DARK MATTER, A NEW PROOF OF THE PREDICTIVE POWER OF GENERAL RELATIVITY

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Without observational or theoretical modifications, Newtonian and general relativity seem to be unable to explain gravitational behavior of large structure of the universe. The assumption of dark matter solves this problem without modifying theories. But it implies that most of the matter in the universe must be unobserved matter. Another solution is to modify gravitation laws. In this article, we study a third way that does not modify gravitation of general relativity and not modify the matter's distribution, by using gravitomagnetism in a new physical context. Compare with Newtonian gravitation, it leads to add a new component without changing the gravity field. As already known, we retrieve that this new component is generally small enough to be undetectable. But we will see that the galaxies clusters can generate a significant component and embed large structure of universe. We show that the magnitude of this embedding component is once again small enough to be in agreement with current experimental results, undetectable at the scale of our solar system, but detectable at the scale of the galaxies and explain dark matter. Mainly, it explains six unexplained phenomena, the rotation speed of galaxies, the rotation speed of dwarf satellite galaxies, the movement in a plane of dwarf satellite galaxies, the decreasing quantity of dark matter with the distance to the center of galaxies' cluster, the expected quantity of dark matter inside galaxies and the expected experimental values of parameters Ω_{dm} of dark matter measured in CMB. This solution implies consequences on the dwarf galaxies (distribution in planes) that just have been observed and differentiate it from dark matter solution. It could explain some others facts (galaxies with two portions of their disk that rotate in opposite directions, galaxies with a truly declining rotation curve, narrowness of galaxy's jets, precocity of organization of galaxies…).

Keywords: gravitation, gravitic field, dark matter, galaxy.

1. Overview

1.1. Current solutions

Why is there a dark matter assumption? Starting with the observed matter, the gravitation theories (Newtonian and general relativity) seem to fail to explain several observations. The discrepancies between the theory and the observations appear only for large astrophysical structure and with orders of magnitude that let no doubt that something is missing in our knowledge of gravitation interaction. There are mainly three situations that make necessary this assumption. At the scale of the galaxies, the rotation's speeds of the ends of the galaxies can be explained if we suppose the presence of more matter than the observed one. This invisible matter should represent more than 90% (RUBIN et al., 1980) of the total matter (sum of the observed and invisible matters). At the scale of the clusters of galaxies, the gravitational lensing observations can be explained with the presence of much more invisible matter (ZWICKY, 1937; TAYLOR et al., 1998; WU et al., 1998), at least 10 times the observed matter. At the scale of the Universe, cosmological equations can very well explain the dynamics of our Universe (for example the inhomogeneities of the microwave background) with the presence of more matter than the observed one. This invisible matter should, in this third situation, represent about 5 times the observed matter (PLANCK Collaboration, 2014).
How can we explain the origin of this dark matter? In fact, a more objective question should be how we can explain these discrepancies between theories and observations. There are two ways to solve this problem, supposing that what we observe is incomplete or supposing that what we idealize is incomplete. The first one is to suppose the existence of an invisible matter (just like we have done previously to quantify the discrepancies). It is the famous dark matter assumption that is the most widely accepted assumption. This assumption has the advantage of keeping unchanged the gravitation theories (NEGI, 2004). The inconvenient is that until now, no dark matter has been observed, WIMPS (ANGLOHER et al., 2014; DAVIS, 2014), neutralinos (AMS Collaboration, 2014), axions (HARRIS & CHADWICK, 2014). More than this the dynamics with the dark matter assumption leads to several discrepancies with observations, on the number of dwarf galaxies (MATEO, 1998; MOORE et al., 1999; KLYPIN et al., 1999) and on the distribution of dark matter (DE BLOK, 2009; HUI, 2001).
The second one is to modify the gravitation theories. The advantage of this approach is to keep unchanged the quantity of observed matter. The inconvenient is that it modifies our current theories that are very well verified at our scale (planet, star, solar system…). One can gather these modified theories in two categories, one concerning Newtonian idealization and the other concerning general relativity. Briefly, in the Newtonian frame, one found MOND (MILGROM, 1983) (MOdified Newtonian Dynamics) that essentially modified the inertial law and another approach that modifies gravitational law (DISNEY, 1984; WRIGHT & DISNEY, 1990). In the general relativity frame, one found some studies that don't modify Einstein's equations but that looking for specific metrics (LETELIER, 2006; COOPERSTOCK & TIEU, 2005; CARRICK & COOPERSTOCK, 2010). And one found some other studies that modified general relativity. Mainly one has a scalar-tensor-vector gravity (MOFFAT, 2006) (MOG), a 5D general relativity taking into account Hubble expansion (CARMELI, 1998; HARTNETT, 2006), a quantum general relativity (RODRIGUES et al., 2011), two generalizations of MOND, TeVeS (BEKENSTEIN, 2004; MCKEE, 2008) and BSTV (SANDERS, 2005), a phenomenological covariant approach (EXIRIFARD, 2010)…

1.2. Solution studied in this paper

In our study, we will try a third way of explanation of these discrepancies. This explanation doesn't modify the quantities of observed matter and doesn't modify general relativity. We are going to use the native metric of linearized general relativity (also called gravitoelectromagnetism) in agreement with the expected domain of validity of our study ($\varphi \ll c^2$ and $v \ll c$). It doesn't modify the gravity field of Newtonian approximation but defines a better approximation by adding a component (called gravitic field, similar to magnetic field of Maxwell idealization) correcting some imperfections of the Newtonian idealization (in particular the infinite propagation speed of gravitation). In a first paragraph, we will recall the linearization of general relativity and give some orders of magnitude of this component, the gravitic field. In a second paragraph, we will focus first on the problem of the rotation of the extremities of galaxies on sixteen galaxies. We will retrieve a published result (LETELIER, 2006), showing that own gravitic field of a galaxy cannot explain dark matter of galaxies. But one will see that clusters' gravitic field could replace dark matter assumption. Before talking about its origin, one will see the orders of magnitude of this component, expected to obtain the "flat" rotation speed of galaxies. Its magnitude will be small enough to be in agreement with the fact that it is undetectable and that the galaxies are randomly oriented. In a third paragraph, we will talk about the possible origins of this component. One will see that the value of the gravitic field explaining dark matter for galaxies could likely come from the cluster of galaxies (in agreement with a recent observation showing that dark matter quantity decreases with the distance to the center of galaxies' cluster). In the next paragraphs, one will use these values of gravitic field on several situations. It will permit to retrieve many observations (some of them not explained). These values (if interpreted in the hypothesis of dark matter) will give the expected quantity of dark matter inside the galaxies. These same values will explain the speed of dwarf satellite galaxies. Its theoretical expression will explain their planar movement (recently observed and unexplained). We will also show that we can obtain a good order of magnitude for the quantity of dark matter Ω_{dm} (obtained from the inhomogeneities of the microwave background, CMB). One will still make some predictions and in particular an original and necessary consequence on the dwarf galaxies that can differentiate our solution compared to the dark matter assumption. We will end with a comparison between our solution and the dark matter assumption. Some recent articles (CLOWE et al., 2006; HARVEY et al., 2015) show that if dark matter is actually matter, it cannot be ordinary matter. We will see in the end that these studies do not invalidate the solution proposed in this paper. Instead they could even allow testing our solution.

The goal of this study is to open a new way of investigation to explain dark matter. This work has not the pretention to produce a definitive proof but it nevertheless provides a body of evidence that confirm the relevance of the proposed solution and leads to a significant prediction that differentiate it from the other explanations.

To end this introduction, one can insist on the fact that this solution is naturally compliant with general relativity because it is founded on a component coming from general relativity (gravitic field), traditionally neglected, on which several papers have been published and some experimental tests have been realized. The only assumption that will be made in this paper is that there are some large astrophysical structures that can generate a significant value of gravitic field. And we will see that the galaxies' cluster can generate it.

2. Gravitation in linearized general relativity

Our study will focus on the equations of general relativity in weak field. These equations are obtained from the linearization of general relativity (also called gravitoelectromagnetism). They are very close to the modeling of electromagnetism. Let's recall the equations of linearized general relativity.

2.1. Theory

From general relativity, one deduces the linearized general relativity in the approximation of a quasi-flat Minkowski space ($g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}$; $|h^{\mu\nu}| \ll 1$). With following Lorentz gauge, it gives the following field equations (HOBSON et al., 2009) (with $\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta$):

$$\partial_\mu \bar{h}^{\mu\nu} = 0 \;\; ; \;\; \Box \bar{h}^{\mu\nu} = -2\frac{8\pi G}{c^4}T^{\mu\nu} \quad (I)$$

With:

$$\bar{h}^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}h \;;\; h \equiv h^\sigma_\sigma \;\;;\;\; h^\mu_\nu = \eta^{\mu\sigma}h_{\sigma\nu} \;;\; \bar{h} = -h \quad (II)$$

The general solution of these equations is:

$$\bar{h}^{\mu\nu}(ct,\vec{x}) = -\frac{4G}{c^4}\int \frac{T^{\mu\nu}(ct - |\vec{x}-\vec{y}|, \vec{y})}{|\vec{x}-\vec{y}|}d^3\vec{y}$$

In the approximation of a source with low speed, one has:

$$T^{00} = \rho c^2 \;\;;\;\; T^{0i} = c\rho u^i \;\;;\;\; T^{ij} = \rho u^i u^j$$

And for a stationary solution, one has:

$$\bar{h}^{\mu\nu}(\vec{x}) = -\frac{4G}{c^4}\int \frac{T^{\mu\nu}(\vec{y})}{|\vec{x}-\vec{y}|}d^3\vec{y}$$

At this step, by proximity with electromagnetism, one traditionally defines a scalar potential φ and a vector potential H^i. There are in the literature several definitions (MASHHOON, 2008) for the vector potential H^i. In our study, we are going to define:

$$\bar{h}^{00} = \frac{4\varphi}{c^2} \;\;;\;\; \bar{h}^{0i} = \frac{4H^i}{c} \;\;;\;\; \bar{h}^{ij} = 0$$

With gravitational scalar potential φ and gravitational vector potential H^i:

$$\varphi(\vec{x}) \equiv -G\int \frac{\rho(\vec{y})}{|\vec{x}-\vec{y}|}d^3\vec{y}$$

$$H^i(\vec{x}) \equiv -\frac{G}{c^2}\int \frac{\rho(\vec{y})u^i(\vec{y})}{|\vec{x}-\vec{y}|}d^3\vec{y} = -K^{-1}\int \frac{\rho(\vec{y})u^i(\vec{y})}{|\vec{x}-\vec{y}|}d^3\vec{y}$$

With K a new constant defined by:

$$GK = c^2$$

This definition gives $K^{-1} \sim 7.4 \times 10^{-28}$ very small compare to G.

The field equations (I) can be then written (Poisson equations):

$$\Delta\varphi = 4\pi G\rho \;\;;\;\; \Delta H^i = \frac{4\pi G}{c^2}\rho u^i = 4\pi K^{-1}\rho u^i \quad (III)$$

With the following definitions of $\vec{g}$ (gravity field) and $\vec{k}$ (gravitic field), those relations can be obtained from following equations:

$$\vec{g} = -\overrightarrow{grad}\varphi \ ; \ \vec{k} = \overrightarrow{rot}\,\vec{H}$$
$$\overrightarrow{rot}\vec{g} = 0 \ ; \ div\,\vec{k} = 0 \ ;$$
$$div\,\vec{g} = -4\pi G\rho \ ; \ \overrightarrow{rot}\vec{k} = -4\pi K^{-1}\overrightarrow{\mathrm{J_p}}$$

With relations (II), one has:

$$h^{00} = h^{11} = h^{22} = h^{33} = \frac{2\varphi}{c^2} \ ; \ h^{0i} = \frac{4H^i}{c} \ ; \ h^{ij} = 0 \quad (IV)$$

The equations of geodesics in the linear approximation give:

$$\frac{d^2x^i}{dt^2} \sim -\frac{1}{2}c^2\delta^{ij}\partial_j h_{00} - c\delta^{ik}\left(\partial_k h_{0j} - \partial_j h_{0k}\right)v^j$$

It then leads to the movement equations:

$$\frac{d^2\vec{x}}{dt^2} \sim -\overrightarrow{grad}\varphi + 4\vec{v}\wedge\left(\overrightarrow{rot}\vec{H}\right) = \vec{g} + 4\vec{v}\wedge\vec{k}$$

One will need another relation for our next demonstration. In agreement with previous Poisson equations (III), we deduce that gravitic field evolves with r^{-2} ($k \propto r^{-2}$). More precisely, just like in electromagnetism, one can deduce from Poisson equation that $\vec{k} \sim \left(-\frac{1}{K}\right)m_p\left(\frac{1}{r^2}\right)\overrightarrow{v_s} \wedge \vec{u}$ but it is its dependence in r^{-2} that is pertinent for our study.

From relation (IV), one deduces the metric in a quasi flat space:

$$ds^2 = \left(1 + \frac{2\varphi}{c^2}\right)c^2dt^2 + \frac{8H_i}{c}cdtdx^i - \left(1 - \frac{2\varphi}{c^2}\right)\sum\left(dx^i\right)^2$$

In a quasi-Minkowski space, one has:

$$H_i dx^i = -\delta_{ij}H^j dx^i = -\vec{H}.\overrightarrow{dx}$$

We retrieve the known expression (HOBSON et al., 2009) with our definition of H_i:

$$ds^2 = \left(1 + \frac{2\varphi}{c^2}\right)c^2dt^2 - \frac{8\vec{H}.\overrightarrow{dx}}{c}cdt - \left(1 - \frac{2\varphi}{c^2}\right)\sum\left(dx^i\right)^2 \quad (V)$$

Remark: Of course, one retrieves all these relations starting with the parametrized post-Newtonian formalism. From (CLIFFORD M. WILL, 2014) one has:

$$g_{0i} = -\frac{1}{2}(4\gamma + 4 + \alpha_1)V_i \ ; \ V_i(\vec{x}) = \frac{G}{c^2}\int\frac{\rho(\vec{y})u_i(\vec{y})}{|\vec{x} - \vec{y}|}d^3\vec{y}$$

The gravitomagnetic field and its acceleration contribution are:

$$\overrightarrow{B_g} = \vec{\nabla}\wedge\left(g_{0i}\overrightarrow{e^i}\right) \ ; \ \overrightarrow{a_g} = \vec{v}\wedge\overrightarrow{B_g}$$

And in the case of general relativity (that is our case):

$$\gamma = 1 \ ; \ \alpha_1 = 0$$

It then gives:

$$g_{0i} = -4V_i \ ; \ \overrightarrow{B_g} = \vec{\nabla}\wedge\left(-4V_i\overrightarrow{e^i}\right)$$

And with our definition:

$$H_i = -\delta_{ij}H^j = \frac{G}{c^2}\int\frac{\rho(\vec{y})\delta_{ij}u^j(\vec{y})}{|\vec{x} - \vec{y}|}d^3\vec{y} = V_i(\vec{x})$$

One then has:

$$g_{0i} = -4H_i \ ; \ \overrightarrow{B_g} = \vec{\nabla}\wedge\left(-4H_i\overrightarrow{e^i}\right) = \vec{\nabla}\wedge\left(4\delta_{ij}H^j\overrightarrow{e^i}\right)$$
$$= 4\vec{\nabla}\wedge\vec{H}$$
$$\overrightarrow{B_g} = 4\overrightarrow{rot}\,\vec{H}$$

With the following definition of gravitic field:

$$\vec{k} = \frac{\overrightarrow{B_g}}{4}$$

One then retrieves our previous relations:

$$\vec{k} = \overrightarrow{rot}\,\vec{H} \ ; \ \overrightarrow{a_g} = \vec{v}\wedge\overrightarrow{B_g} = 4\vec{v}\wedge\vec{k}$$

A last remark: The interest of our notation is that the field equations are strictly equivalent to Maxwell idealization. Only the movement equations are different with the factor “4”. But of course, all the results of our study could be obtained in the traditional notation of gravitomagnetism with the relation $\vec{k} = \frac{\overrightarrow{B_g}}{4}$.

