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Peter Martey Addo, Philippe de Peretti, Hayette Gatfaoui, Jakob Runge. The kiss of information theory that captures systemic risk. 2014. hal-01110712v2

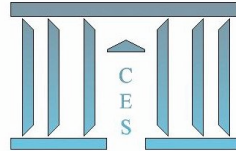
HAL Id: hal-01110712

<https://hal.science/hal-01110712v2>

Submitted on 18 May 2015

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2014.69R



The kiss of information theory that captures systemic risk

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Abstract

We provide a new approach to understanding systemic risk by analysing complex linkages in finance and insurance sectors. The analysis is achieved by using a recently proposed method for quantifying causal coupling strength, which identifies the existence of causal dependencies between two components of a multivariate time series and assesses the strength of their association by defining a meaningful coupling strength. The measure of association is general, causal and lag-specific, reflecting a well interpretable notion of coupling strength and is practically computable. A comprehensive analysis of the feasibility of this approach is provided via simulated and real data.

Keywords: Systemic risk, Causal dependencies, Financial institutions, Linkages, Sovereign Debt

JEL: C40, C32, C51, G12, G29

1. Introduction

The recent financial crisis of 2007–2009 has led policymakers to search for new tools to better understand systemic risk, i.e. contagion, and to try to develop early warning indicators. As the events following the turmoil in financial markets unfolded, it became evident that modern financial systems exhibit a high degree of interdependence making it difficult in predicting the consequences of such an intertwined system. To understand the interdependence of the system taken as a whole entity, network analysis has emerged as a leading econometric tool. For instance, the nature of connections between financial institutions usually results from both the asset and the liability side of their balance sheet. The structure of such connections between financial institutions can be captured by employing a network representation of financial systems (Allen and Babus (2008); Billio et al. (2012); Bisiyas et al. (2012)).

Network analysis is advancing as a common tool for assessing dynamics within the various sectors (nodes) of the financial system; it reveals that a truly systemic perspective needs to combine the focus on various sections of the financial sector with an analysis of the interlinkages

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Preprint submitted to SYRTO Working Paper Series (no.20)

March 9, 2015

among them incorporating the interaction with the real economy (Bisias et al. (2012)). Intuitively, a network structure describes a collection of nodes and the links between them (Lauritzen (1996); Dahlhaus (2000); Eichler (2012)). The nodes can be individuals, institutions or countries, financial assets, or even collections of such entities. The direct relations between these entities of interest are usually referred to as links. These links could be directed or undirected. Network theories can be used to monitor and assess systemic risks, contagion, linkages and vulnerabilities in the financial system. The nature of networks permit us to picture beyond the immediate “point of impact” of a shock and enhances the analysis of spillovers likely to arise from interlinkages. Thus, network perspective would not only account for the various connections that exist within the financial sector or between financial sector and other sectors, but would also consider the quality, in terms of strength, of these links.

Recent works documenting the applications of network analysis to financial systems concentrates on issues such as financial stability and contagion. The literature mostly investigates how financial institutions (or entities) are interconnected and how different network structures react to the breakdown of a single (or an ensemble) institution in order to identify which ones are more fragile (Allen and Babus (2008); Billio et al. (2012)). To our best of knowledge, existing research focused on the network topology of asset returns of financial systems are the works of Billio et al. (2012) and Bisias et al. (2012). Most of these works that study interconnections between subprocess of multivariate process make use of lagged cross-correlation and regression analysis. These concepts are commonly used to gain insights into interaction mechanisms between multivariate processes, in particular to assess time delays and to quantify the strength of a connection. For instance, the noteworthy work of Billio et al. (2012) aims to capture the interconnectedness based on principal components analysis and Granger-causality tests. A drawback of these concepts related to Granger causality network is that a strong dependence on the influence of serial dependencies or autocorrelation is usually observe, which can lead to misleading conclusions about time delays and also obscures a quantification of the interconnections. The question that still stands is how do we create a formal measure of systemic risk that adequately captures complex linkages in the financial system?

Our main aim is to offer a novel information theoretic approach to understanding systemic risk by identify linkages, and assess the strength of such connections. Our analysis exploits the concept of quantifying causal coupling strength, which identifies the existence of causal associations between two components of a multivariate time series and assesses the strength of their association by defining a meaningful coupling strength using time series graphs and information theory. In otherwords, we offer an approach which is based on time series graphs and information theory, which is able to detect and quantify causal dependencies from multivariate time series. The measure introduced in this work gives similar scores to equally noisy dependencies, which is essential for comparisons and ranking of strength of dependencies within subprocesses of the multivariate time series. Unlike network of Granger-causal relations proposed in Billio et al. (2012), we provide a measure that is lag-specific and gives non-zero value only to the dependency between lagged components of a multivariate process that are not independent conditional on the remaining process. The advantage of our approach over Billio et al. (2012) is that the study of interconnectedness is uniquely determined by the interaction of the two components alone and in a way autonomous of their interaction with the remaining process since the misleading influence of autodependency within a process is excluded. In addition, our approach only require that the multivariate time series be stationary. This measure has been successfully use in quantifying the strength and delay of climatic interactions (Runge et al. (2014)). In this work, we provide a comprehensive feasibility of the use of information theory and time series graphs

to capture complex linkages in financial institutions.

Our first empirical illustration of the approach considers the dataset used in Billio et al. (2012), which corresponds to monthly asset- and market-capitalization-weighted return indexes of hedge funds, brokers/dealers, banks, and insurers. These financial institutions represent the financial and insurance sectors as detailed in Billio et al. (2012). Our empirical findings show that there is high coupling strength of linkages between the four sectors, increasing the channels through which shocks can be transmitted throughout the financial system. The results point to important asymmetry in the causal dependencies in the insurance sector and brokers prior to the financial crisis of 2007–2009. Considering the whole time period, we observe that most directed links observed originate from insurers (IN) and in this case a failure of the insurers could cause a significant impact in the financial system. Moreover, given the high coupling strength between banks and insurers, a failure in the banking system could significantly destabilize the financial system. Based on our findings, we claim that the banks were more central to systemic risk. In addition, due to the high coupling strength of the contemporaneous links between the three sectors: banks (BK), Broker/dealers (PB) and insurers (IN), any large shock in one will spread quickly through the system. We find that the returns of insurers have significant impact on returns of brokers, where the time delay of impact is shorter during the crisis. Over time, we find that banks were always contemporaneously linked to brokers with a higher coupling strength observed before year 2000. In addition, our results indicate that insurers did not, in many cases, directly impact hedge funds significantly. After year 2000, we find contemporaneous links between banks and insurers, and also between brokers and hedge funds, where the former exhibits a higher coupling strength. The aftermath of 1998 collapse of the \$5 billion hedge fund Long Term Capital Management (LTCM) could have led to the contemporaneous link between brokers and hedge funds. In general, we find that banks were not directly connected to hedge funds. However, the two sectors were indirectly linked via a contemporaneous chain of dependencies from banks to brokers and then to hedge funds. We remark that just prior to the financial crisis of 2007–2009, the hedge funds did have a significant negative impact on banks. Interestingly, most connections were observed for the time period (1994M01–2007M12), where hedge funds had a negative impact on the financial system especially on the banks and brokers. This increase in size and causal dependencies between these sectors could serve as a significant systemic risk indicator.

In the second empirical application, we focus on investigating the transmissions of the European Sovereign debt crisis by examining the behavior of the 10-year government bond yields of eleven European Monetary Union (EMU 11) countries. The EMU 11 considered in our application is composed of the periphery group of countries: Portugal (Por), Ireland (Ire), Italy (Ita), Greece (Gre), Spain (Spa); and the central group of countries: Austria (AU), Germany (Ger), The Netherlands (Ne), Belgium (Bel), France (Fra), and Finland (Fin). Our findings show an increase in the number of causal dependencies in the bond yields among the countries prior and even much more during the crisis. We remark that the crisis was marked by an increase in negative coupling strengths among sovereigns, which could serve as an indicator of systemic risk. The finding shows that government bond yield of the Netherlands were not linked to any of the periphery countries severely hit by the sovereign debt crisis. In particular, we find that the sovereign bond yields of Germany, the Netherlands, Finland and Austria were more stable compared to other core countries like France and Belgium, which were linked to at least two periphery countries. Our results point to France not being severely hit by the sovereign debt crisis, could be due to its high causal dependence with Germany, the Netherlands, Austria and Finland, although France did have a contagion spillover effects mainly from Ireland and Italy. Based on

our findings, the European Monetary Union was still in crisis as at the end of 2014. In order to safeguard the stability of the european financial system, we suggest that much importance be placed on reducing these causal dependencies and contagion among member countries.

The paper is organised as follows: In Section 2, an overview of time series graphs and information theoretic measure is provided. An outline of the estimation procedure of the approach is also provided for practical purposes. In Section 3 & Section 4, a comprehensive analysis of the feasibility of this approach to simulated and real data is provided. Section 5 concludes.

2. Measure of causal dependence and coupling strength

In this section, we provide an overview of an approach which makes use of time series graphs and information theory. We will start by providing an insight to Granger-causality (Granger (1969)), the definition of time series graph and then the information theoretic measure. Let \mathcal{A} and \mathcal{B} be stationary stochastic processes, $\mathcal{U}_t^- = (\mathcal{U}_{t-1}, \mathcal{U}_{t-2}, \dots)$ be the information in the entire universe until the present. Now let $\mathcal{A}_t^- = (\mathcal{A}_{t-1}, \mathcal{A}_{t-2}, \dots)$ denotes the past until present and $\sigma^2(\cdot)$ denote variance.

Definition 1 (Granger Causality). *We say that \mathcal{A} Granger-causes \mathcal{B} if*

$$\sigma^2(\mathcal{B}|\mathcal{U}^-) < \sigma^2(\mathcal{B}|\mathcal{U}^- \setminus \mathcal{A}^-).$$

This implies that the error variance of predicting the process \mathcal{B} given \mathcal{U}_t^- is less than the error variance of predicting \mathcal{B} when \mathcal{A}^- is excluded (Granger (1969)). We remark that although the prediction method to determine σ^2 could be linear or nonlinear, the use of σ^2 restricts the notion of causality and does not takes into account of the whole distribution of the error. As such, the need for information theory framework since it does considers the whole distribution of the residual (Runge et al. (2012a)).

2.1. Time series graphs

Like graphical models (Lauritzen (1996)), time series graphs are based on the concept of conditional independence and the time-dependence in the time series is used to define directional links in the graph.

Definition 2 (Time Series Graph). *Let \mathfrak{X} be a stationary multivariate time series with a set of subprocesses V at each time $t \in \mathbb{Z}$ and directional links be defined in E . Then*

$$\mathcal{G} = (V \times \mathbb{Z}, E)$$

is the time series graph of \mathfrak{X} , where the set of nodes in the graph are made up of V .

Here each node in the graph represents a single random variable i.e. a subprocess at a certain time t . Given a stationary multivariate discrete-time series stochastic process \mathfrak{X} , where $\mathfrak{X}_t^- = (\mathfrak{X}_{t-1}, \mathfrak{X}_{t-2}, \dots)$ denotes the past until present. We say that the nodes $\mathcal{A}_{t-\tau} \in \mathcal{G}$ and $\mathcal{B}_t \in \mathcal{G}$ are connected by a lag-specific directed link “ $\mathcal{A}_t \xrightarrow{\tau} \mathcal{B}_t$ ” pointing forward in time if and only if $\tau > 0$ and

$$I_{\mathcal{A} \xrightarrow{\tau} \mathcal{B}}^{link}(\tau) \equiv I(\mathcal{A}_{t-\tau}; \mathcal{B}_t \mid \mathfrak{X}^- \setminus \{\mathcal{A}_{t-\tau}\}) > 0. \quad (1)$$

Thus, $\mathcal{A}_{t-\tau}$ and \mathcal{B}_t are connected if they are not independent conditionally on the past of the whole process excluding $\{\mathcal{A}_{t-\tau}\}$ (denoted by the symbol \setminus) which implies a lag-specific causality with respect to \mathfrak{X} . $I(\cdot; \cdot | \cdot)$ denotes conditional mutual information. If $\mathcal{A} \neq \mathcal{B}$ then “ $\mathcal{A}_t \rightarrow^\tau \mathcal{B}_t$ ” represents a coupling at lag τ . An autodependency at lag τ corresponds to $\mathcal{A} = \mathcal{B}$. The nodes $\mathcal{A}_t \in \mathcal{G}$ and $\mathcal{B}_t \in \mathcal{G}$ are connected by an undirected contemporaneous link “ $\mathcal{A} - \mathcal{B}$ ” (Eichler (2012)) if and only if

$$I_{\mathcal{A}-\mathcal{B}}^{\text{link}} \equiv I(\mathcal{A}_t; \mathcal{B}_t | \mathfrak{X}_{t+1}^- \setminus \{\mathcal{A}_t, \mathcal{B}_t\}) > 0. \quad (2)$$

Notice that the contemporaneous present $\mathfrak{X}_t \setminus \{\mathcal{A}_t, \mathcal{B}_t\} \subset \mathfrak{X}_{t+1}^- \setminus \{\mathcal{A}_t, \mathcal{B}_t\}$. In the case of the multivariate autoregressive process, this corresponds to non-zero entries in the inverse covariance matrix of the innovation terms. The equations 1 & 2 involve infinite dimensional vectors and can not be directly computed. However, using the Markov property, this issue can be circumvented by introducing the notion of *parents* and *neighbors* of subprocesses. Let

$$\mathcal{P}_{\mathcal{B}_t} \equiv \{\mathcal{A}_{t-\tau} : \mathcal{A} \in \mathfrak{X}, \tau > 0, \mathcal{A}_{t-\tau} \rightarrow \mathcal{B}_t\} \quad (3)$$

be a finite set of *parents* of \mathcal{B}_t which separates \mathcal{B}_t from the past of the whole process $\mathfrak{X}_t^- \setminus \mathcal{P}_{\mathcal{B}_t}$. The past lags of the subprocess \mathcal{B} in \mathfrak{X} can be part of the parents. The *neighbors* of a process \mathcal{B}_t is defined as

$$\mathcal{N}_{\mathcal{B}_t} \equiv \{\mathcal{A}_t : \mathcal{A} \in \mathfrak{X}, \mathcal{A}_t - \mathcal{B}_t\} \quad (4)$$

The *parents* of all subprocesses in \mathfrak{X} together with the contemporaneous links forms the time series graph. Any node $\mathcal{B}_t \in \mathcal{G}$ in the time series graph is conditionally independent¹ of $\mathfrak{X}_t^- \setminus \mathcal{P}_{\mathcal{B}_t}$ given its *parents* $\mathcal{P}_{\mathcal{B}_t}$.

It is noteworthy that the measure of connectedness based on Granger causality as proposed Billio et al. (2012) is similar to the information theory formulation in equations 1 & 2. However, the drawback here is that it is not uniquely determined by the interactions of the two components alone and does not exclude the misleading influence of autodependencies within processes. In the section that follows, we define the information theoretic measure which is lag-specific, and able to detect and quantify causal relationships between two subprocesses in a way autonomous of their interaction with the remaining process.

