

# Transport coefficients in thermal plasmas

## Numerical calculation in the equilibrium Helium plasma

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### Introduction

The properties of transport in plasmas and/or in gases at high temperature ( $> 4000K$ ) play a key role in several applications, like the area of electric arc, plasma welding, plasma cutting and the field of aerospace problems for example atmospheric entry flights [1] in which we need an accurate computation of the transport properties. The estimation of transport coefficients is very difficult to do experimentally at high temperature, thus the theoretical/numerical calculation is the adequate method.

Two approaches solutions of the Boltzmann integro-differential equation have been proposed by Grad [2] and Chapman-Enskog [3] using the analysis of rigorous kinetic theory. The first method is called the Grad's moment method and second method Chapman-Enskog is the most used in the calculation of transport coefficients and we have opted for this method.

### Transport Coefficients

Gradients in concentration, velocity and temperature, cause a net displacement or a net transport of mass, momentum, and energy, respectively [4]. We can formulate mathematically this notion of transport as general form of a flux:

$$\vec{J} = -\alpha \vec{\nabla} \varphi \quad (1)$$

where  $\vec{J}$  is the flux vector appropriate to quantity  $\varphi$ , and  $\alpha$  is a proportionality constant known as the transport coefficient.

Chapman-Enskog [3] have formulated all transport coefficients in term of collision integrals [2] and we can show these coefficients in their first approximation as follows:

- **Coefficient of Self-diffusion** ( $m^2 s^{-1}$ ):

$$D(T) = 2.628 \times 10^{-7} \left( \frac{T^3}{M} \right)^{\frac{1}{2}} \frac{1}{p < \Omega^{1,1}(T) >} \quad (2)$$

- **Viscosity** ( $kg m^{-1} s^{-1}$ ):

$$\eta(T) = 2.669 \times 10^{-6} (TM)^{\frac{1}{2}} \frac{1}{< \Omega^{2,2}(T) >} \quad (3)$$

- **Thermal Conductivity** ( $J m^{-1} s^{-1} K^{-1}$ ):

$$\kappa(T) = 8.3227 \times 10^{-5} \left( \frac{T}{M} \right)^{\frac{1}{2}} \frac{1}{< \Omega^{2,2}(T) >} \quad (4)$$

where  $M$ ,  $p$ , and  $T$  are the *molecular weight* of the particle, the *pressure* in atmosphere, and *Temperature* in Kelvin, respectively.  $\Omega^{2,2}$  and  $\Omega^{1,1}$  ( $\text{\AA}^2$ ) are the collision integrals which are formulated by Hirschfelder [2] as:

$$\Omega^{\ell,s}(T) = \sqrt{\frac{KT}{2\pi m}} \int_0^\infty e^{-\gamma^2} \gamma^{2s+3} Q^\ell(E) d\gamma \quad (5)$$

$\ell, s$  are the orders of approximation,  $K$  is the Boltzmann constant and  $m$  is the mass of species.  $\gamma^2 = E/KT$  is the reduced energy of the colliding particles and  $Q(E)$  is the integral cross section defined as:

$$Q^\ell(E) = 2\pi \int_0^\pi (1 - \cos^\ell \chi) \sigma(\chi, E) \sin(\chi) d\chi \quad (6)$$

$$= 2\pi \int_0^\infty (1 - \cos^\ell \chi) b db \quad (7)$$

where  $b$  is the impact parameter,  $\sigma(\chi, E)$  is the differential scattering cross section:

$$\sigma(\chi, E) = \frac{b}{\sin(\chi)} \left| \frac{db}{d\chi} \right| \quad (8)$$

$\chi$  is the deflection angle given by the equation:

$$\chi(E, b) = \pi - 2b \int_{rm}^\infty \frac{dr/r^2}{(1 - b^2/r^2 - V(r)/E)} \quad (9)$$

where  $rm$  is the distance of closest approach and  $V(r)$  is the interaction potential between colliding particles and it is the most interested parameter for the study of the collision problem [2].

### Singularity Problems

We distinguish three main singularities when we evaluate numerically the collision cross section.

