



**HAL**  
open science

## Qualitative non-destructive testing of concrete-like materials

Lorenzo Audibert, Housseem Haddar, Alexandre Girard

► **To cite this version:**

Lorenzo Audibert, Housseem Haddar, Alexandre Girard. Qualitative non-destructive testing of concrete-like materials. WAVES 2013, Jun 2013, Tunis, Tunisia. hal-01111875

**HAL Id: hal-01111875**

**<https://hal.science/hal-01111875v1>**

Submitted on 31 Jan 2015

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Qualitative non-destructive testing of concrete-like materials

**L. Audibert<sup>1,2,\*</sup>, H. Haddar<sup>2</sup> and A. Girard<sup>1</sup>**

<sup>1</sup>EDF R&D, STEP Departement

<sup>2</sup>INRIA Saclay Ile de France/Ecole Polytechnique CMAP

\*Email: lorenzo.audibert@edf.fr

## Abstract

In this paper we explore the capacity of Qualitative Inversion Methods to detect macroscopic cracks or a lattice of small cracks in concrete-like materials. These materials are difficult to probe since the heterogeneities size inside the medium and the wavelength of classically used sensors are of the same order of magnitude. We shall demonstrate how this difficulty can be avoided in the case of macroscopic cracks by using so-called differential measurements and application of the Linear Sampling Method. For a lattice of small cracks we rather propose to construct a macroscopic indicator based on the eigenvalues of a suitable transmission problem.

## Introduction

We are interested in using elastic waves to perform complete non destructive testing of concrete-like materials. The main difficulty in controlling concrete is its heterogeneous nature. Concrete is made of cement paste, water and aggregates. After a drying period, this mixture results in a heterogeneous material. As far as waves propagation is concerned, the main characteristics is the difference of celerity between aggregates and cement paste, especially as the wavelength and the size of the aggregates are similar, thus, in what follows, we will model concrete as a biphasic material. This concrete-like material has the following properties: the celerity of pressure wave is  $5700ms^{-1}$  in aggregates and  $4300ms^{-1}$  in cement paste. Defect

which has a length larger than the aggregates appears and grows until it reaches the surface. Those two types of defects are of interest and we will expose our preliminary results on how to detect them. Although waves in concrete are elastic, we start with the simpler case of acoustic waves and postpone the treatment of the elastic one. We therefore assume that the pressure field,  $u$  is solution to the well-known Helmholtz equations:

$$\left\{ \begin{array}{l} \Delta u + k^2 n u = 0, \quad u = u^i + u^s \quad \text{in} \quad \mathbb{R}^2 \setminus \Gamma \\ \lim_{r \rightarrow \infty} \sqrt{r} \left( \frac{\partial u^s}{\partial r} - i k u^s \right) = 0 \\ \frac{\partial u}{\partial \nu} = 0 \quad \text{on} \quad \Gamma \end{array} \right.$$

where  $\Gamma$  is the crack(s) inside the medium,  $\nu$  is a unit normal vector on  $\Gamma$ ,  $n$  is the relative index with respect to the celerity in the air and  $u^i$  is the incident field created by a point source located at the interface air-concrete. For our numerical simulations, in order to simulate the heterogeneities in concrete we used synthetic geometries generated by [4] (See Figure 1, left) and eliminated the aggregates, which have an area smaller than  $\frac{\lambda^2}{10^2}$ , where  $\lambda = \frac{2\pi}{k}$ . This results in the medium represented by Figure 1, right. According to the remarks above, the index of the aggregates equals  $2, 8 \cdot 10^{-3}$  and of the cement equals  $4, 8 \cdot 10^{-3}$ .

## 1 The inverse crack problem

We here investigate the inverse problem of finding cracks in concrete using the framework of the linear sampling method [5]. First we concentrate on the macrocracks and then on the small cracks lattice. We use multistatic measurements with sensor (sources and receivers) located on the interface  $\Sigma$  between air and concrete, namely these measurements are  $u^s(x, x_0)$   $x, x_0 \in \Sigma$ . In the sequel we introduce the subscript  $b$  which indicates the solution of the direct problem without defect and the subscript  $h$  which indicates the direct problem without defects and aggregates. The function  $\Phi(z, \cdot)$  denotes the fundamental outgoing solution with Dirac source at position  $z$ .

