Detection and quantification of causal dependencies in multivariate time series: a novel information theoretic approach to understanding systemic risk

Peter Martey Addo, Philippe de Peretti

UNIVERSITÉ PARIS 1 – PANTHÉON - SORBONNE –

Documents de Travail du
Centre d'Economie de la Sorbonne

CES Working Papers

Detection and quantification of causal dependencies in multivariate time series: a novel information theoretic approach to understanding systemic risk

Peter Martey ADDO, Philippe De PERETTI

2014.69

Maison des Sciences Économiques, 106-112 boulevard de L'Hôpital, 75647 Paris Cedex 13
http://centredeconomiesorbonne.univ-paris1.fr/
ISSN : 1955-611X

Detection and quantification of causal dependencies in multivariate time series: a novel information theoretic approach to understanding systemic risk

Peter Martey ADDO[a,b,*], Philippe De PERETTI[a,b]

[a]*Centre National de la Recherche Scientifique (CNRS)*
[b]*Centre d'Économie de la Sorbonne (CES) - CNRS : UMR8174 - Université Paris I - Panthéon Sorbonne, France*

Abstract

The recent financial crisis has lead to a need for regulators and policy makers to understand and track systemic linkages. We provide a new approach to understanding systemic risk tomography in finance and insurance sectors. The analysis is achieved by using a recently proposed method on quantifying causal coupling strength, which identifies the existence of causal dependencies between two components of a multivariate time series and assesses the strength of their association by defining a meaningful coupling strength using the momentary information transfer (MIT). The measure of association is general, causal and lag-specific, reflecting a well interpretable notion of coupling strength and is practically computable. A comprehensive analysis of the feasibility of this approach is provided via simulated data and then applied to the monthly returns of hedge funds, banks, broker/dealers, and insurance companies.

Keywords: Systemic risk, Financial crisis, Coupling strength, Financial institutions
JEL: G12, C40, C32, G29

1. Introduction

The recent financial crisis of 2007–2009 has lead to a need for the financial industry, regulators and policymakers to develop a meaningful understanding and track of systemic risk. As the events following the turmoil in financial markets unfolded, it became evident that modern financial systems exhibit a high degree of interdependence making it difficult in predicting the consequences of such an intertwined system. The nature of connections between financial institutions results from both the asset and the liability side of their balance sheet (Allen and Babus (2008); Billio et al. (2012); Bisias et al. (2012)) . The structure of such connections between financial institutions can be captured by employing a network representation of financial systems.

Network analysis is advancing as a common tool for assessing dynamics within the various sections of the financial sector; it reveals that a truly systemic perspective needs to combine the focus on various sections of the financial sector with an analysis of the interlinkages among

*Correspondence to: Centre d'Économie de la Sorbonne (CES) - CNRS : UMR8174 - Université Paris I - Panthéon Sorbonne, 106-113 boulevard de l'hopital, 75013, Paris, France.
Email address: `peter.addo@univ-paris1.fr` (Peter Martey ADDO)

Preprint submitted to SYRTO Working Paper Series *October 15, 2014*

them incorporating the interaction with the real economy (Bisias et al. (2012)). Intuitively, a network structure describes a collection of nodes and the links between them. The nodes can be individuals, institutions or countries, financial assets, or even collections of such entities. The direct relation between these entities of interest are usually referred to as links. Network theories can be used to monitor and assess systemic risks, contagion, linkages and vulnerabilities in the financial system. The nature of networks permit us to picture beyond the immediate "point of impact" of a shock and enhances the analysis of spillovers likely to arise from interlinkages. Thus, network perspective would not only account for the various connections that exist within the financial sector or between financial sector and other sectors, but would also consider the quality of these links.

Recent works documenting the applications of network analysis to financial systems concentrates on issues such as financial stability and contagion. The literature mostly investigates how financial institutions (or entities) are interconnected and how different network structures react to the breakdown of a single (or an ensemble) institution in order to identify which ones are more fragile (Allen and Babus (2008); Billio et al. (2012)). To our best of knowledge, existing research focused on the network topology of asset returns of financial systems are the works of Billio et al. (2012) and Bisias et al. (2012). In this article, our aim is to identify linkages, lags of linkages and assess the strength of such connections. Our analysis exploits the concept of quantifying causal coupling strength, which identifies the existence of causal associations between two components of a multivariate time series and assesses the strength of their association by defining a meaningful coupling strength using the momentary information transfer (MIT) (Runge et al. (2012a,b)). In otherwords, our proposed model-free approach which is based on time series graphs and information theory, is able to detect and quantify causal dependencies from multivariate time series.

Our empirical findings show that there is high coupling strength of linkages between the four sectors, increasing the channels through which shocks can be transmitted throughout the financial system. The results point to important asymmetry in the causal dependencies in the insurance sector and brokers prior to the financial crisis of 2007–2009. We find that the returns of insurers have significant impact on returns of brokers, where the time delay of impact is shorter during the crisis. Over time, we find that banks were always contemporaneously linked to brokers with a higher coupling strength observed before year 2000. In addition, our results indicate that insurers did not, in many cases, directly impact hedge funds significantly. After the year 2000, we find contemporaneous link between banks and insurers, and also between brokers and hedge funds, where the former is with a higher coupling strength. The aftermath of 1998 collapse of the $5 billion hedge fund Long Term Capital Management (LTCM) could have lead to the contemporaneous link between brokers and hedge funds. In general, we find that banks were not directly connected to hedge funds. However, the two sectors were indirectly linked via a contemporaneous chain of dependencies from banks to brokers and then to hedge funds. We remark that just prior to the financial crisis of 2007–2009, the hedge funds did have a significant negative impact on banks. Interestingly, most connections were observe for the time period (1994M01–2007M12), were hedge funds had a negative impact on the financial system especially on the banks and brokers. This increase in size and causal dependencies between these sectors could serve as a significant systemic risk indicator.

The paper is organised as follows: In Section 2, an overview of time series graph using momentary information transfer measure is provided. In Section 3 & Section 4, we present a comprehensive analysis of the feasibility of this approach to simulated and real data is provided. Section 5 concludes.