To summarize Newtonian gravitation is a traditional approximation of general relativity. But linearized general relativity shows that there is a better approximation, equivalent to Maxwell idealization in term of field equation, by adding a gravitic field very small compare to gravity field at our scale. And, as we are going to see it, this approximation can also be approximated by Newtonian gravitation for many situations where gravitic field can be neglected. In other words, linearized general relativity explains how, in weak field or quasi flat space, general relativity improves Newtonian gravitation by adding a component (that will become significant at the scales of clusters of galaxies as we will see it).

In this approximation (linearization), the non linear terms are naturally neglected (gravitational mass is invariant and gravitation doesn’t act on itself). This approximation is valid only for low speed of source and weak field (domain of validity of our study).

All these relations come from general relativity and it is in this theoretical frame that we will propose an explanation for dark matter.

2.2. Orders of magnitude

The theory used in this study is naturally in agreement with general relativity because it is the approximation of linearized general relativity. But it is interesting to have orders of magnitude for this new gravitic field.

2.2.1. Linearized general relativity and classical mechanics

In the classical approximation ($\|\vec{v}\| \ll c$), the linearized general relativity gives the following movement equations (m_i the inertial mass and m_p the gravitational mass):

$$m_i\frac{\overrightarrow{dv}}{dt} = m_p[\vec{g} + 4\vec{v}\wedge\vec{k}]$$

A simple calculation can give an order of magnitude to this new component of the force due to the gravitic field. On one hand, gravity field gives $\|\vec{g}\| \propto G\frac{m_p}{r^2} \sim 6.67\times10^{-11}\frac{m_p}{r^2}$, on the other hand, for a speed $\|\overrightarrow{v_s}\| \sim 1m.s^{-1} \ll c$ (speed of the source that generates the field) one has $\|\vec{k}\| \propto \|\overrightarrow{v_s}\|.K^{-1}\frac{m_p}{r^2} \sim 7.4\times10^{-28}\frac{m_p}{r^2} \sim 10^{-17}G\frac{m_p}{r^2}$. That is to say that for a test particle speed $\|\vec{v}\| \sim 1m.s^{-1} \ll c$, the gravitic force $\vec{F}_k$ compared to the gravity force $\vec{F}_g$ is about $\|\vec{F}_k\| \sim 10^{-17}\|\vec{F}_g\|$. This new term is extremely small and undetectable with the current precision on Earth.

2.2.2. Linearized general relativity and special relativity

Linearized general relativity is valid in the approximation of low speed for source only. But, there isn’t this limitation for test particle. One can then consider a test particle of high speed. In the special relativity approximation ($\|\vec{v}\| \sim c$), with $\vec{v}$ the speed of the test particle, the linearized general relativity gives the following movement equations:

$$\frac{\overrightarrow{dp}}{dt} = m_p[\vec{g} + 4\vec{v} \wedge \vec{k}]$$

It is the same equation seen previously, with here the relativistic momentum. We have seen that $\|\vec{k}\| \sim 10^{-17}\|\vec{g}\|$ in classical approach. If $v \sim c$ (speed of the test particle), one always has $\|\vec{v} \wedge \vec{k}\| \sim 10^{-9}\|\vec{g}\|$ which is always very weak compared to the force of gravity ($\|\vec{F_k}\| = 10^{-9}\|\vec{F_g}\|$). Once again, even in the domain of special relativity (high speed of test particle), the gravitic term is undetectable.

To explain dark matter, we are going to see that we need to have $\|\vec{k}\| \sim 10^{-16}$ inside the galaxies. It means that, for our galaxy, our solar system must be embedded in such a gravitic field. For a particle test of speed $v \sim c$, it gives a gravitic acceleration $\|\vec{v} \wedge \vec{k}\| \sim 10^{-8} m.s^{-2}$. Compared to gravity acceleration ($\|\vec{g}\| \sim 300\, m.s^{-2}$) near Sun, it represents an undetectable correction of about 10^{-11} $\|\vec{g}\|$ (on the deviation of light for example).

3. Gravitic field: an explanation of dark matter

One of clues which push to postulate the existence of dark matter is the speed of the ends of the galaxies, higher than what it should be. We will see that the gravitic field can explain these speeds without the dark matter assumption. For that, we will first consider an example (data from "Observatoire de Paris") to demonstrate all our principles. Next, we will apply the traditional computation (KENT, 1987) on sixteen measured curves: NGC 3198, NGC 4736, NGC 300, NGC 2403, NGC 2903, NGC 3031, NGC 5033, NGC 2841, NGC 4258, NGC 4236, NGC 5055, NGC 247, NGC 2259, NGC 7331, NGC 3109 and NGC 224.

3.1. Dark matter mystery

Some examples of curves of rotation speeds for some galaxies are given in Fig. 1.

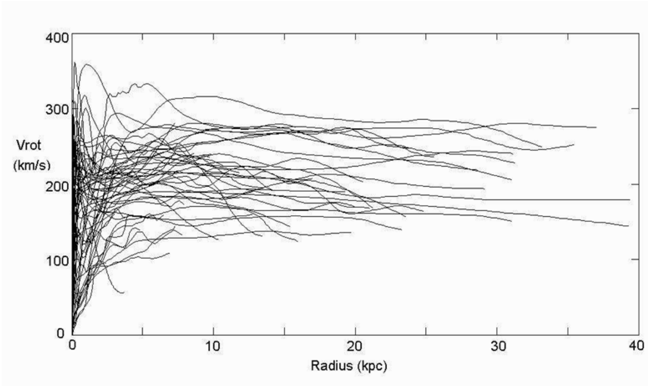


Fig. 1: Superposition of several rotation curves (Sofue et al., 1999)

One can roughly distinguish three zones:

- Zone (I) [0 ; 5 kpc]: close to the center of the galaxy, fast growth rotation speed
- Zone (II) [5 ; 10 kpc]: zone of transition which folds the curve, putting an end to the speed growth
- Zone (III) [10 kpc ; ..[: towards the outside of the galaxy, a "flat" curve with a relative constancy speed (contrary to the decreasing theoretical curve)

Gravitation, for which speed should decrease, failed to explain such curves in zone (III) without dark matter assumption. And these examples are not an exception; it is a general behavior.

3.2. Basis of our computation

The dark matter assumption is a way to increase the effect of the gravity force. To be able to have a gravitic force ($\vec{F_k} = m_p 4\vec{v} \wedge \vec{k}$) that can replace the dark matter, the gravitic field $\vec{k}$ should have a consistent orientation with the speed of the galactic matter to generate a centripetal force (just like the gravity force). This can be performed for example with the situation of the following figure:

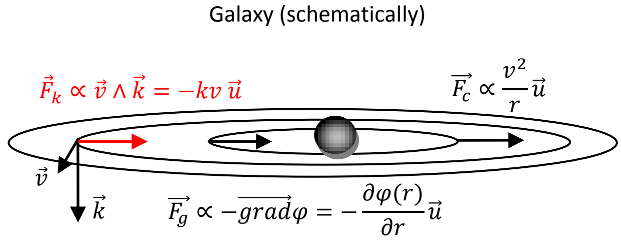


Fig. 2: Simplified representation of the equilibrium of forces in a galaxy

The simplified Fig. 2 can help us to visualize how the two components of the linearized general relativity intervene in the equilibrium of forces. The gravitic field, perpendicular to galaxy rotation plane, with the velocity of the matter generates a centripetal force increasing the Newtonian gravitation.

One can note that the orientation of this $\vec{k}$ (required to have a centripetal gravitic force) is consistent with the gravitic field that the speed of matter of the galaxy can generate. As one can see in fig. 3, if we approximate the galaxy with several coils of matter one obtains an internal gravitic field with the expected orientation.

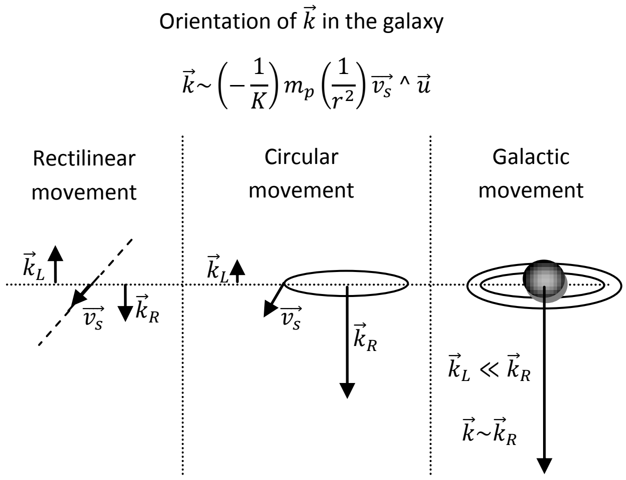


Fig. 3: Explanation of the orientation of $\vec{k}$ in the cases of rectilinear, circular and galactic movement of matter galaxy

The traditional computation of rotation speeds of galaxies consists in obtaining the force equilibrium from the three following components: the disk, the bugle and the halo of dark matter. More precisely, one has (KENT, 1986):

$$\frac{v^2(r)}{r} = \left(\frac{\partial\varphi(r)}{\partial r}\right) \; with \; \varphi = \varphi_{disk} + \varphi_{bulge} + \varphi_{halo}$$

Then total speed squared can be written as the sum of squares of each of the three speed components:

$$v^2(r) = r\left(\frac{\partial\varphi_{disk}(r)}{\partial r}\right) + r\left(\frac{\partial\varphi_{bulge}(r)}{\partial r}\right) + r\left(\frac{\partial\varphi_{halo}(r)}{\partial r}\right)$$
$$= v_{disk}^2(r) + v_{bulge}^2(r) + v_{halo}^2(r)$$

From this traditional decomposition, we obtain the traditional graph of the different contributions to the rotation curve (just like in Fig. 4).

Disk and bulge components are obtained from gravity field. They are not modified in our solution. So our goal is now to obtain only the traditional dark matter halo component from the linearized general relativity. According to this idealization, the force due to the gravitic field $\vec{k}$ takes the following form $\|\overrightarrow{F_k}\| = m_p 4\|\vec{v} \wedge \vec{k}\|$ and it corresponds to previous term $m_p \frac{\partial\varphi_{halo}(r)}{\partial r} = \|\overrightarrow{F_k}\|$. In our first step, to simplify our computation, we idealize a situation where we have the approximation $\vec{v} \perp \vec{k}$. In a second step, we will see some very important consequences due to the vector aspect ($\vec{v} \wedge \vec{k}$), it will lead to several predictions. This first situation gives the following equation:

$$\frac{v^2(r)}{r} = \left(\frac{\partial\varphi_{disk}(r)}{\partial r}\right) + \left(\frac{\partial\varphi_{bulge}(r)}{\partial r}\right) + 4k(r)v(r)$$
$$= \frac{v_{disk}^2(r)}{r} + \frac{v_{bulge}^2(r)}{r} + 4k(r)v(r)$$

Our idealization means that:

$$v_{halo}^2(r) = v^2(r) - v_{disk}^2(r) - v_{bulge}^2(r) = 4rk(r)v(r)$$

The equation of dark matter (gravitic field) is then:

$$v_{halo}(r) = 2\left(rk(r)v(r)\right)^{1/2} \qquad (VI)$$

This equation gives us the curve of rotation speeds of the galaxies as we wanted. The problem is that we don't know this gravitic field $\vec{k}$, but we know the curves of speeds that one wishes to have. We will thus reverse the problem and will look at if it is possible to obtain a gravitic field which gives the desired speeds curves.

From the preceding relation (VI), let us write k according to v, one has:

$$\frac{v_{halo}^2(r)}{4rv(r)} = k(r) \quad (VII)$$

3.3. Computation step I: a mathematical solution

To carry out our computation, we are going to take in account the following measured and theoretical curves due to the "Observatoire de Paris / U.F.E." (Fig. 4).

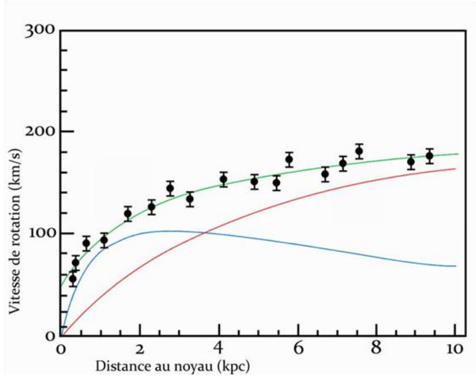


Fig. 4: Rotation curve of a typical spiral galaxy, showing measurement points with their error bar, the curve (green) adjusting the best data (black), the speed of the disk (in blue) and that of a halo of invisible matter needed to account for the observed points (in red). [Crédit "Astrophysique sur Mesure"]

On Fig. 5, one has the gravitic field computed with formula (VII). The numerical approximation used for $v_{halo}(r)$ and $v(r)$ curves are given at the end of the paper in Tab.2:

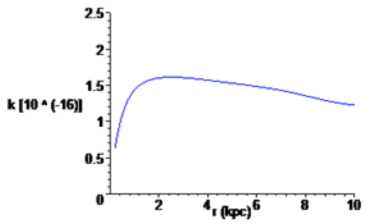


Fig. 5: Gravitic field which gives the expected rotational speed curve of Fig. 4.

This computed gravitic field is the one necessary to obtain the measured rotation speed of this galaxy without dark matter assumption. This curve plays the same role than the distribution of dark matter in the eponym assumption, it explains v_{halo}. But of course one must now study this solution in a more physical way. We will see that if we only consider the internal gravitic field of the galaxy it doesn't physically work but a solution can be imagined. This solution will be able to justify physically this curve, to be in agreement with observations and even to explain some unexplained observations.