2.2. Information theoretic measure

In line with our objective to study the channels through which shocks can be transmitted between financial institutions, we provide in this section an information theoretic measure of causal dependencies and coupling strength of such causal relationships. The definitions of links based on information theoretic together with time series graph will be used to capture complex linkages in the financial system.

Definition 3 (Momentary Information Transfer (MIT) Links). *For two subprocesses \mathcal{A} , \mathcal{B} of a stationary multivariate discrete time process \mathfrak{X} with parents $\mathcal{P}_{\mathcal{A}_t}$ and $\mathcal{P}_{\mathcal{B}_t}$ in the associated time*

¹This is the causal Markov condition.

series graph and $\tau > 0$, the general information theoretic measure MIT (Runge et al. (2012a)) between $\mathcal{A}_{t-\tau}$ and \mathcal{B}_t is given by

$$\begin{aligned} I_{\mathcal{A} \rightarrow \mathcal{B}}^{MIT}(\tau) &\equiv I(\mathcal{A}_{t-\tau}; \mathcal{B}_t \mid \mathcal{P}_{\mathcal{B}_t} \setminus \{\mathcal{A}_{t-\tau}\}, \mathcal{P}_{\mathcal{A}_{t-\tau}}) > 0 \\ &= H(\mathcal{B}_t \mid \mathcal{P}_{\mathcal{B}_t} \setminus \{\mathcal{A}_{t-\tau}\}, \mathcal{P}_{\mathcal{A}_{t-\tau}}) - H(\mathcal{B}_t \mid \mathcal{P}_{\mathcal{B}_t}) \end{aligned}$$

and contemporaneous MIT defined by

$$I_{\mathcal{A} - \mathcal{B}}^{MIT} \equiv I(\mathcal{A}_t; \mathcal{B}_t \mid \mathcal{P}_{\mathcal{B}_t}, \mathcal{P}_{\mathcal{A}_t}, \mathcal{N}_{\mathcal{A}_t} \setminus \{\mathcal{B}_t\}, \mathcal{N}_{\mathcal{B}_t} \setminus \{\mathcal{A}_t\}, \mathcal{P}_{\mathcal{N}_{\mathcal{A}_t} \setminus \{\mathcal{B}_t\}}, \mathcal{P}_{\mathcal{N}_{\mathcal{B}_t} \setminus \{\mathcal{A}_t\}}). \quad (5)$$

where $H(X)$ is Shannon's entropy and $H(X|Y)$ denotes conditional Shannon's entropy (Shannon (1948)).

Definition 4 (MIT Coupling strength). *The partial correlation MIT measure, denoted ρ^{MIT} , associated with equation (5), for the strength of coupling mechanism between $\mathcal{A}_{t-\tau}$ and \mathcal{B}_t is given by*

$$\rho_{\mathcal{A} \rightarrow \mathcal{B}}^{MIT}(\tau) \equiv \rho(\mathcal{A}_{t-\tau}; \mathcal{B}_t \mid \mathcal{P}_{\mathcal{B}_t} \setminus \{\mathcal{A}_{t-\tau}\}, \mathcal{P}_{\mathcal{A}_{t-\tau}}). \quad (6)$$

The measure ρ^{MIT} quantifies how much the variability in \mathcal{A} at the exact lag τ directly influences \mathcal{B} , irrespective of the past of $\mathcal{A}_{t-\tau}$ and \mathcal{B}_t . ρ^{MIT} is the cross-correlation of the residual after $\mathcal{A}_{t-\tau}$ and \mathcal{B}_t have been regressed on both the parents of $\mathcal{A}_{t-\tau}$ and \mathcal{B}_t . In economics, the detection of causal relationships among variables are of great importance (Granger (1969); Billio et al. (2012)). These variables can be viewed as nodes of graph where the links correspond to interactions. Unlike classical statistics, interactions in the framework of information theory are viewed as transfers of information.

The partial correlation MIT is a time-delayed conditional mutual information and a measure of association that is general in the sense that it does not assume a certain model class underlying the process that generates the time series. This measure of association will be useful in understanding the interdependence in multivariate economic and financial time series analysis as the process underlying the generated signals are a priori unknown (Addo et al. (2012)). In addition, the general framework of graphical models² (Lauritzen (1996); Dahlhaus (2000); Eichler (2012)) makes MIT causal as it gives a non-zero value only to lagged components that are not independent conditional on the remaining process. This measure in many cases is able to exclude the misleading influence of autodependency within a process in an information-theoretic way. Runge et al. (2014) successfully used this approach in quantifying the strength and delay of climatic interactions. In this work, we will make use of the partial correlation MIT in quantifying the strength and delay of interactions between multivariate economic and financial time series. As documented in literature (Runge et al. (2012a, 2014)), measures of multivariate dependence should usually fulfill the following properties:

1. Causality – the measure should give non-zero value only to the dependency between lagged components of a multivariate process that are not independent conditional on the remaining process.

²Graphical models provide a framework to distinguish direct from indirect interactions between and within subprocesses of a multivariate process (Lauritzen (1996)).

2. Lag-specific – it should quantify dependence between lagged time series. This enhances better interpretation.
3. Coupling strength autonomy – given dependent components, the measure should provide a causal notion of coupling strength that is well interpretable. In other words, it is uniquely determined by the interaction of the two components alone and in a way autonomous of their interaction with the remaining process.
4. Practical computability – estimation does not for instance require somewhat arbitrary truncation as in the case of other methods like transfer entropy.

These properties are well fulfilled by the MIT measure, which allows to reconstruct interaction networks where not only the links are causal, but also meaningfully weighted and have the attribute of coupling delayed. To our best of knowledge, other methods such as the transfer entropy which is the information-theoretic analogue of Granger causality do not support these properties. For instance, the transfer entropy is not uniquely determined by the interaction of the two components alone and depends on misleading effects such as autodependency and interaction with other process. This violates the coupling strength autonomy. In addition, aside that the transfer entropy requires arbitrary truncation during estimation, this measure can lead to false interpretation since it is not lag-specific. To enhance proper interpretation of results due to links in the time series graph, caution should be taken when interpreting contemporaneous links as it is not clear which direction they correspond to or whether they are entirely due to a common driver. It is worth pointing out that the analytical distribution of the partial is known and equivalent to the distribution of cross-correlation with the degree of freedom reduced by the cardinality of the set of conditions (Fisher (1924)). Thus, statistical inference on the measure ρ^{MIT} is straightforward (Runge et al. (2014)). In the empirical application, Section (4), of the approach, we present results that corresponds to a 95% significance level of a two-sided test on ρ^{MIT} .

2.3. Estimation

To apply the above methodology in practice, we outline a “two-steps” approach for the detection and quantification of causal dependencies in multivariate time series:

1. A modified PC algorithm (Spirtes et al. (2000); Runge et al. (2012a)) is used to estimate the parents of each process. This step can be viewed as a variable selection method. Unlike graphical models, only undirected links are inferred and the second step of the original PC algorithm is omitted (Spirtes et al. (2000)). This step determines the existence or absence of a link, which also provide useful information on the causality between lagged components of the multivariate process \mathfrak{X} . The estimation of these time series graphs is obtained by iteratively inferring *parents* (Runge et al. (2012b)).
2. Next, MIT link definitions are used and all possible links are tested again since some links might still turn out to be non-significant. In this step, the problem of serial dependencies is drastically reduced and the estimation of the coupling measure serves as a meaningful weight for every existing link in the graph (Runge et al. (2012a)).

In estimating the conditional mutual information (CMI), the most commonly used approach is the binning method. In this method, the joint and marginal distributions are estimated and then plugged into the entropy formula to compute the CMI. The estimation is achieved by binned histograms with equiquantile (or some pre-defined) bins such that the marginals distributions are uniform (Paluš (1996); Darbellay and Vajda (1999)). The main challenge with this method is the

issue with curse of dimensionality as the number of bins exponential increase with the dimension of the time series (Hlávačková-Schindler et al. (2007); Runge et al. (2012b)). Alternatively, the nearest neighbor conditional mutual information estimator proposed in Frenzel and Pompe (2007); Runge et al. (2012b,a) can be used. The section that follows will be dedicated to the empirical illustration of the feasibility of the approach on simulated and real data.

3. Simulations experiment

In this section, we provide a simulation study on the identification of interdependences and strength in a simulated multivariate process with a priori knowledge on the network structure. In this respect, consider a simulated 1000 points of a stationary multivariate autoregressive process (displayed in Figure (1a)) made up of four subprocesses $\{X_t, Y_t, Z_t, W_t\}'$ defined by

$$X_t = aX_{t-1} + cZ_{t-4} + \varepsilon_x \quad (7)$$

$$Y_t = kX_{t-1} + hY_{t-1} + \varepsilon_y \quad (8)$$

$$Z_t = dY_{t-2} + bZ_{t-1} + fW_{t-1} + \varepsilon_z \quad (9)$$

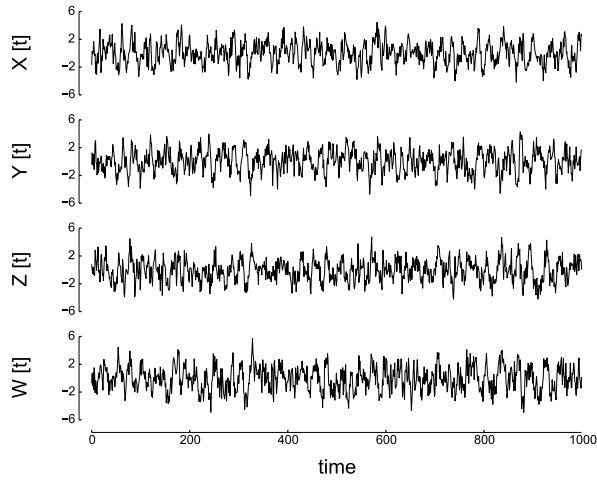
$$W_t = eY_{t-3} + gW_{t-1} + \varepsilon_w \quad (10)$$

and the innovation covariance matrix given by $\Sigma_\varepsilon = \begin{pmatrix} 1 & 0 & d & 0 \\ 0 & 1 & 0 & d \\ d & 0 & 1 & 0 \\ 0 & d & 0 & 1 \end{pmatrix}$, where $a = 0.6, b = 0.4, c =$

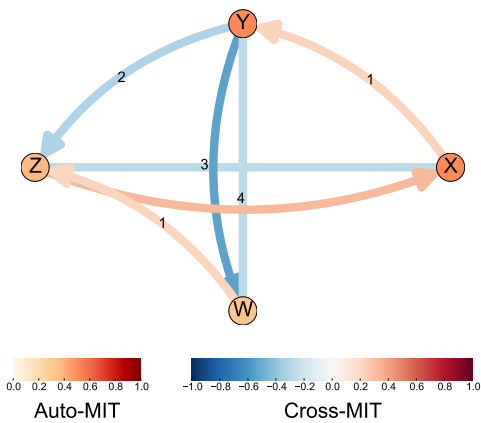
$0.3, d = -0.3, e = -0.6, f = 0.2, g = 0.4, h = 0.6$ and $k = 0.3$. Notice that the lagged causal chain for this process is $X \rightarrow^1 Y \rightarrow^2 Z$ with feedback $Z \rightarrow^4 X$, and $Y \rightarrow^3 W \rightarrow^1 Z$, plus contemporaneous links $X - Z$ and $Y - W$. The results of the time series graph via MIT approach is provided in Figures (1b & 1c). We obtained that the causal dependencies between the subprocesses are well identified and coincide with our a priori knowledge on the network structure. We remark that not only are the causal lags of dependencies between subprocesses identified, the magnitude and sign of the MIT coupling strength of linkages is also obtained (see Figure (1c)). In checking for robustness, we perform the analysis on sliding windows of length 200 and step size of 100. The results on the ensemble graph for windows are displayed in Figure (2). Here the link width denotes the percentage of windows that contain this link in the time series graphs. In otherwords, robust links have a larger link width. We observe that the priori knowledge on causal dependencies among these subprocesses are well captured by the approach. In the section that follows, we will provide an application of this approach to analysing systemic risk.

4. Empirical application

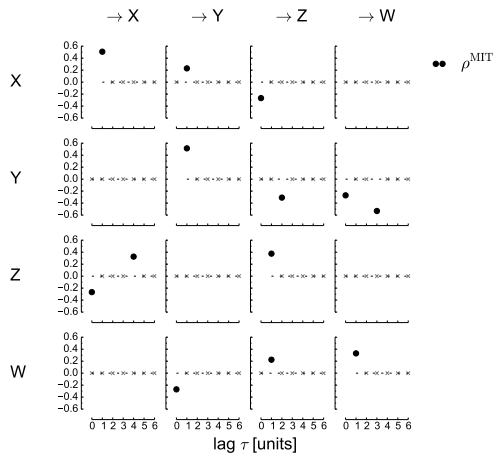
In this section, we provide two empirical applications of the approach to the financial context. The first illustration will focus on studying linkages in four financial and insurance sectors of the economy as done in Billio et al. (2012). The second application explores causal linkages in eleven European Monetary Union (EMU) countries before and during the european sovereign debt crisis. In both illustrations, we apply the proposed methodology to full sample period, expanding windows and then on sliding windows for proper robust conclusions.



(a) Plot of four subprocesses of a stationary multivariate autoregressive process

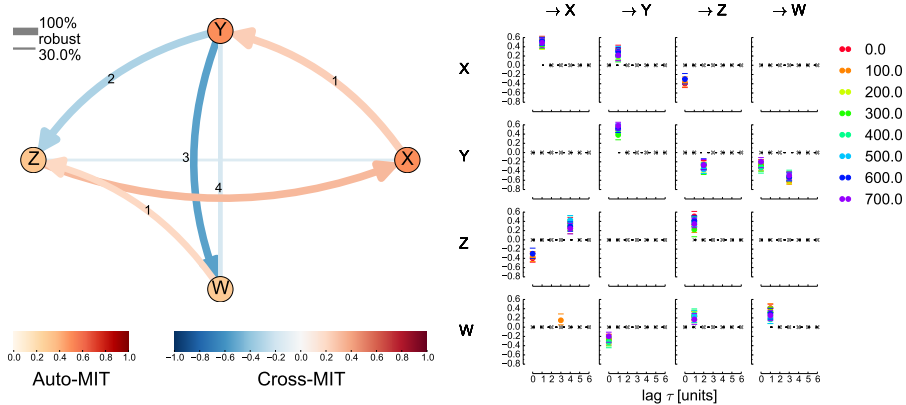


(b) MIT plot for the simulated process.



(c) The plot of significant lags for simulated process associated with the MIT plot.

Figure 1: The detection & quantification of causal dependencies in a simulated multivariate autoregressive process described in equations (7–10) via the MIT approach. The results indicates a lagged causal chain as $X \xrightarrow{1} Y \xrightarrow{2} Z$ with feedback $Z \xrightarrow{4} X$, and $Y \xrightarrow{3} W \xrightarrow{1} Z$, plus contemporaneous links $X - Z$ and $Y - W$. In Figure (1c), the existence of a *dot* at a particular lag τ of a subprocess ξ on the plots located on the diagonals indicates an autoregressive link of the form $\xi_{t-\tau} \rightarrow \xi_t$, which we denote as “ $\xi_t \xrightarrow{\tau} \xi_t$ ”. Contemporaneous links can be read from the plots by observing *dots* at where $\tau = 0$. The vertical axes represent the sign and magnitude of the MIT coupling strength (ρ^{MIT}) of the linkages, which corresponds to the coefficients of the simulated autoregressive process.