1. **Rainbow Scattering**: One can remark in equation (8) that if ( $d\chi/db = 0$ ) then the differential cross section will be infinite ( $\sigma(\chi, E) \rightarrow \infty$ ).
2. **Glory Scattering**: This singularity occurs when the angle of deflection is a multiple of  $\pi$  ( $\chi = n\pi$ ), consequently  $\sin(\chi) = 0$  and the differential cross section will be infinite ( $\sigma(\chi, E) \rightarrow \infty$ ).
3. **Orbiting**: This singularity [5] is the most important anomalous that appears when calculating the total cross section equation (7). It means that when a particle collides with another one it can be that the two particles orbit each other, therefore the total cross section  $Q(E)$  may have many oscillations and the computing of  $Q(E)$  becomes very hard to do.

### Numerical Calculation

The transport coefficients formulated in equation (2), (3) and (4) are expressed in function of collision integrals equation (5), and the collision integral is function of collision cross section equation (7) which is itself function of angle of deflection equation (9). In summary, to compute the transport coefficients returns to evaluate efficiently the three integrals which are included one into the other one. We have developed a numerical program which compute the three integrals in same time using the *Clenshaw-Curtis quadrature* [6], and we have also succeeded to eliminate the singularities that occur principally in cross section integrals by introducing an adequate analytical expression in the regions where occur these singularities. Our program has been used to calculate the transport coefficients of the Helium plasma, the results in table 1 are compared with those of literature [7] and they are in good agreement.

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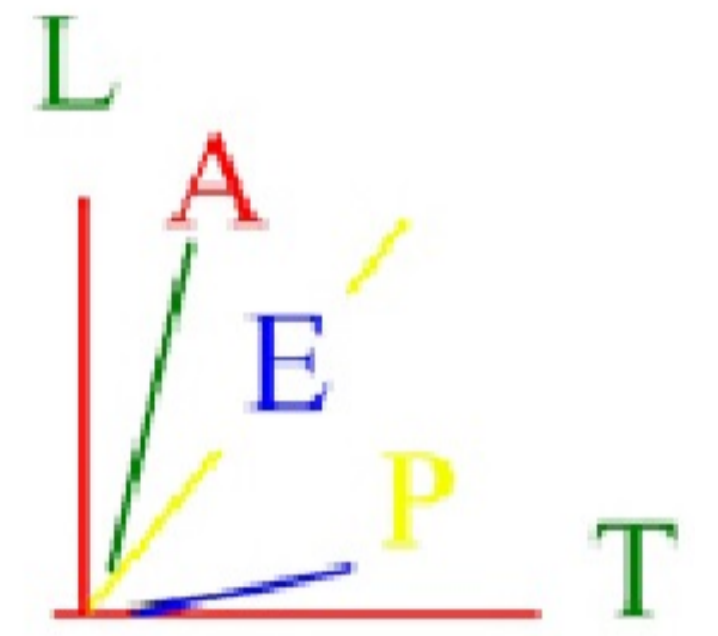
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T(K)	$\eta$ ( $\mu.Pa.s$ )	$\kappa$ ( $mW.m^{-1}.K^{-1}$ )	$D$ ( $cm^2.s^{-1}$ )
100	9.410	73.307	0.262
200	15.061	117.326	0.853
300	19.885	154.904	1.705
400	24.276	189.109	2.793
500	28.385	221.117	4.101
1000	46.613	363.106	13.661
2000	77.922	606.996	46.349
3000	106.248	827.645	95.665
4000	133.049	1036.421	160.755
5000	158.822	1237.182	241.290
7000	208.795	1626.459	447.861
9000	257.533	2006.116	711.852
10000	280.895	2188.097	865.839

Table 1: Transport coefficients of Helium versus temperature

For temperatures below  $10000K$  we have taken account only neutral particles because at such temperature the presence of ions and electrons is negligible. At high temperature ( $T > 10000$ ) we have to take account all interactions which may occur between neutrals, ions and electrons and the results are shown in Figures 1,2,3 and 4.