2. Background

2.1. Overview of Approach

In this section, we provide an overview of a model–free approach which makes use time series graphs and information theory. The approach is based on the momentary information transfer (MIT) which builds on the fundamental concept of source entropy (as detailed in Runge et al. (2012a,b)). The MIT is a time-delayed conditional mutual information and a measure of association that is general in that it does not assume a certain model class underlying the process that generates the time series. This measure of association will be useful in understanding the interdependence in multivariate economic and financial time series analysis as the process underlying the generated signals are a priori unknown (Addo et al. (2012)). In addition, the general framework of graphical models[1] (Lauritzen (1996); Dahlhaus (2000); Eichler (2012)) makes MIT causal as it gives a non-zero value only to lagged components that are not independent conditional on the remaining process. This measure in many cases is able to exclude the misleading influence of autodependency within a process in an information-theoretic way. Runge et al. (2013) successfully used this approach in quantifying the strength and delay of climatic interactions. In order to test hypotheses on the interdependencies between processes underlying data, there is a need for statistical measures of association. We seek for a measure with the following properties:

1. General – it should not be restricted to certain types of association like linear measures.
2. Equitability – it should give similar scores to equally noisy dependencies. This property is essential for comparisions and ranking of strength of dependencies within subprocesses of the multivariate time series.
3. Causality – the measure should give non-zero value only to the dependency between lagged components of a multivariate process that are not independent conditional on the remaining process.
4. Coupling strength autonomy – given dependent components, the measure should provide a causal notion of coupling strength that is well interpretable. In otherwords, it is uniquely determined by the interaction of the two components alone and in a way autonomous of their interaction with the remaining process.
5. "practically computable" – estimation does not for instance require somewhat arbitrary truncation as in the case of other methods like Transfer entropy (TE).

These properties in the MIT approach allows to reconstruct interaction networks where not only the links are causal, but also meaningfully weighted and have the attribute of coupling delayed. To our best of knowledge, other methods such as the transfer entropy (TE) which is the information-theoretic analogue of Granger Causality do not support these properties. For instance, the TE is not uniquely determined by the interaction of the two components alone and depends on misleading effects such as autodependency and interaction with other process. This voilates the coupling strength autonomy. In addition, aside that the TE requires arbitrary truncation during estimation, this measure can lead to false interpretation since it is not lag-specific.

In this work, we outline our proposed "two-steps" approach for the determination of a coupling strength:

[1] Graphical models provide a framework to distinguish direct from indirect interactions between and within subprocesses of a multivariate process (Lauritzen (1996)).

1. This first step determines the existence or absence of a link, which also provide useful information on the causality between lagged components of the multivariate process. The graphical model is estimated as detailed in Runge et al. (2012b).
2. Estimation of ρ^{MIT} as a meaningful weight for every existing link in the graph detailed in Runge et al. (2012a).

2.2. Notations and Definitions

Given a stationary multivariate discrete-time series stochastic process $\mathfrak{X}$. Let the uni or multivariate subprocesses be $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \cdots$ and the random variables at time t as $\mathfrak{X}_t, \mathcal{A}_t, \mathcal{B}_t, \cdots$. $\mathfrak{X}_t^- = (\mathfrak{X}_{t-1}, \mathfrak{X}_{t-2}, \cdots)$ and $\mathcal{A}_t^- = (\mathcal{A}_{t-1}, \mathcal{A}_{t-2}, \cdots)$ denotes the past. For convenience $\mathfrak{X}$, $\mathfrak{X}_t$, $\mathfrak{X}_t^-$ and $\mathcal{A}_t^-$ are treated as sets of random variables, in that $\mathcal{A}_t^-$ can be considered a subset of $\mathfrak{X}_t^-$.

In the graphical model approach (Lauritzen (1996)), each node in the graph represents a single random variable i.e. a subprocess at a certain time t. We say that the nodes $\mathcal{A}_{t-\tau}$ and $\mathcal{B}_t$ are connected by a directed link "$\mathcal{A}_t \longrightarrow^{\tau} \mathcal{B}_t$" pointing forward in time if and only if $\tau > 0$ and

$$I_{\mathcal{A} \xrightarrow{link} \mathcal{B}}(\tau) \equiv I\Big(\mathcal{A}_{t-\tau}; \mathcal{B}_t \;\Big|\; \mathfrak{X}^- \backslash \{\mathcal{A}_{t-\tau}\}\Big) > 0. \tag{1}$$

Thus, $\mathcal{A}_{t-\tau}$ and $\mathcal{B}_t$ are connected if they are not independent conditionally on the past of the whole process excluding $\{\mathcal{A}_{t-\tau}\}$ (denoted by the symbol \) which implies a lag-specific causality with respect to $\mathfrak{X}$. If $\mathcal{A} \neq \mathcal{B}$ then "$\mathcal{A}_t \longrightarrow^{\tau} \mathcal{B}_t$" represents a coupling at lag τ. An autodependency at lag τ corresponds to $\mathcal{A} = \mathcal{B}$. The nodes $\mathcal{A}_t$ and $\mathcal{B}_t$ are connected by an undirected contemporaneous link "$\mathcal{A} - \mathcal{B}$" (Eichler (2012)) if and only if

$$I_{\mathcal{A} \overset{link}{-} \mathcal{B}} \equiv I\Big(\mathcal{A}_t; \mathcal{B}_t \;\Big|\; \mathfrak{X}_{t+1}^- \backslash \{\mathcal{A}_t, \mathcal{B}_t\}\Big) > 0. \tag{2}$$

Notice that the contemporaneous present $\mathfrak{X}_t \backslash \{\mathcal{A}_t, \mathcal{B}_t\}$ is included in the condition. Thus $\mathfrak{X}_t \backslash \{\mathcal{A}_t, \mathcal{B}_t\} \subset \mathfrak{X}_{t+1}^- \backslash \{\mathcal{A}_t, \mathcal{B}_t\}$. In the case of the multivariate autoregressive process, this corresponds to non-zero entries in the inverse covariance matrix of the innovation terms. The equations 1 & 2 involve infinite dimensional vectors and can not be directly computed. Using the Markov property, this issue can be circumvented by introducing the notion of *parents* and *neighbors* of subprocesses. Let

$$\mathcal{P}_{\mathcal{B}_t} \equiv \{\mathcal{Z}_{t-\tau} : \mathcal{Z} \in \mathfrak{X}, \tau > 0, \mathcal{Z}_{t-\tau} \longrightarrow \mathcal{B}_t\} \tag{3}$$

be a finite set of *parents* of $\mathcal{B}_t$ which separates $\mathcal{B}_t$ from the past of the whole process $\mathfrak{X}_t^- \backslash \mathcal{P}_{\mathcal{B}}$. The past lags of the subprocess $\mathcal{B}$ in $\mathfrak{X}$ can be part of the parents. The *neighbors* of a process $\mathcal{B}_t$ is defined as

$$\mathcal{N}_{\mathcal{B}_t} \equiv \{\mathcal{A}_t : \mathcal{A} \in \mathfrak{X}, \mathcal{A}_t - \mathcal{B}_t\} \tag{4}$$

The *parents* of all subprocesses in $\mathfrak{X}$ together with the contemporaneous links forms the time series graph. The estimation of these time series graphs is obtained by iteratively inferring *parents* is well detailed in Runge et al. (2012b). This allows for the determination of the existence or absence of a link and a causality between lagged components of $\mathfrak{X}$.