(a) An ensemble of *MIT plot* for the simulated process (b) The plot of significant lags for sliding window analysis.

Figure 2: An ensemble of *MIT plot* for the simulated process on a sliding window of length 200 and step size of 100. The link width denotes the percentage of windows that contain this link in the time series graph. The result coincide with our a priori knowledge on the causal chains among the variables and do not differ from the results obtained in Figure (1).

4.1. Financial and insurance sector

The recent global financial crisis (2007–2009) has clearly demonstrated the need for the financial industry, regulators and policymakers to develop a better understanding of systemic risk. In particular, measuring the financial systemic importance of financial institutions is crucial in identifying linkages and the potentially destabilizing constituents of the global financial system. In this section, we consider the dataset used in Billio et al. (2012), with special attention on identifying the linkages, lags of linkages and to assess the strength of such linkages among the four main sectors: hedge funds (HR), banks (BK), Broker/dealers (PB) and insurers (IN). These financial institutions represents the financial and insurance sectors described in more detail by Billio et al. (2012). In this respect, we make use of the monthly asset- and market-capitalization-weighted return indexes of these sectors as inputs for the MIT approach in analysing systemic risk. The time period for the data set in Jan 1994–Dec 2008. Our analysis is performed on the full sample period and also considers the following different time windows : 1994M01–2004M12, 1994M01–2005M12, 1994M01–2006M12, 1994M01–2007M12, to study possible changes in the systemic risk network structure.

The results of the proposed MIT approach when applied to the multivariate process $\{HF_t, PB_t, BK_t, IN_t\}'$ for the full time period is shown in Figure (3). Lag-specific causal dependencies between the four sectors can be easily visualized in Figures (3a & 3b). The sign and magnitude of the coupling strength can be read from Figure (3b). The time series graph (network structure) provides an understanding of the possible channels through which financial crisis were transmitted. In Figure (3a), we detect three contemporaneous links: $BK - PB$, $PB - HF$ and $BK - IN$, where the latter has a higher coupling strength ($\rho^{MIT} \approx 0.6$) as seen in Figure (3b). Most directed links observed in Figure (3a) originates from insurers (IN) and in this case, a failure of the insurers could cause a significant impact in the financial system. Moreover, the given the high coupling strength between banks and insurers, a failure in the banking system could significantly destabilize the financial system. In addition, due to the high coupling strength of the contemporaneous

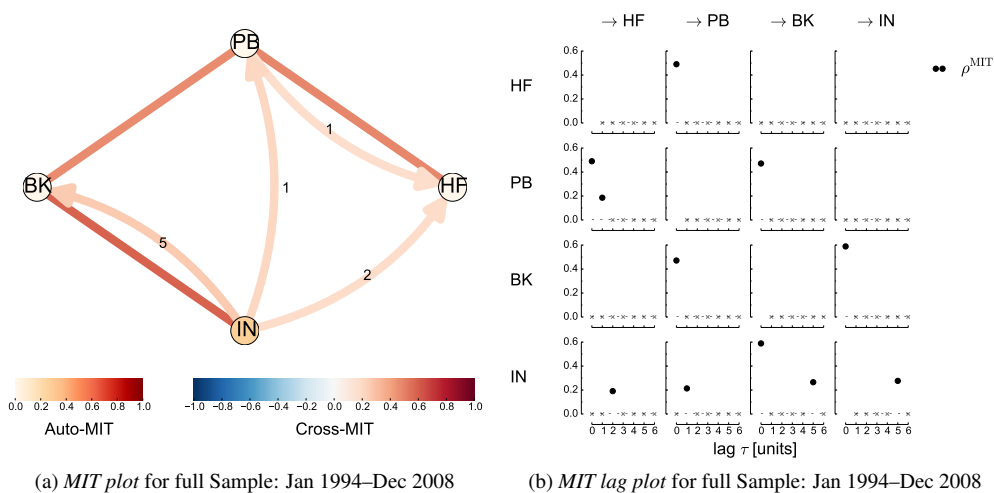


Figure 3: The *MIT Plots* showing both the linkages among the four sectors and the strength of such connections for the full study period, Jan 1994–Dec 2008. The second representation Figure (3b) of the *MIT plots* provides additional information on autoregressive linkages by inspecting the diagonals. The existence of a *dot* at a particular lag τ of sector ξ on the plots located on the diagonals indicates an autoregressive link of the form $\xi_{t-\tau} \rightarrow \xi_t$, which we denote as “ $\xi_t \rightarrow^\tau \xi_t$ ”.

links between the three sectors: banks (BK), Broker/dealers (PB) and insurers (IN), any large shock in one will spread quickly through the system.

We now provide results for four time periods: (1994M01–2004M12, 1994M01–2005M12, 1994M01–2006M12, 1994M01–2007M12), in Figures (4& 5). We make use of expanding window size to allow for the detection of changes in network structure across time. For all the time periods considered, we identified three contemporaneous links: $BK - PB$, $PB - HF$ and $BK - IN$ with the latter link having a higher coupling strength $\rho^{MIT} \simeq 0.65$ as shown in Figure (5). Moreover, there exists a causal autodependency in insurers at lag $\tau = 5$ ($IN_t \rightarrow^5 IN_t$) and a direct link $IN_t \rightarrow^2 HF_t$ with $|\rho^{MIT}| \simeq 0.25$. We obtain that hedge funds were only linked with banks ($HF_t \rightarrow^2 BK_t$) prior (1994M01–2006M12,–2007M12) to and during the financial crisis of 2007–2009, which was not evident at the end of the full sample period. Notice that right before and during the financial crisis of 2007–2009 (1994M01–2007M12), the interlinkages between the sectors increased. One of the lagged causal chain for this sample period is given by $HF_t \rightarrow^2 PB_t \rightarrow^5 BK_t$, $HF_t \rightarrow^2 BK_t$, and $IN_t \rightarrow^6 PB_t$. We find that the lag of the direct link between insurers and brokers observed for time period (1994M01–2006M12,–2007M12) changed from $\tau = 6$ to $\tau = 1$ at the end of full sample period. This means that it did take less time for shocks in the insurance sector to be transmitted to brokers during the crisis. We also find that time series graph was unchanged for the time period (1994M01–2004M12) and (1994M01–2005M12) as shown in Figures (4a& 4b) except for the exclusion of $IN_t \rightarrow^6 PB_t$ in the latter.

In order to account for the possibility of nonlinearity in the form of breaks or rare events in the multivariate signal, we make use of the unthresholded recurrence plot, which is well documented in literature (Iwanski and Bradley (1998); Marwan et al. (2007); Addo et al. (2013)). We identify breaks at the dates: 1998M08, 2000M03 and 2008M11, as shown on Figure (6). Based on these

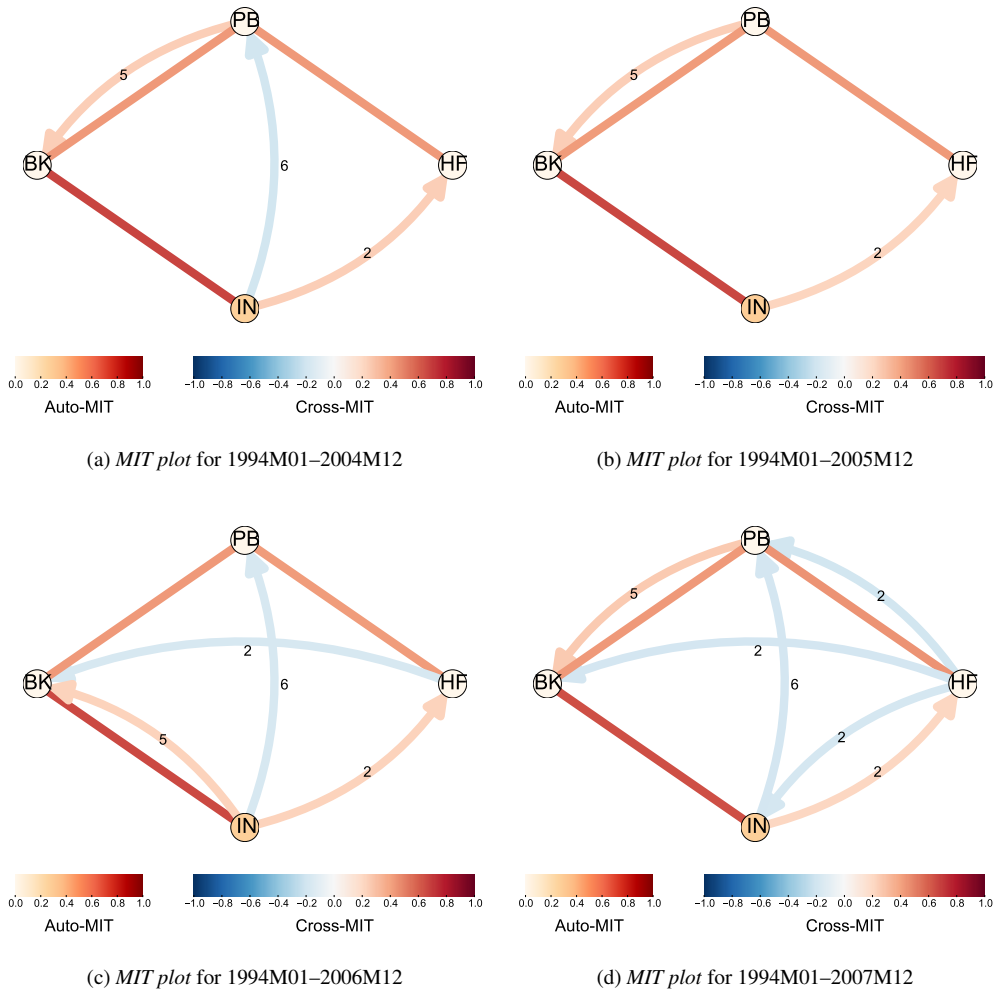


Figure 4: The *MIT Plots* displays how the four sectors are linked and the strength of the linkages. It shows autoregressive links, contemporaneous links and directed (i.e. across sectors) links among sectors. The numbers at some ends of directed arrows (links) corresponds to the significant lag of the linkage. Thus, a directed link “ $\mathcal{A} \rightarrow^\tau \mathcal{B}$ ” means that \mathcal{B} is influenced by \mathcal{A} at lag(s) τ . Contemporaneous links between two sectors: \mathcal{A} and \mathcal{B} , is denoted “ $\mathcal{A} - \mathcal{B}$ ”.

dates, we define three time windows: 1994M01–1998M08, 1994M01–2000M03, and 1998M08–2008M11, for the MIT analysis. The results of the time series graphs for each time period is provided in Figure (7) and Figure (8). For the time period (1994M01–1998M08 and 1994M01–2000M03), we obtain the same network structure and coupling strengths. The time series graph for this period is composed of a high coupling strength of a contemporaneous link between brokers and banks ($\rho^{MIT} \simeq 0.75$), and $IN_t \rightarrow^6 HF_t$ ($\rho^{MIT} = -0.4$). For the time period (1998M08–2008M11), the coupling strength for the contemporaneous link $BK - PB$ reduced from $\rho^{MIT} \simeq 0.75$ to 0.5 (see Figure (8c)). We also find that the causal autodependency in

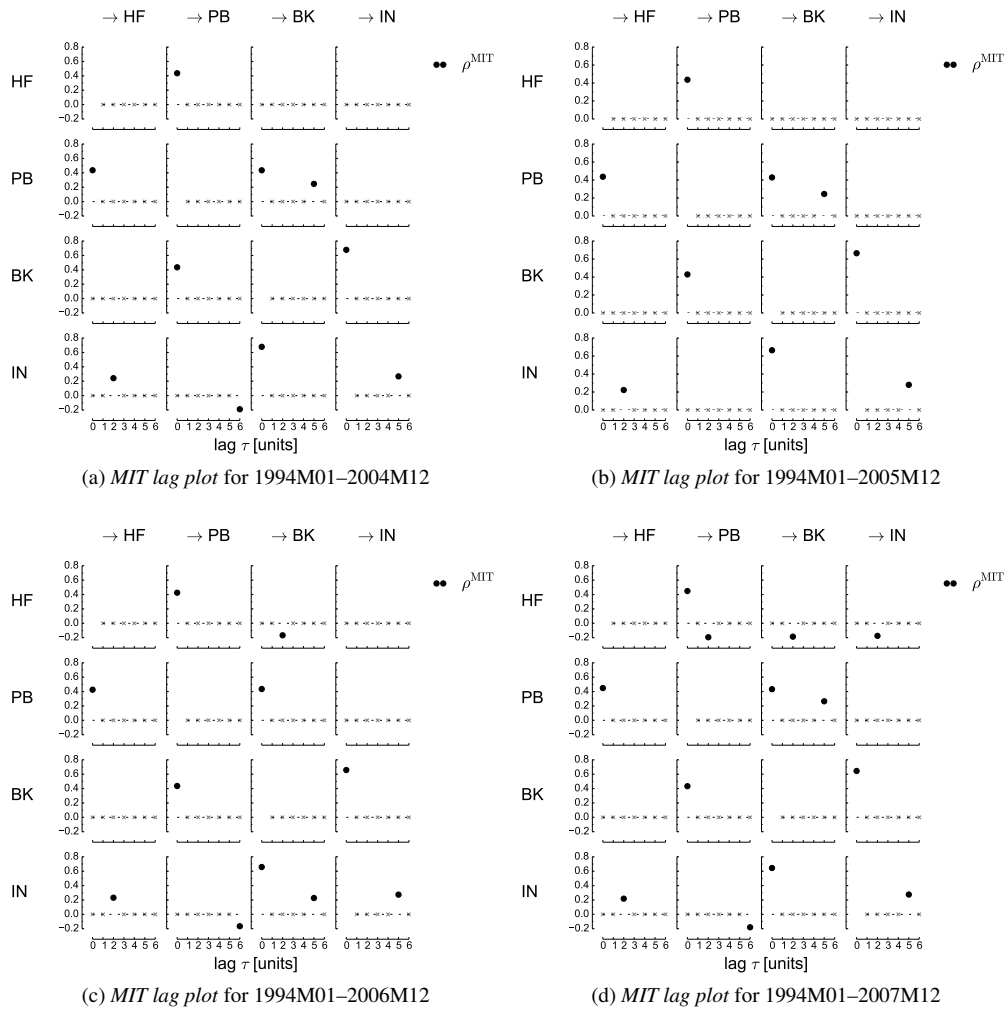


Figure 5: This is an alternative representation of the MIT Plots in Figure 4. A directed link “ $\mathcal{A} \rightarrow^\tau \mathcal{B}$ ” means that \mathcal{B} is influenced by \mathcal{A} at lag(s) τ . The existence of a dot at a particular lag τ of sector ξ on the plots located on the diagonals indicates an autoregressive link of the form $\xi_{t-\tau} \rightarrow \xi_t$, which we denote as “ $\xi_t \rightarrow^\tau \xi_t$ ”. Contemporaneous links can be read from the plots by observing dots at where $\tau = 0$. The vertical axes represent the MIT coupling strength of the linkages.

insurers at lag $\tau = 5$ ($IN_t \rightarrow^5 IN_t$) and two other contemporaneous links: $PB - HF$ and $BK - IN$ identified (see Figures (4)) in the previous time windows, emerged after year 2000 (see Figures (7b& 7c)). These contemporaneous links could be due to the early 2000’s recession (i.e 2001 to 2003: the collapse of the Dot Com Bubble and September 11th attacks).

