Modeling control valves in water distribution systems using a continuous state formulation

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Modeling Control Valves in Water Distribution Systems Using a Continuous State Formulation

by Olivier Piller[1] and Jakobus E. van Zyl[2] M.ASCE

ABSTRACT

Control valves are commonly used for the operation of water distribution systems. Modeling these devices typically requires that their operating states are known, or that a computationally expensive search is undertaken over all possible operating states. This paper presents a novel method of modeling control valves (including flow control, pressure sustaining, pressure reducing and check valves) in extended-period simulations of water distribution systems. Instead of the normal discrete control problem formulation, it is approached with the Karush-Kuhn-Tucker equations for an optimization problem with constraints.

The proposed method does not pre-require the operating state (open, closed, active) of each valve to be determined, as this is done implicitly. Pipe and valve flow rates and nodal heads are determined by (1) minimizing deviations from targets at control valves and (2) satisfy the state equations (conservation of mass and energy) by solving a constrained least-square problem.

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Sensitivity equations with respect to the control variables (valve settings) are derived from the state equations, and the control variables are updated using Levenberg-Marquardt iterations. The results of simple problems and case studies are presented to demonstrate the effectiveness of the approach.

Keywords: Water distribution systems; Hydraulic models; Control valves; Algorithms; Least-squares optimization; Penalty method; Flow control; Pressure control

INTRODUCTION

Water distribution systems have to provide a high level of service under widely varying conditions. To achieve this, engineers often employ control valves to manage flows and pressures. Control valves can operate mechanically (such as check valves) or through hydraulic circuits (such as flow control and pressure regulating valves), and can be controlled by local conditions or an external signal.

The common algorithms used for modeling the hydraulics of a water distribution system do not model the time-varying behavior of the system continuously, but calculate ósnapshotsô of the systemôs hydraulic state at certain points in the simulation period. At each time step, the snapshot solver has to solve the hydraulic network equations while simultaneously calculating the settings of all the control valves in the system. Tank levels are updated between snapshot simulations using a simple Euler integration scheme.

The commonly used open source software, Epanet (Rossman, 2000), uses a set of control rules to calculate control valve settings. Although the Epanet method works well in practice and is widely accepted in the hydraulic modeling community, there is no guarantee that its heuristic algorithm will be able to find the correct control valve settings in all cases. In fact,

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Simpson (1999) illustrated this through a number of control valve problems for which the Epanet hydraulic engine could not find a solution, or produced incorrect results.

Alternative methods for modeling control valves have developed in recent years. Piller and Bremond (2001) proposed a least-squares global optimization approach to determine the control valve state by minimizing the differences between the target settings and calculated values. Piller *et al.* (2005) applied the same optimization framework with an attempt to model time-varying behavior of the system continuously using slow transients (or rigid column without water hammer). This allowed them to model the continuous changes in the system state until an equilibrium (steady) state is achieved. The reaction speed of the control valve can be incorporated in the calculations by adding a constraint in the optimization solver. The authors noted that certain solutions that are infeasible using a demand-driven approach are in fact possible in real life, and can be solved correctly if a pressure-driven approach is followed.

Deuerlein *et al.* (2005) proposed a method based on Nash Equilibrium to determine the correct settings of pressure control valves. The valve head losses were taken as optimization variables and were estimated with a gradient-based algorithm that minimizes the corresponding convex variational problem. This method simultaneously solves as many constrained convex minimization problems as the number of pressure regulating valves plus one. The derived system is composed of the steady state equations (reduced to the loop energy balances) with one additional equation for each pressure regulating valve and complementary slackness condition. This system employs nonnegative Lagrange multipliers and its Jacobian is non-symmetrical, which may lead to a reduced solving efficiency. This reflects the fact that the system is not derived from a single optimization problem. It is worth

noting that the authors found their method to be robust and to produce good results based on several example problems. In a further paper, Deuerlein *et al*. (2008) used the same approach, but with the residual squared between the predicted value and the target value. This represents a more direct objective function similar to that used by Piller and Bremond (2001). Moreover, the authors described some simple examples for which no solutions or no unique solutions could be found.

Another method to handle flow control and check valves was proposed by Deuerlein *et al.* (2009). They use the content and co-content theory to define conditions that guarantee the existence and uniqueness of the solution before simultaneously solving the network hydraulics and valve settings. Subdifferential analysis is used to deal with the non-differentiable flow versus headloss relationships of flow control and check valves, and the combined equations are solved as a constrained nonlinear programming problem. An interesting result was the interpretation of the flow rate inequality multiplier as the head loss over the flow control valve.

In this study, different approaches are used to solve flow and pressure control valves in a hydraulic network. Flow control valves are handled by applying an external penalty function to the valveôs headloss equation in the vicinity of the valve setting. Check valves are handled as special flow control valves with a minimum flow rate setting of zero. The flow control valves are then solved with the other network hydraulic equations using a standard network solver. Pressure control valves are solved externally to the hydraulic solver by employing a Newton Projection Minimization algorithm, for which global convergence is guaranteed.

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An overview of the snapshot hydraulic equations is presented before describing the proposed algorithms for handling flow and pressure control valves. The proposed method is illustrated on a number of example problems for which Epanet is not currently able to find correct solutions.

HYDRAULIC MODEL

Hydraulic equations. Equations describing the hydraulics of water distribution systems are based on the principles of conservation of mass and energy for an incompressible fluid. These equations are solved to obtain the unknown flow rates in pipes, and hydraulic heads at nodes. The hydraulic network equations are described by:

$$\begin{cases} \mathbf{A}\mathbf{Q}^* + \mathbf{d} = \mathbf{0}_{nu} \\ \mathbf{h}^* - \mathbf{A}^T\mathbf{H}^* - \mathbf{A}_f{}^T\mathbf{H}_f = \mathbf{0}_{np} \\ \mathbf{h}^* = \mathbf{h}(\mathbf{Q}^*, \mathbf{r}) \end{cases} \quad (1)$$

Where $\mathbf{Q}^*$ is the vector of link flowrates with size *np* (number of links), **d** the vector of nodal demands with size *nu* (number of unknown-head nodes), **A** an *nu* x *np* incidence matrix representing unknown-head node connectivity, $\mathbf{A}_f$ an *nf* (number of fixed-head nodes) x *np* incidence matrix of fixed-head nodes, $\mathbf{H}^*$ the vector of hydraulic heads for the unknown-head nodes, $\mathbf{H}_f$ the vector of hydraulic heads for the fixed-head nodes, $\mathbf{h}^*$ is a vector of link head losses. A_{ij} = +1 if the pipe j leaves node i and i is an unknown head node; A_{ij} = -1 if it enters node i and i is an unknown head node; and A_{ij} = 0 otherwise. The same definition applies to A_f but with i a fixed head node. The first two equations describe the conservation of mass and energy respectively, and are linear. The last is a nonlinear equation that describes the relationship between the link flow rates and head losses, typically based on the Darcy-Weisbach or still Hazen-Williams formulae.

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Solving the hydraulic equations. Various methods have been proposed for solving water distribution system hydraulics. The first Loop method proposed by Cross (1936) updates the value of loop flow rates for the corresponding loop energy equation subject while fixing all other flow rates (a similar Node method was also proposed by Cross). It can be shown (Piller, 1995) that the Hardy Cross Loop method corresponds to a cyclic relaxation for the minimization of an energy function. The convergence of the latter can be drastically improved by simultaneously considering all the loops and nodes.