Definition 1. *For two subprocesses $\mathcal{A}$, $\mathcal{B}$ of a stationary multivariate discrete time process $\mathfrak{X}$ with parents $\mathcal{P}_{\mathcal{A}_t}$ and $\mathcal{P}_{\mathcal{B}_t}$ in the associated time series graph and $\tau > 0$, the general information theoretic measure MIT (described in Runge et al. (2012a)) between $\mathcal{A}_{t-\tau}$ and $\mathcal{B}_t$ is given by*

$$I_{\mathcal{A} \xrightarrow{MIT} \mathcal{B}}(\tau) \equiv I\Big(\mathcal{A}_{t-\tau}; \mathcal{B}_t \;\Big|\; \mathcal{P}_{\mathcal{B}_t} \backslash \{\mathcal{A}_{t-\tau}\}, \mathcal{P}_{\mathcal{A}_{t-\tau}}\Big) > 0 \tag{5}$$

and contemporaneous MIT defined by

$$I_{\mathcal{A} \overset{MIT}{-} \mathcal{B}} \equiv I\Big(\mathcal{A}_t; \mathcal{B}_t \;\Big|\; \mathcal{P}_{\mathcal{B}_t}, \mathcal{P}_{\mathcal{A}_t}, \mathcal{N}_{\mathcal{A}_t} \backslash \{\mathcal{B}_t\}, \mathcal{N}_{\mathcal{B}_t} \backslash \{\mathcal{A}_t\}, \mathcal{P}_{\mathcal{N}_{\mathcal{A}_t} \backslash \{\mathcal{B}_t\}}, \mathcal{P}_{\mathcal{N}_{\mathcal{B}_t} \backslash \{\mathcal{A}_t\}}\Big). \tag{6}$$

Definition 2. *For two subprocesses $\mathcal{A}$, $\mathcal{B}$ of a stationary multivariate discrete time process $\mathfrak{X}$ with parents $\mathcal{P}_{\mathcal{A}_t}$ and $\mathcal{P}_{\mathcal{B}_t}$ in the associated time series graph and $\tau > 0$, the partial correlation measure, denoted ρ^{MIT}, associated with equation* (5)*, for the strength of coupling mechanism between $\mathcal{A}_{t-\tau}$ and $\mathcal{B}_t$ is given by*

$$\rho_{\mathcal{A} \xrightarrow{MIT} \mathcal{B}}(\tau) \equiv \rho\Big(\mathcal{A}_{t-\tau}; \mathcal{B}_t \;\Big|\; \mathcal{P}_{\mathcal{B}_t} \backslash \{\mathcal{A}_{t-\tau}\}, \mathcal{P}_{\mathcal{A}_{t-\tau}}\Big). \tag{7}$$

The measure ρ^{MIT} quantifies how much the variability in $\mathcal{A}$ at the exact lag τ directly influences $\mathcal{B}$, irrespective of the past of $\mathcal{A}_{t-\tau}$ and $\mathcal{B}_t$. ρ^{MIT} is the cross-correlation of the residual after $\mathcal{A}_{t-\tau}$ and $\mathcal{B}_t$ have been regressed on both the parents of $\mathcal{A}_{t-\tau}$ and $\mathcal{B}_t$. In economics, the detection of causal relationships among variables are of great importance (Granger (1969); Billio et al. (2012)). These variables can be viewed as nodes of graph where the links correspond to interactions. Unlike classical statistics, interactions in the framework of information theory are viewed as transfers of information and thus our approach is model-free. The section that follows will be dedicated to the empirical illustration of the feasibility of the approach on simulated and real data.

3. Simulations experiment

In this section, we provide a simulation study on the identification of interdependences and strength in a simulated multivariate process with a priori knowledge on the network structure. In this respect, consider a simulated 1000 points of a multivariate autoregressive process (displayed in Figure (1a)) made up of four subprocesses $\{X_t, Y_t, Z_t, W_t\}'$ defined by

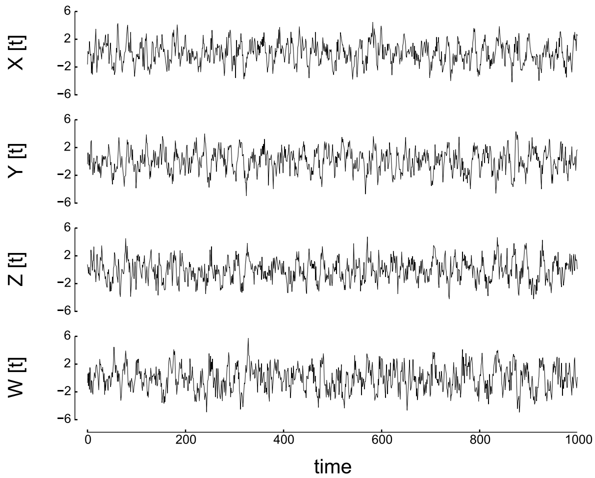
$$X_t = aX_{t-1} + cZ_{t-4} + \varepsilon_x \tag{8}$$

$$Y_t = cX_{t-1} + aY_{t-1} + \varepsilon_y \tag{9}$$

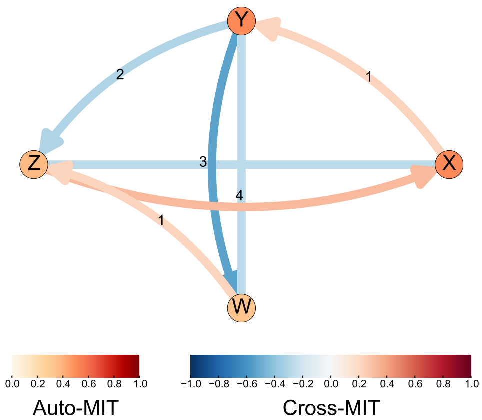
$$Z_t = dY_{t-2} + bZ_{t-1} + fW_{t-1} + \varepsilon_z \tag{10}$$