Finally, we perform analysis on the whole sample period over a sliding window of length 36 months and a step size of 12 months to check for robustness on the linkages among these four institutions. The results on the ensemble graph and significant lag plots is reported in Figure (9). This figure shows the overall significant causal dependence among the four institutions for all

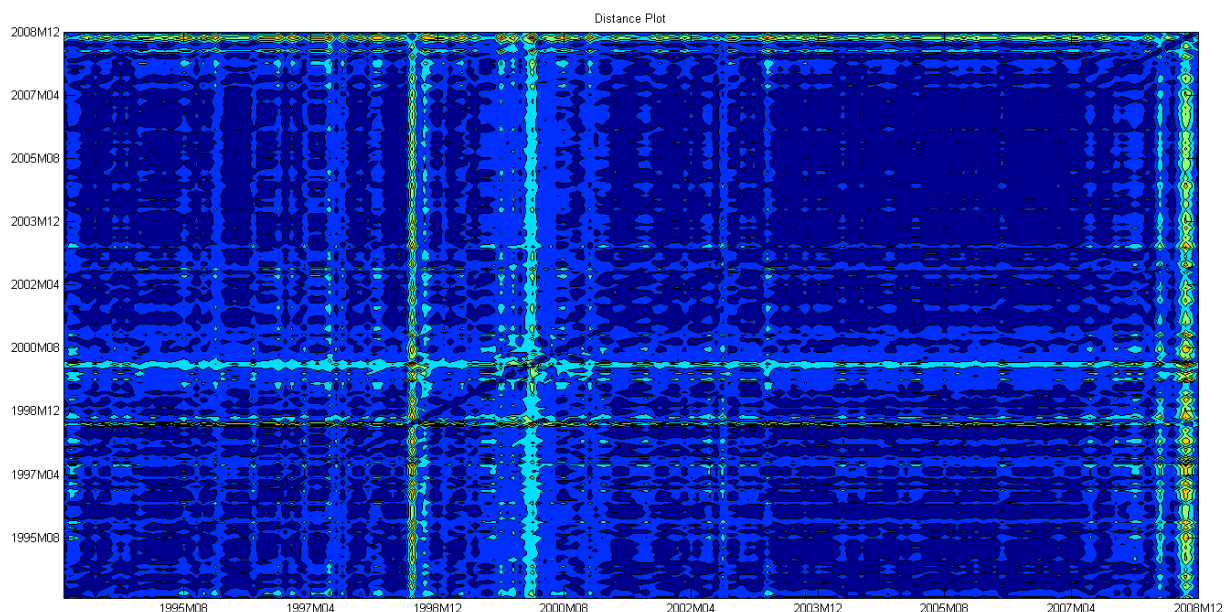


Figure 6: A contour plot (heat map) of the unthresholded recurrence matrix, which is also referred to as the distance matrix in Addo et al. (2014). Considering the entire sample, we identify breaks at the dates: 1998M08, 2000M03 and 2008M11. We define time windows: 1994M01–1998M08, 1994M01–2000M03, 1998M08–2008M11, based on these dates for the MIT analysis.

time series graphs over the sliding windows. Our analysis reveals three contemporaneous links: $BK - PB$, $BK - IN$ and $PB - HF$, that were previous identified in Figures (4& 5). We remark that the fraction of graphs for all windows that contained the contemporaneous link between brokers and hedge funds was less in comparison with the other two links. This accounts for the difference in link width in Figure (9a). Contrary to the view expressed in Billio et al. (2012), our findings provide support to claim that both banks and the so-called shadow banking system were more central to systemic risk. In particular, due to the high coupling strength of contemporaneous links between banks, Broker/dealers and insurers, any large shock in one will spread quickly through the system.

In general, our results point to important asymmetry in the causal dependencies in the insurance sector and brokers prior to the financial crisis of 2007–2009. We find that the returns of insurers have significant impact on returns of brokers, where the time delay of impact is shorter during the crisis (see Figures (4c & 4d) and Figure (3a)). For all time periods considered, we find that banks were always contemporaneously linked to brokers with a higher coupling strength observed before year 2000. In addition, our results indicate that insurers did not, in many cases, directly impact hedge funds significantly. After the year 2000 (see Figure (7b)), we find contemporaneous link between banks and insurers, and also between brokers and hedge funds, where the former is with a higher coupling strength. The aftermath of 1998 collapse of the \$5 billion hedge fund Long Term Capital Management (LTCM) could have lead to the contemporaneous link between brokers and hedge funds. In general, we find that banks were not directly connected to hedge funds. However, the two sectors were indirectly linked via a contemporaneous chain of dependencies from banks to hedge funds: $BK - PB - HF$, as seen Figure (4). We remark that

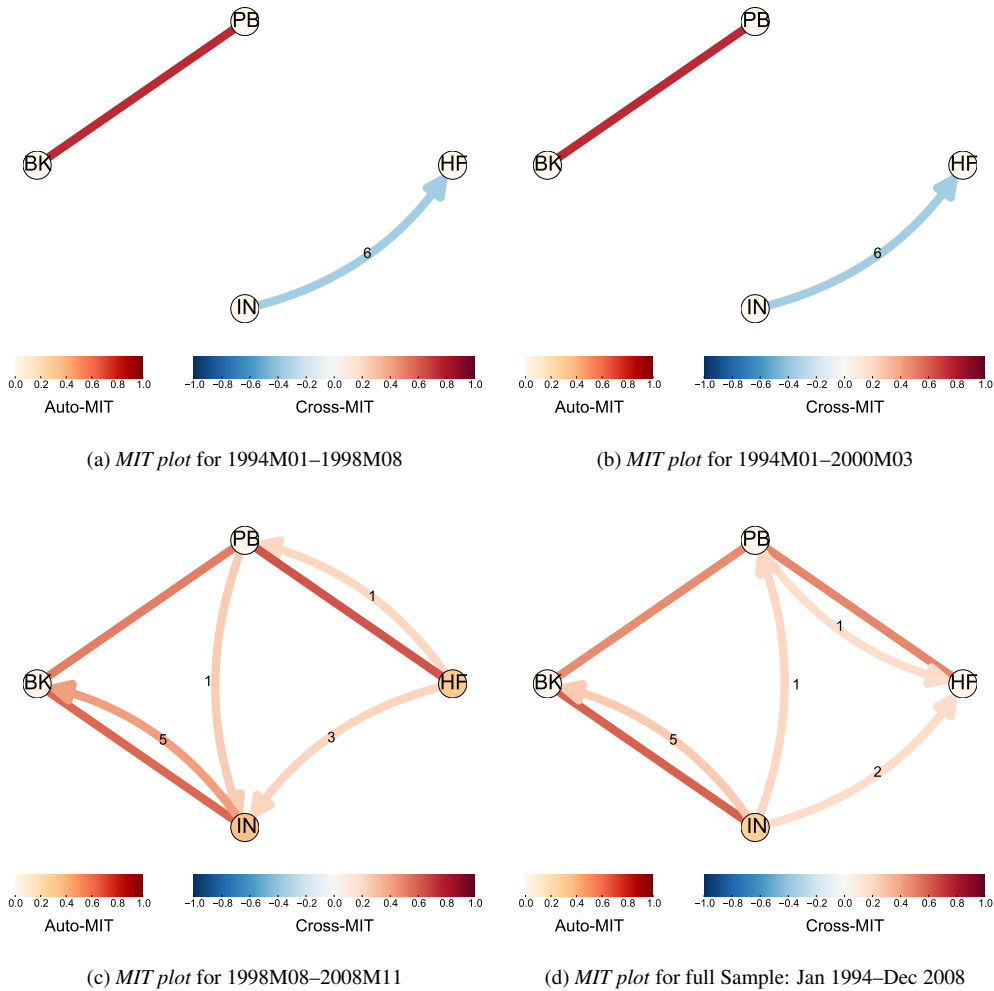


Figure 7: A directed link “ $\mathcal{A} \rightarrow^\tau \mathcal{B}$ ” means that \mathcal{B} is influenced by \mathcal{A} at lag(s) τ . The existence of a *dot* at a particular lag τ of sector ξ on the plots located on the diagonals indicates an autoregressive link of the form $\xi_{t-\tau} \rightarrow \xi_t$, which we denote as “ $\xi_t \rightarrow^\tau \xi_t$ ”. Contemporaneous links can be read from the plots by observing *dots* at where $\tau = 0$. The vertical axes represent the MIT coupling strength of the linkages. We define time windows: 1994M01–1998M08, 1994M01–2000M03, 1998M08–2008M11, based on the distance matrix plot in Figure (6).

just prior to the financial crisis of 2007–2009, the hedge funds did have a significant negative impact on banks (see Figures (5c & 5d)). In this period, as seen in Figures (4d & 5d), hedge funds had a significant bilateral relationships with insurers, brokers and banks. We remark that most connections were observe for the time period (1994M01–2007M12), shown in Figure (4d), were hedge funds had a negative impact on the financial system especially on the banks and brokers. This increase in number of connections and causal dependencies between these sectors just before and in the beginning of the recent crisis could serve as a significant systemic risk indicator. Our results have showed that there is high coupling strength of linkages between the

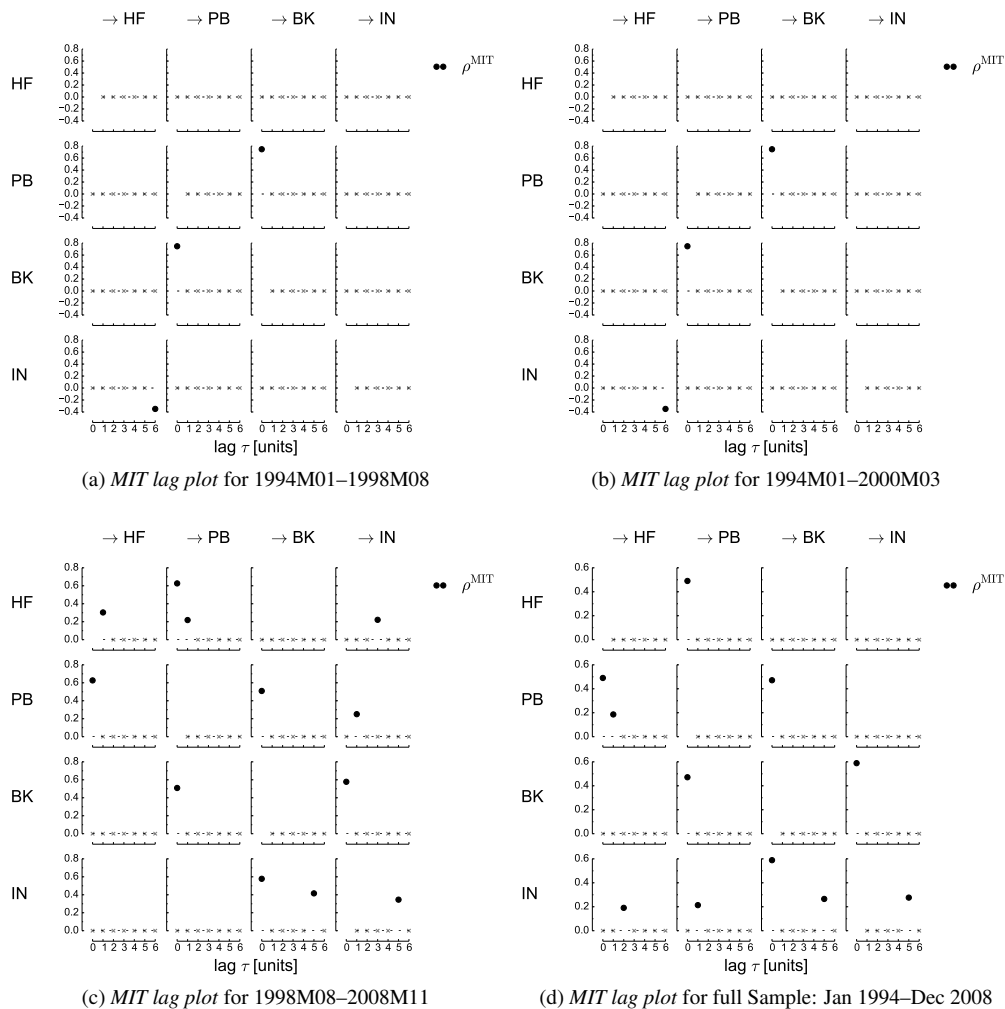
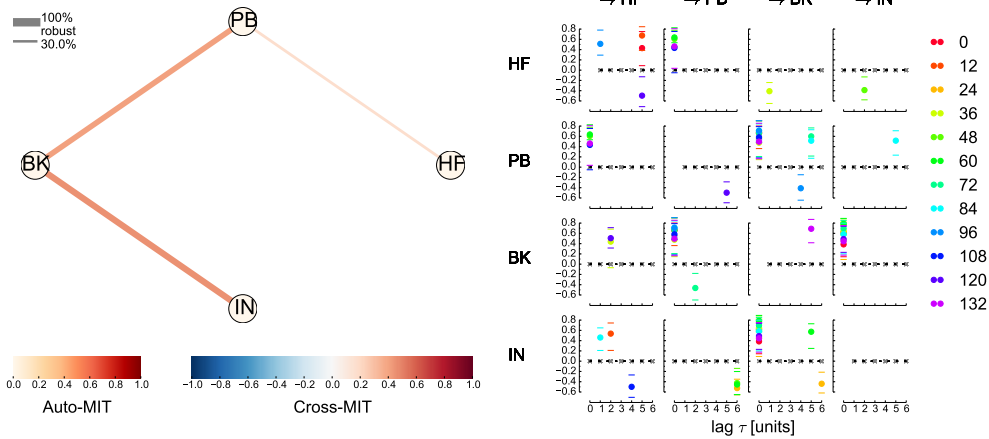


Figure 8: The lag plot indicating significant causal dependencies and coupling strength between subprocesses for the time windows: 1994M01–1998M08, 1994M01–2000M03, 1998M08–2008M11. In otherwords, this is an alternative representation of Figure (7) with information on the magnitude and sign of the coupling strength.

four sectors, increasing the channels through which shocks can be transmitted throughout the financial system.

4.2. European Sovereign debt crisis

The data used in our empirical analysis consist of daily 10-year government bond yields for eleven European Monetary Union (EMU 11) countries. The EMU 11 used in this work is composed of the periphery group of countries: Portugal (Por), Ireland (Ire), Italy (Ita), Greece (Gre), Spain (Spa); and the central group of countries: Austria (AU), Germany (Ger), The Netherlands (Ne), Belgium (Bel), France (Fra), and Finland (Fin). The idea is to empirical investigate



(a) An ensemble of *MIT* plot for full sample: Jan 1994–Dec 2008 on a sliding window. (b) The *MIT* lag plot for sliding window analysis on the full sample period.

