Time--dependent Bragg diffraction and short--pulse reflection by one-dimensional photonic crystals

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Abstract

The time-dependence of the Bragg diffraction by one-dimensional photonic crystals and its influence on the short pulse reflection are studied in the framework of the coupled-wave theory. The indicial response of the photonic crystal is calculated and it appears that it presents a time-delay effect with a transient time conditioned by the extinction length. A numerical simulation is presented for a Bragg mirror in the x-ray domain and a pulse envelope modelled by a sine-squared shape. The potential consequences of the time-delay effect in time-dependent optics of short-pulses are emphasized.

Keywords: *time-dependent diffraction, short-pulse reflection, photonic crystal*

1. Introduction

The propagation of short pulses is an old problem which started with the works by Sommerfeld and Brillouin [1,2]; they demonstrated the occurrence of precursors and their works have given rise to a lot of studies relative to the physics of these forerunners in homogeneous dielectric media [3] and also in one-dimensional photonic crystal (1D-PC) [4,5] we are interested in. Beside the particular physics of the precursors, the study of the propagation of the main pulse has been mainly developed in the context of femtosecond lasers [6–8]. At the present time, there is a renewal of the optics of ultra-short and short pulses with the advent of femto-, subfemto- and attosecond sources in the high energy domain of the electromagnetic spectrum : isolated [9,10] or train [11,12] attopulses are delivered by high harmonic generation (HHG) sources in the X-UV range, subfemto- and femtopulses are generated by X-ray free electron lasers (X-FEL) [13] or other new sources (Thomson scattering, …) in the x-ray domain [14,15].

These sources required optical devices such as monochromators or mirrors and the reflection properties of multilayer reflectors [16–18] and crystals [19,20] dedicated for this kind of sources have been treated by means of methods implemented for the frequency domain, the problem being that reflection of a short pulse differs in many aspects from the one of monochromatic continuous radiation since the pulse is a superposition of many harmonics according to the uncertainty principle; the harmonics propagate with different velocities because of the wavelength dispersion which results in pulse distortion. The strategy for the calculation generally follows a Fourier mathematical approach : the time structure of the incident pulse is expanded in its Fourier components and each frequency component is treated by the Fresnel formalism or Darwin-Prins dynamical theory, the time shape of the reflected pulse being occasionally built by inverse Fourier transform; conclusions are drawn concerning the time evolution of the reflectivity of the reflectors and distortion of the pulse. In this context, the Green impulse response corresponding to a Dirac-delta incident pulse is generally calculated since it allows calculating an arbitrary incident pulse response by means of the convolution theorem [19].

To compensate for the wavelength dispersion, the concept of chirped mirrors has been applied, initially in the infrared and visible domains for femtosecond pulses [21,22], then transposed [23] and successfully tested [24] in the X-UV spectral range and

attosecond domain. The design of these mirrors lies also generally on a Fourier approach.

The study of the dynamical diffraction of short pulses by crystals in the time domain, by solving the time-dependent propagation equation (Tagaki-Taupin equation), was done by Chukhovskii and Förster [25], then Wark and Lee [26]. In the present work, we adopt a rather similar approach for a 1D-PC ; the 1D-PC is modelled by a periodic stack of bilayers. The electric field of the pulse is written as a carrier wave modulated by a slow-varying envelope; starting from Maxwell equations, the propagation time-dependent equation is deduced for the envelope and after some assumptions (such as the two-waves approximation and the instantaneous response of the media), a coupled system of partial differential equations (PDEs) is obtained for the two Fourier components of the two wave fields in the spirit of the coupled wave theory (CWT). This is done in Section 2. In Section 3, we present an analytic solution of the coupled system by using a matrix formalism and we calculate the reflectivity versus time for a constant intensity source abruptly switched on, or in other words, we determine the indicial response of the 1D-PC for a Heaviside unit-step input. We show in Section 4, from a numerical example that the peak reflectivity versus time of the 1D-PC displays a transient period during which the reflectivity starts from a null value to reach the steady-state value with a rise-time determined by the extinction length; using Strejc's method, we then deduce from the computed indicial response, the transfer function which is the Laplacian transform of the impulse response. The knowledge of the transfer function gives access to the dynamics of reflection of a pulse of any shape by means of the convolution theorem ; the case of a sine-squared pulse is examined. In conclusion, Section 5, we discuss the implications of this time-delay effect, in particular for the x-ray regime and we give the perspectives.