$$W_t = eY_{t-3} + gW_{t-1} + \varepsilon_w \tag{11}$$

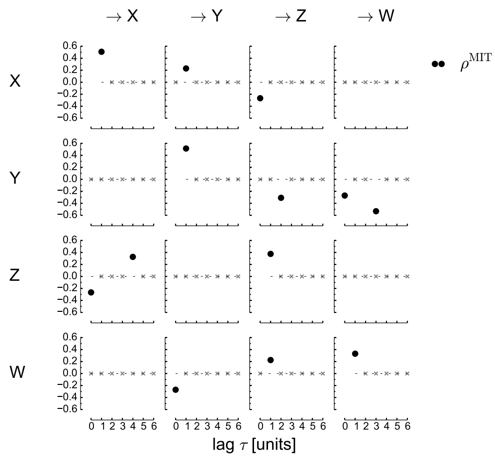
and the innovation covariance matrix given by $\Sigma_\varepsilon = \begin{pmatrix} 1 & 0 & d & 0 \\ 0 & 1 & 0 & d \\ d & 0 & 1 & 0 \\ 0 & d & d & 1 \end{pmatrix}$, where $a = 0.6, b = 0.4, c = 0.3, d = -0.3, e = -0.6, f = 0.2$, and $g = 0.4$. Notice that the lagged causal chain for this process is $X \longrightarrow^1 Y \longrightarrow^2 Z$ with feedback $Z \longrightarrow^4 X$, and $Y \longrightarrow^3 W \longrightarrow^1 Z$, plus contemporaneous links $X - Z$ and $Y - W$. The results of the time series graph via MIT approach is provided in Figures (1b & 1c). We obtained that the causal dependencies between the subprocesses are well identified and coincide with our a priori knowledge on the network structure. We remark that not only are the causal lags of dependencies between subprocesses identified, the magnitude and sign of the MIT coupling strength of linkages is also obtained (see Figure (1c)). In the section that follows, we will provide an application of this approach to analysing systemic risk.



(a) Plot of four subprocesses of a multivariate autoregressive process



(b) *MIT plot* for the simulated process.



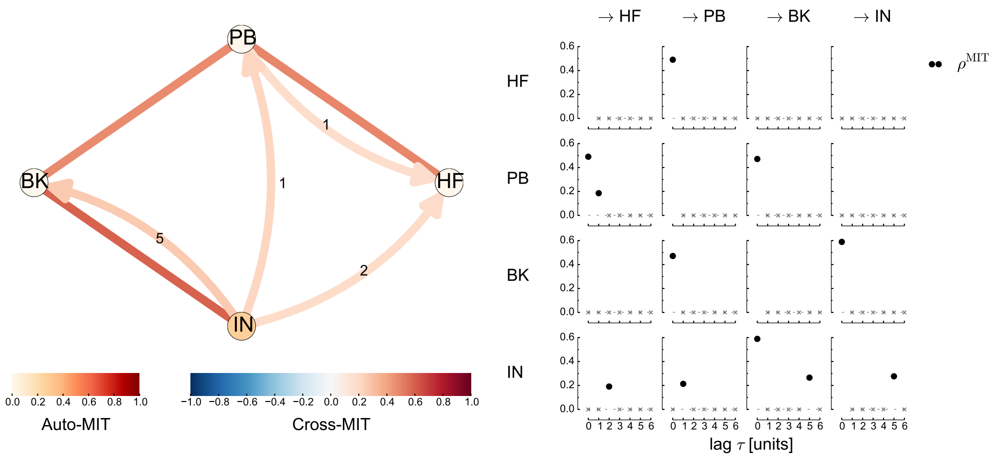
(c) The plot of significant lags for simulated process associated with the *MIT plot*.

Figure 1: The detection & quantification of causal dependencies in a simulated multivariate autoregressive process described in equations (8–11) via the MIT approach. The results indicates a lagged causal chain as $X \xrightarrow{1} Y \xrightarrow{2} Z$ with feedback $Z \xrightarrow{4} X$, and $Y \xrightarrow{3} W \xrightarrow{1} Z$, plus contemporaneous links $X - Z$ and $Y - W$. In Figure (1c), the existence of a *dot* at a particular lag τ of a subprocess ξ on the plots located on the diagonals indicates an autoregressive link of the form $\xi_{t-\tau} \longrightarrow \xi_t$, which we denote as "$\xi_t \xrightarrow{\tau} \xi_t$". Contemporaneous links can be read from the plots by observing *dots* at where $\tau = 0$. The vertical axes represent the sign and magnitude of the MIT coupling strength (ρ^{MIT}) of the linkages, which corresponds to the coefficients of the simulated autoregressive process.

4. Application to multivariate financial time series

The recent global financial crisis (2007–2009) has clearly demonstrated the need for the financial industry, regulators and policymakers to develop a better understanding of systemic risk.

In particular, measuring the financial systemic importance of financial institutions is crucial in identifying linkages and the potentially destabilizing constituents of the global financial system. In this section, we consider the dataset used in Billio et al. (2012), with special attention on identifying the linkages, lags of linkages and to assess the strength of such linkages among the four main sectors: hedge funds (HR), banks (BK), Broker/dealers (PB) and insurers (IN). These financial institutions represents the financial and insurance sectors described in more detail by Billio et al. (2012). In this respect, we make use of the monthly returns of these sectors as inputs for the MIT approach in analysing systemic risk. The time period for the data set in Jan 1994–Dec 2008. Our analysis is performed on the full sample period and also considers the following different time windows : 1994M01–2004M12, 1994M01–2005M12, 1994M01–2006M12, 1994M01–2007M12, to study possible changes in the systemic risk network structure.



(a) *MIT plot* for full Sample: Jan 1994–Dec 2008 (b) *MIT lag plot* for full Sample: Jan 1994–Dec 2008

Figure 2: The *MIT Plots* showing both the linkages among the four sectors and the strength of such connections for the full study period, Jan 1994–Dec 2008. The second representation Figure (2b) of the MIT plots provides additional information on autoregressive linkages by inspecting the diagonals. The existence of a *dot* at a particular lag τ of sector ξ on the plots located on the diagonals indicates an autoregressive link of the form $\xi_{t-\tau} \longrightarrow \xi_t$, which we denote as "$\xi_t \longrightarrow^{\tau} \xi_t$".

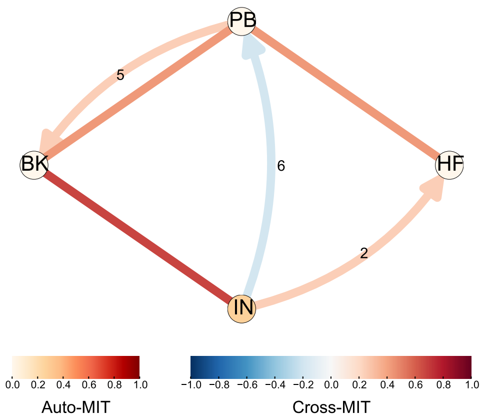
The results of the proposed MIT approach when applied to the multivariate process $\{HF_t, PB_t, BK_t, IN_t\}'$ for the full time period is shown in Figure (2). Lag-specific causal dependencies between the four sectors can be easily visualized in Figures (2a & 2b). The sign and magnitude of the coupling strength can be read from Figure (2b). The time series graph (network structure) provides an understanding of the possible channels through which financial crisis were transmitted. In Figure (2a), we detect three contemporaneous links: $BK - PB$, $PB - HF$ and $BK - IN$, where the latter has a higher coupling strength ($\rho^{MIT} \simeq 0.6$) as seen in Figure (2b). We remark that the results on the multivariate process $\{HF_t, PB_t, BK_t, IN_t\}'$ displayed in Figures (2a & 2b) can also be represented by the set of equations:

$$HF_t = 0.5PB_t + 0.2PB_{t-1} + 0.2IN_{t-2} \quad (12)$$
$$PB_t = 0.5HF_t + 0.5BK_t + 0.2IN_{t-1} \quad (13)$$
$$BK_t = 0.5PB_t + 0.6IN_t + 0.3_I N_{t-5} \quad (14)$$
$$IN_t = 0.6BK_t + 0.3IN_{t-5}. \quad (15)$$