Figure 9: Sliding window analysis on the full sample period: Jan 1994–Dec 2008. The sliding window is of length 36 months and a step size of 12 months. The link width denotes the percentage of windows that contain this link in the time series graph.

transmissions of the sovereign debt crisis by examining the behavior of the government bond yields of these countries for the period (01/01/2007–31/12/2014). Our daily data comes from the Bloomberg. The start date of the sample is chosen in order to study dynamics prior and during the Sovereign debt crisis. Given that the 10-year bond yield a country is denoted B_t . In order to ensure stationarity, we work with the time series $\mathfrak{B}_t = \log(B_t) - \log(B_{t-1})$, which correspond to the growth rate of the sovereign bond yields, see Figure (10).

The time series graph for the year 2007 is shown in Figure (11). In this graph, we identify causal linkages during the start of the 2007–2009 global financial crisis. At the end of the global financial crisis in 2009 (see Figure (12c)), we observe an increase in the number of causal connections, with most previous links maintained. In general, the number of linkages did start increasing from the year 2007 to the end of 2009. Figures (12 & 13) provide information on the evolution of the causal dependencies among countries across the time frame before and during the European sovereign crisis. The significant lag plots of the time series are presented in Figures (A.15–A.22). The number of linkages among the sovereign bond yields increases across time as the window expands from 2007. We also observe more linkages with negative flows over the expanding windows at the start and during the sovereign debt crisis, as shown in Figure (13). It is clearly seen that there are more connections and negative flows at the end of the 2014, which signals that the EMU 11 is not yet out of the sovereign debt crisis. In Figure (12d), we find that the Netherlands were not linked to any of the periphery countries at the start of the sovereign debt crisis. The sovereign bond yields of the Netherlands and Germany were highly coupled during the crisis, see Figure (13).

Finally, we perform our analysis on sliding windows to enhance robust conclusions on the identification and quantification of causal dependencies among sovereign bond yields for the whole sample period. We present in Figure (14) the results for sliding window of length 1 year with a step size of 6 months, and the sliding window of length 1 year with a step size of 1 year.

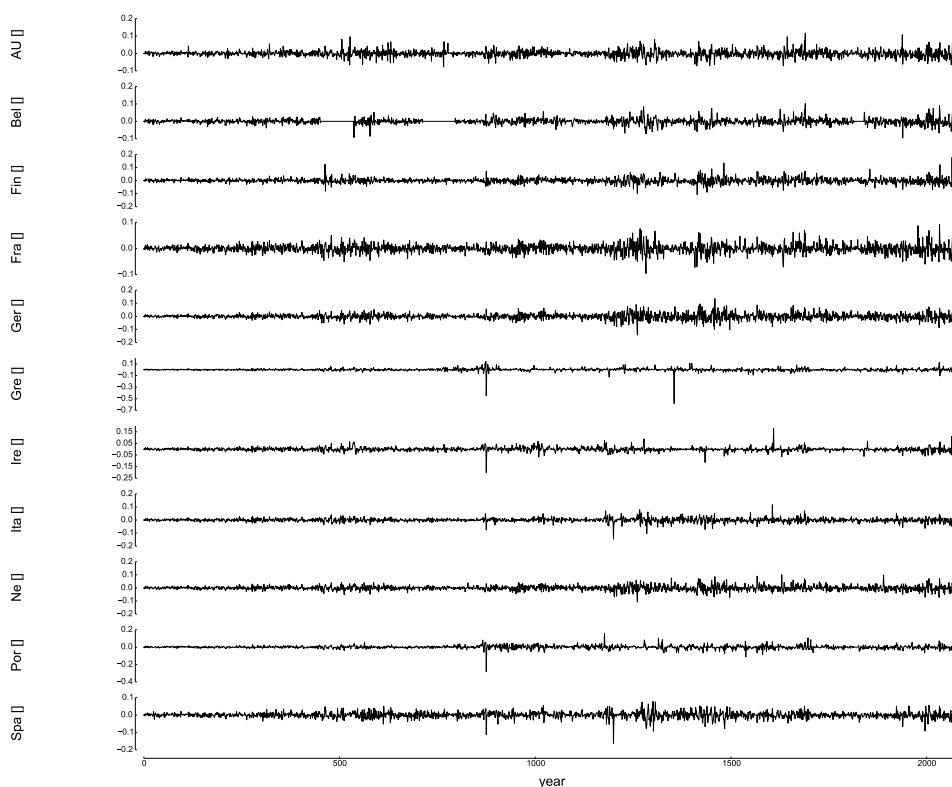


Figure 10: Time series plot of the daily 10-year European Sovereign bond yields, \mathfrak{Y}_t , the period from 1/01/2007–31/12/2014. The time indexes correspond to the following dates: “1”= 1/01/2007, “500”= 11/28/2008, “1000” = 10/29/2010, “1500”= 9/28/2012, and “2000”= 8/29/2014.

The associated significant lag plots are provided in Figure (B.23) and Figure (B.24). We remark that our findings do not differ when we use sliding windows of length 2 years with step the results with sizes 1 year or 2 years. The results for these sliding window analysis can be made available upon request from the authors. The link width for the sliding window analysis denotes the percentage of windows that contain this link in the time series graph. In general, the graphs point to higher coupling between the Netherlands and Germany, the Netherlands and France, France and Germany, Belgium and Austria, and then Italy and Spain. Our finding shows that government bond yield of the Netherlands were not linked to any of the periphery countries severely hit by the sovereign debt crisis. In particular, the sovereign bond yields of Germany, the Netherlands, Finland and Austria were more stable compared to other core countries like France and Belgium, which were linked to at least two periphery countries. For instance, France did have a contagion

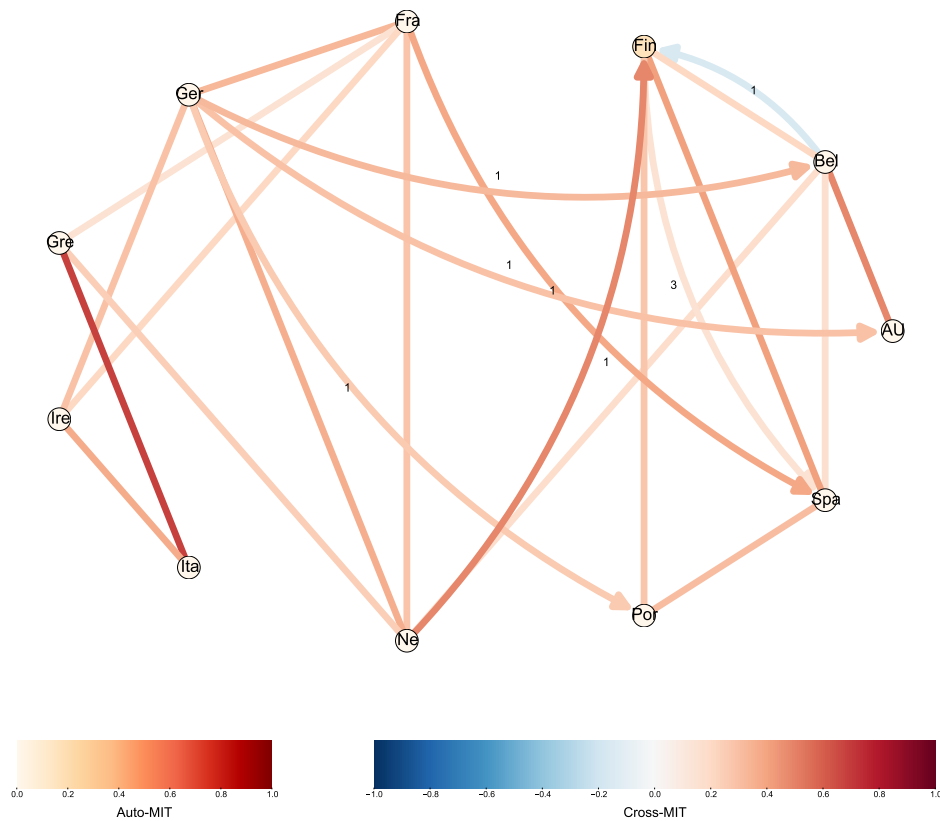
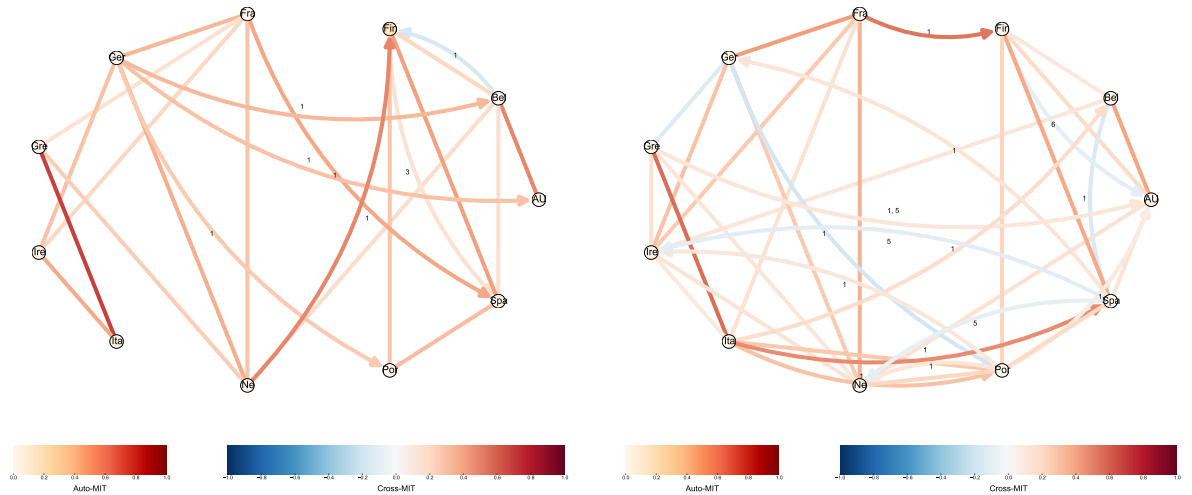


Figure 11: Time series graph of the 10-year European Sovereign bond yields for 2007. The year “2007” denotes the period from 1/01/2007–31/12/2007.

spillover effects mainly from Ireland and Italy. However, it’s causal dependence with Germany, the Netherlands, Austria and Finland, could probably explain why France was not severely hit by the sovereign debt crisis. The graphs in Figure (14) points to transmissions of risk effects from Greece to the periphery countries hit by the crisis.

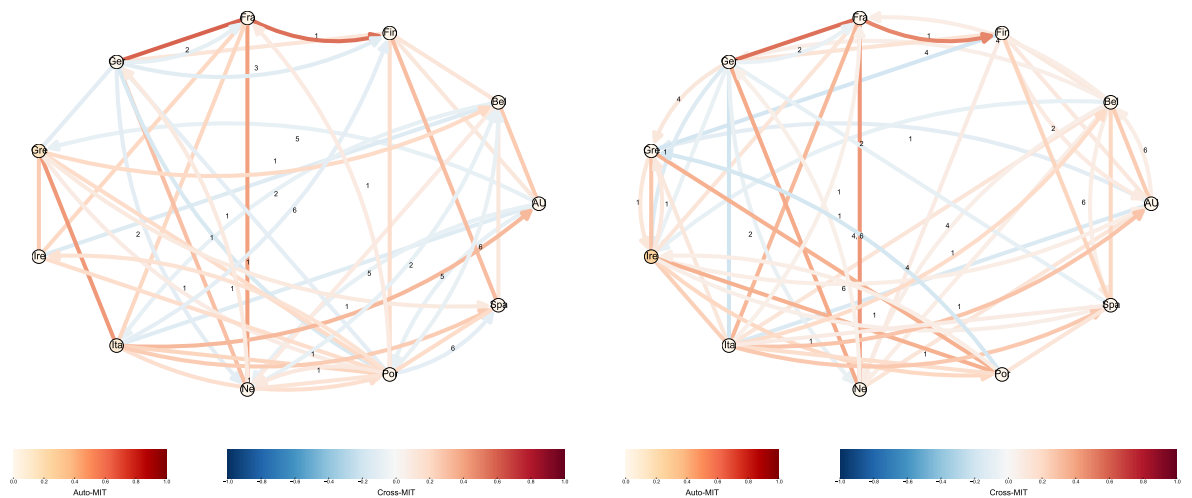
5. Concluding Remarks

In this study we have introduced and provided a novel methodology for analysing complex linkages among financial institutions. In particular, we have presented the drawback of using the original framework of Granger causality (linear or nonlinear) tests in measuring connectedness among subprocesses of a multivariate process. We offer a novel information theoretic approach for capturing and studying complex linkages in the financial system based on time series graphs



(a) 2007

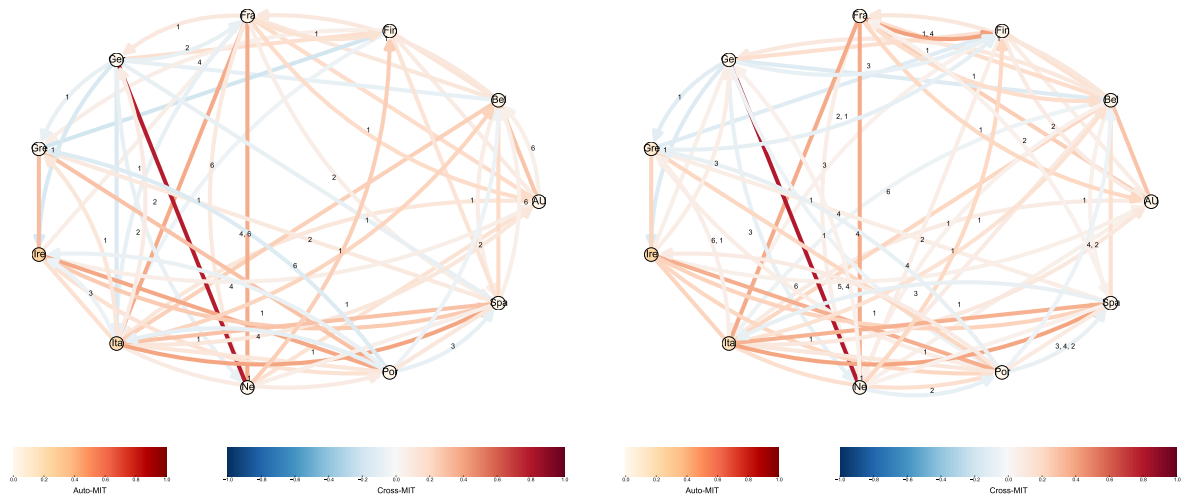
(b) 2007–2008



(c) 2007–2009

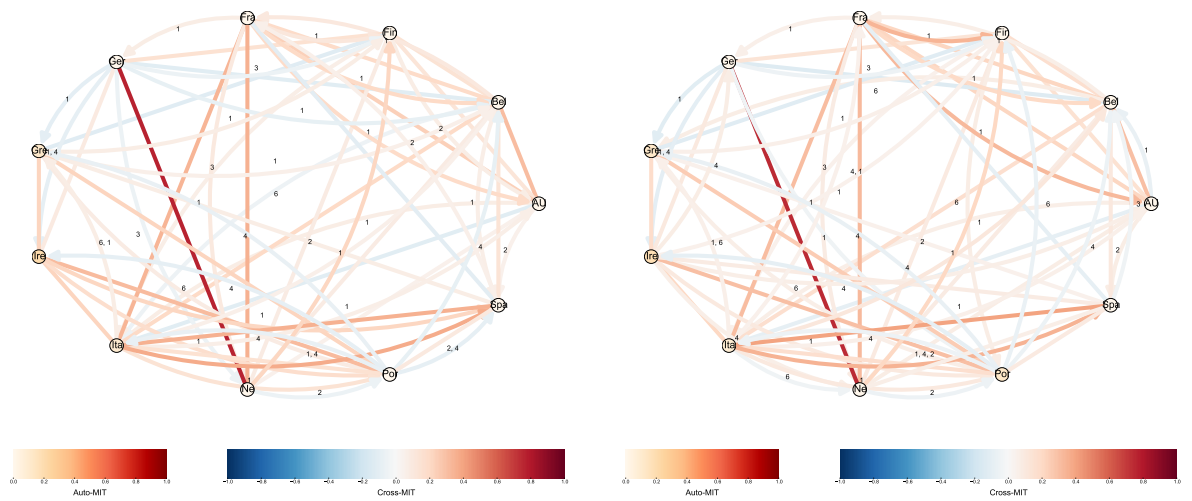
(d) 2007–2010

Figure 12: Time series graph of daily 10-year European Sovereign bond yields over expanding windows. The year “2007” means the period from 1/01/2007–31/12/2007. The time window of the format “20AA”–“20BB” denotes the period from the start date of year “20AA” to the end date of year “20BB”.