Subsequently, several Newton-Raphson based algorithms have been proposed. These algorithms may be classified as:

- Nodal methods, which are based on the nodal mass balances and describe the system state with head variables, e.g. (Chandrashekar and Stewart 1975; Lam and Wolla 1972; Martin and Peters 1963).
- Loop or simultaneous path methods, which are based on loop energy balances and describe the system state with loop flow rate variables, e.g. (Carpentier *et al.*, 1985; Epp and Fowler 1970).
- The Linear method proposed by Wood and Charles (1972), which is based on mass and energy balances and describes the system with link flow rate variables.
- Hybrid methods, which are based on mass and energy balances and describe the system with both link flow rates and nodal head variables, e.g. (Carpentier *et al*., 1985; Todini and Pilati, 1988; Piller 1995).

The Loop, Linear and Hybrid method classes result in equations that are the best conditioned for fast convergence (they converge in the same number of iterations from the same starting

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point), but the Hybrid method generally has fewer computational overheads than the other two methods, and is thus preferred. The Global Gradient Algorithm method by Todini and Pilati (1988) was implemented in the public domain Epanet software (Rossman 2000), which has become the standard method used in research and industry. Alternative Hybrid formulations are employed in software packages such as Piccolo (2013) and Porteau (2013). A problem with the Newton-Raphson based algorithms is that the global convergence of the method is only guaranteed if the initial solution is sufficiently close to the final solution (see e.g. the global damped Newton theorem in Ortega and Rheinboldt; 1970).

For global convergence to be guaranteed, it is necessary to adopt an optimization approach. Such formulations were proposed by Collins *et al.* (1978), Carpentier *et al.* (1985) and Piller (1995). An optimization approach allows correction made to the solution at each iteration to be tested for effectiveness, thus allowing numerical instabilities to be avoided. In addition, the existence and uniqueness of a solution to the equations can be proven, and thus convergence on a unique solution is guaranteed.

The proposed hydraulic solver is derived from the Content formulation by Collins, which describes the principle of least action for the hydraulic network, and can be written as:

$$\min_{\mathbf{Q}} f(\mathbf{Q}) = \mathbf{Q}^T \bar{\mathbf{h}}(\mathbf{Q}) - \mathbf{Q}^T \mathbf{A}_f{}^T \mathbf{H}_f \quad \text{subject to} \quad -\mathbf{A}\mathbf{Q} - \mathbf{d} = \mathbf{0}_{nu} \tag{22}$$

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Where $f(\mathbf{Q})$ is called the Content function of the system. The units of the Content function are that of power per unit weight. It is expressed as the sum of two terms with the first term representing the power dissipated in the network to reach the final steady state and the second the external power available to the system. $\mathbf{Q}$ is a vector of the link flowrates that complies

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with the conservation of mass, but not necessarily with the conservation of energy (i.e. the link flowrates in the solver before convergence has been achieved). $\bar{\mathbf{h}}$ is the vector of average headlosses between flowrates of zero to **Q** and its i^{th} component is:

$$\bar{h}_i(Q_i) = (1/Q_i)\int_0^{Q_i} h_i(u),\ Q_i \neq 0 \text{ and } \bar{h}_i(0) = 0$$

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$\mathbf{Q}^*$ is used for the correct values of the flowrates after convergence (that complies with both mass and energy balance). Thus $f(\mathbf{Q})$ is at a minimum when $\mathbf{Q} = \mathbf{Q}^*$.

Moreover, the headloss vector function **h** is modified near zero to ensure that f(**Q**) is strongly convex and twice continuously differentiable. The modification is required to ensure that the first derivative of the headloss function does not become zero at a flowrate of zero ï for details, see Piller (1995). This smoothing process is not necessary to guarantee the existence and uniqueness of the solution, but avoids numerical problems experienced by hydraulic solvers when flows in pipes are close to zero (all solver algorithms have to deal with this problem in some way).

The Content model minimization problem (2) is solved with a Lagrange-Newton method that is obtained by applying a Newton method to find a saddle point of the problem Lagrangian. At each iteration, updated heads and flowrates, as well as the Content function are calculated. If the descent criterion (similar to the Wolfe conditions) is not satisfied (i.e. the Content function does not decrease sufficiently), the flowrate adjustment is diminished by a factor ρ_k to ensure that global minimum of the Content function is found. At each iteration, the updated heads and flowrates are calculated by:

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187
$$\mathbf{H}^{k+1} = \left(\mathbf{A}\mathbf{D}_k^{-1}\mathbf{A}^T\right)^{-1}\left[\mathbf{A}\mathbf{D}_k^{-1}\left\{\mathbf{h}(\mathbf{Q}^k) - \mathbf{A}_f{}^T\mathbf{H}_f\right\} - \left\{\mathbf{A}\mathbf{Q}^k + \mathbf{d}\right\}\right]$$
$$\mathbf{Q}^{k+1} = \mathbf{Q}^k - \rho_k\mathbf{D}_k^{-1}\left[\mathbf{h}(\mathbf{Q}^k) - \mathbf{A}_f{}^T\mathbf{H}_f - \mathbf{A}^T\mathbf{H}^{k+1}\right] \quad (33)$$

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188 with $\mathbf{D}_k = \mathbf{D}(\mathbf{Q}^k) = \partial_Q \mathbf{h}(\mathbf{Q}^k)$ the Jacobian matrix of **h** in $\mathbf{Q}^k$ and $\mathbf{A}\mathbf{D}_k^{-1}\mathbf{A}^T$ is a symmetric,
189 positive definite Jacobian matrix associated to the unknown head update.

190

191 A convergence criterion on the energy balance on the pipes is used by stopping when
192 $\max_i \left\{\left(\mathbf{h}(\mathbf{Q}^k) - \mathbf{A}_f{}^T\mathbf{H}_f - \mathbf{A}^T\mathbf{H}^{k+1}\right)_i,\ \text{i=1, } \cdots, \text{n}_p\right\} \le \varepsilon$, with Ůa small value.

193

194 **PROPOSED MODEL FOR FLOW AND PRESSURE REGULATING DEVICES**

195 In this study, the optimization approach used in the hydraulic solver was expanded to handle
196 control valves. Different approaches are used for the modeling of flow and pressure control
197 valves. The headloss function of a flow control valve is adjusted by adding a penalty when
198 the valve setting is violated. This modified headloss function is then treated like any other
199 link in the hydraulic solver. Check valves are handled as special flow control valves with a
200 minimum flow rate setting of zero. Pressure control valves are solved externally to the
201 hydraulic solver by employing a Newton Projection Minimization algorithm, for which
202 global convergence is guaranteed. The target flowrates and heads are subjected to constraints
203 in the form of the conservation laws and the hydraulic behavior of elements in the system.

204

205 ***Flow Regulating Devices***

206 Flow regulating valves include flow control valves that prevent the flow rate through the
207 valve exceeding the target value, and non-return or check valves that allow flow to occur in
208 only one direction. Flow control valves are modeled as part of existing links in the model,

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since this results in a simpler model and avoids numerical problems in valves with very small head losses.