3.4. Computation step II: a physical solution

At this step, we only mathematically solved the problem of the dark matter by establishing the form that the gravitic field $\vec{k}$ should take. The only physical assumption that we have made is that the force due to the gravitic field is written in the form $\overrightarrow{F_k} \propto \vec{v} \wedge \vec{k}$. And under this only constraint, we just come to show that it is possible to obtain a speeds curve like that obtained in experiments. To validate this solution, it is necessary to check the physical relevance of this profile of gravitic field and in particular to connect it to our gravitic definition of the field. Furthermore, at this step, with our approximation, this computed gravitic field doesn't represent only the gravitic field of the galaxy but also the others internal effects, in particular near the center (as frictions for example). But our goal will be to obtain a good physical approximation far from the galaxy center where the dark matter becomes unavoidable.

To physically study this solution far from galaxy center, the galaxy can be approximated as an object with all its mass at the origin point and some neglected masses around this punctual center. Such an idealization is an approximation in agreement with the profile of the mass distribution of a galaxy. Far from galaxy center, it's also the domain of validity of linearized general relativity. Far from galaxy center, one then should retrieve some characteristics of our "punctual" definition ($\vec{k} \approx \left(-\frac{1}{K}\right) m_p \left(\frac{1}{r^2}\right) \vec{v} \wedge \vec{u}$ in agreement with Poisson equations) in particular the three following characteristics: decreasing curve, curve tending to zero and $\vec{k} \propto \left(\frac{1}{r^2}\right)$.

__Positive point__: To be acceptable physically, it is necessary at least that this field is decreasing far from its source. For a galaxy, the large majority of its mass is in the neighborhoods of the center (zone (I)). Thus one must obtain a gravitic field which decreases far from the central area (zone (III)). The field obtained is globally in conformity with this expectation.

__Negative points__: Compared to the characteristic of the "punctual" definition ($\vec{k} \approx \left(-\frac{1}{K}\right) m_p \left(\frac{1}{r^2}\right) \vec{v} \wedge \vec{u}$), the previous gravitic field reveals two problems. First, our curve doesn't decrease near the galaxy's center. Secondly, our curve doesn't decrease to zero.

Let's see the first problem: The gravitic field graph does not always decrease. The curve starts to decrease only around 2 kpc. This problem can certainly be solved because our approximation take in account only gravitation effect. More precisely, gravitic field curve is computed starting with real value of rotation speed. But this measured rotation speed is due to gravitation but also to others phenomena (frictions for example). And our computation procedure takes in account only gravitation. So, previous computed "k" curve (obtained from measured speeds) doesn't represent only gravitation, in particular in zone (I) and (II). In these zones, the dynamic is dominated by others phenomena due to density of matter, in particular frictions and collisions, that could explain the graph. More than this, in these zones, the condition of validity of the approximation of general relativity in a quasi flat space is certainly not verified. So in this zone, our approximation underestimates the real internal gravitic field ("computed value = real value - dissipation due to the friction's effects").

Let's see now the second problem which will lead us to a possible explanation of the dark matter. The end of previous computed gravitic field curve should tend to our punctual definition of gravitic field. That is to say that gravitic field should decrease to zero. It clearly does not. More precisely, it should decrease to zero with a curve in $\frac{1}{r^2}$. One can try to approximate previous computed curve with a curve in $\frac{1}{r^2}$ (Fig. 6). But unfortunately, the approximation is so bad that it cannot be an idealization of the real situation as one can see it on the following graph.

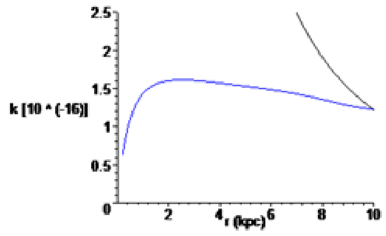


Fig. 6: Approximation of gravitic field with a curve in r^{-2}. It cannot represent reality.

This result means that the "internal" gravitic component ("internal" because due to the own galaxy) cannot explain the disk rotation speed at the end of the galaxies. One can note that this result is in agreement with the studies in (LETELIER, 2006), (CLIFFORD M. WILL, 2014) or (BRUNI et al., 2013) that also take into account non linear terms. It also means that, far from the center of the galaxy, even the non linear term cannot explain the dark matter.

But this gravitic field is generated by all moving masses. The clusters of galaxies, the clusters of clusters (superclusters) and so on have a gravitic field. The previous calculation only invalidates the galactic gravitic field as an explanation of dark matter. And as a galaxy is embedded in such astrophysical structures (cluster, supercluster…), one has to study if the gravitic field of these large structures could explain the rotation speeds of the ends of the galaxies. Instead of studying each structure one by one (that would be difficult because we don't know the value of their own gravitic fields) one simplifies the study by looking at, in a more general way, what value of gravitic field one should have to explain the rotation speed of the ends of galaxies. By this way, one will have the order of magnitude of the required gravitic field. Then, one will be able to show if large astrophysical structures could give such a value or not. We will see that the cluster is a very good candidate. And also, from these values of gravitic field, one will be able to obtain lot of very interesting observational results.

So, because general relativity implies that all large structures generate a gravitic field and because observations imply that galaxies are embedded in these large structures, we make the unique assumption of our study (we will see that this assumption will be more likely a necessary condition of general relativity):

Assumption (I):

- Galaxies are embedded in a non negligible external gravitic field
- This external gravitic field, as a first approximation, is locally constant (at the scale of a galaxy).

Remarks: The assumption is only on the fact that the gravitic field is large enough (non negligible) to explain rotation curve and not on the existence of these gravitic fields that are imposed by general relativity. And one can also note (to understand our challenge) that the constraints imposed by the observations imply that, in the same time, it must be small enough not to be directly detectable in our solar system and not to impose the orientation of the galaxies but large enough to explain dark matter. All these requirements will be verified. Furthermore, when we are going to look at the origin of this embedding gravitic field, this assumption

will become more an unavoidable condition imposed by general relativity at the scale of clusters than a hypothesis. The second point is mainly to simplify the computation and we will see that it works very well.
But of course, this external gravitic field should be consistently derived from the rotation of matter within general relativity (it will be seen in §4.1). But we will also see (§6.2) that this approximation of a uniform gravitic field is compliant with linearized general relativity (just like it is in Maxwell idealization for electromagnetism).

Mathematically, this assumption means that $\vec{k} = \overrightarrow{k_{punctual}} + \overrightarrow{k_0}$ with the "internal" gravitic component of the galaxy $\left\|\overrightarrow{k_{punctual}}\right\| = \frac{K_1}{r^2}$ ("punctual" definition of the linearized general relativity) and an "external" gravitic component $\overrightarrow{k_0}$, the approximately constant gravitic field of the close environment (assumption (I)). If we take $\left\|\overrightarrow{k_0}\right\| = 10^{-15.97}$ and $K_1 = 10^{24.19}$ (values obtained to adjust this curve on the previous computed one), one has the curves on Fig. 7:

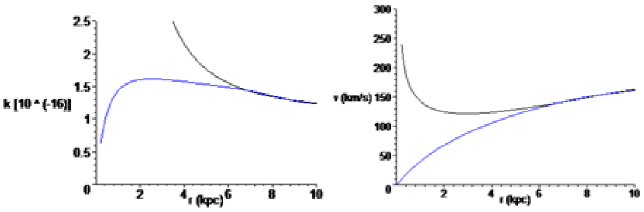


Fig. 7: Gravitic field approximation ($k = \frac{10^{24.19}}{r^2} + 10^{-15.97}$) and the rotation speed computed with it.

Left curves represent gravitic field and right curves the rotation speed obtained from formula (VI). In the last part of the graph (domain of validity of linearized general relativity), the curves are indistinguishable. From about 6 kpc to the end, the evolution in $\frac{1}{r^2}$ is excellent.

To summarize our dark matter explanation, in the frame of linearized general relativity, the speeds of rotation at the ends of the galaxy can be obtained with:

- An internal gravitic field that evolves like r^{-2} far from the center of galaxy (in agreement with the "punctual" definition of linearized general relativity)
- A constant external gravitic field embedding the galaxy.

One can write these contributions:

$$k(r) = \left(\frac{K_1}{r^2} + k_0\right) \qquad (VIII)$$

Remark: The use of the approximation $\frac{K_1}{r^2}$ helps to extract the pure internal gravitic field by excluding the other non gravitic effects from the central area of the galaxy. It then enhances our approximation of the pure internal gravitic field. But as it is obtained by fitting our previous experimental curve that underestimates this pure internal gravitic field, it must always underestimate it. We will quantify this discrepancy. I recall that our main goal is to obtain an approximation in the ends of the galaxy (that is to say k_0), area where the other non gravitic effects vanish. Our approximation should be sufficient for this goal.
One can note that this contribution to be compliant with general relativity implies two constraints. First, the term $\frac{K_1}{r^2}$ that is the own (internal) gravitic field of the galaxy should be retrieved from simulations ever done in several papers (for example BRUNI et al., 2013). It will validate our approximation. Secondly, for each gravitic field, there should be a gravity field. For example, in our computation, associated to $\frac{K_1}{r^2}$, we take into account the gravity field for the galaxy (the term $\frac{\partial \varphi_{disk}(r)}{\partial r}$). But for our external gravitic field k_0, there should also be a gravity field. So to be compliant with our assumption, this gravity field should be negligible compare to k_0 (because we don't take it into account). These constraints will be verified in this study.

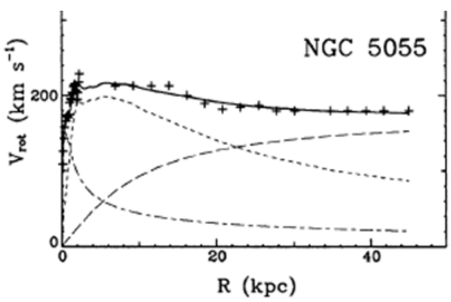
3.5. Application on several galaxies

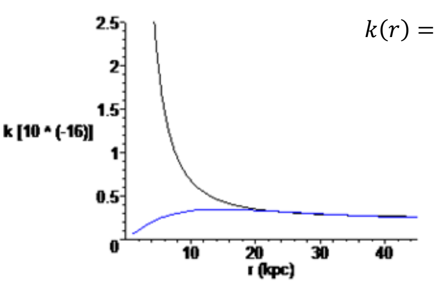
We are now going to test our solution on different galaxies studied in (KENT, 1987). Because the linearized general relativity doesn't modify the components $v_{disk}(r)$ and $v_{bulge}(r)$, one can focus our study on the relation $v_{halo}^2(r) = 4k(r)v(r)r$.
For that we are going to determine $k_{exp}(r)$ curve, from experimental data with the relation, formula (VII):

$$\frac{v_{halo}^2(r)}{4rv(r)} = k_{exp}(r)$$

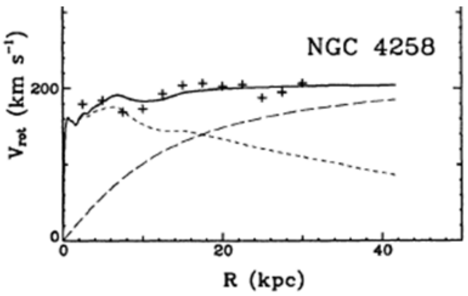
I recall that we know the curve $v(r)$ which is given by experimental data and we know $v_{halo}(r)$, the resultant component obtained from relation $v_{halo}^2(r) = v^2(r) - v_{disk}^2(r) - v_{bulge}^2(r)$ where $v_{disk}^2(r)$ and $v_{bulge}^2(r)$ are also deduced from experimental data. The numerical approximation used for $v_{halo}(r)$ and $v(r)$ curves are given at the end of the paper in Tab. 3.

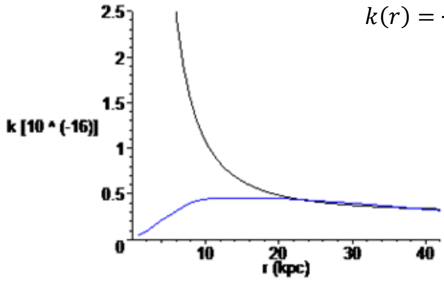
Then we are going to approach this $k_{exp}(r)$ curve (blue curves in Fig. 8) with our expected expression $k(r) = \left(\frac{K_1}{r^2} + k_0\right)$ which explains dark matter (black curves of right graphs in Fig. 8).



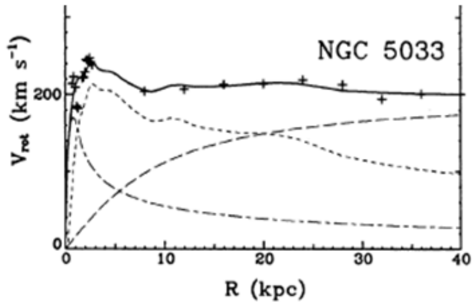


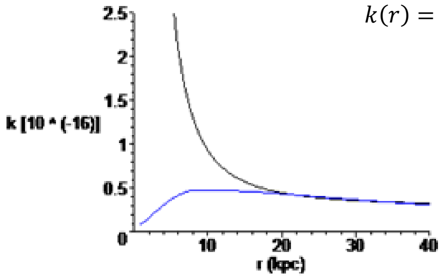
$$k(r) = \frac{10^{24.6}}{r^2} + 10^{-16.62}$$



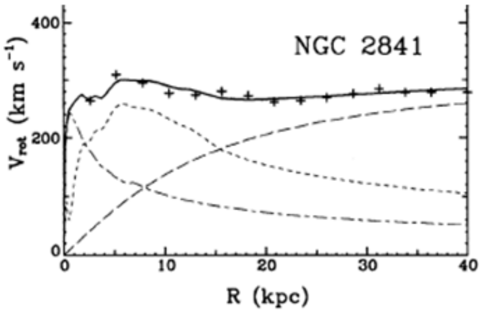


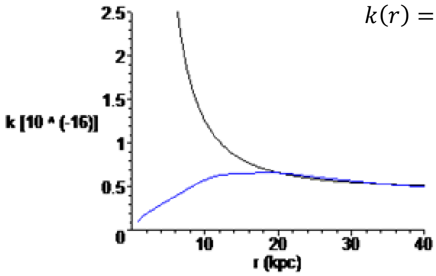
$$k(r) = \frac{10^{24.85}}{r^2} + 10^{-16.54}$$



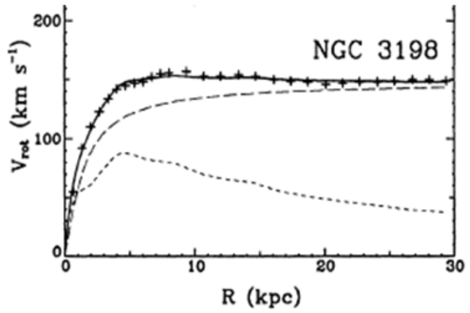


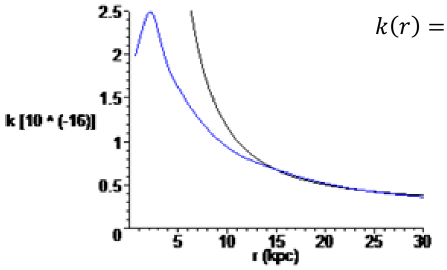
$$k(r) = \frac{10^{24.76}}{r^2} + 10^{-16.54}$$



