet size. The parameters $m, h, A, N, W_{j}$ are characteristics of the $A C_{i}$. For example, $W_{j}$ depends on its initial value $W_{0}$ (also denoted $C W_{\text {min }}$ ), which is a variable that differs from an $A C_{i}$ to another one. The values of $P_{e}$ depends on the transmission environment.
Let $\tau$ be the probability that an $A C_{i}$ accesses a channel. It corresponds to the sum of the probabilities to be in one of the final state of backoff, which allows transmitting on the medium, then:

$$
\begin{aligned}
\tau_{i} & =\sum_{j=0}^{m+h} b_{j, 0,0,0_{i}}=\sum_{j=0}^{m+h} P_{f i}^{i} \times b_{0,0,0,0_{i}} \\
& =\frac{1-P_{f i}^{m+h+1}}{1-P_{f i}} \times b_{0,0,0,0_{i}}
\end{aligned}
$$

Giving that a STA includes $4 A C_{i}$, the probability that a STA transmits equals the probability that at least one of the $A C_{i}$ transmits, so:

$$
\begin{equation*}
\tau=1-\prod_{i=1}^{3}\left(1-\tau_{i}\right) \tag{13}
\end{equation*}
$$

Considering that the channel is occupied by the $A C_{i}$, if the transmission, collision or error is related to this $A C_{i}$. Let $v_{i}$ be the probability that the channel is occupied by $A C_{i}$

$$
\begin{equation*}
v_{i}=\sum_{d=1}^{\left[T_{s}\right]} b_{-2,0, d, 0}+\sum_{j=0}^{m+h} \sum_{d=1}^{\left[T_{c}\right]} b_{j, 0, d, 0}+\sum_{j=0}^{m+h} \sum_{d=1}^{\left[T_{e}\right]} b_{j, 0, d, 1} \tag{14}
\end{equation*}
$$

Then :

$$
\begin{align*}
& v_{i}=b_{0,0,0,0} \\
& \quad \times\left(1-P_{f i}^{m+h+1}\right)  \tag{15}\\
& \\
& \quad \times\left(T_{s}+\frac{P_{i} \times T_{c}+\left(1-P_{i}\right) p_{e} \times T_{e}+2}{1-P_{f i}}\right)
\end{align*}
$$

And $v$ the probabilility that the channel is occupied by a station

$$
\begin{equation*}
v=1-\prod_{i=1}^{3}\left(1-v_{i}\right) \tag{16}
\end{equation*}
$$

The probability that the channel is busy is given by:

$$
\begin{equation*}
P_{b}=1-(1-v)^{M} \tag{17}
\end{equation*}
$$

M stands for the total number of active stations. The collision probability is given by the probability that at least one other STA transmits at the same time (called external collision) or an other $\mathrm{ACi}^{\prime}$ in the same STA (virtual internal collision).

$$
\begin{equation*}
P_{i}=1-(1-\tau)^{M-1} \times \prod_{i^{\prime}>i}\left(1-\tau_{i^{\prime}}\right) \tag{18}
\end{equation*}
$$

where i' means that $A C_{i^{\prime}}$ has a higher priority than ACi .

## IV. Performance evaluation: <br> The throughput

The standardized throughput for a given AC is derived as the ratio between the effective time to transmit the data and the average time between two successive transmissions. This average time takes into account the time spent in the contention process, the time possibly wasted in collision and/or error as well as time to successfully transmit the packet, including transmission times of the protocol overheads.

Let $S_{i}$ be the throughput for the $A C_{i}$

$$
\begin{equation*}
S_{i}=\frac{P_{s i} P}{E[I]+\sum_{i^{\prime}=0}^{3} P_{s i^{\prime}}\left(T_{s}+A I F S\left[A C_{i^{\prime}}\right]\right)+\sum_{i^{\prime}=0}^{3} P_{i^{\prime}} T_{c}+P_{e} T_{e}} \tag{19}
\end{equation*}
$$

Where P stands for the data payload and $\mathrm{E}[\mathrm{I}]$ is the average time where the channel is idle. We have:

$$
\begin{equation*}
E[I]=\frac{1}{P_{b}}-1 \tag{20}
\end{equation*}
$$

$P_{s i}$ and $P_{s i^{\prime}}$ correspond to the probability that the transmission succeeds resp. for $A C_{i}$ and $A C_{i^{\prime}}$.
$P_{s i}$ is given by the following:
With

$$
\begin{align*}
& P_{s i}=\frac{M \times P_{t i}(1-v)^{M-1} \times \prod_{i^{\prime}>i}\left(1-v_{i^{\prime}}\right)}{1-(1-v)^{M}}  \tag{21}\\
& P_{t i}=T s \times b_{0,0,0,0_{i}} \times\left(1-\left(P_{f i}\right)^{m+h+1}\right) \tag{22}
\end{align*}
$$

For $T_{s}, T_{c}$, and $T_{e}$, we use the values given by the standard [10].

Below, we give the equations to calculate each of those times. Since there are two modes, namely the basic mode and the RTS/CTS mode, we indicate by a "b" the values for the basic mode and " $r$ " those for the RTS/CTS mode.

For the basic mode

$$
\begin{aligned}
T s b= & P H Y \text { header }+ \text { MACheader }+ \text { Tp }+\gamma \\
& + \text { SIFS }+A C K+\gamma \\
T e b= & T s b \\
T c b= & P H Y \text { header }+ \text { MACheader }+ \text { Tp }+\gamma \\
& +A C K+S I F S
\end{aligned}
$$

And for the RTS/CTS mode:

$$
\begin{aligned}
T s r= & R T S+\gamma+S I F S+C T S+\gamma \\
& +S I F S+\text { PHYheader }+ \text { MACheader } \\
& +T p+\gamma+\text { SIFS }+ \text { ACK }+\gamma \\
\text { Ter }= & T s r \\
T c r= & R T S+\gamma+S I F S+C T S+\gamma \\
& + \text { SIFS }
\end{aligned}
$$

Where $\gamma$ stands for the propagation time, which is often neglected in the models, and $T p$ stands for the payload's transmission time which obviously depends on the nominal throughput R (e.g.: 54 mbps for 802.11a).

## V. Conclusion

In this article, we have presented an accurate EDCA model based on a four dimensional Markov chain. We consider a nonideal channel, under different traffic scenarios (saturated and unsaturated). Our model is very rich, which makes it on the one hand, more accurate and closer to reality, but on the other hand, it requires more complex calculations. The main result of the model lies in the computation of the throughput which leads to the capacity of the system. This result is essential in order to design a planification tool for WiFi QoS-enabled networks. Further extensions of this model will also compute the average delay and study networks in which packet sizes may be variable.

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Fig. 4. The full Markov chain