In the proposed approach, the problem is not solved with hard inequality constraints (like in Deuerlein *et al.,* 2009) but by adding penalties to the Content function. This exterior penalty method facilitates the satisfaction of constraints while being robust and simple to implement. The hydraulic solver used is based on an optimization approach, and this facilitates the simultaneous handling of system links and flow control valves. The modified Content optimization problem (2) now becomes:

$$\min_{\mathbf{Q}} \tilde{f}(\mathbf{Q},\mathbf{r}) = f(\mathbf{Q}) + \tfrac{1}{3} \sum_{j \in J_{FCV}} r_j \max\left(0, Q_j - Q_j^{set}\right)^3$$
$$\text{subject to } -\mathbf{A}\mathbf{Q} - \mathbf{d} = 0_{nu}$$

Where J_{FVC} is the index set of pipes with a flow control valve; the r_j are positive resistance coefficients; and the last term penalizes violations of flow control valve settings. In general the penalty function method requires that $\tilde{f}(\mathbf{Q},\mathbf{r}^k)$ is minimized for a sequence of $\mathbf{r}^k$ until a suitable solution is found. However, for this application it was found good results are obtained with a large identical scalar $r_{\max}$ value. The corresponding headloss penalty is obtained as:

$$h_j^{FCV}(Q_j, r_{\max}) = r_{\max} \max\left(0, Q_j - Q_j^{set}\right)^2 = h_0 \max\left(0, \frac{Q_j - Q_j^{set}}{\Delta Q}\right)^2 \quad (44)$$

With h_0 the headloss penalty for a flow rate violation of ΔQ; Q_j is the flowrate in the pipe j; Q_j^{set} is the setting value; and $\mathbf{h}^{\mathrm{FCV}}$, the headloss penalty function of $\mathbf{Q}$, is the gradient of the additional term to the Content function.

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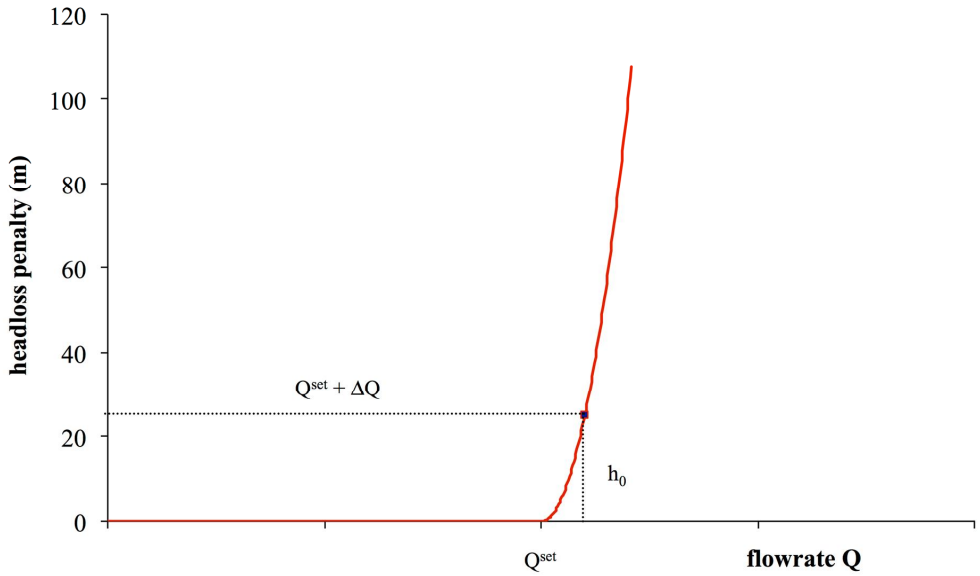
230 The penalized headloss function for a flow control valve is a smooth quadratic function
231 whose general form is presented in **Figure 1~~Figure 1~~**.



232

233 **Figure 1.** Headloss modeling of a flow control valve by external penalty.

234 Check valves can be modeled by considering them as a special type of flow control valve
235 with the constraint that $Q_k \geq 0$. The penalty function for check valve is described by:

236 $$h_k^{CV}(Q_k, r_{\max}) = -r_{\max} \max\left(0, -Q_k\right)^2 = -h_0 \max\left(0, \frac{-Q_k}{\Delta Q}\right)^2 \quad (55)$$

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237 The corresponding curve is described in Figure 2~~Figure 2~~.

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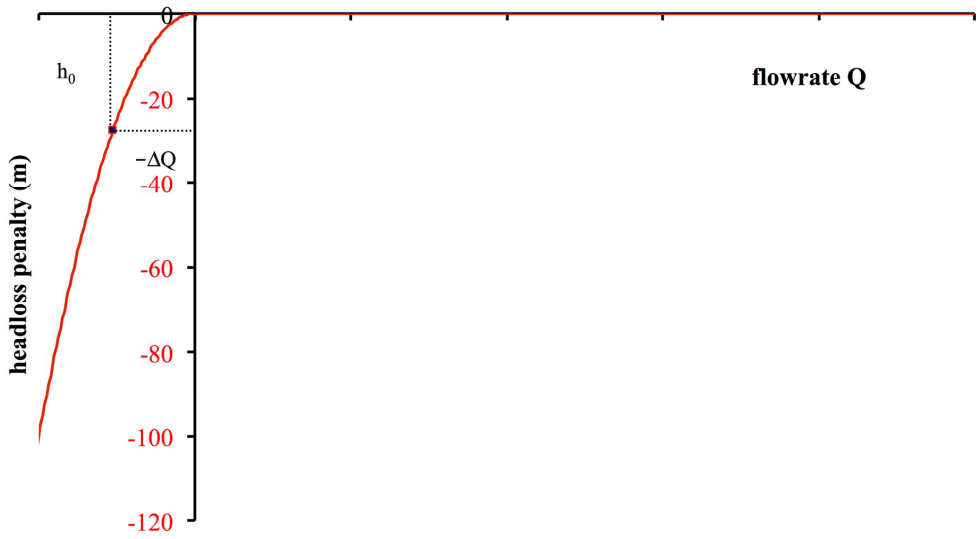

Figure 2. Headloss modeling of a check valve by external penalty.

The generalized Content minimization problem for handling FCVs and CVs is then:

$$\min_{\mathbf{Q}} \tilde{f}(\mathbf{Q},\mathbf{r}) = f(\mathbf{Q}) + \tfrac{1}{3} r_{\max} \left[\sum_{j \in J_{FCV}} \max\left(0, Q_j - Q_j^{set}\right)^3 + \sum_{k \in K_{CV}} \max\left(0, -Q_k\right)^3 \right] \quad (66)$$

$$\text{subject to } \mathbf{-AQ-d} = 0_{nu}$$

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Pressure Regulating Devices (PRDs)

Formulation

A pressure-reducing valve (PRV) aims to maintain a certain maximum pressure on the downstream side of the valve. PRVs are often used at the supply points of pressure zones to ensure that pipes are not overloaded and leakage is minimized. On the other hand, a pressure-sustaining valve (PSV) is used to maintain a minimum pressure on the upstream side of the valve.

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Piller and Van Zyl (2009) used dummy pressure sustaining valves as a modeling trick to correct hydraulic predictions for network section supplied via a high-lying node experiencing negative pressure. This may occur if the normal supply pipe to the network section has failed. In practice, air will enter the system at the elevated node (*e.g.* through air valves, leaks or open taps), and thus the supply to the network section will likely be interrupted.