2. Time-dependent coupled-wave theory

We consider a 1D-PC consisting in a periodic stack of bilayers as shown in Figure 1 which displays the geometry of the problem. We consider only the *s*-polarization case. The propagation of the optical pulse in the 1D-PC is governed by the following wave equation deduced from Maxwell's equations

$$\frac{\partial^2 \boldsymbol{E}(\rho,z,t)}{\partial \rho^2}+\frac{\partial^2 \boldsymbol{E}(\rho,z,t)}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}(\rho,z,t)}{\partial t^2}=\frac{4\pi}{c^2}\frac{\partial^2 \boldsymbol{P}(\rho,z,t)}{\partial t^2}$$

(1)

where $\boldsymbol{E}$ is the electric field, $\boldsymbol{P}$ the polarization vector, c is the speed of light in vacuum. The geometry and some notations of the problem are presented in Figure 1.

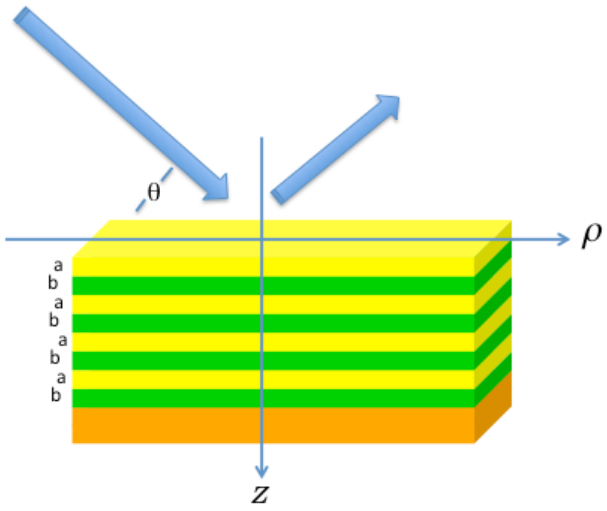

Figure 1: Sketch of the 1D-PC consisting in a periodic stack of $\mathcal{N}$ bilayers with thickness d; the bilayer is made up of a material **a** *and material* **b** *of thickness $d_a = \gamma d$ and $d_b = (1-\gamma)d$ respectively . The incoming radiation strikes the system under a glancing angle θ.*

The optical pulse is formed of a carrier wave with frequency ω modulated by an envelope $\boldsymbol{E_0}(z,t)$

$$\boldsymbol{E}(\rho,z,t) = \boldsymbol{E_0}(z,t)\, e^{-i\,\omega\, t}\, e^{\,i\, k_\rho\, \rho} \tag{2}$$

A sine-squared limited in a time interval is usually adopted to model the envelope, especially for X-FEL single pulses [27], but let us emphasize that our approach does not depend on the pulse shape. Inserting Eq.(2) in Eq.(1) leads to the following equation for the envelope

$$\frac{\partial^2 \boldsymbol{E_0}(z,t)}{\partial z^2} - {k_\rho}^2\, \boldsymbol{E_0}(z,t) + \frac{1}{c^2}\left(\omega^2\, \boldsymbol{E_0}(z,t) + 2\, i\, \omega\, \frac{\partial \boldsymbol{E_0}(z,t)}{\partial t} - \frac{\partial^2\, \boldsymbol{E_0}(z,t)}{\partial t^2}\right)$$
$$= \frac{4\,\pi}{c^2}\, e^{+i\,\omega\, t}\, e^{-\,i\, k_\rho\, \rho}\, \frac{\partial^2 \boldsymbol{P}(\rho,z,t)}{\partial t^2} \tag{3}$$