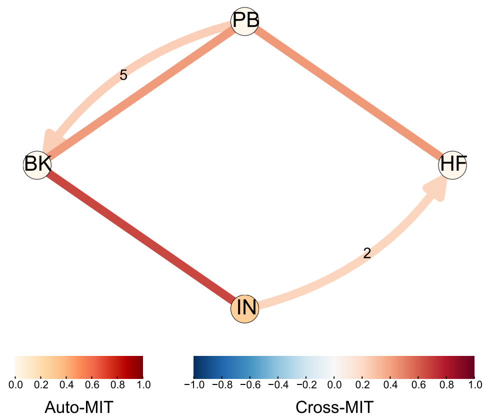
Most directed links observed in Figure (2a) originates from insurers (IN) and in this case, a failure of the insurers could cause a significant impact in the financial system. Moreover, the given the high coupling strength between banks and insurers, a failure in the banking system could significantly unstabilize the financial system. In addition, due to the high coupling strength of the contemporaneous links between the three sectors: banks (BK), Broker/dealers (PB) and insurers (IN), any large shock in one will spread quickly through the system.

We now provide results for four time periods: (1994M01–2004M12, 1994M01–2005M12, 1994M01–2006M12, 1994M01–2007M12), in Figures (3& 4). We make use of expanding window size to allow for the detection of changes in network structure across time. For all the time periods considered, we identified three contemporaneous links: $BK - PB$, $PB - HF$ and $BK - IN$ with the latter link having a higher coupling strength $\rho^{MIT} \simeq 0.65$ as shown in Figure (4). Moreover, there exists a causal autodependency in insurers at lag $\tau = 5$ ($IN_t \longrightarrow^5 IN_t$) and a direct link $IN_t \longrightarrow^2 HF_t$ with $|\rho^{MIT}| \simeq 0.25$. We obtain that hedge funds were only linked with banks ($HF_t \longrightarrow^2 BK_t$) prior (1994M01–2006M12,–2007M12) to and during the financial crisis of 2007–2009, which was not evident at the end of the full sample period. Notice that right before and during the financial crisis of 2007–2009 (1994M01–2007M12), the interlinkages between the sectors increased. One of the lagged causal chain for this sample period is given by $HF_t \longrightarrow^2 PB_t \longrightarrow^5 BK_t$, $HF_t \longrightarrow^2 BK_t$, and $IN_t \longrightarrow^6 PB_t$. We find that the lag of the direct link between insurers and brokers observed for time period (1994M01–2006M12,–2007M12) changed from $\tau = 6$ to $\tau = 1$ at the end of full sample period. This means that it did take less time for shocks in the insurance sector to be transmitted to brokers during the crisis. We also find that time series graph was unchanged for the time period (1994M01–2004M12) and (1994M01–2005M12) as shown in Figures (3a& 3b) except for the exclusion of $IN_t \longrightarrow^6 PB_t$ in the latter.

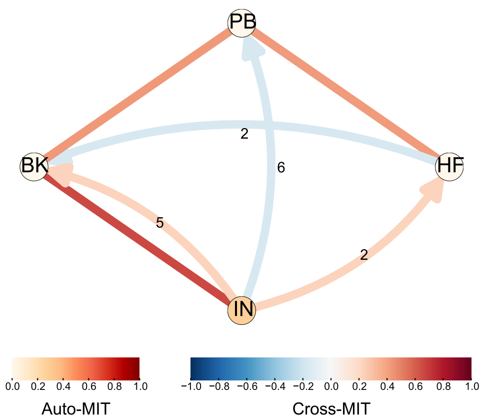
Finally to account for the possibility of nonlinearity in the form of breaks or rare events in the multivariate signal, we make use of the unthresholded recurrence plot, which is well documented in literature (Iwanski and Bradley (1998); Marwan et al. (2007); Addo et al. (2013)). We identify breaks at the dates: 1998M08, 2000M03 and 2008M11, as shown on Figure (5). Based on these dates, we define three time windows: 1994M01–1998M08, 1994M01–2000M03, and 1998M08–2008M11, for the MIT analysis. The results of the time series graphs for each time period is provided in Figure (6) and Figure (7). For the time period (1994M01–1998M08 and 1994M01–2000M03) , we obtain the same network structure and coupling strengths. The time series graph for this period is composed of a high coupling strength of a contemporaneous link between between brokers and banks ($\rho^{MIT} \simeq 0.75$), and $IN_t \longrightarrow^6 HF_t$ ($\rho^{MIT} = -0.4$). For the time period (1998M08–2008M11), the coupling strength for the contemporaneous link $BK - PB$ reduced from $\rho^{MIT} \simeq 0.75$ to 0.5 (see Figure (7c)). We also find that the causal autodependency in insurers at lag $\tau = 5$ ($IN_t \longrightarrow^5 IN_t$) and two other contemporaneous links: $PB - HF$ and $BK - IN$ identified (see Figures (3)) in the previous time windows, emerged after year 2000 (see Figures (6b& 6c)). These contemporaneous links could be due to the early 2000's recession (i.e 2001 to 2003: the collapse of the Dot Com Bubble and September 11th attacks).