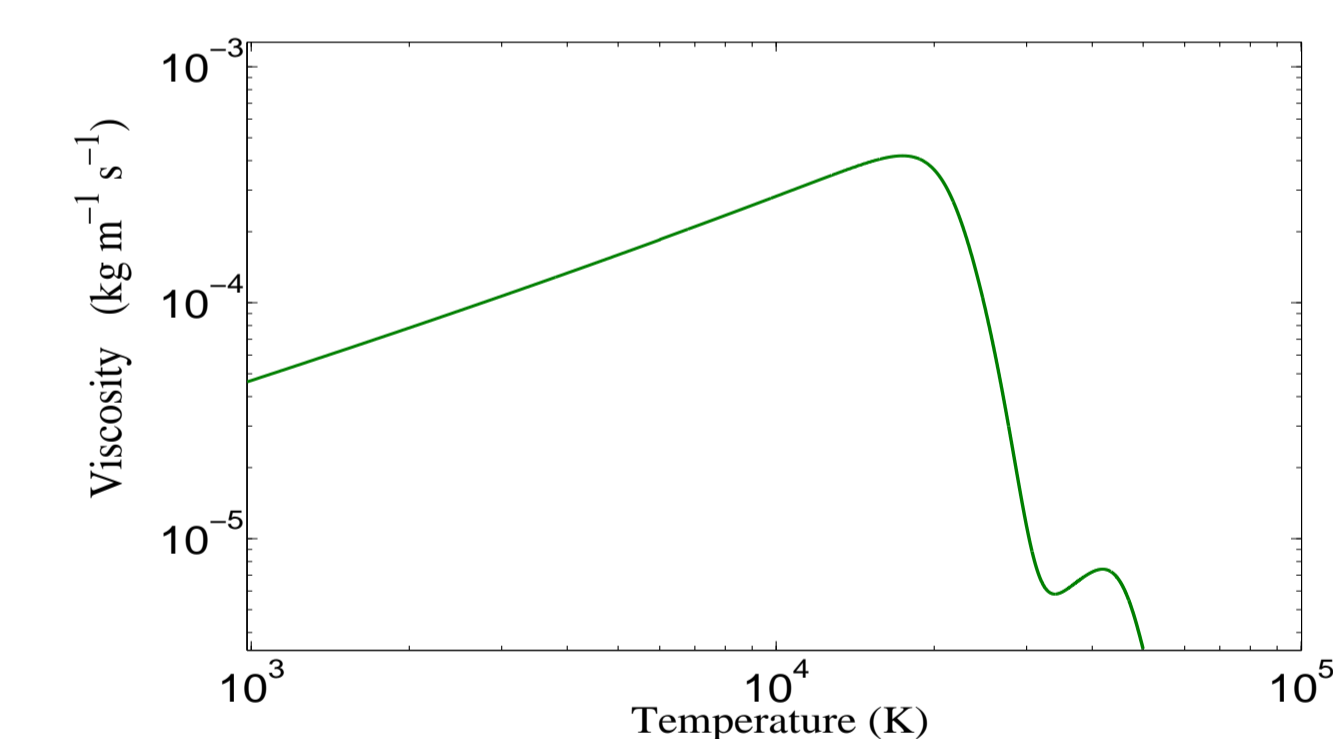


Figure 1: Viscosity of equilibrium helium plasma

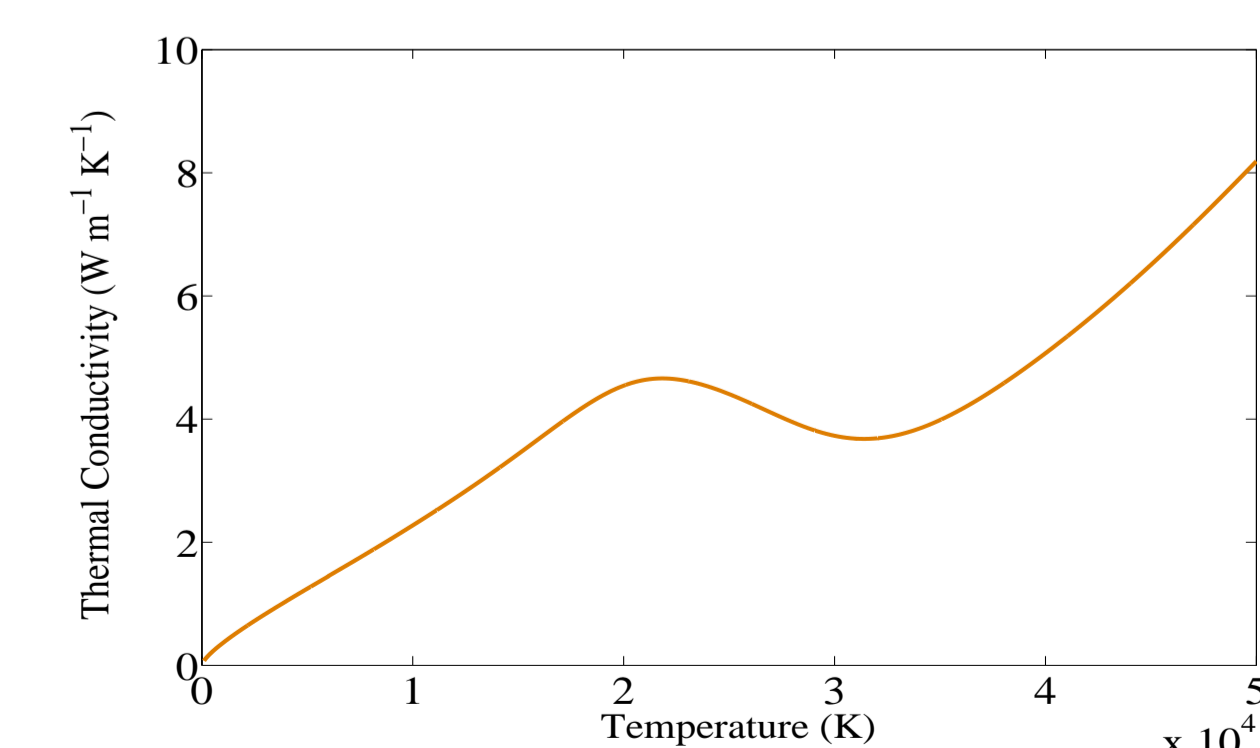


Figure 2: Thermal conductivity of equilibrium helium plasma

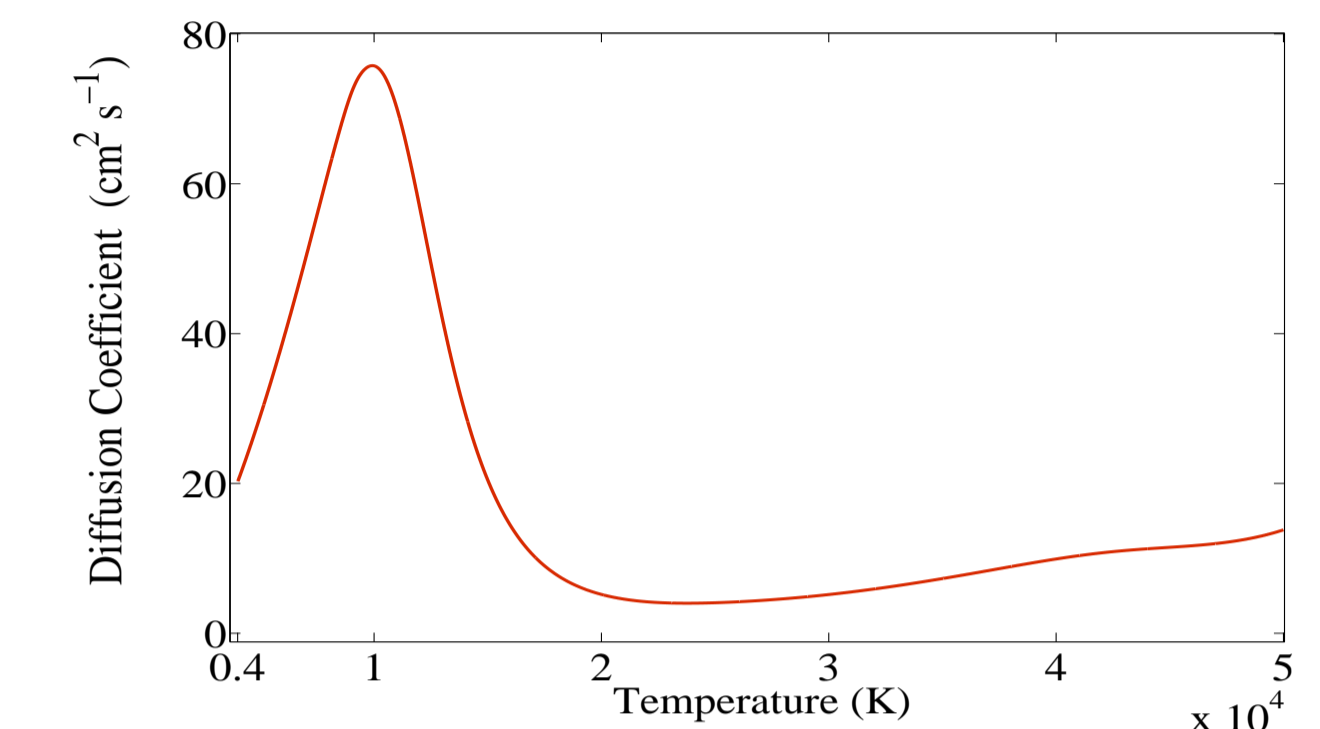