(a) 2007–2011

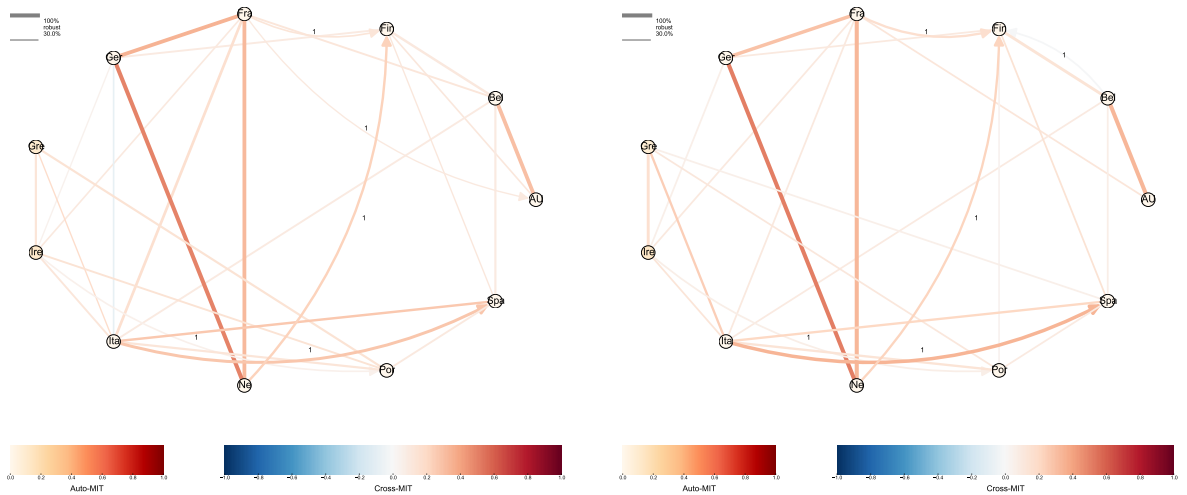
(b) 2007–2012



(c) 2007–2013

(d) 2007–2014

Figure 13: Time series graph of daily 10-year European Sovereign bond yields over expanding windows. The year “2007” means the period from 1/01/2007–31/12/2007. The time window of the format “20AA”–“20BB” denotes the period from the start date of year “20AA” to the end date of year “20BB”.



(a) Sliding window of length 260 and step size 130 days.

(b) Sliding window of length 260 and step size 260 days.

Figure 14: Time series graphs on daily 10-year European Sovereign bond yields over sliding windows. The length of 130 days correspond to half a year, and 260 days is a year.

and information theory. In particular, we show that this approach identifies linkages, transmission channels, lags of linkages and assess the strengths of such connections. Unlike existing methodologies with assumptions on stationarity of time series, model structure and parameter space, our approach only requires that the multivariate time series be stationary. Our approach excludes the misleading influence of autodependency within a process by assigning a non-zero value only to lagged components that are not independent conditional on the remaining process. We have applied this methodology to simulated data and presented a comprehensive feasibility of the approach in capturing lag-specific causal dependencies in multivariate process. We have also applied this approach to real data to analyse interconnectedness among financial institutions and also to study the European Sovereign debt crisis considering different time frames and sliding windows. Further studies should look into enhancing this methodology by defining a nonlinear equivalent measure of the coupling strength of interconnectedness.

Acknowledgement

The authors are grateful to participants of the 2014 Meeting of the Consortium for Systemic Risk Analytics (CSRA) held at Massachusetts Institute of Technology (MIT), for their valuable suggestions, and comments that have significantly improved this contribution. This project has received funding from the European Union's Seventh Framework Programme (FP7-SSH/20072013) for research, technological development and demonstration under grant agreement n^o320270 (SYRTO). This documents reflects only the author's view. The European Union is not liable for any use that may be made of the information contained therein.

Appendix A. Significant lag plots for 10-year European Sovereign bond yields

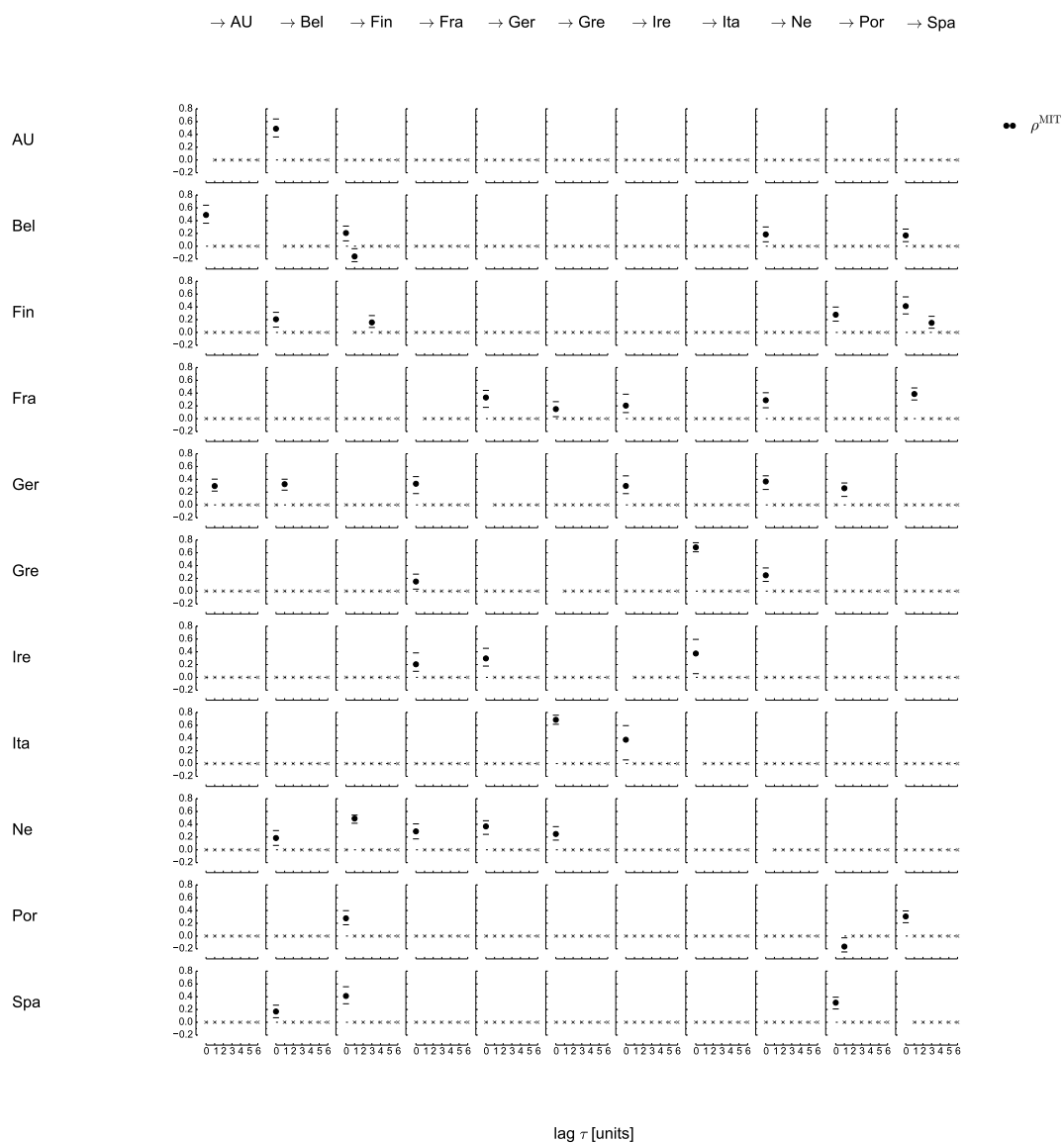


Figure A.15: Significant lag plots for 10-year European Sovereign bond yields for 2007. This corresponds to the time series graph in Figure (12a).

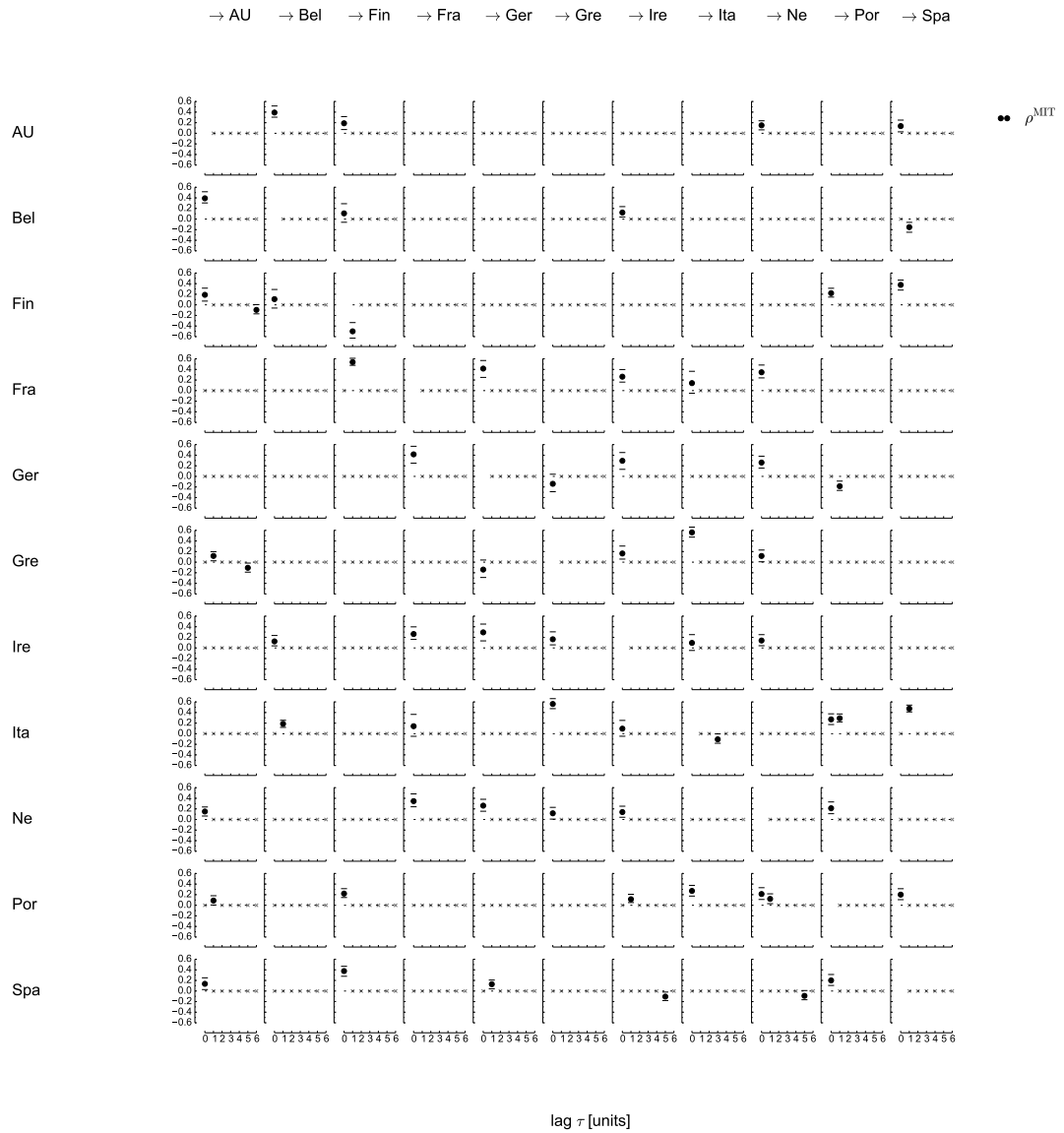


Figure A.16: Significant lag plots for 10-year European Sovereign bond yields for 2007–2008. This corresponds to the time series graph in Figure (12b).