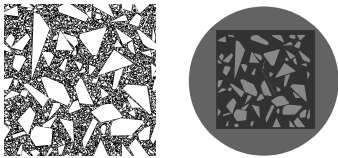


Figure 1: Simulated concrete and numerical set-up for direct and inverse problem

appears in concrete materials mainly in two forms. First a lattice of small cracks which are located at the interface between aggregates and cement paste. When this lattice is too dense, a macroscopic crack

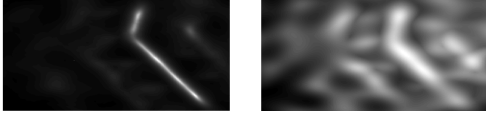


Figure 2: Identification of cracks using  $\Phi_b$ (left) and  $\Phi_h$ (right)

### 1.1 Macroscopic Cracks

We introduce the near field operator for differential-measurements (i.e. measurements of  $u^s$  and  $u_b^s$ ),

$$[\mathcal{N}_b g](\cdot) = \int_{\Sigma} [u^s(\cdot, x_0) - u_b^s(\cdot, x_0)]g(x_0)ds(x_0)$$

Following the framework of the Linear Sampling Method for cracks developed in [2], one can find the support of the scattering crack, by looking for every  $z$  at the value of  $\|g_z\|$ , where  $g_z$  is the (regularized) solution to  $[\mathcal{N}_b g_z](\cdot) = \frac{\partial \Phi_b}{\partial \nu(z)}(z, \cdot)$  (We refer to [2] for the details on how one copes with the unknown normal  $\nu(z)$ ). Using this algorithm in our configuration yields to a quite good reconstruction as demonstrated by Figure 2 (left). However, although in practice  $u_b^s$  can be known in a differential measurement framework,  $\Phi_b$  will always be unknown. Considering  $g_z$  the solution of  $[\mathcal{N}_b g_z](\cdot) = \frac{\partial \Phi_h}{\partial \nu(z)}(z, \cdot)$  shows (Figure 2, right) that the quality of the reconstruction considerably deteriorates, although one can still distinguish the existence of a defect different from aggregates shape. This can be enough for qualitative inspection. We shall explore in the near future how one can improve the reconstruction by optimizing the choice of the background as in [3]. We shall also numerically analyze the feasibility of control with non-differential measurements. Finally, on the theoretical level, sensors at the interface are a realistic set up which is not yet justified for the LSM and we shall seek for a formal justification.

### 1.2 The case of lattice of microscopic cracks

When the cracks are small and numerous the LSM algorithm would not be able to locate them even in a differential setting. We shall rather use a macroscopic indicator based on the interior transmission eigenvalues which are computable from the measurements using following near field operator:

$$[\mathcal{N}_h g](\cdot) = \int_{\Sigma} [u^s(\cdot, x_0) - u_h^s(\cdot, x_0)]g(x_0)ds(x_0)$$

With this operator, since it also contains the contribution of the aggregates in the scattering effect, LSM will produce a support that covers almost all the heterogeneous material. It is well known [5] that the LSM fails for some frequencies which are, in our case, the eigenvalues of the following transmission problem:

$$\begin{cases} \Delta v + k^2 v = 0 & \text{in } \mathcal{D} \\ \Delta u + nk^2 u = 0 & \text{in } \mathcal{D} \setminus \Gamma \\ u = v, \frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} & \text{on } \partial \mathcal{D} \\ \frac{\partial u}{\partial \nu} = 0 & \text{on } \Gamma \end{cases}$$

where  $\mathcal{D}$  will be the scatterer (including the aggregates convex support and the cracks). Motivated by [1], we shall investigate the evolution of those frequencies with respect to the presence of cracks. In order to ensure that first transmission eigenvalues stay in the frequency range of the sources, we may introduce an artificial contrast in the equation solved by  $v$ , (characterized by an index  $n_a \neq n$ ) localized in a region of size comparable to the incident wavelength. We finally recall that our interior transmission problem is still open in terms of existence and evolution of the eigenvalues with respect to  $\Gamma$ .

## References

- [1] F. Cakoni, A. Cossonnière and H. Haddar, *Transmission eigenvalues for inhomogeneous media containing obstacles*, Inverse Problems and Imaging, 6, no 3, 373-398 (2012).
- [2] F. Ben Hassen, Y. Boukari and H. Haddar, *Application of the linear sampling method to identify cracks with impedance boundary conditions*, Inverse Problems in Science and Engineering, pp. 1-25 (2012).
- [3] H. Haddar and G. Migliorati, *Numerical analysis of the Factorization Method for EIT with piece wise constant uncertain background*, 2012 (preprint)
- [4] J. Escoda and al., *Estimation of local stresses and elastic properties of a mortar sample by FFT computation of fields on a 3D image*, Cement and Concrete Research, 41(5):542556 (2011).
- [5] F. Cakoni and D. Colton, *Qualitative Methods in Inverse Scattering Theory*, 1st edition, Springer-Verlag, 2006.