We assume that the polarization is essentially electronic, and it follows instantly the change on the electric field, which is a good approximation on a time scale of a few femtosecond, and that the media have a linear behaviour. Hence we write

$$\boldsymbol{P}(\rho,z,t) \simeq \chi(z,t)\, \boldsymbol{E_0}(z,t)\, e^{-i\,\omega\, t}\, e^{\,i\, k_\rho\, \rho}$$

(4)

where $\chi(z,t)$ is the electronic susceptibility. It leads to

$$\frac{\partial^2 \boldsymbol{E_0}(z,t)}{\partial z^2} - {k_\rho}^2 \boldsymbol{E_0}(z,t) + \frac{1}{c^2}\left(\omega^2 \boldsymbol{E_0}(z,t) + 2\, i\, \omega \frac{\partial \boldsymbol{E_0}(z,t)}{\partial t} - \frac{\partial^2 \boldsymbol{E_0}(z,t)}{\partial t^2}\right)$$
$$= \frac{4\pi}{c^2} e^{+i\omega t} e^{-i k_\rho \rho} \frac{\partial^2 (\chi(\rho,z,t)\, \boldsymbol{E_0}(z,t)\, e^{-i\omega t}\, e^{i k_\rho \rho}}{\partial t^2})$$

(5)

At this stage, we will precise the framework of our analysis ; we adopt the slowly varying amplitude approximation (SVA) both in time and space : the second derivatives with respect to space and time are neglected, and the following conditions are fulfilled

$$\left|\frac{\partial \boldsymbol{E_0}(z,t)}{\partial z}\right| \ll \frac{\omega}{c} \left|\boldsymbol{E_0}(z,t)\right|$$

(6)

$$\left|\frac{\partial \boldsymbol{E_0}(z,t)}{\partial t}\right| \ll \omega \left|\boldsymbol{E_0}(z,t)\right|$$

(7)

The nature of these usual approximations is very different ; the SVA in space holds if the variation of the electric field amplitude is small upon travelling a distance equal to the carrier wavelength, whilst the SVA in time is valid provided that the electric field amplitude varies little within the carrier period at any position. Moreover we consider that the time scale of the changes in the susceptibility and the electric field are long compared to the period of the optical pulse ; that is we adopt the first-order approximation.

$$\frac{\partial \chi(\rho,z,t)}{\partial t} \boldsymbol{E_0}(z,t) \ll \frac{\partial \boldsymbol{E_0}(z,t)}{\partial t}$$

(8)

As a result of the different approximations, Eq.(5) can be reduced to a time-dependent Schrödinger equation

$$\frac{\partial^2 \boldsymbol{E_0}(z,t)}{\partial z^2} + k_\perp(z)^2 \boldsymbol{E_0}(z,t) + 2\, i\, \epsilon(z) \frac{\omega}{c^2} \frac{\partial \boldsymbol{E_0}(z,t)}{\partial t} = 0$$

(9a)

with

$$\epsilon(z) = \left(1 - \chi(z)\right)^2$$

(9b)

and

$$k_{\perp}(z)^2 = \epsilon(z)\,\frac{\omega^2}{c^2} - {k_{\rho}}^2$$

(9c)

Eqs.(9) could be treated by means of numerical codes developed to solve the time-dependent Schrödinger equation. We choose in this work to use instead the approach of CWT which proved to be very efficient in this kind of optical problem [28]. In a 1D-PC, the electric field can be written as the superposition of two waves propagating in opposite directions along the z-axis, so that we write in the spirit of the CWT

$$\boldsymbol{E_0}(z,t) = F(z,t)\,e^{+\,i\,\kappa\,z} + B\,(z,t)\,e^{-\,i\,\kappa\,z}$$

(10a)

with

$$\kappa = k\,sin\theta = k_{\perp @\,\epsilon(z)=1}$$

(10b)

