

Addressing Constraint Robustness to Torque Errors in Task-Space Inverse Dynamics

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Abstract—This appendix reports the analytical expressions of the gradients and the Hessians of the cost functions resulting from the robust TSID formulations proposed in the paper. This expressions can speed up the resolution of these optimization problems by means of a Newton’s method.

APPENDIX

A. Individual Inequalities

We report hereafter the expressions of the gradient and the Hessian of (13), that is R_{ind} :

$$\nabla R_{ind}(\tau) = - \sum_{i=1}^m G_i^\top \frac{f_{G_i}(G_i\tau + g_i)}{F_{G_i}(G_i\tau + g_i)},$$

where $f_{G_i}(x)$ is the pdf of $e_{G_i} = G_i e$. In case we want to use a Newton’s method we also need the Hessian of R_{ind} , which is:

$$\nabla^2 R_{ind}(\tau) = - \sum_{i=1}^m G_i^\top G_i f_{G_i}(\cdot) \left(\frac{G_i\tau + g_i}{F_{G_i}(\cdot)\sigma_i^2} + \frac{f_{G_i}(\cdot)}{F_{G_i}(\cdot)^2} \right),$$

where we omitted the argument of f_{G_i} and F_{G_i} (which is always $G_i\tau + g_i$) for readability.

B. Hyper-Rectangle

We report hereafter the expressions of the gradient and the Hessian of the cost function of problem (16). The gradient is:

$$\begin{aligned} \frac{\partial c}{\partial \tau} &= D^\top D\tau - D^\top d \\ \frac{\partial c}{\partial s_i} &= -w \frac{f_i(s_i)}{\beta_i} \quad i = 1 \dots n \\ \frac{\partial c}{\partial s_{i+n}} &= w \frac{f_i(s_{i+n})}{\beta_i} \quad i = 1 \dots n, \end{aligned}$$

where f_i is the pdf of e_i and $\beta_i = F_i(s_i) - F_i(s_{i+n})$. The Hessian is a sparse matrix and its nonzero entries are:

$$\begin{aligned} \frac{\partial^2 c}{\partial \tau^2} &= D^\top D \\ \frac{\partial^2 c}{\partial s_i^2} &= w f_i(s_i) \frac{s_i \sigma_i^{-2} \beta_i + f_i(s_i)}{\beta_i^2} \\ \frac{\partial^2 c}{\partial s_{i+n}^2} &= -w f_i(s_{i+n}) \frac{s_{i+n} \sigma_i^{-2} \beta_i - f_i(s_{i+n})}{\beta_i^2} \\ \frac{\partial^2 c}{\partial s_i \partial s_{i+n}} &= -w \frac{f_i(s_i) f_i(s_{i+n})}{\beta_i^2} \end{aligned}$$

Note that we could also exploit the sparsity of the Hessian to speed up its decomposition.