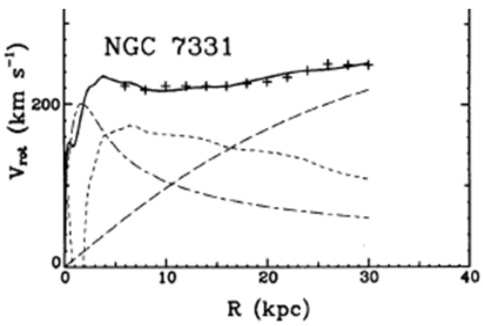


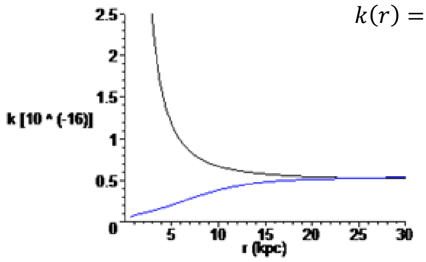
$$k(r) = \frac{10^{24.85}}{r^2} + 10^{-16.33}$$



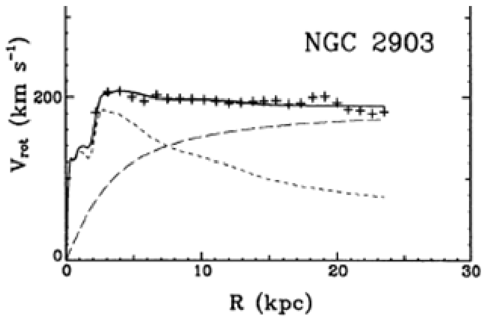


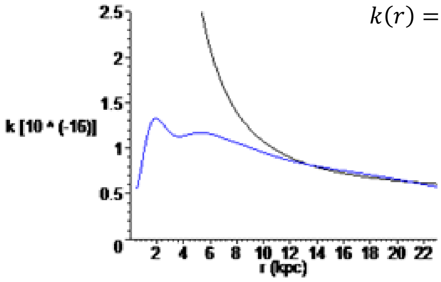
$$k(r) = \frac{10^{24.9}}{r^2} + 10^{-16.55}$$



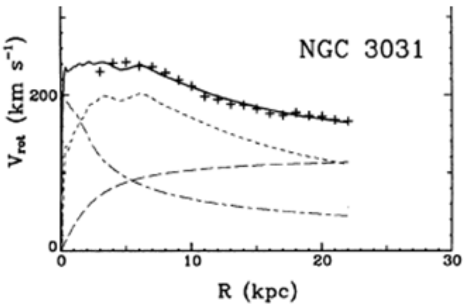


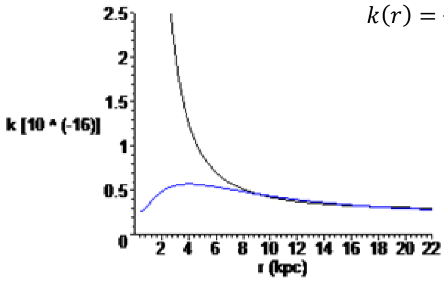
$$k(r) = \frac{10^{24.18}}{r^2} + 10^{-16.3}$$



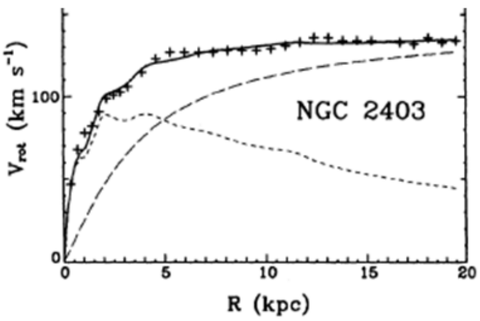


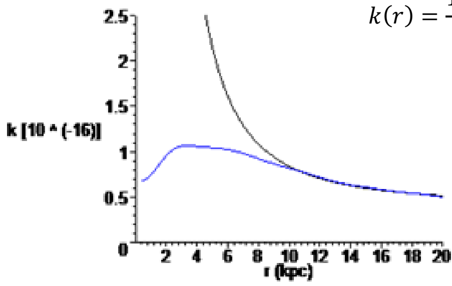
$$k(r) = \frac{10^{24.71}}{r^2} + 10^{-16.3}$$



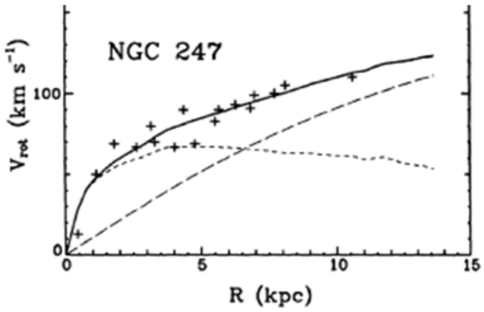


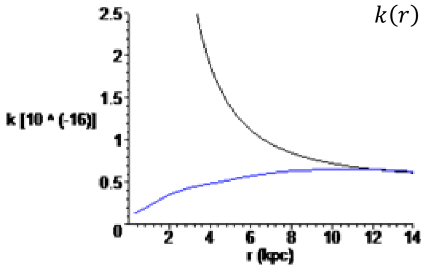
$$k(r) = \frac{10^{24.15}}{r^2} + 10^{-16.57}$$



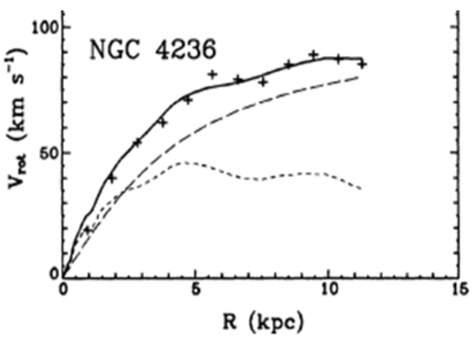


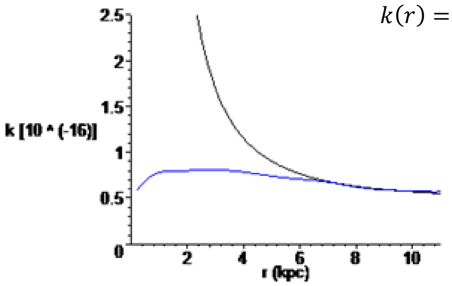
$$k(r) = \frac{10^{24.59}}{r^2} + 10^{-16.39}$$





$$k(r) = \frac{10^{24.3}}{r^2} + 10^{-16.3}$$





$$k(r) = \frac{10^{24.}}{r^2} + 10^{-16.34}$$

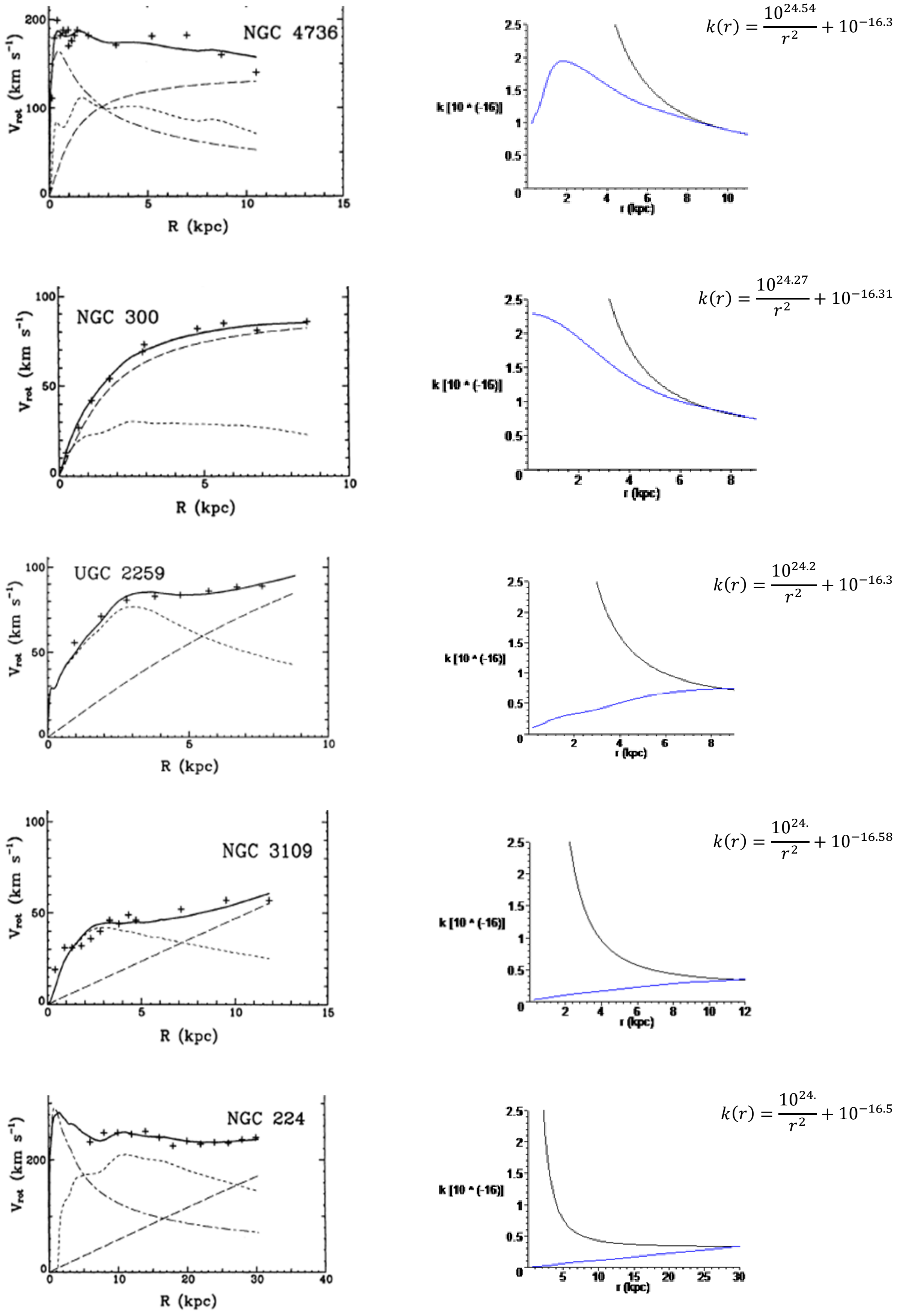


Fig. 8: Left graphs are the basis data from (KENT, 1987); right graphs represent $k_{exp}(r)$ (in blue) and $k(r) = \left(\frac{K_1}{r^2} + k_0\right)$ (in black) explaining ends of rotation speed curves (analytic expression of $k(r)$ is given).

Discussion on these results:

1) Our solution makes the assumption of a constant asymptotic behavior of gravitic field (tending to the external gravitic field k_0 at the ends of galaxies). These results confirm the tendency to get a curve $k(r)$ flattens for large r. However, there are two exceptions. The last two curves do not flatten in the studied interval of distance. These results also show that the external gravitic field values are in the interval $10^{-16.62} < k_0 < 10^{-16.3}$.
2) Our solution makes the more precise assumption of an evolution of the field as $\frac{K_1}{r^2} + k_0$ far from the center of the galaxy. This hypothesis is verified because theoretical $k(r)$ and experimental $k_{exp}(r)$ curves are equivalent on more than 30 % of their definition (up to 50 % for some), always with the exception of the last two curves. Furthermore, our idealization comes from linearized general relativity approximation that is valid only in zone (III) far from the center of the galaxy and then nearly 100% of the zone (III) is explained. This is the first main result of our study. These results also show that, with our approximation, the internal gravitic field values are in the interval $10^{24.} < K_1 < 10^{24.9}$. We are going to compare these values with published studies.
3) One can note this remarkable agreement with observations. The order of magnitude of k_0 is small enough to be undetectable in our solar system (cf. the calculation in paragraph 2) and at the same time it is high enough to explain rotation speed of galaxies. Dark matter has the disadvantage of being almost all the matter of our universe and to be undetectable until now, meaning that all our theories would explain (and would be founded on) only a negligible part of our reality.
4) Validity of our approximation: The value K_1 is associated to the internal gravitic field of galaxies. As we said before, it has ever been computed in several studies. One can then compare our approximation to these more accurate studies. For example, NGC 3198 gives the value $K_1 \sim 10^{24.9}$ for a visible mass of about $m_p \sim 6 \times 10^{40} kg$ (VAN ALBADA et al., 1985). One then has:

$$\frac{\|\vec{k}\|}{\|\vec{g}\|} \sim \frac{K_1}{r^2}\left(G\frac{m_p}{r^2}\right)^{-1} \sim \frac{10^{24.9}}{6.67 \times 10^{-11} \times 6 \times 10^{40}} \sim 2 \times 10^{-6}$$

As seen at the beginning of our study, in term of gravitomagnetism, one has $\overrightarrow{B_g} = 4\vec{k}$, it leads to a ratio $\frac{\|\overrightarrow{B_g}\|}{\|\vec{g}\|} = 4\frac{\|\vec{k}\|}{\|\vec{g}\|} \sim 0.8 \times 10^{-5}$. In (BRUNI et al., 2013), the simulation gives a ratio between the vector and scalar potential of about 10^{-5}. As we said previously, our deduced internal gravitic field must be underestimated compare to the real value because our idealization doesn't take into account the friction's effects ("computed value = real value - dissipation due to the friction's effects"). And secondarily, our idealization doesn't take into account the non linear terms. It then explains that our ratio is slightly inferior to a more accurate simulation. But even with that, our approximation gives a good order of magnitude of the relative values of the gravity and gravitic fields. If one looks at the absolute value of our gravitic field, at $r \sim 1.3 Mpc \sim 10^{22} m$, one has $\|\overrightarrow{B_g}\| = 4\frac{K_1}{r^2} \sim \frac{10^{24.9}}{(1.3 \times 10^{22})^2} \sim 10^{-18.7}$. In (BRUNI et al., 2013), the simulation gives for the power spectra the value $P_B(r \sim 1.3 Mpc) \sim 10^{-18}$. This comparison confirms that our approximation gives a gravitic field that is underestimated. With our example, the factor of correction (due mainly to friction's effects) is about $5 \sim 10^{0.7}$. Its effective gravitic field is around 5 times greater than our approximation. But the order of magnitude is still correct (even without the non linear terms). Let's make a remark. Our explanation of dark matter is focused on the external term k_0. The next results will be obtained far from the galaxies' center. We are going to see that far from the center of galaxies only k_0 acts. And far from the center of galaxies, the non linear terms (of internal gravitic field) and friction's effects are negligible. For these reasons, with our approximation, the value of external gravitic field k_0 should be a more accurate value.
5) A first reaction could be that the external gravitic field should orient the galaxies in a same direction, which is at odds with observations. With our analysis, one can see that the internal gravitic field of galaxies ($\frac{K_1}{r^2} \sim \frac{10^{24.9}}{r^2}$) is bigger than the embedding gravitic field ($k_0 \sim 10^{-16.55}$) required to explain dark matter for $r < 17kpc$ ($\frac{10^{24.9}}{r^2} > 10^{-16.55} \Rightarrow r^2 < 10^{41.45}$). And if one takes in account the factor of correction seen previously on K_1, the internal gravitic field imposes its orientation until $r \sim 40kpc$. With these orders of magnitude, we understand that galaxies' orientation is not influenced by this gravitic field k_0 of the environment. The randomly distribution of the galaxies' orientation is then completely in agreement with our solution.
6) Influence of internal gravitic field far from the galaxy's center: One can note that, for only the internal gravitation, one has in the galaxies $\frac{\|\overrightarrow{F_k}\|}{\|\overrightarrow{F_g}\|} \sim 4v\frac{\|\vec{k}\|}{\|\vec{g}\|} \sim 10^{-5}v$. And for $r > 10kpc$, one has in the galaxies $v \sim 10^5 m.s^{-1}$. It implies that $\frac{\|\overrightarrow{F_k}\|}{\|\overrightarrow{F_g}\|} \sim 1$ in this area. So, far from galaxies center, the gravitic force is of the same magnitude than the gravity force. And then, just like the internal gravity force, the internal gravitic force decreases and vanishes far from galaxies center explaining why it cannot explain dark matter.