The susceptibility in the 1D-PC can be expanded in Fourier series

$$\chi(z,t) = \bar{\chi} + \sum_{p=-\infty}^{+\infty} \Delta\chi\,u_p\,e^{i\,pg\,z}$$

(11a)

with

$$\bar{\chi} = \chi_1\,\gamma + \chi_2\,(1-\gamma)$$

(11b)

$$\Delta\chi = \chi_1 - \chi_2$$

(11c)

$$u_p = \frac{-i}{2\,p\,\pi}\,(1 - e^{-2i\pi p\gamma})$$

(11d)

$$g = \frac{2\,\pi}{d}$$

(11e)

In order to get a system of differential equations with constant terms, it is convenient to introduce auxiliary amplitude terms

$$f(z,t) = F(z,t)\,\exp\left[-i\,\left(\frac{p\,g}{2\,\pi} - \kappa\right)z\,\right]$$

(12a)

and

$$b(z,t) = B(z,t)\exp\left[+i\left(\frac{p\,g}{2\,\pi} - \kappa\right)z\right]$$

(12b)

In the vicinity of the *pth* Bragg diffraction resonance $\frac{p\,g}{2\,\pi} - \kappa \approx 0$ and $f(z,t) = F(z,t)$, $b(z,t) = B(z,t)$. Combining Eqs.(9)-(12) and performing the calculation in the framework of the assumptions given above, it comes after some algebra the following system of time-dependent coupled PDEs satisfied by the varying amplitudes $f(z,t)$ and $b(z,t)$ closed to the *pth* Bragg diffraction ; using the matrix formalism, this system reads

$$\frac{\partial}{\partial z}\,\overline{\mathcal{E}}(z,t) + \overline{T}\frac{\partial}{\partial t}\,\overline{\mathcal{E}}(z,t) = i\,\overline{\mathcal{M}}\,\overline{\mathcal{E}}(z,t)$$

(13a)

where $\overline{\mathcal{E}}(z,t)$ is the column amplitude vector

$$\overline{\mathcal{E}}(z,t) = \begin{pmatrix} f(z,t) \\ b\,(z,t) \end{pmatrix}$$

(13b)

$\overline{\mathcal{M}}$ is the propagation matrix in space

$$\overline{\mathcal{M}} = \begin{pmatrix} -\alpha & K^{+} \\ K^{-} & \alpha \end{pmatrix}$$

(13c)

with

$$\alpha = \frac{p\,g}{2\,\pi} + \frac{k^2}{2\,\kappa} - \kappa$$

(13d)

$$K^{+} = \frac{k^2}{2\,\kappa}\,\Delta\chi\,u_p$$

(13c)

$$K^{-} = \frac{k^2}{2\,\kappa}\,\Delta\chi\,u_{-p}$$

(13d)

$\overline{T}$ is the propagation matrix in time

$$\overline{T} = \begin{pmatrix} -\frac{k}{c\,\kappa} & 0 \\ 0 & \frac{k}{c\,\kappa} \end{pmatrix}$$

(13e)

3. Matrix formalism for the CWT

We first consider the time-independent case

$$\frac{\partial}{\partial z}\,\overline{\mathcal{E}}(\mathrm{z}) = i\,\overline{\mathcal{M}}\,\overline{\mathcal{E}}(\mathrm{z}) \tag{14}$$

The solution can be obtained by substituting

$$\overline{\mathcal{E}}(\mathrm{z}) = \begin{pmatrix} A \\ B \end{pmatrix} e^{\,i\,\psi\,z} \tag{15}$$

into Eq.(14). It comes

$$\overline{(\mathcal{M} - \psi\,\bar{I})}\,\begin{pmatrix} A \\ B \end{pmatrix} = 0 \tag{16}$$