Figure 3: Multicomponent diffusion coefficient between He and He<sup>+</sup>

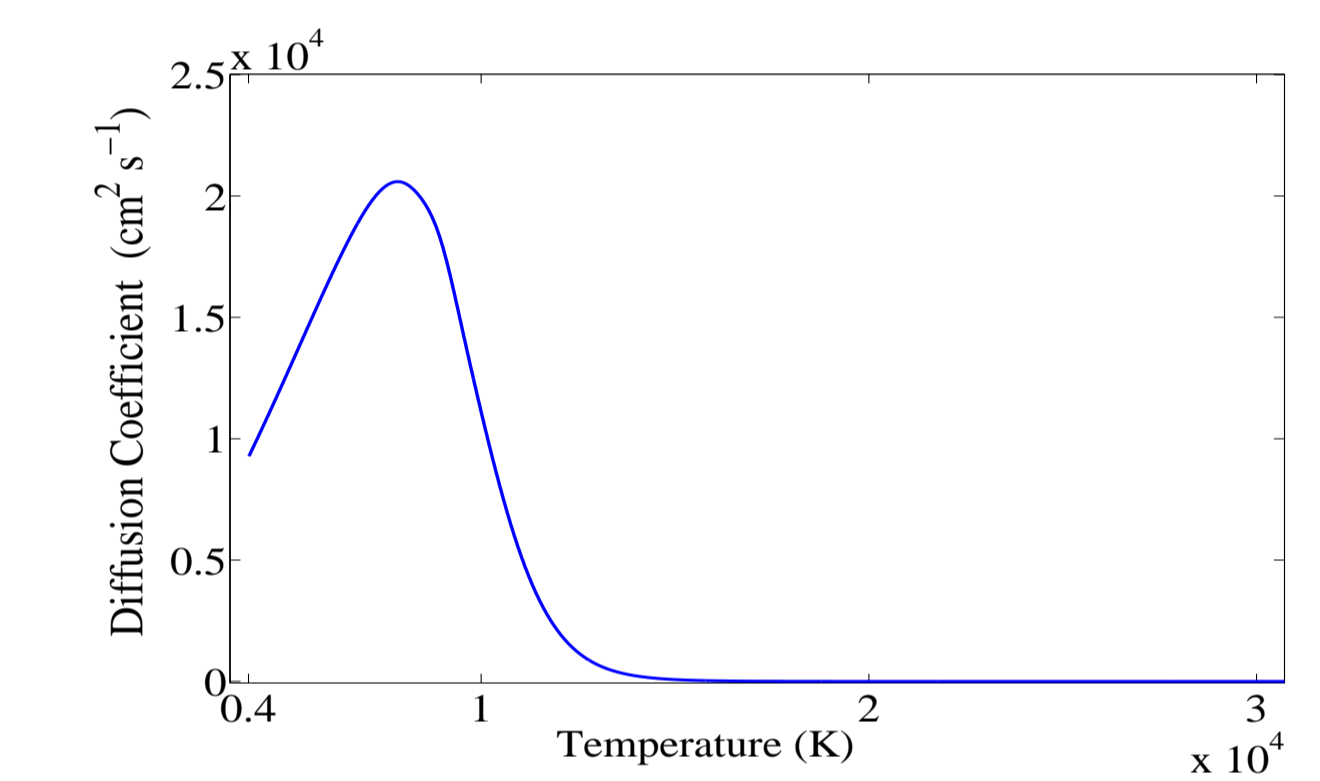


Figure 4: Multicomponent diffusion coefficient between He and e<sup>-</sup>

### Conclusion

We have performed a new efficient method to calculate numerically the transport coefficients at high temperature. Viscosity, diffusion coefficient and thermal conductivity were computed successfully for the equilibrium helium plasma which are in good agreement with those in the literature.

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