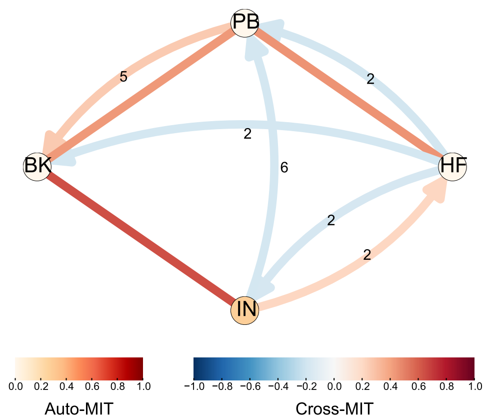
(a) *MIT plot* for 1994M01–2004M12



(b) *MIT plot* for 1994M01–2005M12



(c) *MIT plot* for 1994M01–2006M12

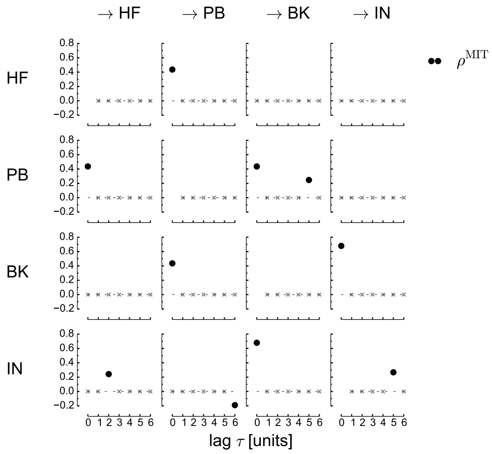


(d) *MIT plot* for 1994M01–2007M12

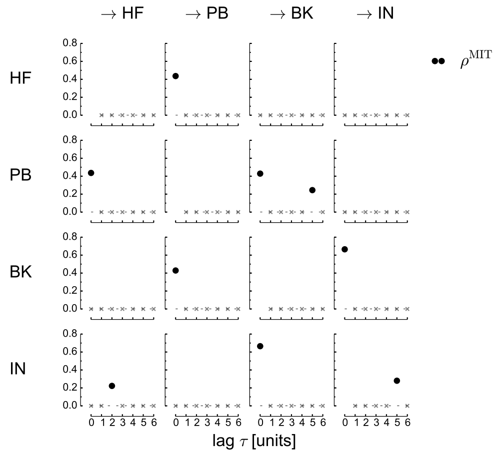
Figure 3: The *MIT Plots* displays how the four sectors are linked and the strength of the linkages. It shows autoregressive links, contemporaneous links and directed (i.e across sectors) links among sectors. The numbers at some ends of directed arrows (links) corresponds to the significant lag of the linkage. Thus, a directed link “$\mathcal{A} \xrightarrow{\tau} \mathcal{B}$” means that $\mathcal{B}$ is influenced by $\mathcal{A}$ at lag(s) τ. Contemporaneous links between two sectors: $\mathcal{A}$ and $\mathcal{B}$, is denoted “$\mathcal{A} - \mathcal{B}$”.

5. Concluding Remarks

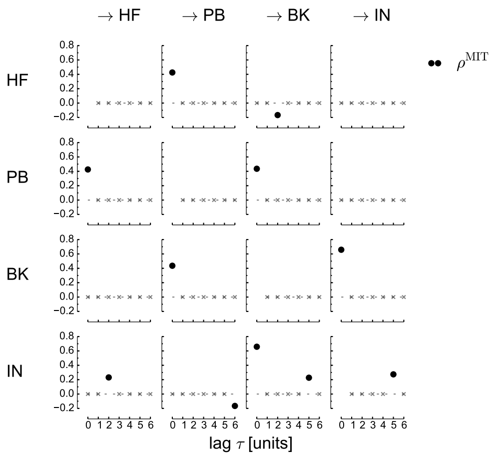
The global financial crisis of 2007–2009 has created a renewed interest to develop a better understanding of systemic risk. As documented in literature, the financial system has become considerably more complex and interconnected over the past two decades. In this work, we propose a novel information theoretic approach to understanding systemic risk. We show that this model–free approach identifies linkages, transmission channels, lags of linkages and assess the strengths of such connections. Unlike existing methodologies with assumptions on stationarity of time series, model structure and parameter space, our approach only requires that the



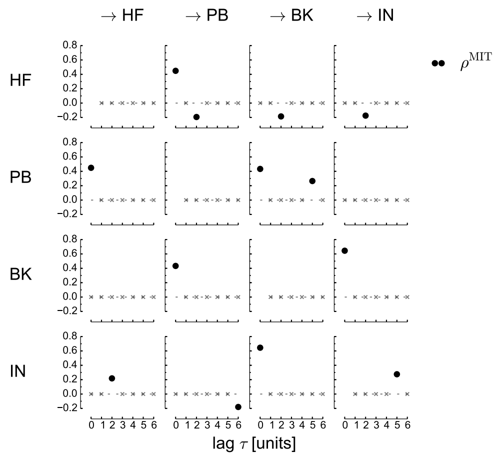
(a) *MIT lag plot* for 1994M01–2004M12



(b) *MIT lag plot* for 1994M01–2005M12



(c) *MIT lag plot* for 1994M01–2006M12



(d) *MIT lag plot* for 1994M01–2007M12

Figure 4: This is an alternative representation of the *MIT Plots* in Figure3. A directed link "$\mathcal{A} \longrightarrow^{\tau} \mathcal{B}$" means that $\mathcal{B}$ is influenced by $\mathcal{A}$ at lag(s) τ. The existence of a *dot* at a particular lag τ of sector ξ on the plots located on the diagonals indicates an autoregressive link of the form $\xi_{t-\tau} \longrightarrow \xi_t$, which we denote as "$\xi_t \longrightarrow^{\tau} \xi_t$". Contemporaneous links can be read from the plots by observing *dots* at where $\tau = 0$. The vertical axes represent the MIT coupling strength of the linkages.

multivariate time series be stationary. Our approach excludes the misleading influence of autodependency within a process by assigning a non-zero value only to lagged components that are not independent conditional on the remaining process. Using monthly returns data for insurers, brokers/dealers, hedge fund indexes and portfolios of publicly traded banks, we detect and quantify causal dependencies in these financial institutions to provide an understanding of systemic risk.

Our results point to important asymmetry in the causal dependencies in the insurance sector and brokers prior to the financial crisis of 2007–2009. We find that the returns of insurers have