ψ are eigenvalues of $\overline{\mathcal{M}}$

$$\psi^{\pm} \equiv \pm q\,;\, q = \sqrt{K^{+}K^{-} + \alpha^{2}} \tag{17}$$

To the two eigenvalues $\psi^{\pm}$ are associated two eigenvectors $\bar{V}^{\pm}$

$$\bar{V}^{\pm} = \begin{pmatrix} K^{+} \\ \pm q \end{pmatrix} \tag{18}$$

The solutions for $\overline{\mathcal{E}}(\mathrm{z})$ can be derived using the eigenmatrix

$$\bar{P} = (\bar{V}^{+} \quad \bar{V}^{-}) = \begin{pmatrix} K^{+} & K^{+} \\ +\,q - \alpha & -q - \alpha \end{pmatrix} \tag{19}$$

that is

$$\overline{\mathcal{E}}(z) = \bar{P}\,\begin{pmatrix} e^{+\,i\,q\,z} & 0 \\ 0 & e^{-\,i\,q\,z} \end{pmatrix}\bar{P}^{-1}\,\overline{\mathcal{E}}(0)$$

Since the general solution is a linear combination of the eigensolutions

$$\overline{\mathcal{E}}(\mathrm{z}) = C^{+}\,e^{+\,i\,q\,z}\,\bar{V}^{+} + C^{-}\,e^{-\,i\,q\,z}\,\bar{V}^{-} = \bar{P}\begin{pmatrix} e^{+\,i\,q\,z} & 0 \\ 0 & e^{-\,i\,q\,z} \end{pmatrix}\begin{pmatrix} C^{+} \\ C^{-} \end{pmatrix} \tag{20}$$

At z = 0, one has

$$\begin{pmatrix} C^{+} \\ C^{-} \end{pmatrix} = \bar{P}^{-1}\,\overline{\mathcal{E}}(0) \tag{21}$$

Putting Eq.(21) in Eq (20), it follows that

$$\overline{\mathcal{E}}(\mathrm{z}) = \bar{S}\,(z)\,\overline{\mathcal{E}}(0)$$

(22)

where $\bar{S}\,(z)$ is obtained by the product $\bar{P}\begin{pmatrix} e^{+\,i\,q\,z} & 0 \\ 0 & e^{-\,i\,q\,z} \end{pmatrix}\bar{P}^{-1}$:

$$\bar{S}\,(z) = \begin{pmatrix} \cos qz - \; i\;\dfrac{\alpha}{q}\sin qz & i\;\dfrac{K^{+}}{q}\;\sin qz \\ i\;\dfrac{K^{-}}{q}\;\sin qz & \cos qz + \; i\;\dfrac{\alpha}{q}\sin qz \end{pmatrix}$$

(23)

Returning to the time-dependent case, one searches the solution by analogy with the time-independent case in the following form :

$$\overline{\mathcal{E}}(z\,,t) = \begin{pmatrix} A(t) \\ B(t) \end{pmatrix} e^{\,i\,\psi\,z}$$

(24)

Inserting in Eq.(13a) gives after derivation with respect space

$$\frac{\partial}{\partial t}\begin{pmatrix} A(t) \\ B(t) \end{pmatrix} = -\bar{G}\;\begin{pmatrix} A(t) \\ B(t) \end{pmatrix}$$

(25)

with

$$\bar{G} = \; i\;\overline{T}^{-1}(\psi\,\bar{I} - \bar{\mathcal{M}})$$

(26)

Integration on time gives

$$\begin{pmatrix} A(t) \\ B(t) \end{pmatrix} = \exp(-\bar{G}\;t)\begin{pmatrix} A(0) \\ B(0) \end{pmatrix}$$

(27)

Finally it comes by following a way similar to the one presented for the time-independent case

$$\overline{\mathcal{E}}(z\,,t) = \mathrm{R}(z,t)\;\overline{\mathcal{E}}(0\,,0)$$

(28)

where

$$\mathrm{R}(z,t) = \exp(-\bar{G}\;t)\;\bar{S}\,(z)$$

(29)