At this step, to compare in the main lines and in a simplified way our study with already published papers, one can say that in terms of linearized general relativity the already published papers idealize the gravitic field like $k = \frac{K_1}{r^2}$ (but more accurately, taking into account non linear terms) and in our approach we idealize the gravitic field like $k = \frac{K_1}{r^2} + k_0$. The already published papers deduce their expression from mass distribution of galaxies, giving them only the gravitic field of galaxies. Conversely, we deduce our expression from the rotation speeds of galaxies, giving us the gravitic field of galaxies and a supplemented external gravitic field. The rest of our study is going to justify this expression by showing that the computed values (for k_0) can be deduced from larger structures than galaxies; that the computed values (for k_0) can explain unexplained observations and retrieve some known results; by showing that our expression is consistent with general relativity. It means to show that the uniform k_0 verifies the field equations of linearized general relativity (that is our domain of validity) and that k_0 has to be associated with a negligible gravity field g_0.

4. Cluster as possible origin of the gravitic field $\mathbf{k_0}$

We are now going to use these values of k_0 ($10^{-16.62} < k_0 < 10^{-16.3}$). It will allow obtaining the expected rotation speed of satellite dwarf galaxies and retrieving the expected quantities of "dark matter" in the galaxies. We will also demonstrate that the theoretical expression of gravitic field can explain the dynamic of satellite dwarf galaxies (movement in a plane) and allow obtaining an order of magnitude of the expected quantitiy of "dark matter" in the CMB. But just before we are going to show that the clusters of galaxies could generate these values of k_0.

4.1. Theoretical evidences

Before beginning this paragraph, I would like to precise that the goal of this paragraph is not to obtain a value of the gravitic field for large structures but to determine which large structure(s) could be a good candidate(s) to generate our embedding gravitic field. Inside our theoretical frame, we determined the values of the internal gravitic fields of the galaxies. From these values and from the definition of the gravitic field, we are going to apply a change of scale to estimate the value of the internal gravitic field of objects larger than galaxies. Of course, this way of approximating can be criticized. But the same criticism can be assigned to each calculation on each large structure. One can then hope, by this way, obtaining the relative contribution of these large structures (but not necessarily their actual values). This process will show that the galaxies' cluster is certainly the main contributor to our embedding gravitic field (and astonishingly this process will also give the good order of magnitude of the expected gravitic field, i.e. not only in relative terms).

What can be the origin of this embedding gravitic field? In our theoretical frame, the gravitic field (just like magnetic field of a charge in electromagnetism) can come from any moving mass. Because of the orders of magnitude seen previously (for particle, Earth, Solar system and galaxies) one can expect that the embedding gravitic field (k_0) should be due to a very large astrophysical structure (with a sufficient internal gravitic field "$\frac{K_1}{r^2}$" to "irradiate" large spatial zone). Furthermore, just like magnetic field of magnets (obtained as the sum of spins), this external gravitic field could also come from the sum of several adjacent large structures. Our theoretical solution then implies that our embedding gravitic field can come from the following possibilities:

Case A:
  Internal gravitic field of the galaxy
Case A bis:
  Sum of internal gravitic fields of several close galaxies
Case B:
  Internal gravitic field of the cluster of galaxies
Case B bis:
  Sum of internal gravitic fields of several close clusters
Case C:
  Internal gravitic field of larger structure (cluster of cluster,…)
Case C bis:
  Sum of internal gravitic fields of several close larger structures
…
CaseD:
  Internal gravitic field of our Universe

This expected external gravitic field should be consistently derived from the rotation of matter within general relativity. It means that if it should come from an internal gravitic field of a large structure, it should evolve like $\frac{K_1}{r^2}$ far from the source (in agreement with linearized general relativity because of Poisson equations (III)). So, let's see how large can this term "$\frac{K_1}{r^2}$"be for these large structures.

Case A: We have seen that our solution (in agreement with some other published papers) rejects the possibility of an internal gravitic field of galaxy sufficient to explain dark matter. More precisely, we are going to see that, in our approximation, around $r \sim 15\ kpc$, the internal and external gravitic fields have similar magnitudes. But for $r \gg 15\ kpc$, the internal gravitic field of the galaxy can't explain k_0 because it becomes too small. We are going to see that the value of k_0 can explain the rotation speed of satellite dwarf galaxies (for $r > 100\ kpc$). At $100\ kpc$ internal gravitic field of galaxy represents only about 1% of k_0 (for average values $k_0 \sim 10^{-16.5}$, $K_1 \sim 10^{24.5}$ give at $r \sim 100 kpc \sim 3 \times 10^{21} m$ the value $\frac{K_1}{r^2} \sim 10^{-18.5}$).

Case A bis: Even the sum of several close galaxies can't generate our required k_0. Roughly in a cluster of galaxies, there are about $N \sim 10^3$ galaxies for a diameter of $D \sim 10^{23} m \sim 5 Mpc$. It leads to an average distance between galaxies of about $d \sim 10^{22} m$ (for a spherical local idealization : $N * (\frac{d}{2})^3 \sim (\frac{D}{2})^3$). We have seen that internal gravitic field of a galaxy is about $K_1 \sim 10^{24.5}$. The contribution to the gravitic field of an adjacent galaxy at the distance $d \sim 10^{22} m$ is then $\frac{K_1}{d^2} \sim \frac{10^{24.5}}{10^{44}} = 10^{-19.5}$. If we suppose that a galaxy has about ten close neighbors (for example in a cubic distribution of galaxies) it leads to a gravitic field of about $10^{-18.5}$. It only represents 1% of the expected external gravitic field.

Case B: Without an explicit calculation of the internal gravitic field of a cluster, it is difficult to give its contribution to the embedding gravitic field k_0. One can try a roughly approximation. The mass of a cluster is about $10^{44} kg$ (without dark matter). It is about 1000 times the mass of a typical galaxy ($10^{41} kg$) and the velocities of galaxies are about 10 times greater than matter in galaxies. By definition, the gravitic field is proportional to the mass and to the speed ($\vec{k} \approx \left(-\frac{1}{K}\right) m_p \left(\frac{1}{r^2}\right) \vec{v_s} \wedge \vec{u}$). The internal gravitic field of a cluster could then be about $10^3 \times 10 \times K_1 = 10^4 K_1 \sim 10^{28.5}$. At a distance of $D \sim 10^{23} m \sim 5 Mpc$ (size of a cluster) it would give $\frac{10^4 K_1}{D^2} \sim \frac{10^{28.5}}{10^{46}} = 10^{-17.5}$. It then represents about 10% of the expected external gravitic field. And more we are close to the cluster center, more the external garvitic field could be entirely explained by internal gravitic field of the cluster (for example at 1Mpc it represents about 100%).

Case B bis: Furthermore, the sum of the internal gravitic field of adjacent clusters could maintain this embedding gravitic field on very large distance. Approximately, one found the same ratio between the cluster and its galaxies than between the supercluster and its clusters. Roughly in a supercluster, there can be about $N \sim 10^3$ cluster for a diameter of $D \sim 10^{24} m \sim 50 Mpc$. It leads to an average distance between clusters of about $d \sim 10^{23} m$ (for a spherical idealization : $N * (\frac{d}{2})^3 = (\frac{D}{2})^3$). We have seen that roughly internal gravitic field of a cluster would be about $10^4 K_1 \sim 10^{28.5}$. The contribution to the gravitic field of an adjacent cluster at the distance $d \sim 10^{23} m$ is then $\frac{10^4 K_1}{d^2} \sim \frac{10^{28.5}}{10^{46}} = 10^{-17.5}$. If we suppose that a cluster has about ten close neighbors (for example in a local cubic distribution of clusters) it leads to a gravitic field of about$10^{-16.5}$. It represents 100% of the required external gravitic field.

Case C: The mass of a supercluster is about $10^{46} kg$ (with dark matter, meaning that real mass should be less). It is about 100 times the mass of a typical cluster ($10^{44} kg$) and the velocities are about 10 times greater than matter in clusters. By definition the gravitic field is proportional to the mass and to the speed. It gives then roughly that the internal gravitic field of a supercluster can be about $10^2 \times 10 \times K_1 = 10^3 K_1 \sim 10^{27.5}$. At a distance of $D \sim 10^{24} m \sim 50 Mpc$ (size of a supercluster) it would give $\frac{10^3 K_1}{D^2} \sim \frac{10^{27.5}}{10^{48}} = 10^{-20.5}$. It then represents about 0.01% of the expected external gravitic field. And even at a distance of $d \sim 10^{23} m$, one has only about 1% and with an overestimated of the mass. The internal gravitic field of a supercluster seems not to be able to explain embedding gravitic field of the galaxies.

With these calculations, we see that general relativity requires that some large astrophysical structures generate significant gravitic fields ($k_0 = 10^{-16.5}$ is significant because sufficient to explain dark matter). And with our roughly calculation, the own gravitic field of a cluster cannot be neglected and even it seems to give the right order of magnitude to explain dark matter. Furthermore, by definition, because k_0 depends on the matter speed of the source ($\vec{k} \sim \left(-\frac{1}{K}\right) m_p \left(\frac{1}{r^2}\right) \overrightarrow{v_s} \wedge \vec{u}$) and because the gravitic force also depends on the speed of matter that undergoes the gravitic field ($\vec{F}_k \sim m_p 4\vec{v} \wedge \vec{k}$), the influence of gravitic field (compare to gravity field) must become more and more important with matter speed ($\frac{\|\vec{F_k}\|}{\|\vec{F_g}\|} \propto v^2$ for$v_s \sim v$). In general, higher is the scale, higher is the typical speed. In other words, higher is the scale, greater is the influence of the gravitic field compared with the gravity field. It is therefore likely that our assumption (I) is less an assumption than a necessary condition that must be taken into account at the scale of galaxies and beyond. We have seen before that at lower scales than galaxies, the gravitic force was negligible but at the scale of galaxies, it begins to be significant ($\frac{\|\vec{F_k}\|}{\|\vec{F_g}\|} \sim 1$).

The cases B and B bis are important because it allows justifying our second constraint on assumption (I). We have seen that there are two constraints on our assumption to be compliant with general relativity. The first one (to retrieve the already calculated and published term $\frac{K_1}{r^2}$) was verified previously. The second one is on the fact that there should be a negligible gravity field g_0 associated with the gravitic field k_0. We have seen that the influence of the gravitic field evolves like $\frac{\|\vec{F_k}\|}{\|\vec{F_g}\|} \propto v^2$ and that $\frac{\|\vec{F_k}\|}{\|\vec{F_g}\|} \sim 1$ at the scale of galaxies. In the case B of a single cluster, the speed is about ten times greater than in the galaxies. So, it gives $\frac{\|\vec{F_k}\|}{\|\vec{F_g}\|} \sim 100$. It is sufficient to neglect the gravity field. Furthermore, associated with the case B bis, one can imagine another mechanism (that also neutralize the gravity field) but that could give also an interesting property on the spatial extension of k_0. If we take into account the neighboring clusters, the gravity field must greatly decrease and the gravitic field can increase, as one can see on this simplified representation (Fig. 9).

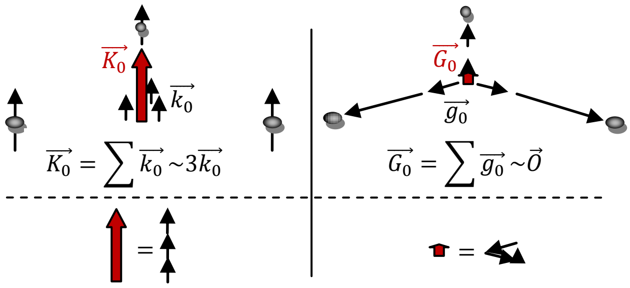


Fig. 9: Influence of the neighbors on the gravitic field (increase of the norm of $\|\overrightarrow{k_0}\|$) and on the gravity field (neutralization of $\|\overrightarrow{g_0}\|$).

The sum of these vectors allows justifying (once again) that the associated gravity field $\overrightarrow{g_0}$ could be neglected compared to$\overrightarrow{k_0}$. But such a situation means that the internal gravitic fields of neighboring clusters mustn't have a randomly distributed orientation. Our study is not enough accurate to define how large must be the spatial extension of a same value of k_0 but as we are going to see it for the satellite dwarf galaxies, it should maintain for $r > 100\ kpc$. This mechanism could help to explain such a spatial extension (the second point of our assumption (I). In this case, it would mean that for very large astrophysical structures, there could be coherent orientations. This possibility could be in agreement with some published papers (HUTSEMEKERS, 1998; HUTSEMEKERS et al., 2005) that reveal coherent orientations for large astrophysical structures. This situation is similar, in electromagnetism, to the situation of magnetic materials, from which the atomic spins generate a magnetic field at the upper scale of the material. And I recall that linearized general relativity leads to the same field equation than Maxwell idealization. So, this case B bis could allow completely justifying our assumption (I) that explains dark matter in the frame of general relativity.

One can note that, because of this origin, the value of external gravitic field k_0 could be a signature of a cluster (it could be a way to know if a galaxy is or not in a cluster).

A last remark on the case D, which is beyond the scope of our paper, the gravitic field at the scale of the universe could also lead to an explanation of dark energy but it implies a new fundamental physical assumption (LE CORRE, 2015). Some experiments at CERN are testing the possibility of such a fundamental assumption.

4.2. Experimental evidences

If we consider the own gravitic field of the cluster, the maximal value of this field must be around the center of the mass's distribution of the cluster. So, with our previous deduction on the origin of the external gravitic field, one can deduce that k_0 should be maximal at the center of the cluster. In other words, it means that external gravitic field (our "dark matter") should decrease with the distance to the center of the cluster. Recent experimental observations (JAUZAC et al., 2014) in MACSJ0416.1-2403 cluster reveals that the quantity of dark matter decreases with the distance to the center of the cluster, in agreement with our origins of external gravitic field. This is the second main result of our study.