Just like flow control valves, pressure regulating devices (PRDs) are modeled as part of existing links with a given target pressure on the downstream (i_D) or upstream (i_U) PRD sides. Thus for a PRV:

$$H_{i_D} \le H_{i_D}^{set}$$

and for a PSV:

$$H_{i_U} \ge H_{i_U}^{set}$$

If **S** is the selection matrix of the *nt* nodes with pressure setting targets, the complete set of constraints can be written in matrix form as:

$$\mathbf{SH} \le \mathbf{H}^{set}$$

When in use, PRVs and PSVs create local headlosses to get the network pressures as close as possible to the head set point vector H^{set} (target pressure + ground level). These local headlosses are added to the total headlosses **h**(**Q**) for links with such devices:

$$\mathbf{h}^{PRD}(\mathbf{Q},\mathbf{r}) = \mathbf{B}(\mathbf{Q})\mathbf{r} \text{ with } \mathbf{B}(\mathbf{Q}) = \mathbf{C}\mathbf{S}_{\mathbf{Q}}{}^{T},\ r_i = \frac{8K_i}{g\pi^2 D_i^4},\ i = 1,\cdots,nt \qquad (7)$$

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Where r_i is the secondary resistance factor of the pipe with K_i the dimensionless corresponding secondary headloss coefficient, D_i the diameter of the pipe i; C is the diagonal matrix of element $C_{ii} = \max\left(0, Q_i\right)^2$ and $\mathbf{S}_Q$ is the matrix of size nt x np for identifying the pipes with PRVs. Control valves only act in one direction - depending on how they are defined, these valves will either stop flow reversal, or, if negative flow is allowed, behave as pipes with known secondary headloss coefficients.

Pressure regulating devices are handled with a function that penalizes deviations from the target settings, as described by the first term in this function:

$$\min_{\mathbf{r}} c(\mathbf{r}) = \tfrac{1}{2}\left(\mathbf{SH}(\mathbf{r}) - \mathbf{H}^{set}\right)^T \mathbf{I}^+ \left(\mathbf{SH}(\mathbf{r}) - \mathbf{H}^{set}\right) + \tfrac{1}{2} m \left(\mathbf{r} - \mathbf{r}_0\right)^T \mathbf{I}^+ \left(\mathbf{r} - \mathbf{r}_0\right) \quad \text{subject to: } \mathbf{0}_{nt} \le \mathbf{r} \le r_{\max} \mathbf{1}_{nt} \tag{8}$$

The second term in the equation is a Tikhonov regularization term used to ensure that numerical problems are avoided in converging to a unique solution. The value of r_{max} is the same as that used for flow control valves: $r_{max} = h_0/\Delta Q^2$ in Figs. 1 and 2. $\mathbf{I}^+$ is the indicator matrix to implement the pressure control valve behavior for positive flowrates only, $\mathbf{r}_0$ is the value of $\mathbf{r}$ from the previous iteration (or an initial estimate), and m is the Tikhonov factor used to control the convexity of the function. The Tikhonov regularization term is only included when the function has insufficient convexity, and in our experience it is mostly equal to zero.

The function c is not differentiable at r_i such as $Q_i(r_i) = 0$. However, since control valves are active in a very small positive range of Q_i, and is either open or closed outside this range, it wasn't necessary to modify the $\mathbf{I}^+$ term.

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According to the Weierstrass theorem, there exists a solution for the problem by continuity of *c* on a non-empty compact (closed and bounded) constraint set. Because of a suitable *m* coefficient, the strict convexity of *c* guarantees the uniqueness of the solution.

For different pressure settings of a single PRV, Figure 3~~Figure 3~~ illustrates general form, in relation to the *K* (dimensionless) coefficients, of the sum of squares of the residuals that are obtained:

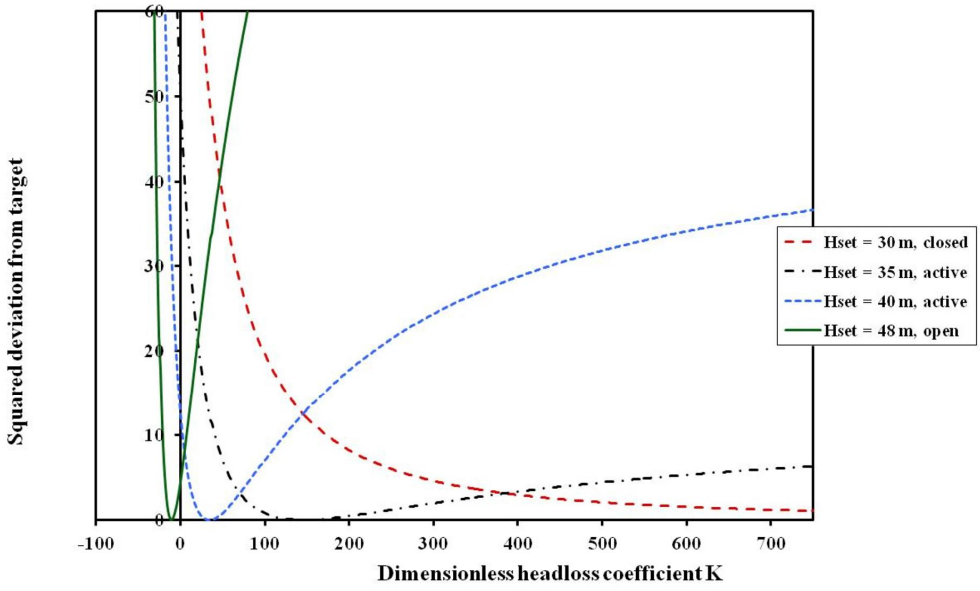

Figure ~~3~~3. General form of criterion c(*T*) to be minimized with m taken at zero.

For the four curves shown, the objective function c has a horizontal asymptote and a finite limit when *K* tends to infinity. There is a head setting H^{set}, below which the objective c does not have a minimum, in Figure 3~~Figure 3~~ when H^{set} Ò 30 m. In such cases, the solution of (8) is K_{max} if *m* = 0 and correspondingly the valve status is ócloseddô. For intermediary head settings between 30 and 45 m, a minimum global solution exists in the first (positive) quadrant, and the valve will be active if the strict inequality holds. For head setting values

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greater than or equal to 45 m, the valve status is óopenô and the unconstrained minimum occurs in the second (negative) quadrant. The solution of (8) for an open valve is $K = 0$. Note that the four curves are smooth, they each have no more than one minimum in the feasible set. From Figure 3~~Figure 3~~, it is clear that the criterion described by Eq. (8) without a regularization term (*i.e.*, m taken at zero) may be concave and its curve may possess inflexion points.

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Optimality Criteria to determine the correct status of PRD valves

With the convexity and the differentiability of the least-squares criterion *c*, it is possible to define necessary and sufficient optimality conditions that are the associated Karush, Kuhn and Tucker (KKT) equations (a generalization of Lagrange condition for inequality constraints). These optimality conditions are useful to determine the correct **status** of the valves (rather than the valve settings). Once correct statuses of the PRD valves are determined, a second-stage least-squares problem Eq. (8) should be formulated with only the deviations from the target settings for PRDs that are active to determine the exact local head losses created by the control valves in order to meet the pressure targets.

To solve the KKT equations, the gradient of *c* has to be determined. Using the implicit function theorem, we can show that y = **H**(**r**) is a continuous differentiable function with regards to the **r** variables. This gives an expression for the gradient of *c*:

$$\nabla c(\mathbf{r}) = \mathbf{J}^T \left(\mathbf{SH}(\mathbf{r}) - \mathbf{H}^{set} \right) + m\left(\mathbf{r} - \mathbf{r}_0 \right) \tag{9}$$

Where

$$\mathbf{J} = \mathbf{S}\partial_{\mathbf{r}}\mathbf{H} = \mathbf{S}\left(\mathbf{AD}^{-1}\mathbf{A}^T \right)^{-1} \mathbf{AD}^{-1}\mathbf{B}$$

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331 is the Jacobian matrix of the **H** function at PRD target nodes with respect to the **r**
332 coefficients, **B** is the same as in Eq. (7) and **D** is the derivative of **h** with respect to the flow
333 rate **Q**. The total headloss function **h**(**Q**,**r**) includes friction losses, FCVs, CVs and PRDs.
334 The calculation of the gradient of c is immediate, as the matrix to be inverted is very sparse
335 and its Cholesky decomposition is known from the current hydraulic solution of system (Eq.
336 1) with the previous values of **r**.