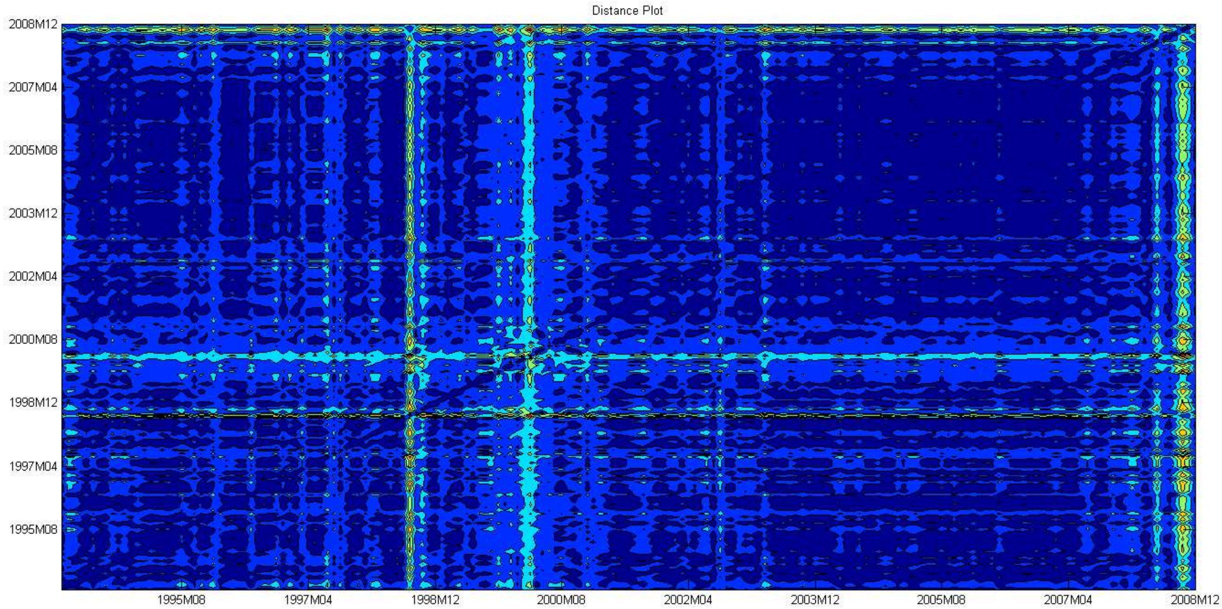
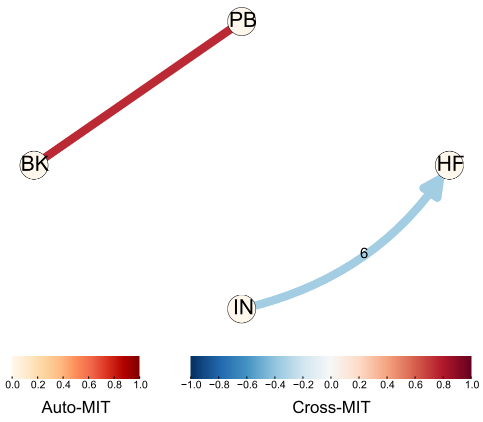
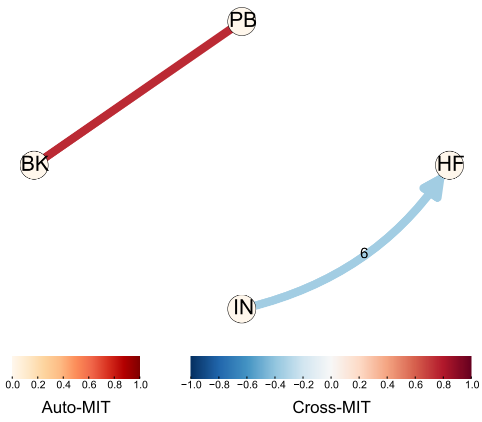


Figure 5: A contour plot (heat map) of the unthresholded recurrence matrix, which is also referred to as the distance matrix in Addo et al. (2014). Considering the entire sample, we identify breaks at the dates: 1998M08, 2000M03 and 2008M11. We define time windows: 1994M01–1998M08, 1994M01–2000M03, 1998M08–2008M11, based on these dates for the MIT analysis.

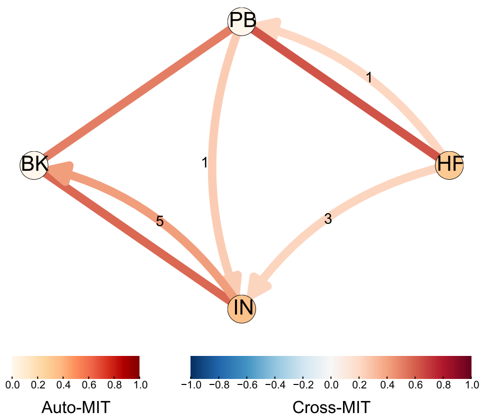
significant impact on returns of brokers, where the time delay of impact is shorter during the crisis (see Figures (3c & 3d) and Figure (2a)). For all time periods considered, we find that banks were always contemporaneously linked to brokers with a higher coupling strength observed before year 2000. In addition, our results indicate that insurers did not, in many cases, directly impact hedge funds significantly. After the year 2000 (see Figure (6b)), we find contemporaneous link between banks and insurers, and also between brokers and hedge funds, where the former is with a higher coupling strength. The aftermath of 1998 collapse of the $5 billion hedge fund Long Term Capital Management (LTCM) could have lead to the contemporaneous link between brokers and hedge funds. In general, we find that banks were not directly connected to hedge funds. However, the two sectors were indirectly linked via a contemporaneous chain of dependencies from banks to hedge funds: $BK-PB-HF$, as seen Figure (3). We remark that just prior to the financial crisis of 2007–2009, the hedge funds did have a significant negative impact on banks (see Figures (4c & 4d)). Most connections were observe for the time period (1994M01–2007M12), shown in Figure (3d), were hedge funds had a negative impact on the financial system especially on the banks and brokers. This increase in size and causal dependencies between these sectors could serve as a significant systemic risk indicator. Our results have showed that there is high coupling strength of linkages between the four sectors, increasing the channels through which shocks can be transmitted throughout the financial system.



