

# Addressing Constraint Robustness in Task-Space Inverse Dynamics

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**Abstract**—This appendix reports the analytical expressions of the gradients and the Hessians of the cost functions resulting from the robust TSID formulations proposed in the paper. This expressions can speed up the resolution of these optimization problems by means of a Newton’s method.

## APPENDIX

### A. Individual Inequalities

We report hereafter the expressions of the gradient and the Hessian of (15), that is  $R_{ind}$ :

$$\nabla R_{ind}(\tau) = - \sum_{i=1}^m G_i^\top \frac{f_{G_i}(G_i\tau + g_i)}{F_{G_i}(G_i\tau + g_i)},$$

where  $f_{G_i}(x)$  is the pdf of  $e_{G_i} = G_i e$ . In case we want to use a Newton’s method we also need the Hessian of  $R_{ind}$ , which is:

$$\nabla^2 R_{ind}(\tau) = - \sum_{i=1}^m G_i^\top G_i f_{G_i}(\cdot) \left( \frac{G_i\tau + g_i}{F_{G_i}(\cdot)\sigma_i^2} + \frac{f_{G_i}(\cdot)}{F_{G_i}(\cdot)^2} \right),$$

where we omitted the argument of  $f_{G_i}$  and  $F_{G_i}$  (which is always  $G_i\tau + g_i$ ) for readability.

### B. Hyper-Rectangle

We report hereafter the expressions of the gradient and the Hessian of the cost function of problem (19). The gradient is:

$$\begin{aligned} \frac{\partial c}{\partial \tau} &= D^\top D\tau - D^\top d \\ \frac{\partial c}{\partial s_i} &= -w \frac{f_i(s_i)}{\beta_i} \quad i = 1 \dots n \\ \frac{\partial c}{\partial s_{i+n}} &= w \frac{f_i(s_{i+n})}{\beta_i} \quad i = 1 \dots n, \end{aligned}$$

where  $f_i$  is the pdf of  $e_i$  and  $\beta_i = F_i(s_i) - F_i(s_{i+n})$ . The Hessian is a sparse matrix and its non-zero entries are:

$$\begin{aligned} \frac{\partial^2 c}{\partial \tau^2} &= D^\top D \\ \frac{\partial^2 c}{\partial s_i^2} &= w f_i(s_i) \frac{s_i \sigma_i^{-2} \beta_i + f_i(s_i)}{\beta_i^2} \\ \frac{\partial^2 c}{\partial s_{i+n}^2} &= -w f_i(s_{i+n}) \frac{s_{i+n} \sigma_i^{-2} \beta_i - f_i(s_{i+n})}{\beta_i^2} \\ \frac{\partial^2 c}{\partial s_i \partial s_{i+n}} &= -w \frac{f_i(s_i) f_i(s_{i+n})}{\beta_i^2} \end{aligned}$$

Note that we could also exploit the sparsity of the Hessian to speed up its decomposition.