To corroborate this origin, one can even give an explanation for the very few cases of galaxies with a truly declining rotation curve, for example NGC 7793 (CARIGNAN & PUCHE, 1990). Because our explanation doesn't modify general relativity, the solution of a declining rotation curve is always possible. For such a situation, the galaxy must not be under the influence of the external gravitic field. This means that the galaxy must be at the ends of the cluster (far from its center) or isolated. As written in (CARIGNAN & PUCHE, 1990), NGC 7793 is effectively "the most distance member of the Sculptor group" in agreement with our solution. Furthermore, one can also note that if the superclusters were the origin of the external gravitic field, the spatial extension of such structures would make nearly impossible to detect galaxies not under the influence of the expected k_0. So, finding few cases of galaxies with a truly declining rotation curve is also in agreement with the fact that the superclusters must not contribute to the external gravitic field.

5. Gravitic field and galaxies

5.1. Application on satellite dwarf galaxies

We are now going to look at satellite dwarf galaxies and retrieve two unexplained observed behaviors (rotation speed values and movement in a plane). First, one deduces an asymptotic expression for the component of gravitic field (our "dark matter"). We know that, to explain rotation speed curve, we need two essential ingredients (internal gravitic field $\frac{K_1}{r^2}$ and embedding external gravitic field k_0). Let's calculate how far one has $\frac{K_1}{r^2}\sim k_0$ for our previous studied galaxies.

Tab. 1: Distance r_0 where internal gravitic field $\frac{K_1}{r^2}$ becomes equivalent to external gravitic field k_0.

	K_1	k_0	$r_0\left[\frac{K_1}{r^2}\sim k_0\right]$	$r_0[kpc]$
NGC 5055	$10^{24.6}$	$10^{-16.62}$	$10^{20.61}$	13
NGC 4258	$10^{24.85}$	$10^{-16.54}$	$10^{20.695}$	16
NGC 5033	$10^{24.76}$	$10^{-16.54}$	$10^{20.65}$	15
NGC 2841	$10^{24.85}$	$10^{-16.33}$	$10^{20.59}$	13
NGC 3198	$10^{24.9}$	$10^{-16.55}$	$10^{20.725}$	18
NGC 7331	$10^{24.18}$	$10^{-16.3}$	$10^{20.24}$	6
NGC 2903	$10^{24.71}$	$10^{-16.3}$	$10^{20.505}$	11
NGC 3031	$10^{24.15}$	$10^{-16.57}$	$10^{20.36}$	8
NGC 2403	$10^{24.59}$	$10^{-16.39}$	$10^{20.49}$	10
NGC 247	$10^{24.3}$	$10^{-16.3}$	$10^{20.3}$	7
NGC 4236	$10^{24.}$	$10^{-16.34}$	$10^{20.17}$	5
NGC 4736	$10^{24.54}$	$10^{-16.3}$	$10^{20.42}$	9
NGC 300	$10^{24.27}$	$10^{-16.31}$	$10^{20.29}$	6
NGC 2259	$10^{24.2}$	$10^{-16.3}$	$10^{20.25}$	6
NGC 3109	$10^{24.}$	$10^{-16.58}$	$10^{20.29}$	6
NGC 224	$10^{24.}$	$10^{-16.5}$	$10^{20.25}$	6

The previous values mean that, in our approximation, for $r \gg r_0 \sim 15\ kpc$ the galactic dynamic is dominated by the external embedding gravitic field k_0.

And, very far from the center of galaxies, formula $(VIII)$ can be written $k(r)\sim k_0$. So far, one can only consider external gravitic field k_0, formula (VI) gives:

$$v_{halo}{}^2(r)\sim 4k_0 v(r) r$$

Very far from center of galaxies, $v_{disk}(r)$ the speed component due to internal gravity tends to zero. If we neglect $v_{bulge}(r)$, one has $v_{halo}^2(r) = v^2(r) - v_{disk}^2(r) - v_{bulge}^2(r) \sim v(r)$

In our solution, very far from galaxies center ($r > 100\ kpc$), one has (if we neglect v_{bulge}):

$$v(r)\sim 4k_0 r \qquad (IX)$$

One can then deduce the rotational speed of satellite galaxies for which the distances are greater than $100\ kpc$. In this area, the own gravitation of galaxies is too weak to explain the measured rotational speed, but the external gravitic field embedding the galaxy can explain them. Our previous study gives:

$$10^{-16.62} < k_0 < 10^{-16.3}$$

With the relation $v(r)\sim 4k_0 r$, one has (in kilo parsec):

$$2.88 \times 10^3 r[kpc] < v(r) < 6.03 \times 10^3 r[kpc]$$

It then gives for $r \sim 100\ kpc$:

$$288\ km.s^{-1} < v(r \sim 100\ kpc) < 603\ km.s^{-1}$$

This range of rotation speed is in agreement with experimental measures (ZARITSKY et al., 1997) (on satellite dwarf galaxies) showing that the high values of the rotation speed continues far away from the center. This is the third main result of our study.

5.2. About the direction of $\overrightarrow{k_0}$

Our previous study reveals that, inside the galaxy, external gravitic field k_0 is very small compare to the internal gravitic field $\frac{K_1}{r^2}$. But this internal component decreases sufficiently to be neglected far from galaxy center, as one can see in the following simplified representation (Fig. 10).

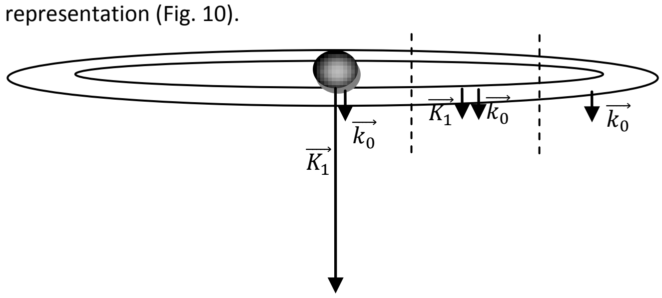


Fig. 10: Evolution of internal gravitic field $\frac{K_1}{r^2}$ compare with external gravitic field k_0 defining three different areas.

One can then define three areas with specific behavior depending on the relative magnitude between the two gravitic fields; very far from center of galaxy for which only direction of $\overrightarrow{k_0}$ acts, close around center of galaxy for which only direction of $\overrightarrow{K_1}$ acts and a transition zone for which the two directions act. Just like in electromagnetism with magnetic field, a gravitic field implies a movement of rotation in a plane perpendicular to its direction. With these facts, one can make several predictions.

Very far from center of galaxy, there are the satellite dwarf galaxies. One can then make a first prediction:

The movement of satellite dwarf galaxies, leaded by only external $\overrightarrow{k_0}$, should be in a plane (perpendicular to $\overrightarrow{k_0}$).

This prediction has been recently verified (IBATA et al., 2014) and is unexplained at this day. In our solution, it is a necessary consequence. This is the fourth main result of our study.

One can also make statistical predictions. The external gravitic field for close galaxies must be relatively similar (in magnitude and in direction). So, close galaxies should have a relatively similar spatial orientation of rotation's planes of their satellite dwarf galaxies. The second prediction is:

Statistically, smaller is the distance inside a pair of galaxies; smaller is the difference of orientation of their satellite dwarf galaxies' planes.

We saw previously that the cluster of galaxies is likely the good candidate to generate our external gravitic field $\overrightarrow{k_0}$. If this assumption is true, one can expect that the orientations of the satellite dwarf galaxies' planes should be correlated with the direction of the internal gravitic field of the cluster (which is our $\overrightarrow{k_0}$). It is quite natural (just like for the galaxies) to assume that this direction is perpendicular to the supergalactic plane.

Statistically, inside a cluster, the orientation of their satellite dwarf galaxies' planes should be close to the supergalactic plane.

These predictions are important because it can differentiate our solution from the dark matter assumption. Because the dark matter assumption is an isotropic solution, it cannot predict à priori such an anisotropic behavior of dwarf galaxies (correlation between dwarf galaxies' directions of two close galaxies and correlation with supergalactic planes). As said before, these coherent orientations could be another clue of a more general property on very large structures at upper scales. One can recall that there are some evidences of coherent orientations for some large astrophysical structures (HUTSEMEKERS, 1998; HUTSEMEKERS et al., 2005). Furthermore, in our solution, there is no alternative. Such correlations are imposed by our "dark matter" explanation.

Very recent observations verify these three predictions (TULLY et al., 2015). The conclusion is:"The present discussion is limited to providing evidence that almost all the galaxies in the Cen A Group lie in two almost parallel thin planes embedded and close to coincident in orientation with planes on larger scales. The two-tiered alignment is unlikely to have arisen by chance". It means that the satellites in the Centaurus A group are distributed in planes (and then also their movement). It is our first prediction. It means that the two planes have very close orientations. It is our second prediction. And it means that these planes have very close orientations to supergalactic plane. It is our third prediction and by this observation it validates that the cluster is likely the good candidate to generate our external $\overrightarrow{k_0}$. These observations are certainly one of the main evidences of the relevance of our solution. And one can go further to compare our solution with the non baryonic dark matter assumption. As we have seen previously, $\overrightarrow{k_0}$ is too small to influence the orientation of the rotation plane of a galaxy. It means that our solution implies a decorrelation between the galaxy's rotation planes and the satellite dwarf galaxies' planes. While the galaxy's rotation planes are distributed randomly, the satellite dwarf galaxies' planes have a specific orientation imposed by the cluster's orientation. For example in the Centaurus A group, the rotation planes of the galaxies are distributed randomly (as everywhere in the universe) contrary to their satellite dwarf galaxies' planes. This situation imposed by our solution and which is verified by observation is more difficult to justify in the traditional dark matter assumption. Because, à priori, if non baryonic matter is distributed as ordinary matter, the gravitation laws should lead to the same solution, i.e. the same orientation of the both planes (plane of the galaxy and plane of the satellite dwarf galaxies). In fact the plane of the satellite dwarf galaxies should be in the continuity of the plane of the galaxy. The previous observations contradict this theoretical expectation of dark matter. Inversely, if we assume a possible specific distribution of dark matter to explain the plane of the satellite dwarf galaxies, it leads to several problems. The first one is that one must define a specific distribution for each galaxy (all the orientations between the two planes are possible). For a galaxy, then what is the constraint that would favor one configuration over another? The second one is how we can explain that baryonic and non baryonic matters which undergo the same gravitation laws converge to two different mathematical solutions (two different physical distributions)? I have no doubt that non trivial complex solutions could be imagined to explain these situations, but our solution is very natural to explain them. To sum up, these observations are unexpected in the dark matter's assumption but required by our solution.

One can imagine two others configurations. When the internal gravitic field of the galaxy and the external $\overrightarrow{k_0}$ is in the opposite direction:

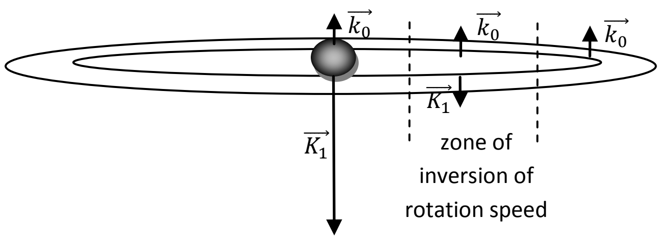


Fig. 11: Internal gravitic field $\frac{K_1}{r^2}$ in opposite direction with external gravitic field k_0 implying two zones of opposite rotation speed.

With this situation (Fig. 11) there are two zones inside the galaxy with two opposite directions of gravitic field. It leads to the third prediction:

A galaxy can have two portions of its disk that rotate in opposite directions to each other.

This prediction is in agreement with observations (SOFUE & RUBIN, 2001).

The other configuration leads to a new possible explanation of the shape of some galaxies:

The warped galaxies could be due to the difference of direction between internal and external gravitic field, as one can see in this other simplified 2D representation (Fig. 12).

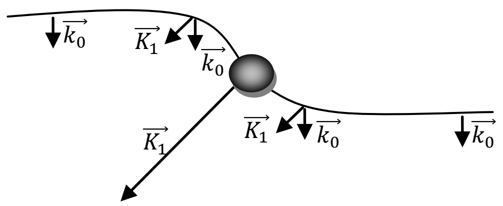


Fig. 12: Warped galaxy due to the difference of direction between internal gravitic field $\frac{K_1}{r^2}$ and external gravitic field k_0.

5.3. Galaxy as an equilibrium solution over time

Our solution, for the galaxies, represents an equilibrium state of the physical equations. If we look into details this equilibrium, one can note that the galaxies are then composed of two kind of equilibrium. Near the center of galaxies, the matter is directly maintained by the gravitational forces of the galaxies. But at the ends of the galaxies, the matter is maintained differently. It is essentially due to the external gravitic field. The consequence is then that, in this area, matter can have à priori any speed at any position because, just like in a magnetic field, the radius of curvature of the trajectory is imposed by the speed of the particle, but any speed is possible. So, one must now explain how one has the good speed at the good position. This can be explained by a filtering over the time. If the speed of the matter is smaller than the equilibrium, then the curvature's radius is also smaller than the equilibrium. The matter will be more and more close to the galaxy and will be caught by the internal gravitation field of the galaxy over the time. If the speed of the matter is greater than the equilibrium, then the radius is also greater than the equilibrium. The matter will be more and more far from the galaxy and will escape from the galaxy over the time. Ultimately, there is only matter in equilibrium (with the adequate speed and radius).
From this specificity in the external area, one can deduce another prediction:

More a galaxy is young; more the dispersion of the speeds of the satellite dwarf galaxies should increase.

6. Gravitic field and quantity of dark matter

We are now going to look at several situations that will give us some values of the quantities of dark matter in agreement with experimental observations.

6.1. 1st approximation

As one has seen it before, gravitic field is very weak compared to gravity field at our scale. In first approximation, one can keep only gravity field and neglected gravitic field for many situations (in fact, all situations in which dark matter assumption is useless). This approximation makes the linearized general relativity equivalent to the Newtonian laws (except for the concept of propagation of gravitational wave). Effectively, from the relation (V) of the linearized general relativity:

$$ds^2 = \left(1+\frac{2\varphi}{c^2}\right)c^2dt^2 - \frac{8\vec{H}.\overrightarrow{dx}}{c}cdt - \left(1-\frac{2\varphi}{c^2}\right)\sum(dx^i)^2$$

One retrieves the Newtonian approximation that is in low speed ($v \ll c$) and in neglecting gravitic field ($\|\vec{H}\|\sim 0$):

$$ds^2 \sim c^2dt^2\left(1+\frac{2\varphi}{c^2}\right)$$

In these approximations, the linearized general relativity gives then the same Newtonian expression of the component g_{00} of the metric ($g_{00}\sim 1+2\frac{\varphi}{c^2}$).

So, if one can neglect gravitic field, all current computation is always valid and even identical. We are going to see that effectively for many situations gravitic field can be neglected, in particular in solar system (Mercury precession, deviation of light near the sun…).