337 Since *c* is continuously differentiable, and the constraints are linear, the first order KKT
338 optimality conditions are met (***e.g.***, see Bazaraa, 1993). Therefore there exist two positive
339 multiplier vectors $\mathbf{M}^1$ and $\mathbf{M}^2 \geq \mathbf{0}_{nt}$ such that:

340 $$\nabla c(\mathbf{r}) = \mathbf{M}^1 - \mathbf{M}^2 \tag{10}$$

341 This consists of the Dual Feasibility conditions while

342 $$\left(\mathbf{M}^1\right)^T \mathbf{r} = 0 \text{ and } \left(\mathbf{M}^2\right)^T \left(\mathbf{r} - r_{\max}\mathbf{1}_{nt}\right) = 0 \tag{11}$$

343 are the Complementary Slackness conditions with $\mathbf{r}^*$ being the optimal solution. The dual
344 feasibility condition states that the gradient is no longer sign-constrained.
345
346 These two conditions are used to check whether the KKT conditions have been met, ***i.e.***
347 whether the correct solution for valve status has been found.
348
349 If $r_i = 0$ (the i^{th} pressure control valve status is open), then by (11) it is necessary that $M_i^2 = 0$
350 and Eq. (10) indicates that the i^{th} component of the gradient must be positive or zero. In
351 identical manner, if $r_i = r_{max}$ (the i^{th} pressure control valve status is closed), then $M_i^1 = 0$ and
352 the corresponding gradient component must be negative or zero. Finally, if $0 < r_i < r_{max}$ (the i^{th}

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pressure control valve is active), then necessarily $M_i^1 = M_i^2 = 0$ and the ith gradient component should be zero.

Projected Levenberg-Marquardt Solution Algorithm

The solution method used is a slight modification of the Levenberg-Marquardt (LM) algorithm that accounts for all the constraints.

At the start of the solution algorithm, initial control valve statuses are obtained from the previous solution or initial settings. The system hydraulics is then solved, and the control valve **r** resistances estimated with the iterative formula:

$$\mathbf{r}^{i+1} = \mathbf{r}^{i} - \mathbf{P}_i \left[\mathbf{J}_i^T \mathbf{J}_i + \mu \mathbf{I}_{nt} + \varepsilon_i diag\left(\mathbf{J}_i^T \mathbf{J}_i \right) \right]^{-1} \nabla c^i \qquad (12)$$

Where ε_i is the LM damping factor, P_i is the projection matrix for bounded primary constraints in (8) (i.e. valves that have fixed óclosedô or óopenô statuses). The value of ε_i is increased if the primal feasibility conditions (PF) are not complied with, if $\mathbf{J}_i^T\mathbf{J}_i$ is an ill-conditioned matrix or if there is no descent. In each step i, the hydraulic system is solved to determine $\mathbf{Q}(\mathbf{r}^i)$ and $\mathbf{H}(\mathbf{r}^i)$. The operation of the pressure regulating devices is modeled by adjusting the local head loss coefficients to satisfy the optimality of a least squares problem.

The algorithm (Eq. (12)), which is a projected method on the box constraints of Eq. (8), will not change valves with status óopenô or óclosedô during iterations. A valve that is initially óactiveô may becomes óopenô if K_i Ò 1E-3 and óclosedô if r_i Ó r_{max} ï 1E-3*r_{max}.

The KKT conditions (Eqs. (10) and (11)) are checked after the iterative scheme Eq. (12) has converged. If all the Lagrange multipliers are non-negative, the KKT conditions are satisfied

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and the first-step optimization of (8) can be terminated. If the KKT optimality conditions are not met, the pressure control valve with the most negative KKT multiplier is released, i.e. its status is changed from ónon-activeô (either ópenô or ôclosedô) to óactiveô.

Once the statuses of the PRD valves have been determined, a second-stage least-squares problem (Eq. (8)) is solved with the same constraints, but with deviations from the target settings only for óactiveô PRDs in the objective function: this is done to determine the exact local head losses created by active control valves in order to meet the pressure targets. This second-stage of the solution has to be done to remove any biases that fully open or closed valves have introduced in the solution.

VALIDATION TESTS

The proposed method was applied to a number of example networks:

- A simple network with a pressure-reducing valve on a pipeline between two tanks. This example was provided to illustrate the good convergence of the method when there is no interaction between valves.
- A simple network consisting of a flow control valve and a pressure-reducing valve in series on a pipeline between two tanks. This network posed a problem for early versions of Epanet.
- A simple network consisting of pressure-sustaining and pressure-reducing valves in series. This network poses a problem for the current version of Epanet.
- A simple network consisting of two valves in parallel that strongly interacts with each other.

The simulations results are discussed with an emphasis on the convergence characteristics of the method.

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A PRV between two tanks.

This example network is shown in Figure 4~~Figure 4~~ and was originally proposed by Simpson (1999).

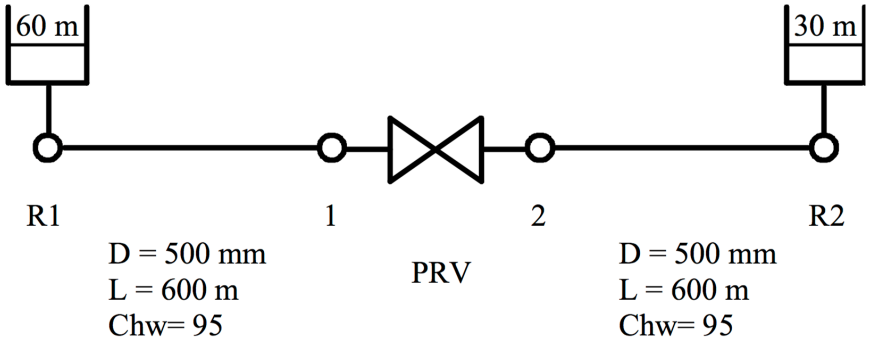

Figure 4~~4~~. Network 1 with 1 PRV between two tanks.

The ground levels of nodes 1 and 2 are zero. Three target heads on node 2 were tested: Hset = a) 60 m, b) 40 m and c) 20 m. They correspond to the following three situations: a) valve open and impossible to achieve the valve target setting because it is unrealistic for the hydraulic grade line, b) device active (an equivalent local head loss is created with coefficient $K^* = 34.5$), c) valve closed, target too low to achieve due to presence of tank R2. The three situations are presented in Figure 5~~Figure 5~~. In all cases, the algorithm converges in no more than 6 iterations.

Situation a): the initial solution is $K^0 = 0$. Next, $K^1 = -46$ does not satisfy the non-negativity constraint, therefore the damping parameter e_1 in Eq. (12) is increased; the step size is reduced and, for k = 4, K^4 is close to zero and the algorithm stops; gradient in Eq. (9) is positive and has a value of $\nabla c(0) = 2{,}547{,}090$; the Karush Kuhn and Tucker equations (10) and (11) are satisfied.