The reflection coefficient follows from the initial and boundary conditions : at $z = 0$, a Heaviside unit-step $\Theta(\mathrm{t})$ is applied, so that $f(0,t) = \Theta(\mathrm{t})$, and at $z = L$ there is no incoming wave, so that $b(L,t) = 0$, which gives

$$\begin{pmatrix} f(L,t) \\ b(L,t) = 0 \end{pmatrix} = \begin{pmatrix} \mathrm{R}_{11}(L,t) & \mathrm{R}_{12}(L,t) \\ \mathrm{R}_{21}(L,t) & \mathrm{R}_{22}(L,t) \end{pmatrix} \begin{pmatrix} \Theta(0^+) \\ b(0,0^+) \end{pmatrix} \tag{30}$$

Then

$$b(0,0^+) = -\frac{\mathrm{R}_{21}(L,t)}{\mathrm{R}_{22}(L,t)} \tag{31}$$

To calculate the reflection coefficient, we need $b(0,t)$. From Eq.(31)

$$\begin{pmatrix} f(0,t) \\ b\,(0,t) \end{pmatrix} = \bar{R}(0,t) \begin{pmatrix} f(0,0^+) = \Theta(0^+)) \\ b(0,0^+) \end{pmatrix} \tag{32}$$

or

$$b(0,t) = \mathrm{R}_{21}(0,t) + \mathrm{R}_{22}(0,t)\, b(0,0^+) \tag{33}$$

Combining Eqs.(31) and (33)

$$b(0,t) = \mathrm{R}_{21}(0,t) - \mathrm{R}_{22}(0,t)\, \frac{\mathrm{R}_{21}(L,t)}{\mathrm{R}_{22}(L,t)} \tag{34}$$

Consequently using the definition of the reflection coefficient, one finds the indicial response R_Θ in terms of reflectivity at the time t, after switching on the pulse at $t = 0$:

$$R_\Theta(\mathrm{t}) \equiv \left| b(0,t) \right|_0^{\mathrm{t}} \Big|^2 \tag{35}$$

4. Numerical simulations

Since development of short pulse sources is rapidly growing in the high energy range of the electromagnetic spectrum, we choose to carry out our simulation in the x-ray domain for a 1D-PC consisting in a Bragg reflector formed by a Fe/C periodic stack of $\mathcal{N}$ bilayers ; the period d is equal to 5.0 nm and the γ ratio is equal to 0.5, that is to say the thicknesses of the Fe and C layers are the same. The energy of the incident radiation is 8 keV which gives a Bragg angle of 0.93°. Figure 2 displays the steady-state reflectivity versus θ angle for $\mathcal{N} = 100$; the value of the peak reflectivity (PR) is around 0.64. In all calculations we use for the optical indices, the values tabulated in the CXRO database [29].

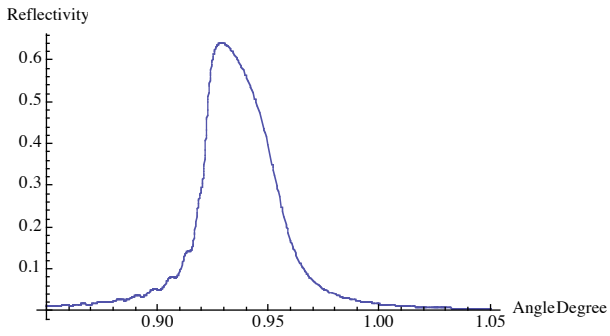

Figure 2: Steady-state reflectivity versus the glancing angle θ at 8 keV of the 1D-PC, the parameters of which are given in the main text.