6.2. 2nd approximation

The precedent "Newtonian" approximation completely neglected gravitic field. Let's make an approximation which considers that gravitic field is weak compared to gravity field but not sufficiently to be neglected. We are going to search for an expression of g_{00} containing the new term g_{0i}. This approximation will be a simplified way to take into account the term g_{0i} in the traditional relation containing only g_{00}. It will allow obtaining orders of magnitude to continue to test the relevancy of our solution.

Our previous study shows that an external gravitic field $\overrightarrow{k_0}$, uniform at the scale of a galaxy, explains the flat rotation's speed. And we have seen that for $r \gg 15kpc$, one can only consider this uniform $\overrightarrow{k_0}$ (the internal gravitic field becomes too small). It is the domain of validity of the linearized general relativity. In electromagnetism, when an atom is embedded in a constant and uniform magnetic field $\vec{B}$, one can take for the potential vector $\vec{A}=\frac{1}{2}\vec{B}\wedge\vec{r}$ (BASDEVANT, 1986). So let's take for the potential vector $\vec{H}=\frac{1}{2}\vec{k}\wedge\vec{r}$. One can note that this definition implies that $\overrightarrow{rot}\,\vec{H}=\vec{k}$ in agreement with Maxwell equations of linearized general relativity, as one can see it in the following calculation of a specific configuration ($\vec{k}\perp\vec{r}$, $\vec{v}\perp\vec{k}$ and $\vec{v}\perp\vec{r}$):

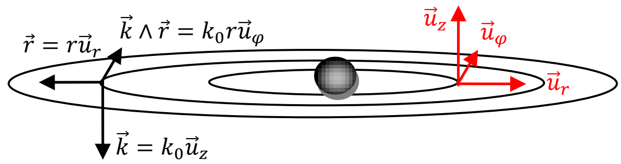


Fig. 13: The potential vector $\vec{H}=\frac{1}{2}\vec{k}\wedge\vec{r}$

Explicitly, in the cylindrical coordinate system $(\vec{u}_r;\vec{u}_\varphi;\vec{u}_z)$ one has the external gravitic field and its potential vector:

$$\vec{k}\sim\overrightarrow{k_0}=\begin{pmatrix}0\\0\\k_0\end{pmatrix}$$

$$\vec{H}=\begin{pmatrix}H_r\\H_\varphi\\H_z\end{pmatrix}=\frac{1}{2}\vec{k}\wedge\vec{r}=\frac{1}{2}\begin{pmatrix}0\\0\\k_0\end{pmatrix}\wedge\begin{pmatrix}r\\0\\0\end{pmatrix}=\frac{1}{2}\begin{pmatrix}0\\k_0 r\\0\end{pmatrix}$$

$$\overrightarrow{rot}\vec{H} = \begin{pmatrix} \frac{1}{r}\frac{\partial H_z}{\partial \varphi} - \frac{\partial H_\varphi}{\partial z} \\ \frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} \\ \frac{1}{r}\left(\frac{\partial (rH_\varphi)}{\partial r} - \frac{\partial H_r}{\partial \varphi}\right) \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -\frac{\partial (k_0 r)}{\partial z} \\ 0 \\ \frac{1}{r}\left(\frac{\partial (r k_0 r)}{\partial r}\right) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ k_0 \end{pmatrix}$$

That shows that one effectively has $\overrightarrow{rot}\vec{H} = \vec{k}$. As announced in our assumption (I), the approximation of a uniform gravitic field $\overrightarrow{k_0}$ is compliant with linearized general relativity.

If we assume that, in the previous cylindrical coordinate system, we have a particle speed $\vec{v} = v\vec{u}_\varphi$ with v constant (one can note that it is approximately the case for the matter in the galaxy for $r \gg 15kpc$), one has $\overrightarrow{grad}(\vec{H}.\vec{v}) = \frac{1}{2}k_0 v \vec{u}_r$:

$$\vec{H}.\vec{v} = \frac{1}{2}\begin{pmatrix} 0 \\ k_0 r \\ 0 \end{pmatrix}.\begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix} = \frac{1}{2}k_0 r v$$

and

$$\overrightarrow{grad}(\vec{H}.\vec{v}) = \begin{pmatrix} \frac{\partial}{\partial r} \\ \frac{1}{r}\frac{\partial}{\partial \varphi} \\ \frac{\partial}{\partial z} \end{pmatrix}\frac{1}{2}k_0 r v = \begin{pmatrix} \frac{1}{2}k_0 v \\ 0 \\ 0 \end{pmatrix}$$

Another explicit calculation gives $\vec{v} \wedge (\overrightarrow{rot}\vec{H}) = k_0 v \vec{u}_r$:

$$\vec{v} \wedge (\overrightarrow{rot}\vec{H}) = \vec{v} \wedge \vec{k} = \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 0 \\ k_0 \end{pmatrix} = \begin{pmatrix} v k_0 \\ 0 \\ 0 \end{pmatrix}$$

Finally, in this configuration, in the galaxy (for $r \gg 15kpc$) with $\vec{H} = \frac{1}{2}\vec{k} \wedge \vec{r}$ and $\vec{v} = v\vec{u}_\varphi$ (v and k constant), one has:

$$\vec{v} \wedge (\overrightarrow{rot}\vec{H}) = 2\overrightarrow{grad}(\vec{H}.\vec{v})$$

By this way, the movement equations become:

$$\frac{d^2\vec{x}}{dt^2} \approx -\overrightarrow{grad}\varphi + 4\vec{v} \wedge (\overrightarrow{rot}\vec{H}) \approx -\overrightarrow{grad}\varphi + 8\overrightarrow{grad}(\vec{H}.\vec{v})$$

$$\frac{d^2\vec{x}}{dt^2} \approx -\overrightarrow{grad}(\varphi - 8\vec{H}.\vec{v})$$

In this configuration, the linearized general relativity modifies the Newtonian potential as:

$$\varphi \to \varphi - 8\vec{H}.\vec{v}$$

And then it leads to an approximation of g_{00} containing g_{0i}:

$$g_{00} \sim 1 + 2\frac{\varphi}{c^2} - 16\frac{\vec{H}.\vec{v}}{c^2}$$

Remarks: This relation, valid in our specific configuration ($\vec{k} \perp \vec{r}$, $\vec{v} \perp \vec{k}$ and $\vec{v} \perp \vec{r}$) can be still a good approximation when the discrepancies to these angles are small.

6.3. Gravitic field and light deviation

Even if this relation is not valid for all situations, we can use it to obtain an order of magnitude of the deviation of light due to gravitic field (our quantities of dark matter). From this new relation $g_{00} \sim 1 + \frac{2\varphi}{c^2} - 16\frac{\vec{v}.\vec{H}}{c^2} = 1 + \frac{2}{c^2}(\varphi - 8\vec{v}.\vec{H})$ the traditional photon deviation expression due to gravitation $\delta_c = \frac{4\varphi}{c^2} = \frac{4GM}{c^2 r}$ is then modified by adding the term $\delta_t = 32\frac{\vec{v}.\vec{H}}{c^2}$ in our specific configuration.

For a photon, one can compute, with $v = c$:

$$\delta_t = 32\frac{\vec{v}.\vec{H}}{c^2} \sim 32\frac{\|\vec{H}\|}{c}$$

As said before, one can take for the potential vector $\vec{H} = \frac{1}{2}\vec{k} \wedge \vec{r}$. The photon deviation expression is then:

$$\delta_t \sim 32\frac{\|\vec{H}\|}{c} \sim 16\frac{kr}{c}$$

Let's apply this relation to our previous studied galaxies. Roughly, at about $r \sim 20kpc \sim 60 \times 10^{19}m$, one has, for nearly all the galaxies, the following value of gravitic field $k \sim 10^{-16.5}$. It gives the order of magnitude of the correction due to gravitic field:

$$\delta_t \propto 16\frac{10^{-16.5} * 60 \times 10^{19}}{3 \times 10^8} \propto 6.3 \times 10^{-5}$$

And the curvature (due to gravity field) is about (for a typical mass of $10^{41}kg$):

$$\delta_c \propto 4\frac{7 \times 10^{-11} * 10^{41}}{9 \times 10^{16} * 60 \times 10^{19}} \propto 0.05 \times 10^{-5}$$

It means that δ_t represents about 99.2% of the deviation ($\delta_t/(\delta_t + \delta_c)$).

In our solution, gravitic field generates nearly all the curvature of galaxies. In the frame of the dark matter assumption, it means that galaxies would be essentially composed of dark matter, in agreement with experimental data (NEYMAN et al., 1961) which gives at least 90%. This is the fifth main result of our study. One can note that this calculation explains that the phenomenon of gravitational lensing is extremely sensitive to "dark matter", much more than the ordinary matter (the gravity component) for large astrophysical structures.

<u>*About light deviation in solar system:*</u> With solar mass $M = 2 \times 10^{30}kg$ and solar radius $r = 7 \times 10^9$ and with the same previous value of galaxy gravitic field for the sun (which is certainly overestimated because Sun mass ~ 10^{-11} Galaxy mass) it gives:

$$\delta_t \propto 16\frac{10^{-16.5} * 7 \times 10^9}{3 \times 10^8} \propto 10^{-14}$$

and

$$\delta_c \propto 4\frac{7 \times 10^{-11} * 2 \times 10^{30}}{9 \times 10^{16} * 7 \times 10^9} \propto 10^{-6}$$

It means that δ_t represents about 10^{-6}% of the deviation ($\delta_t/(\delta_t + \delta_c)$). In solar system, the gravitic field of Sun does not modify the light deviation. There is no detectable "dark matter".

6.4. Gravitic field and Ω_{dm}

So far, we have treated the problem of dark matter in the context of galaxies. But dark matter is also needed in the description of the CMB. We will always address this problem with linearized general relativity. Despite this approximation, we will see that once again the resulting magnitudes are surprisingly good.

Einstein's equations, with the impulse-energy tensor T_{kp} and the sign convention of (HOBSON et al., 2009), are:

$$G_{kp} = R_{kp} - \frac{1}{2}g_{kp}R = -\frac{8\pi G}{c^4}T_{kp}$$

Let's write these equations in the equivalent form:

$$R_{kp} = -\frac{8\pi G}{c^4}(T_{kp} - \frac{1}{2}g_{kp}T)$$

In weak field and low speed ($T_{00} = \rho c^2 = T$), one can write

$$-\frac{1}{2}\Delta g_{00} = -\kappa(T_{00} - \frac{1}{2}g_{00}T)$$

With the traditional Newtonian approximation:

$$g_{00} = 1 + \frac{2}{c^2}\varphi$$

It gives:

$$\frac{1}{c^2}(\Delta\varphi) = \frac{8\pi G}{c^4}\rho c^2\left(1 - \frac{1}{2}\left(1 + \frac{2}{c^2}\varphi\right)\right)$$

$$\Delta\varphi = 8\pi G\rho\left(\frac{1}{2} - \frac{1}{c^2}\varphi\right)$$

$$\Delta\varphi = 4\pi G\rho\left(1 - \frac{2}{c^2}\varphi\right)$$

In this approximation ($\left|\frac{2}{c^2}\varphi\right| \ll 1$), it gives the Newtonian approximation (HOBSON et al., 2009):

$$\Delta\varphi = 4\pi G\rho$$

Now let's use the Einstein equations with our linearized general relativity approximation:

$$\frac{1}{2}\Delta g_{00} = \kappa\left(T_{00} - \frac{1}{2}g_{00}T\right) \text{ and } g_{00} = 1 + \frac{2}{c^2}(\varphi - 8\vec{v}.\vec{H})$$

It gives:

$$\frac{1}{c^2}\left(\Delta\varphi - 8\Delta(\vec{v}.\vec{H})\right) = \frac{8\pi G}{c^4}\rho c^2\left(1 - \frac{1}{2}\left(1 + \frac{2}{c^2}(\varphi - 8\vec{v}.\vec{H})\right)\right)$$

With the assumption of a uniform $\vec{v}$ (ie $\partial_i\vec{v}\sim 0$) and with Poisson equation (III) ($\Delta\vec{H} = \frac{4\pi G}{c^2}\rho\vec{u}$ with $\vec{u}$ the speed of the source), this equation becomes:

$$\left(\Delta\varphi - 32\pi G\rho\frac{\vec{v}.\vec{u}}{c^2}\right) = 8\pi G\rho\left(\frac{1}{2} - \frac{1}{c^2}(\varphi - 8\vec{v}.\vec{H})\right)$$

$$\Delta\varphi = 4\pi G\rho\left(1 - \frac{2}{c^2}(\varphi - 8\vec{v}.\vec{H})\right) + 32\pi G\rho\frac{\vec{v}.\vec{u}}{c^2}$$

In our approximation ($\left|\frac{2}{c^2}(\varphi - 8\vec{v}.\vec{H})\right| \ll 1$), it gives the linearized general relativity approximation:

$$\Delta\varphi = 4\pi G\rho + 32\pi G\rho\frac{\vec{v}.\vec{u}}{c^2}$$

We have then an equation in linearized general relativity approximation that can be interpreted as an idealization of the influence of visible matter ρ_b ("baryonic matter") for the first term "$4\pi G\rho$" and of the gravitic field ρ_{dm} (our dark matter explanation) for the second term "$32\pi G\rho\frac{\vec{v}.\vec{u}}{c^2}$".

$$\Delta\varphi = 4\pi G\left(\rho + 8\rho\frac{\vec{v}.\vec{u}}{c^2}\right) = 4\pi G(\rho_b + \rho_{dm})$$

It is a very interesting result. Even if it is an approximation, our idealization implies naturally to add a component similar to the traditional "ad hoc" dark matter term. One can then try to obtain an approximation of the Ω_{dm} term. Because of the disparities of the distribution of matter and its speed, one cannot use easily this relation to compute the equivalent dark matter quantities of gravitic field in our universe at current time. But at the time of CMB, distribution of matter can be considered homogeneous and speed of particles can be close to celerity of light $\|\vec{v}\|\sim\|\vec{u}\|\sim\alpha c$ with α a factor that must be close to 1. In this approximation, previous equation gives:

$$\Delta\varphi = 4\pi G(\rho + 8\rho\alpha^2) = 4\pi G(\rho_{b,CMB} + \rho_{dm,CMB})$$

It gives the very interesting ratio of gravitic field (equivalent to dark matter) compared to baryonic matter at the time of CMB:

$$\rho_{dm,CMB} \approx 8\alpha^2\rho_{b,CMB}$$

In term of traditional Ω_b, it means that in the approximation of linearized general relativity, one has:

$$\frac{\Omega_{dm}}{\Omega_b} \approx 8\alpha^2$$

The observations give (PLANCK Collaboration, 2014):

$$\frac{\Omega_b}{\Omega_{dm}} = \frac{\Omega_b h^2}{\Omega_{dm}h^2}\sim\frac{0.022}{0.12}\sim 5.45 \quad => \quad \alpha\sim 0.8$$

With our approximation, this ratio can be obtained with the particles speed $\|\vec{v}\|\sim\|\vec{u}\|\sim 0.8c$, about 80% of the light celerity.