Appendix B. Significant lag plots for window analysis on Bond yields

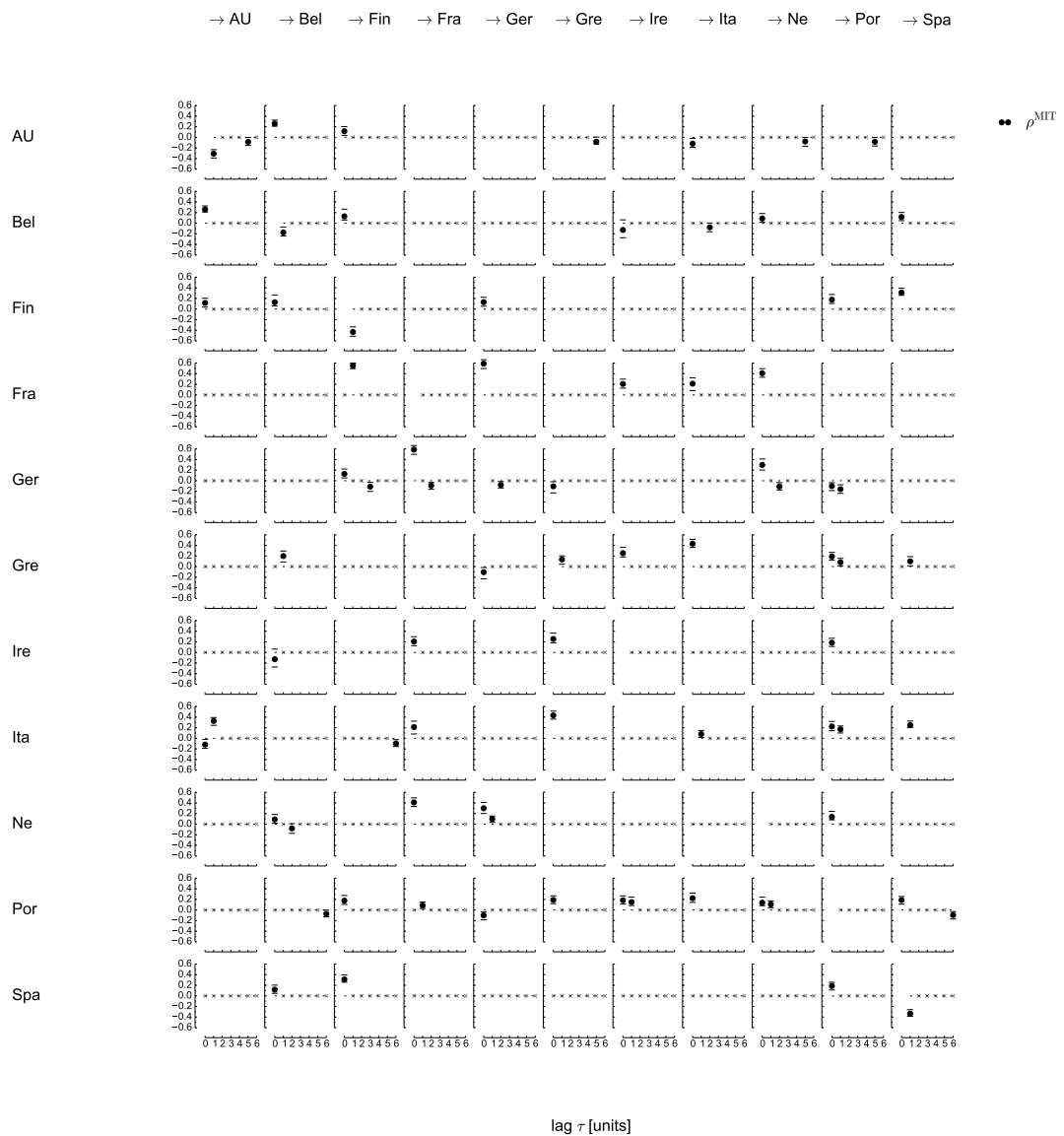


Figure A.17: Significant lag plots for 10-year European Sovereign bond yields for 2007–2009. This corresponds to the time series graph in Figure (12c).

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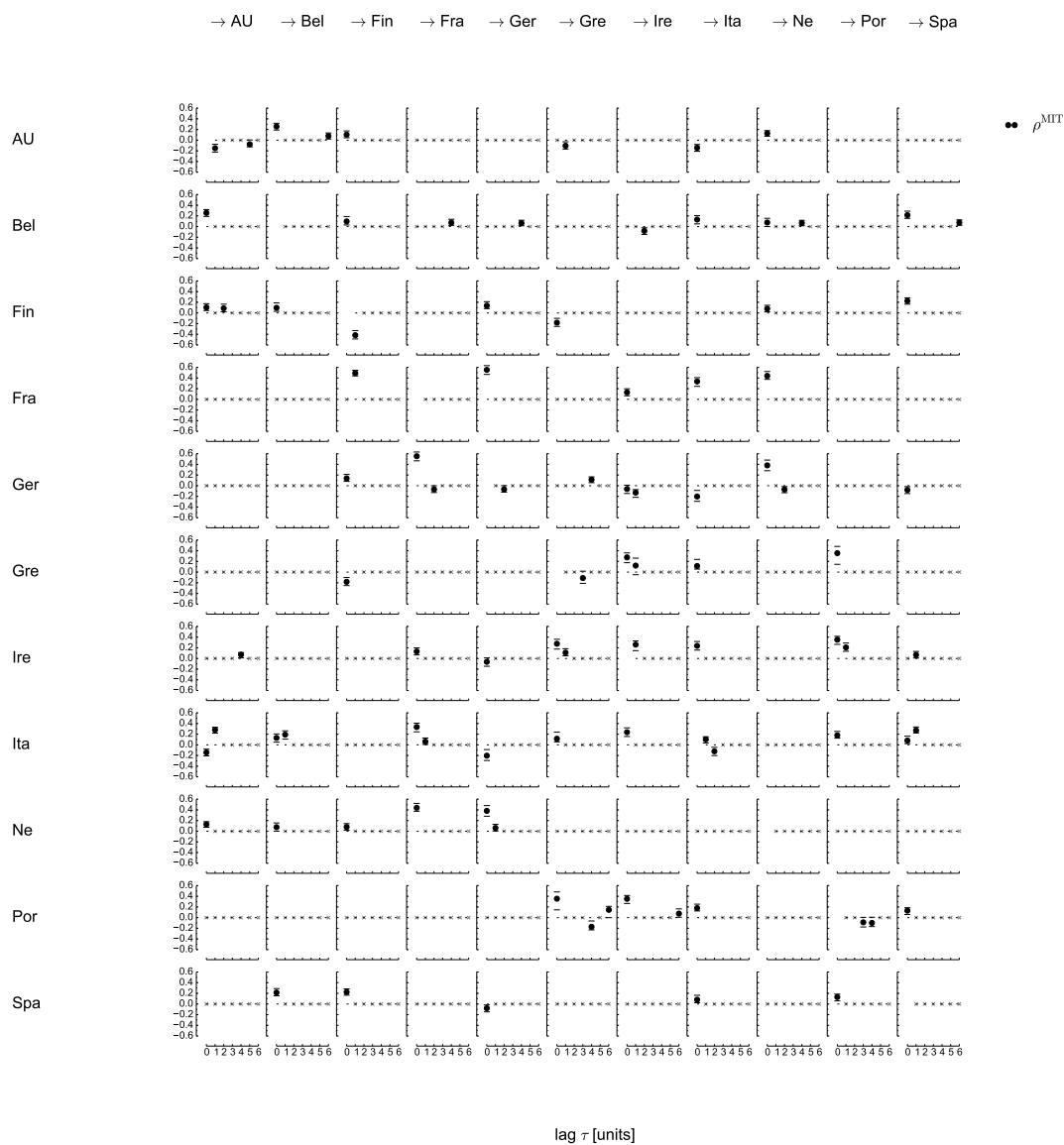


Figure A.18: Significant lag plots for 10-year European Sovereign bond yields for 2007–2010. This corresponds to the time series graph in Figure (12d).

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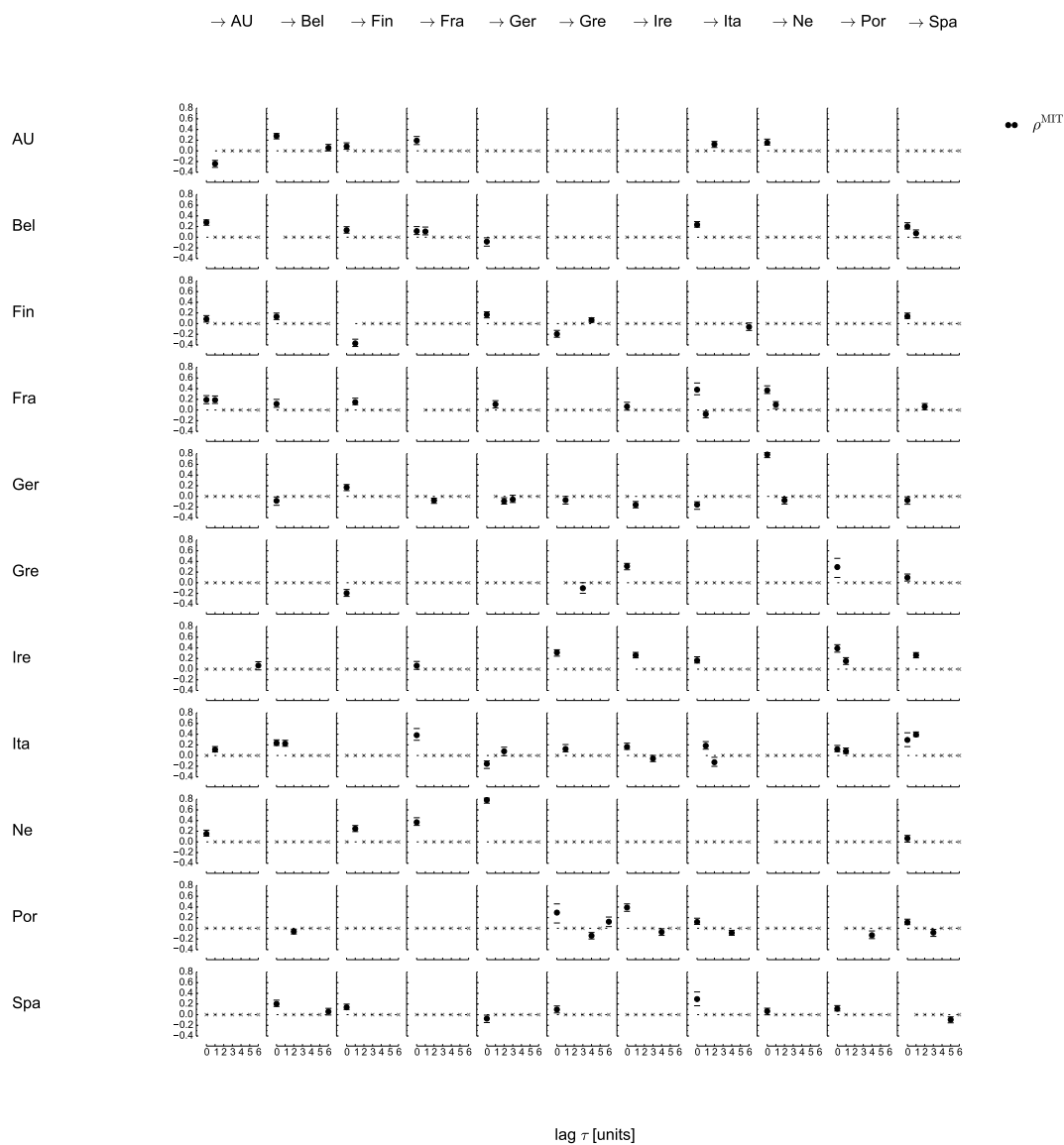


Figure A.19: Significant lag plots for 10-year European Sovereign bond yields for 2007–2011. This corresponds to the time series graph in Figure (13a).

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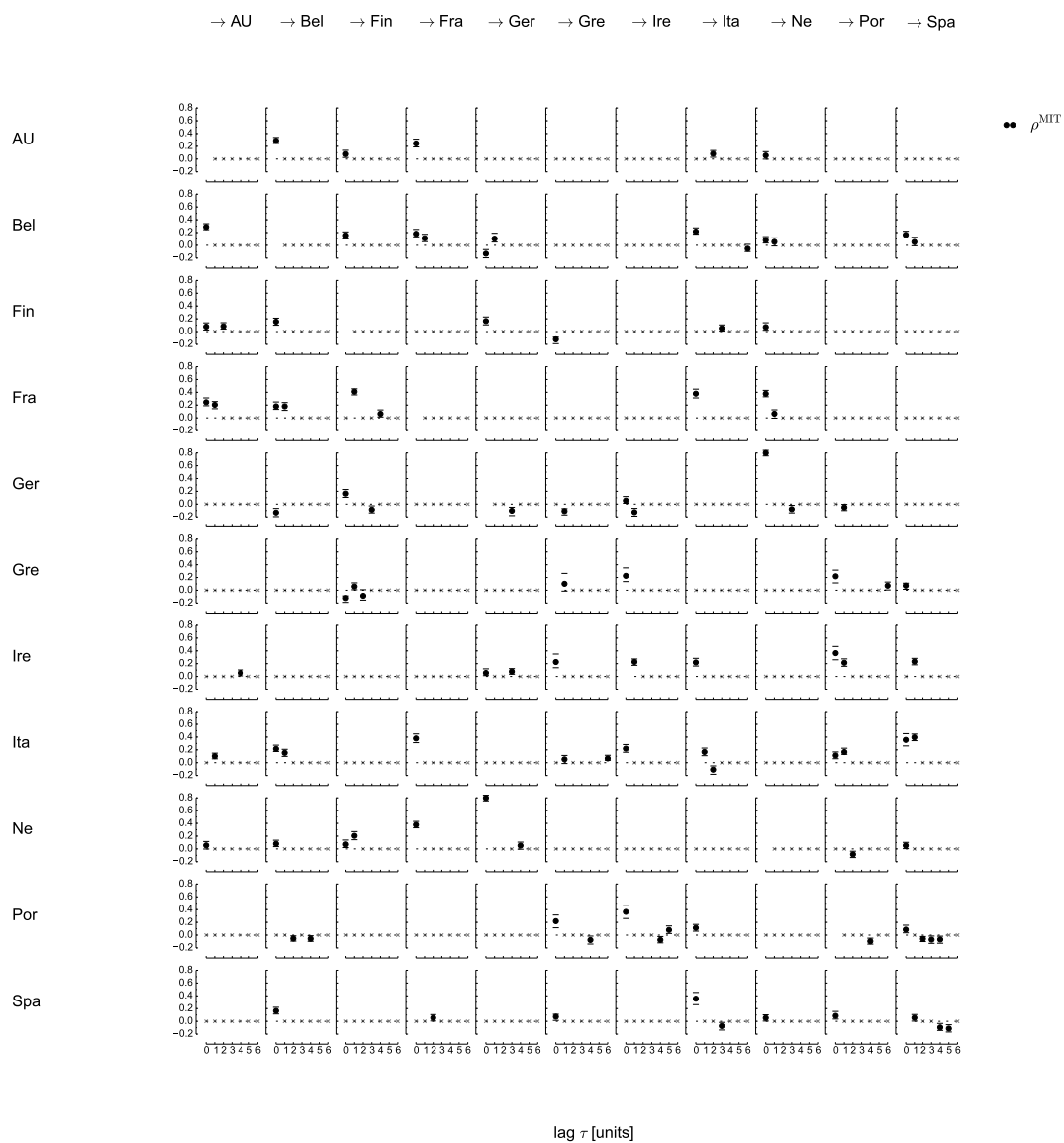


Figure A.20: Significant lag plots for 10-year European Sovereign bond yields for 2007–2012. This corresponds to the time series graph in Figure (13b).