Figure 3 shows the instantaneous PR, hereafter denoted $\hat{R}_{\Theta}(t)$, calculated at 8 keV for a constant intensity source switched on abruptly at $t = 0$; this quantity is the indicial response in terms of peak reflectivity. Let us note that the superscript $\widehat{\ }$ will be used to denote the PR obtained at the Bragg angle which must be distinguished from the reflectivity without superscript obtained at any glancing angle θ. One observes that $\hat{R}_{\Theta}(t)$ starts from zero then grows and tends asymptotically to the steady-state value (0.64) after a transient period of duration τ_t estimated to be around 42 fs. We define the transient time τ_t as the rise-time required to go from 10% to 90% of the asymptotic PR. For the 1D-PC and in the *s*-polarisation case, the extinction length L_{exc} is given by the formula

$$L_{exc} = \frac{\lambda}{\sqrt{Re(\Delta\chi^2 u_{-p} u_p)}} \tag{36}$$

In our example, L_{exc} takes a value close to 2570 nm, so that the characteristic time $\tau_c = L_{exc}/c$ is around 8.5 fs. The time-dependent diffraction in crystals shows that the saturation time τ_s [24] is of the order of a few τ_c: $\tau_s = \sigma\,\tau_c$ with σ around 4 as mentioned in Figure 1 of Ref. [24]. The latter formula gives 34 fs for τ_s in agreement with our transient time value τ_t.

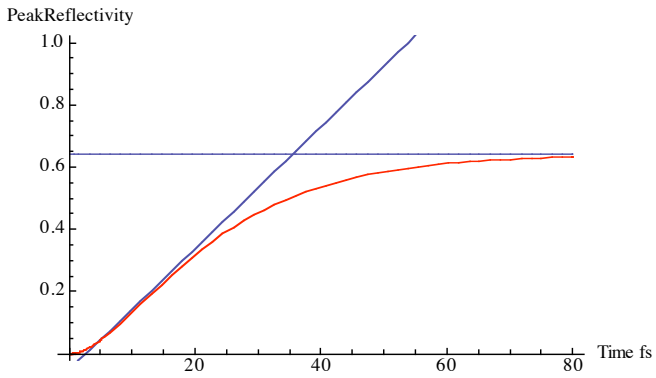

Figure 3: Instantaneous peak reflectivity $\hat{R}_\Theta(t)$ at 8 keV of the 1D-PC, the parameters of which are given in the main text (red curve), steady-state peak reflectivity (dashed blue curve), tangent line at the inflection point of the instantaneous reflectivity (solid blue line).

Let us now consider the transfer function $\hat{R}_\delta(s)$ in terms of peak reflectivity that is the Laplacian transform of the impulse response $\hat{R}_\delta(t)$. This impulse response corresponds to the instantaneous peak reflectivity obtained when the system is struck by a Dirac pulse at $t = 0$ and must not be confused with the indicial response $\hat{R}_\Theta(t)$. There are a lot of methods to deduce the transfer function $\hat{R}_\delta(s)$ from the indicial response $\hat{R}_\Theta(t)$. In a first basic approach, we apply the graphic method introduced by Strejc [30]. We briefly present the method; the transfer function $\hat{R}_\delta(s)$ is written on the following form for an aperiodic system without delay :

$$\hat{R}_\delta(s) = \frac{K}{(1 + \tau\, s)^n} \tag{37}$$

where K, n and τ are parameters to be determined from the curve $\hat{R}_\Theta(t)$: K is given by the asymptotic value $\lim_{t\to\infty} \hat{R}_\Theta(t)$, n is a tabulated value depending on the ratio $\frac{t_1}{t_2 - t_1}$ and τ is also a tabulated value depending on n and the difference $t_2 - t_1$; the quantities t_1, t_2 are the intersections of the tangent line at the inflection point of $\hat{R}_\Theta(t)$ with the abscissa (time) axis and the asymptotic line respectively (see Figure 3). In our example $t_1 \approx 0.0025$ fs, $t_2 \approx 35.34$ fs and consequently from the tables, one finds $n = 1$ and $\tau = 35.34$ fs . Finally in our case :

$$\hat{R}_\delta(s) = \frac{K}{1 + \tau\, s} \approx \frac{0.64}{1 + 35.34\, s} \tag{38}$$