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Situation b): As it is shown in Figure 5~~Figure 5~~, the algorithm is stabilized rapidly near the true value; here, the direction of Levenberg-Marquardt is equivalent to the direction of the Newton-Raphson method; e_i tends rapidly towards zero; the gradient is cancelled out.

Situation c): the PRV is gradually closed to achieve the equivalent head loss of coefficient $K^* = 483{,}738{,}332$ in 5 iterations; the gradient is cancelled out at this point, showing that it is asymptotic on the x-axis at $c(K)$.

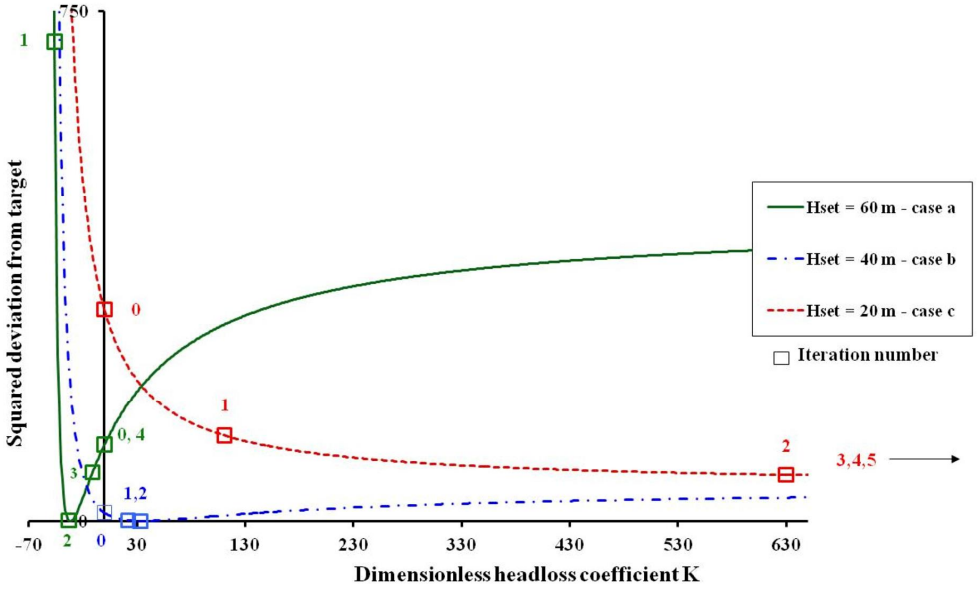

Figure 5~~5~~. Algorithm performance at each iteration.

A FCV and a PRV in series.

The second example network shown in Figure 6~~Figure 6~~ was also proposed by Simpson (1999). This network consists of a flow control valve (FCV) and a pressure-reducing valve in series between two tanks. The tank T1 fills the tank T2 by gravity only. For non-valve configurations (or equivalently the two control valves both inactive and having no minor head losses), the flow rate and the piezometric head at the middle of the path would be approximately 614 l/s and 45 m respectively. The FCV is made inactive (the flow set point is

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2,000 L/s) with a setting far in excess of the gravity flow rate. The PRV is operating and yields a local headloss of 20 m with a dimensionless coefficient K^* = 131.7. The flow rate is 339 L/s. For the three pipes the diameter is 500 mm and the Hazen-Williams coefficient is 100. All the elevations at unknown head nodes are set to zero.

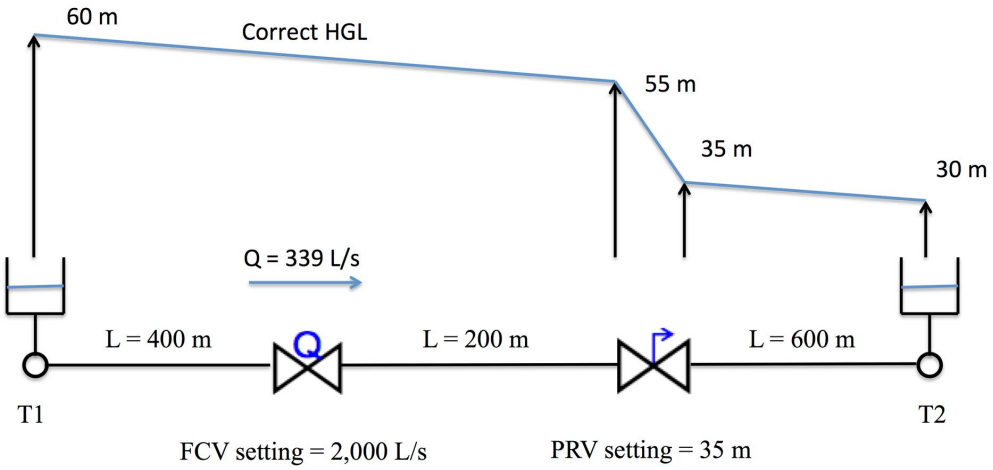

Figure ~~6~~6. Network 2 with a FCV and a PRV in series between two tanks.

The main reason why previous versions of Epanet (e.g., version 2.00.10) may fail to converge or converge to an incorrect solution, for this simple configuration with control valves is that the algorithm may fail to determine the correct statuses of the valves. In the proposed method, a continuous approach is used for both FCVs and PRDs. A flow control valve is modeled as a local headloss that penalizes all violation of the flow set point. With this approach a FCV and a PRV in series will pose no problem. The Table 1~~Table 1~~ summarizes the iterations of the Levenberg-Marquardt method for estimating the pressure valve settings. With CV-Count is the number of pressure control valve components that has converged; RSS is the residual sum of squares *i.e.*, $c(K_i)$ with m=0 ; GRAD/SD is the norm of the gradient along the search direction; LM factor e_i is the Levenberg-Marquardt damping factor; and the last column give the dimensionless friction factor that creates the local headloss $0.5\ K^*\ V^2/g$.

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Table 11. Convergence for the network with one FCV and one PRV in series.

Iteration #	CV_Count	RSS	GRAD/SD	LM factor e_i	K PRV
0	0	50.00000	1881507.19753004	0.00010	0.00000
1	0	8.58045	269899.11571403	0.00004	40.19445
2	0	0.89534	40111.55927071	0.00002	88.28532
3	0	0.02955	4884.57232911	0.00000	122.05247
4	0	0.00006	206.63313520	0.00000	131.20525
5	0	0.00000	0.48852255	0.00000	131.67368
6	0	0.00000	0.00000380	0.00000	131.67480
7	1	0.00000	0.00000007	0.00000	131.67480

The row iteration 0 corresponds to an initialization with K = 0, with the PRV assumed fully open. The head at node 2 is $H_2(K=0)$ = 45 m, which is 10 m above the set point. The RSS is 50 m^2. The reductions of the RSS and of the Gradient are quadratic. The damping factor e_i decreases by 60 % at each iteration. Since there is no open or closed pressure control valve, there is no need to solve the second-stage problem. Moreover, modeling flow control valves with Eq. (4), penalizing the head loss if the flow setting is violated, requires no special treatment for an open FVC. This is a clear advantage.

A PSV and a PRV in series

The third example network consists of two separate PSV and PRV in series as shown in Figure 7Figure 7. Water flows from a reservoir to a Tank for a distance of 2 km. Halfway between the reservoir and tank are two separate devices in series configured to model a combination of pressure sustaining and pressure reducing (PSV/PRV) valves. The PSV/PRV

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valve simultaneously controls the pressure on both sides, maintaining a certain minimum pressure on the upstream side and another (lower) maximum pressure on the downstream side. Both settings cannot be satisfied simultaneously and are active at different times of the day. The initial level in the tank is zero. All elevations at junction nodes are zero. This simple layout poses a problem in the latest version of Epanet (2.00.12). There is no convergence and several warning are generated, such as ñvalve PSV causes ill conditioningò and ñPSV open but cannot deliver pressure at 0:00:00ò.