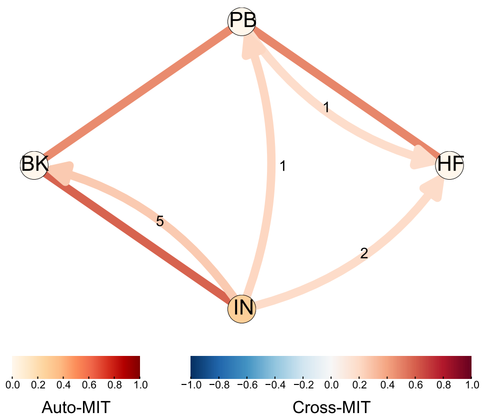
(a) *MIT plot* for 1994M01–1998M08



(b) *MIT plot* for 1994M01–2000M03



(c) *MIT plot* for 1998M08–2008M11

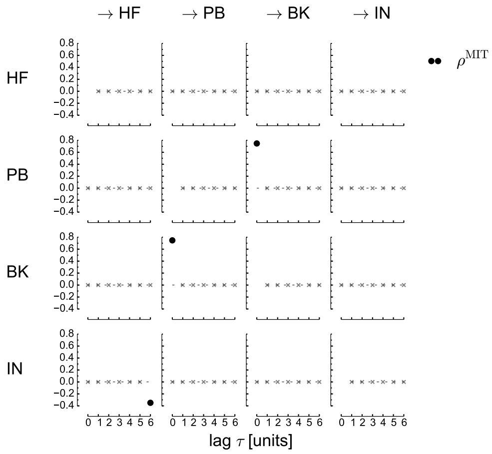


(d) *MIT plot* for full Sample: Jan 1994–Dec 2008

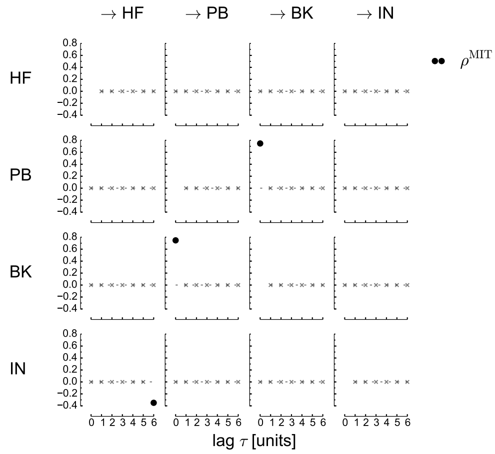
Figure 6: A directed link "$\mathcal{A} \xrightarrow{\tau} \mathcal{B}$" means that $\mathcal{B}$ is influenced by $\mathcal{A}$ at lag(s) τ. The existence of a *dot* at a particular lag τ of sector ξ on the plots located on the diagonals indicates an autoregressive link of the form $\xi_{t-\tau} \longrightarrow \xi_t$, which we denote as "$\xi_t \xrightarrow{\tau} \xi_t$". Contemporaneous links can be read from the plots by observing *dots* at where $\tau = 0$. The vertical axes represent the MIT coupling strength of the linkages. We define time windows: 1994M01–1998M08, 1994M01–2000M03, 1998M08–2008M11, based on the distance matrix plot in Figure (5).

Acknowledgement

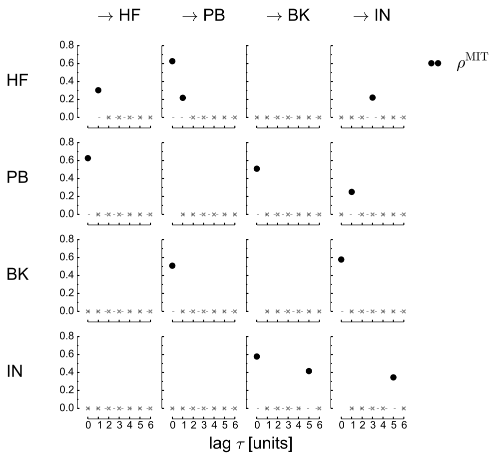
This project has received funding from the European Union's Seventh Framework Programme (FP7-SSH/20072013) for research, technological development and demonstration under grant agreement n^o320270 (SYRTO). This documents reflects only the author's view. The European Union is not liable for any use that may be made of the information contained therein.



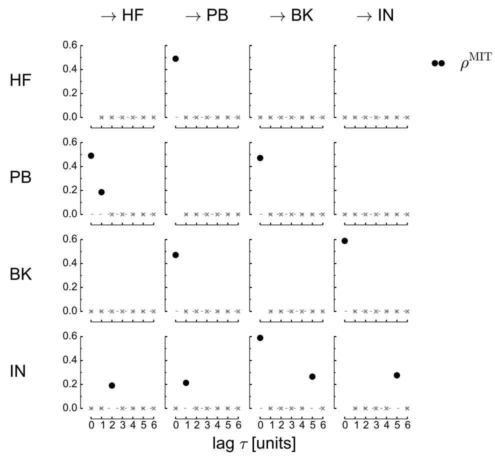
(a) *MIT lag plot* for 1994M01–1998M08



(b) *MIT lag plot* for 1994M01–2000M03



(c) *MIT lag plot* for 1998M08–2008M11



(d) *MIT lag plot* for full Sample: Jan 1994–Dec 2008

Figure 7: The lag plot indicating significant causal dependencies and coupling strength between subprocesses for the time windows: 1994M01–1998M08, 1994M01–2000M03, 1998M08–2008M11. In otherwords, this is an alternative representation of Figure (6) with information on the magnitude and sign of the coupling strength.

References

Addo, P. M., Billio, M., Guégan, D., 2012. Understanding exchange rate dynamics. In A. Colubi, K. Fokianos, & E. J. Kontoghiorghes (Eds.), Proceedings of the 20th International Conference on Computational Statistics, 1–14.

Addo, P. M., Billio, M., Guégan, D., 2013. Nonlinear dynamics and recurrence plots for detecting financial crisis. The North American Journal of Economics and Finance 26, 416–435.

Addo, P. M., Billio, M., Guégan, D., 2014. Turning point chronology for the euro area: A distance plot approach. OECD Journal: Journal of Business Cycle Measurement & Analysis, 1–14.

Allen, F., Babus, A., 2008. Networks in finance. Wharton Financial Institutions Center Working Paper (08–07).

Billio, M., Getmansky, M., Lo, A. W., Pelizzon, L., 2012. Econometric measures of connectedness and systemic risk in the finance and insurance sectors. Journal of Financial Economics 104, 535–559.

Bisias, D., Flood, M., Lo, A. W., Valavanis, S., 2012. A survey of systemic risk analytics. Annual Review of Financial Economics 4:, 255–296.

Dahlhaus, R., 2000. Graphical interaction models for multivariate time series. Metrika 51 (2), 157–172.

Eichler, M., 2012. Graphical modelling of multivariate time series. Probability Theory and Related Fields 1, 233.

Granger, C., 1969. Investigating causal relations by econometric models and cross-spectral methods. Econometrica: Journal of the Econometric Society 37 (3), 424–438.

Iwanski, J. S., Bradley, E., 1998. Recurrence plots of experimental data: To embed or not to embed? Chaos 8 (4), 861–871.

Lauritzen, S. L., 1996. Graphical models. Vol. 16. Oxford University Press.

Marwan, N., Romano, M. C., Thiel, M., Kurths, J., 2007. Recurrence plots for the analysis of complex systems. Physics Reports 438 (5–6), 237–329.

Runge, J., Heitzig, J., Marwan, N., Kurths, J., 2012a. Quantifying causal coupling strength: A lag-specific measure for multivariate time series related to transfer entropy. Physical Review E 86, 061121.

Runge, J., Heitzig, J., Petoukhov, V., Kurths, J., 2012b. Escaping the curse of dimensionality in estimating multivariate transfer entropy. Physical Review Letters 108, 258701.

Runge, J., Petoukhov, V., Kurths, J., 2013. Quantifying the strength and delay of climatic interactions: the ambiguities of cross correlation and a novel measure based on graphical models. Journal of Climate.