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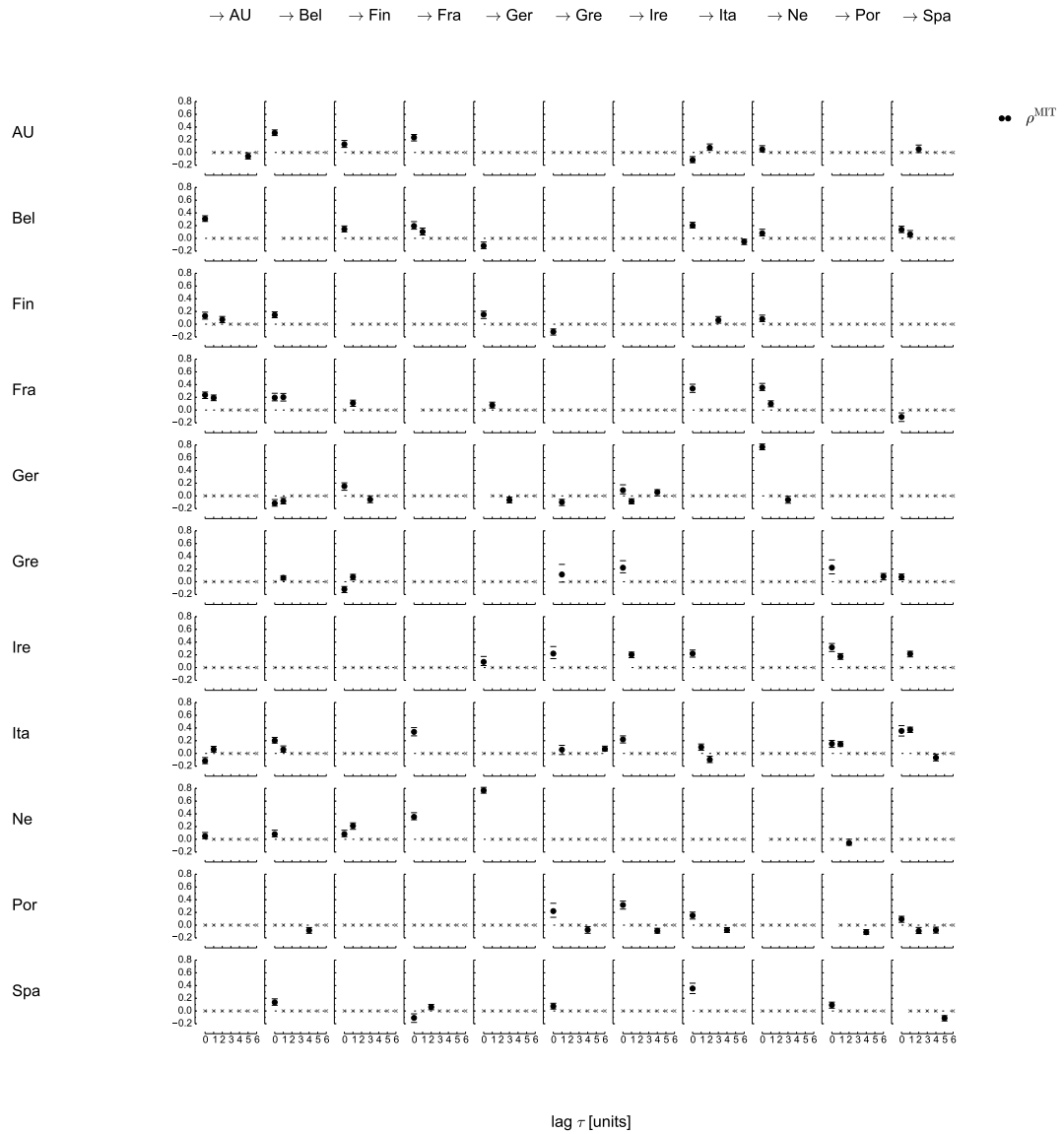


Figure A.21: Significant lag plots for 10-year European Sovereign bond yields for 2007–2013. This corresponds to the time series graph in Figure (13c).

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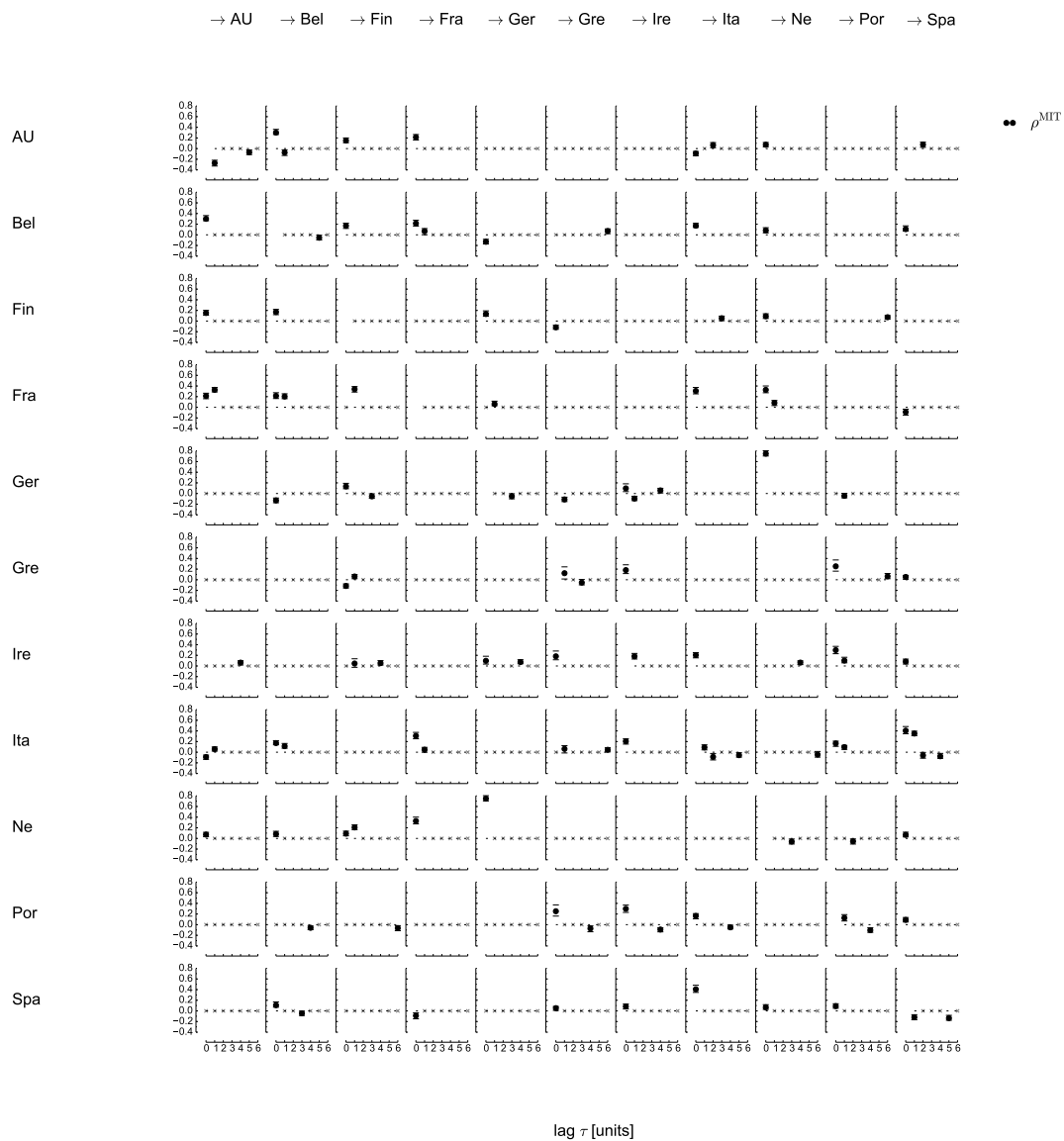


Figure A.22: Significant lag plots for 10-year European Sovereign bond yields for 2007–2014. This corresponds to the time series graph in Figure (13d).

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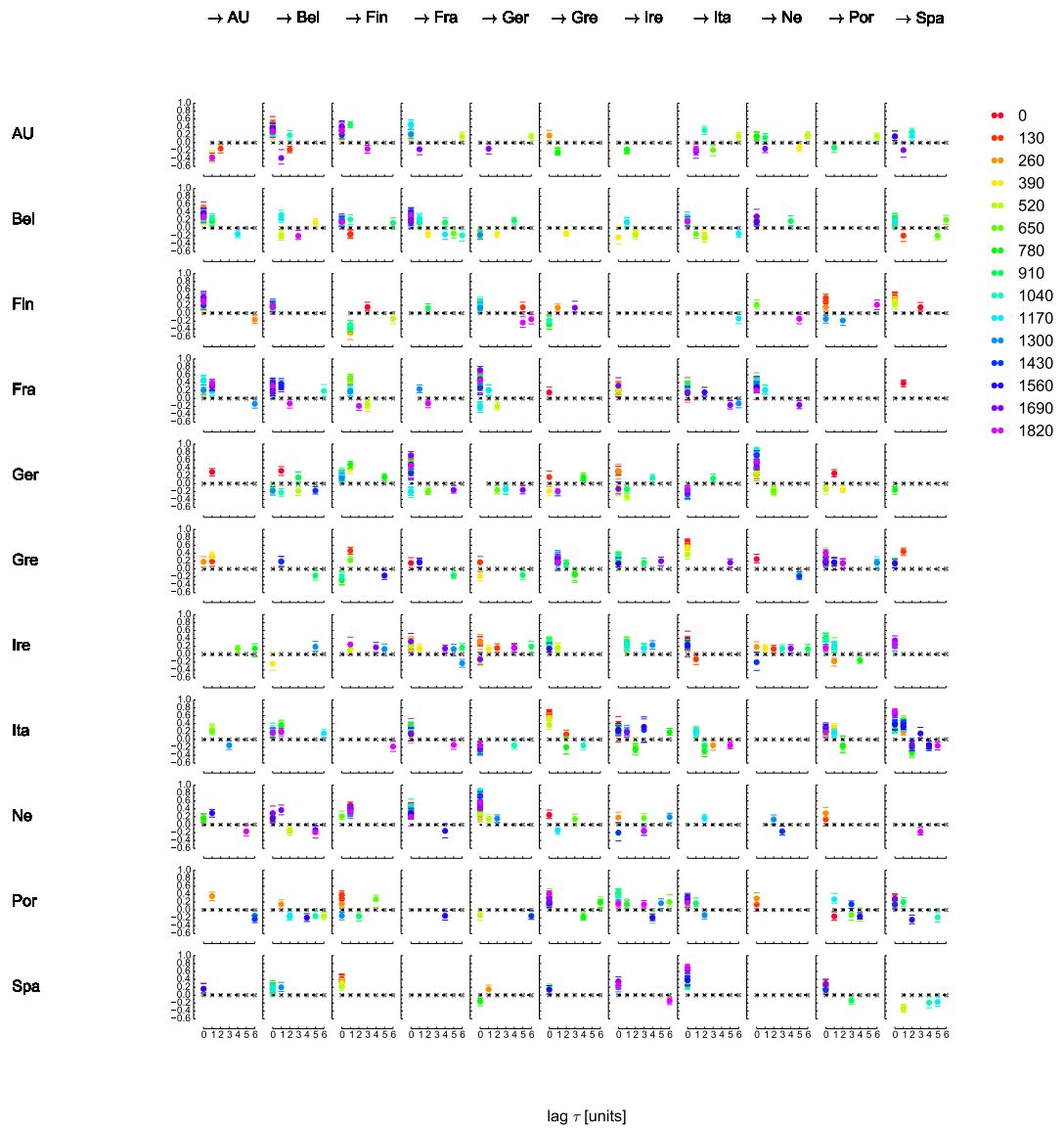


Figure B.23: Sliding window of length 260 and step size 130 days. The time series graph corresponding to this plot is shown in Figure (14a).

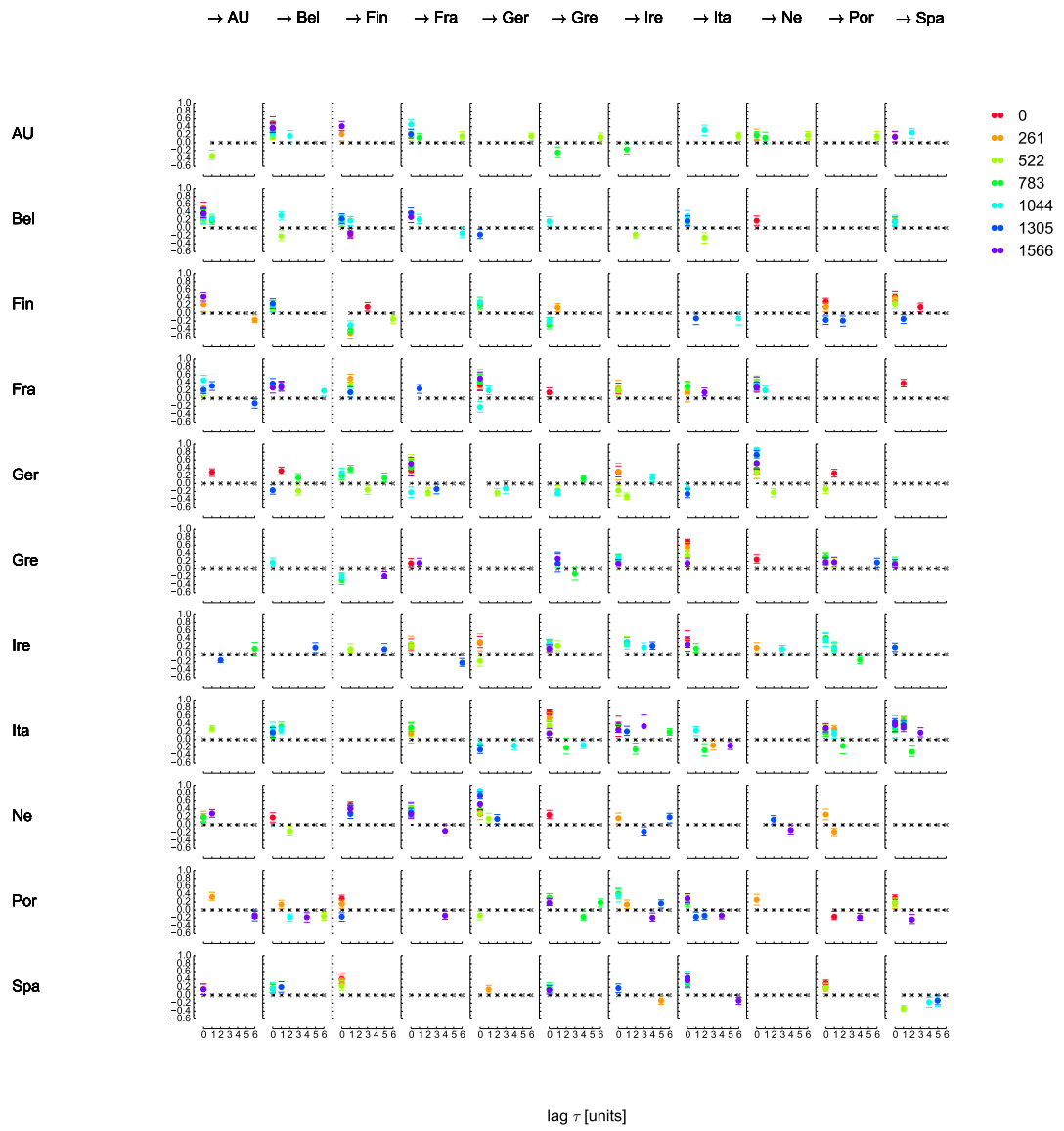


Figure B.24: Sliding window of length 260 and step size 260 days. The time series graph corresponding to this plot is shown in Figure (14b).