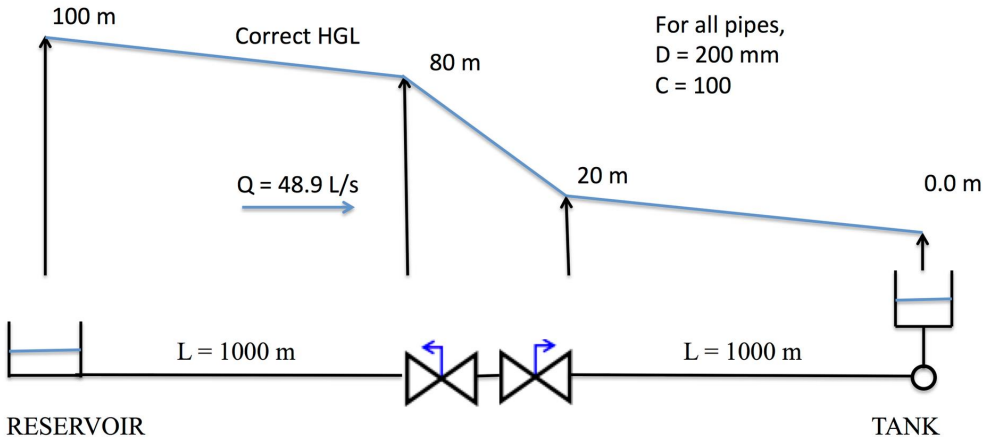

Figure ~~7~~7. Network 3 with a PSV and a PRV in series between a reservoir and a tank.

At the end of the simulation run, the solution should be as shown in the Figure 7~~Figure 7~~. The PSV is active and yields a local headloss of 60 m with a dimensionless coefficient K^* = 486.5. The PRV is open and the pressure at the downstream end of the valve is 10 m below its setting. The flow rate is 48.9 L/s.

Table 2~~Table 2~~ summarizes the iterations of the method (Eq. 12). It converges in 6 iterations to a local minimum solution that is not the correct solution - the two valves both have head losses of 25 m. The flow rate is 55.12 L/s, which is too high. Any additional simultaneous valve closure will raise locally the least-squares criterion c. The pressure upstream the PSV is

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75 m (5 m below the setting). The pressure downstream the PRV is 25 m. The PRV is active with a pressure constraint that is satisfied. This situation is not physically correct but can be explained with the fact that the 2 valves interact strongly with each other.

Table 2~~2~~. Initial convergence for the network with one PSV and one PRV in series.

Iteration #	CV_Count	RSS	GRAD/SD	LM factor e_i	K PSV	K PRV
0	0	649.75005	227019.02185376	0.00010	0.00000	0.00000
1	0	100.36013	33791.61183555	0.00004	75.35540	75.35540
2	0	28.46590	4479.20812686	0.00002	136.42557	136.42557
3	0	25.01620	265.86395946	0.00000	157.61510	157.61510
4	0	25.00000	1.40547468	0.00000	159.28421	159.28421
5	0	25.00000	0.00004447	0.00000	159.29323	159.29323
6	2	25.00000	0.00000079	0.00000	159.29323	159.29323

To move the solution away from the local minimum solution, the recommended method is to penalize the least-squares criterion with a Tikhonov term (Eqs. 8 and 12, $m > 0$ and $r_0 = 0$ for the PRV). In addition the following rule is applied: after convergence, if a situation with active or closed valve, but satisfied constraint occurs the valve with the highest residual is opened. This applies with combined PSV/PRV valves but also to other valve configurations with strong interaction.

The 6 first iterations are then followed by 6 further iterations with the PRV open as shown in Table 3~~Table 3~~. The algorithm converges to the correct valve status solution (active PSV and open PRV).

In addition, the solution at iteration 12 is equivalent to the solution at iteration 6: not only the criterion cost RSS = 25 m^2 but also the hydraulic grade line with 75 m upstream the PSV, and

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25 m downstream the PRV. The head loss of the PSV is 50 m with a dimensionless coefficient K = 318.6, which is equal to the sum of the head losses in iteration 6. However, the algorithm has to continue to the second stage to determine the correct setting of the PSV. The RSS contribution for the PRV is removed from the total RSS as the valve is open. These iterations are summarized in Table 4~~Table 4~~.

Table 3~~3~~. Valve status solution for the network with one PSV and one PRV in series.

Iteration #	CV_Count	RSS	GRAD/SD	LM factor e_i	K PSV	K PRV
7	0	90.32851	30282.48742520	0.00010	159.29323	0.00000
8	1	27.76165	3935.48041489	0.00004	277.43718	0.00000
9	1	25.01060	214.55220639	0.00002	315.86958	0.00000
10	1	25.00000	0.92318209	0.00000	318.57460	0.00000
11	1	25.00000	0.00002352	0.00000	318.58644	0.00000
12	2	25.00000	0.00002352	0.00000	318.58644	0.00000

The exact setting of the PSV (K* = 486.5) is obtained after 6 additional iterations.

Table 4~~4~~. Second-stage solving for the network with one PSV and one PRV in series.

Iteration #	CV_Count	RESID_SS	GRAD/SD	LM factor e_i	K PSV
13	1	12.50000	3652.53528151	0.00010	318.58644
14	1	0.37937	434.37774607	0.00004	451.11238
15	1	0.00068	16.82032793	0.00002	484.93499
16	1	0.00000	0.03264390	0.00000	486.49091
17	1	0.00000	0.00000033	0.00000	486.49396
18	2	0.00000	0.00000033	0.00000	486.49396

Figure 8~~Figure 8~~ shows the nodal heads for the 9 first hours. The tank is cylindrical with 20 m for the maximum level and 10 m for the diameter. J1 is a junction node just upstream the PSV and J3 another one just downstream the PRV. The tank level was updated every hour

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using the forward Euler method. It may be seen than for the first 4 hours the PSV is operating and the PRV is open. Then, starting from time = 5 h the PSV is open and the PRV is active. The 9-hour simulation requires a total of 112 iterations.

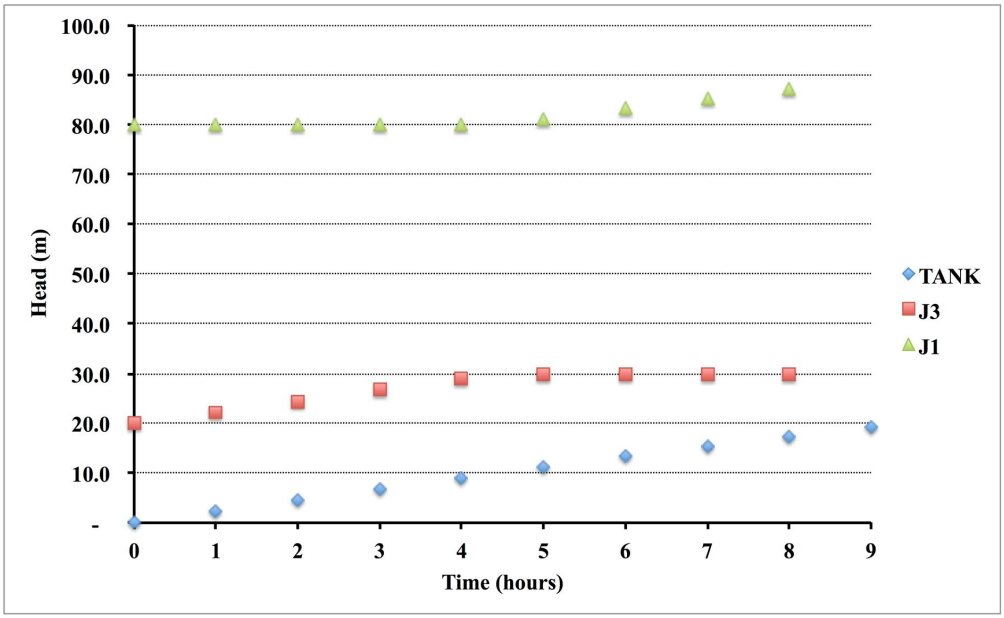

Figure ~~8~~8. Head time series for selected nodes in network 3.