By performing an inverse Laplacian transform, one finds that the indicial response can be approximated by

$$\hat{R}_{\Theta}(t) \approx K\,(1 - e^{-\frac{t}{\tau}}) \tag{39}$$

If we consider an incident pulse of shape $E(t)$, the time-depend reflected pulse *S(t)* is obtained from the convolution product involving the impulse response $R_{\delta}(t)$

$$S(t) = E(t) \star \hat{R}_{\delta}(t) = \int_{-\infty}^{+\infty} E(\theta)\hat{R}_{\delta}(t-\theta)d\theta \tag{40}$$

that is by virtue of the convolution theorem

$$S(s) = E(s)\,R(s) \tag{41}$$

where as usual, $Y(s)$ stands for the Laplacian transform of a function $Y(t)$. If $E(t)$ has a sine-squared shape, $E(t) = \sin^2\left(\pi\,\frac{t}{T}\right)$, in the interval [*0,T*] and *0* outside, as shown in the inset of Figure 4, then the Laplacian transform reads

$$S(s) = \frac{2\,K\,\pi^2\,(1 - e^{-s\,T})\Theta(T)}{(1 + s\,\tau)(4\,\pi^2\,s + s^3\,T^2)} \tag{42}$$

where $\Theta(T)$ stands for the Heaviside unit-step function. The formula for the inverse Laplacian transform of $S(s)$ is too long to be explicitly given in the text, but Figure 4 displays $S(t)$ for an incident pulse of width T = 10 fs, for the system under consideration.

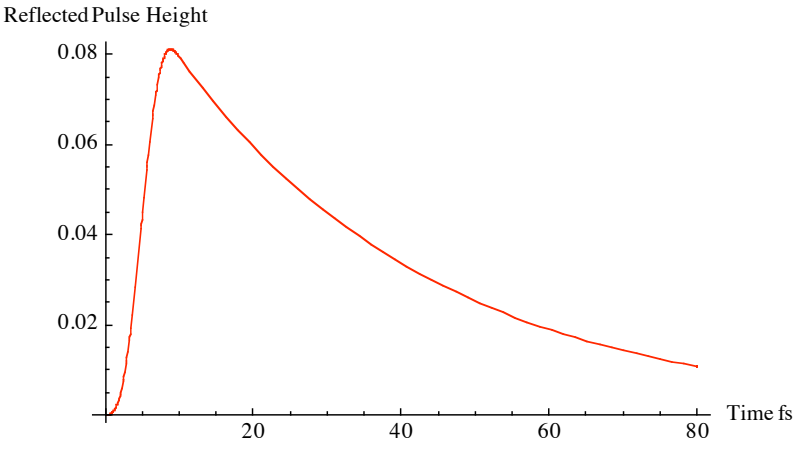

Figure 4: Time dependence of the reflected pulse height S(t) with a sine-squared incident pulse of width T = 10 fs, for the system under consideration (see main text).

One observes clearly the time-delay effect : at t = 10 fs corresponding to the width of the incident pulse, the reflected pulse reaches its maximum intensity which is only 8% of its initial value. Let us also note that the full width at half maximum is about 25 fs, much larger than the initial pulse width.

5. Conclusion

The coupled-wave theory has been successfully implemented to treat the problem of time-dependent Bragg reflection by a 1D-PC. In the framework of this approach, we have shown that the instantaneous peak reflectivity of 1D-PC under unit-step striking presents a time-delay effect : this reflectivity starts from a zero value at the switching-on time and tends asymptotically towards the steady-state value. The transient time seems to be conditioned by the extinction length. This result is in agreement with the conclusions drawn from the study of the time-dependent diffraction by crystals [25,26]. Our conclusion has potential consequences in time-dependent optics, in particular x-ray optics because x-ray pulse sources such as X-FEL have time duration comparable with the transient time of the multilayer Bragg mirrors.

An important but no trivial extension of the present study is to incorporate a pertinent description of the possible distortion of the structure due to the large power which can be deposited in the materials. Indeed short pulses sources, because they generally deliver high power (high energy in short duration), can distort or even damage the optics [31,32].

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