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Two PRVs in parallel.

The fourth example network consists of two PRVs in parallel as shown in Figure 9~~Figure 9~~. The system uses a tank or a pumping station to supply consumers at node JCons. When the pumps are operating the tank is filling through a top inlet. When the pumps are switched off, the tank supplies water to the system. The latter situation is represented in Figure 9~~Figure 9~~. Water flows from the tank to node J1. Then, the flow separates in two different paths with a PRV on each. The PRVs are situated 1 km upstream the node JCons. The pressure settings are 50 m for both PRVs. All node elevations are 200 m. The level in the Tank is 3m above its bottom level of 292 m. The lengths of all three pipes are 1,000 m and their Hazen-Williams coefficients are 95. The demand at JCons is 8.5 L/s.

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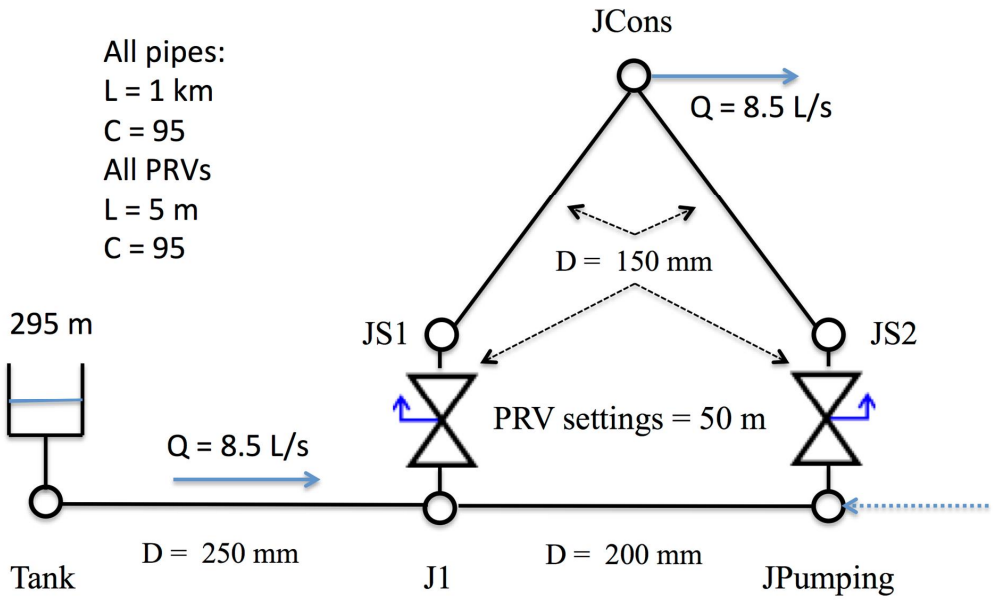

Figure 99. Network 4 with two PRVs in parallel.

The Table 5~~Table 5~~ summarizes the iterations of the Levenberg-Marquardt method. It converges in 12 iterations to a global optimum (RSS = 0.). It may be seen that at iterations 2 and 3 the damping factor e_i has been multiplied by ten. This was done to correct the search direction in order to have an effective descent for the RSS criterion. The two valves are operating and produce a local head loss of 44.71 m for PRV1, and 44.47 m for PRV2. The difference 0.24 m corresponds to the linear head loss of the 1 km long pipe J1/JPumping. A flow rate of 4.25 L/s flows through this pipe. The dimensionless coefficients are relatively large: K^*_{PRV1} = 15,164 and K^*_{PRV2} = 15,083. These two valves are strongly linked, which plays a role for the weaker, linear convergence rate.

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546 **Table 55.** Convergence for the network with two PRVs in parallel.

Iteration #	CV_Count	RSS	GRAD/SD	LM factor e_i	K PRV1	K PRV2
0	0	1988.96816	1185.48303586	0.00010	0.00000	0.00000
1	0	0.01086	0.26839385	0.00004	13517.19639	17036.82993
2	0	0.17871	0.26839385	0.00040	13517.19639	17036.82993
3	0	0.04189	0.26839385	0.00400	13517.19639	17036.82993
4	0	0.00772	0.32911957	0.00160	13763.48414	16695.22089
5	0	0.00463	0.89014765	0.00064	14189.38031	16134.05340
↓						
12		0.00000	0.00000000	0.00000	15164.07956	15083.08428

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548 The residual Jacobian matrix J at iteration 10 is as follows:

549 $$\mathbf{J} = \begin{pmatrix} 163.1268 & 163.0637 \\ 163.0637 & 163.2535 \end{pmatrix}$$

550 It may be seen than the 2 PRVs influence each other to the same extent and when one of
551 PRVs experiences a small change, a similar change is expected in the other valve.

552

553 **CONCLUSION**

554 A new method is presented for handling control valves (including check valves) in hydraulic
555 network modeling. In this method, the behavior of check and control valves are described by
556 continuous functions rather than the mixed discrete-continuous formulation commonly used.

557

558 In this method, flow control valves are handled by imposing a penalty on the valveôs headloss
559 function when the flow setting is violated. Check valves are modeled as special flow control
560 valves with a minimum flow rate setting of zero. The modified headloss functions for flow
561 control valves are incorporated into the hydraulic network equations and the resulting

562 governing equations are solved using a standard solver algorithm. In this work, a Lagrange-
563 Newton-type algorithm by Piller (1995) was used.
564
565 Pressure control valves (such as pressure reducing and pressure sustaining valves) are
566 handled externally to the hydraulic solver through an optimization routine. The goal of the
567 optimization is to find the secondary loss coefficients of the pressure control valves that will
568 minimize the differences between the nodal and the target pressures.
569
570 The proposed method has several benefits compared to the current discrete-continuous
571 formulation. The derivatives of the heads in relation to the input parameters can be written
572 explicitly as functions of the flow rates, and the gradients of the functions with respect to the
573 optimization variables can be obtained analytically. This, in conjunction with the fact that
574 only linear constraints are required in the optimization process, provides good conditions for
575 fast convergence of the method. The robustness of the optimization is ensured by using a
576 Levenberg-Marquardt projection minimization algorithm, for which global convergence is
577 guaranteed.
578
579 It is shown with several case studies than the method finds interesting solutions to control
580 valve problems without resorting to ómodeling tricksô. It is proved to be efficient on
581 problematic case studies.
582
583 While only control valves were considered in this paper, variable speed pumps and handling
584 of high-lying nodes with negative pressures (Piller and Van Zyl 2009) can also be included in
585 the proposed algorithm.
586

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