The important result is not the accuracy of the value (because of our approximation, but one can note that it is not so bad) but it is the order of magnitude. This order of magnitude is impressive. This is the sixth main result of our study (and I recall that our explanation comes from native components of general relativity, without modification).

Remark: One cosmological assumption is to consider the Universe as an isotropic space. The gravitic field may appear as a contradiction with that. But, in fact, it is a contradiction only if all the individual gravitic "spins" are aligned. With the agitation of the Universe in the first moments, there is no chance that they are all aligned. The isotropy's assumption remains a priori valid.

7. A way to measure the gravitic fields

Some recent papers (CLOWE et al., 2006; HARVEY et al., 2015) seem to prove the existence of an exotic matter. The main assumption of these studies is that dark matter is either invisible ordinary matter (not yet detected gases) or either exotic matter (non-baryonic). With this assumption, assuming there is no exotic matter one deduces that the gravitational lens effect is mainly due to the invisible ordinary gas. They then show that the spatial offset between the distributions of the gas (which would explain the dark matter) and the observed mass (visible) is so great that they cannot coincide. One then conclude that dark matter cannot be invisible ordinary matter. Finally, what is shown is that:

If dark matter is either invisible ordinary matter or either non-baryonic matter then there is no doubt that it can be only non-baryonic.

The second result of these studies is that if dark matter is exotic matter (previous deduction) then the possibilities of an exotic particle will greatly reduce. Somehow, while supporting the assumption of dark matter, they make it more unlikely (but not actually impossible).

What about our solution? These studies do not address this solution. Indeed, the fundamental assumption of all these studies is that the gravitational lens effect is mainly due to the mass (the quantity of matter). That is why the previous deduction was:" the gravitational lens effect is mainly due to the invisible ordinary gas". But, as one shows in §6.3, at the scale of the galaxies, the light deviation is mainly due to the gravitic field. So, what has been attributed previously to an invisible gas must be, in our solution, attributed to the gravitic field (without added mass). The previous deduction is then:" the gravitational lens effect is mainly due to the gravitic fields of galaxies and clusters". It leads to two important consequences. First, because the gravitational lens effect is mainly explain without new mass, there is no more problem about an asymmetry of centroids. There is only one kind of matter, the ordinary one. Secondly (certainly the main consequence), is that the gravitational lens effect become the main way to measure the gravitic field. Just like in electromagnetism, the spectral shift can give several information (the value of the magnetic field for example), the gravitational lens can give several information (the value of the gravitic field for example). So the previous papers, instead of invalidate our solution, open a way to test our solution. To address our solution, the gravitational lens effect has just need to be interpreted not in term of new quantity of mass but in term of gravitic field (the second component of gravitoelectromagnetism required by general relativity). So, from the gravitational lens effect, one can expect to measure the gravitic field of the galaxies and the gravitic field of the clusters. From our study, one obtained the following approximation $\frac{10^{24.}}{r^2} < \frac{K_1}{r^2} < \frac{10^{24.9}}{r^2}$ for the internal gravitic field of galaxies. But as said, our approximation should underestimate the effective value (certainly by a factor around 5). So, from the gravitational lens effect, one can expect to measure a value of $\frac{10^{24.7}}{r^2} < \frac{K_1}{r^2} < \frac{10^{25.6}}{r^2}$. A second contribution should represent the gravitic field of the clusters.

8. Some possible experimental tests of gravitic field

Only direct measures could be a proof of the effective existence of the external gravitic field. On Earth the weakness of this term makes it difficult to detect it before a long time. Certainly, experiments on Lense-Thirring effect with very high precision could lead to direct measures of our gravitic field explaining dark matter (by adding, in a classical point of view, a force "$k_0 . v$" with $10^{-16.62} < k_0 < 10^{-16.3}$).

But right now, its existence could be indirectly tested by using the expected computed values of the gravitic fields « $\frac{K_1}{r^2}$» and $\overrightarrow{k_0}$ (explaining dark matter) to explain others phenomena. For example, gravitic field could play a role in the collisions between galaxies but also in the following situations:

The dynamics of internal organization of galaxies: By studying galaxies at different steps of evolution, one could idealize the evolution of this field $\vec{k}$ according with time for the same galaxy and then check that it is coherent with the evolution observed (for example precocity of organization according to the mass, because gravitic field should accelerate galaxies organization). In fact, gravity field implies a general precocity of organization for all large structures (for example why not for early super massive black holes)

The jets of galaxies: One could also try to correlate, statistically, the evolution of the field $\vec{k}$ within the galaxy and thermal agitation towards its center with the size of the jets of galaxies. A priori these matter jets should appear where the gravitic and gravity forces become sufficiently weak energetically compared to thermal agitation. Very close to the galaxy rotational axis, the gravitic force ($\overrightarrow{F_k} \propto \vec{v} \wedge \vec{k}$) becomes small (even if k grows, v is null on the axis), letting matter escape along this rotational axis (explaining the narrowness of the jets, the position of the jets at the galaxy center of rotation and the two opposite direction along the rotational axis). In fact, even the existence of jets can also be more easily explained with gravitic force. More the matter is close to the center, more it accumulates thermal agitation to compensate gravity and gravitic forces (to avoid collapsing). But very close to the center, gravitic force decreases (contrary to gravity force) because of the low speed in the center. Then this very energetic matter cannot be kept by only gravity force. The only exit for this matter very close to the center is to escape explaining the existence of galactic jets.

Influence on the "redshift": With the trio {mass, rotation speed and redshift}, one could compare our solution with dark matter assumption. Briefly, with φ_M the potential scalar associated with the measured mass of an object, v its rotation speed, φ_{DM} its potential scalar associated with the dark matter, $\vec{k}$ and $\|\vec{H}\|$ its gravitic field and its potential vector, one has for the theory with dark matter assumption (forces equilibrium and redshift definition):

$$\nabla\varphi_M + \nabla\varphi_{DM} = \frac{v^2}{r}$$

$$1 + z = \frac{\nu_E}{\nu_R} = \left(\frac{g_{00}(r_R)}{g_{00}(r_E)}\right)^{1/2} = \left(\frac{1 + \frac{2\varphi_M(r_R)}{c^2} + \frac{2\varphi_{DM}(r_R)}{c^2}}{1 + \frac{2\varphi_M(r_E)}{c^2} + \frac{2\varphi_{DM}(r_E)}{c^2}}\right)^{1/2}$$

And for our gravitic field (forces equilibrium and redshift definition):

$$\nabla\varphi_M + 4kv = \frac{v^2}{r}$$

$$1 + z = \frac{\nu_E}{\nu_R} = \left(\frac{g_{00}(r_R)}{g_{00}(r_E)}\right)^{1/2} = \left(\frac{1 + \frac{2\varphi_M(r_R)}{c^2} - \frac{16\vec{v}(r_R).\vec{H}(r_R)}{c^2}}{1 + \frac{2\varphi_M(r_E)}{c^2} - \frac{16\vec{v}(r_E).\vec{H}(r_E)}{c^2}}\right)^{1/2}$$

In our solution, there is a coupling between rotation speed and gravitation. In the theory with dark matter assumption, there is not this coupling. And between the two relations (forces equilibrium and redshift definition) the introduction of the corrective term seems not to be exactly the same. On one hand we have $\nabla\varphi_{DM}//kv = \|\vec{v} \wedge (\vec{\nabla} \wedge \vec{H})\|$ and on the other hand we have $\varphi_{DM}(r)//\vec{v}(r).\vec{H}(r)$ (and $\vec{\nabla}(\vec{v}.\vec{H}) \neq \vec{v} \wedge (\vec{\nabla} \wedge \vec{H})$ in general). These differences could make appear distinguishable situations with speed rotation.

9. External gravitic field versus dark matter

It is now interesting to make the comparison between the more widely accepted assumption (dark matter) and our solution (significant gravitic field).

In traditional approximation (that neglects gravitic field), general relativity can explain very well the inhomogeneities of CMB, but by introducing two new ad hoc unexplainable terms. The most immediate way to interpret them is to postulate, for one of them, the existence of a matter that is not visible (dark matter) and, for the other, an energy embedding our Universe. But these interpretations pose some problems. Let's see about dark matter, subject of our study. One of them is that dark matter should be sensitive only to gravitation (and strangely not to electromagnetism). A second one is about its distribution that is significantly different from the ordinary matter (subject of the same gravitation). A third one is about its quantity (five to six times more abundant than visible matter) in contradiction with the experimental results which show no dark matter. Sometimes, one compares the assumption of dark matter with the one that Le Verrier had made for the existence of a new planet (Neptune). This comparison is very interesting because it effectively can emphasize two fundamental differences between these two situations. The first is that in the case of Le Verrier gravitation worked very well for the other planets. There was therefore no reason to modify the idealization of gravitation for one particular planet. In the assumption of dark matter, it is different. Gravitation does not work for nearly all galaxies. The second is that, for the path of Uranus, the discrepancies were sufficiently small to be explained by the presence of a single planet (slight perturbation of the planet trajectory). This explanation was consistent with the observations and the fact that this planet could not yet be detected (small enough effect). For galaxies, the discrepancy with idealization is so great that the visible matter is almost nothing compared to the dark matter, which is surprisingly at odds with current observations. The solution I propose leads to a new interpretation of these terms, entirely explained by current general relativity. And furthermore it will avoid the previous problems (in agreement with current experimental results):

Ad hoc assumption solved: The assumption of dark matter is an "ad hoc" assumption and the origin of dark matter is, until now, unexplainable. In our study, we have seen that the gravitic field (that is a native general relativity component) of clusters is large enough not to be neglected and to explain “dark matter”. Furthermore, we have seen that several observations corroborate (and can be explained by) this origin of the external gravitic field.

Strange behavior solved: On one hand, dark matter makes the assumption of a matter with a very strange behavior because, contrary to all known matter, it doesn’t interact with electromagnetism. On the other hand, general relativity implies the existence of a significant gravitic field for clusters (and not a “dark” gravitic field with strange behavior). Because gravitic field is a component of gravitation, it explains why it doesn’t concern electromagnetism. The problem of a strange behavior of dark matter is then solved.

Contradiction with experimental observations solved: The dark matter assumption implies that we are embedded in nearly exclusively dark matter (ordinary matter should represent only a little part of the matter). Until now no experimental observation has revealed the main part of our universe. Inversely, external gravitic field (explaining the dark matter with $10^{-16.62} < k_0 < 10^{-16.3}$) is sufficiently small to be undetectable at our scale and sufficiently large to explain dark matter at large scale, in agreement with observations. In a dual interpretation (more theoretical than experimental) one can say that all our known theories have been built on ordinary matter. The dark matter assumption means that even if we are embedded in nearly exclusively dark matter, it doesn’t influence the theories at our scale. Astonishingly our theories only need dark matter for very large structure. It is a strange theoretical consistency. The gravitic field allows retrieving this theoretical consistency.

Dark matter distribution solved: Despite the fact that dark matter, just like visible matter, undergoes gravitation interaction, dark matter distribution is astonishingly different from visible matter. With gravitic field there is no more this problem. Gravitic field is applied on current distribution of visible matter. This component increases the effect of gravity field. It then doesn’t modify the distribution of matter, but it accelerates its organization. It even emphasizes the effect of a distribution in a plane perpendicular to gravitic field. As we have seen it, some observations seem to confirm these tendencies.

10. Conclusion

The linearized general relativity leads to an approximation of the gravitation, equivalent (in term of field equations) to electromagnetism. And just like the atomic spins can explain the magnetic field of magnets without adding a new term to the Maxwell idealization (or a new “dark charge”), the own gravitic field of large astrophysical structures can explain the dark matter (and certainly the dark energy (LE CORRE, 2015)) without adding new ad hoc terms to the Einstein idealization. Rather, the dark matter (and certainly the dark energy) would reveal this traditionally neglected component of general relativity. With our solution, dark matter and dark energy become two wonderful new proofs of general relativity idealization. Indeed, the classical tests of general relativity are usually associated with space curvature. But with our solution, general relativity reveals another specificity of its idealization, the gravitic field that becomes essential at large scale (about 25% of dark matter and 70% of dark energy).

To summarize, one can say that our solution, instead of adding matter, consists in taking into account gravitic fields. We have shown that to explain rotation speed of the ends of the galaxies, a gravitic field of about $10^{-16.5}$ is necessary. This value would be compliant with the expected gravitic field of clusters. This origin is also in agreement with the recent observation of a decreasing quantity of dark matter with the distance to the center of galaxies’ cluster. This value applied to satellite dwarf galaxies gives the expected rotation speed and applied to light deviation is equivalent to the expected quantity of “dark matter” inside the galaxies. The theoretical expression also allows explaining the rotation in a plane of the satellite dwarf galaxies and retrieving a quantity of “dark matter” in the CMB with a good order of magnitude. One can add that this value is small enough to explain why it has not been detected until now at our scale.

This component certainly intervenes in galaxies organization (precocity according to the mass, narrowness of the basis of the jets, collisions…). We have seen that it could explain the existence of galaxies with two portions of their disk that rotate in opposite directions and the existence of galaxies with a truly declining rotation curve. And in particular, our solution implies an important consequence that can differentiate our solution from the dark matter assumption. Statistically, there should be a correlation between the rotation’s planes’ orientation of dwarf galaxies and the distance of galaxies in which these dwarf galaxies belong.

Of course, this solution need to be further tested and developed but these first results are very encouraging. One can wonder why the traditional calculations that take into account this term of gravitic field never revealed this solution (and we saw that this solution is a solution of general relativity, just like the magnetic field of magnets is a solution of Maxwell equations). An explanation can be that the simulations are not enough detailed or complete. One can be skeptical of this explanation but first one cannot exclude this possibility. Secondly, to clarify my thinking, if we look at the situation of the magnetic field of the Earth, one has a similar problem. In a first approximation, the Earth is globally neutral and the matter's movement doesn't change its direction (rotation around the Sun and on itself). A conclusion could be that the magnetic field of the Earth should be negligible and it should never change its direction. Actually, the magnetic field is not negligible and its direction evolves with time. In fact, it can be explained by the complex movement of the matter inside the Earth (that is then not easy to detect and to idealize). And third, it can depend on another parameter, just like in a phase's transition. For example with magnetic materials, depending on the agitation, the spins can be in a state where there is no global field and when the temperature decreases they can make appear a global field. In one case, the simulations don't give any embedding magnetic field, in the other they do. So, one cannot exclude that we haven't enough knowledge of the complex movement of internal matter for the large astrophysical structure to make completely relevant simulations. I think it would be a mistake to dismiss the proposed solution only on that fact.

Taking into account significant gravitic fields should deeply change cosmology at greater scales than the galaxies, as the magnetic field at the microscopic level. For example, one may wonder what role the gravitic field could have on the geometry at the origin of the universe (flatness of the universe). One can also imagine that the value of this external gravitic field could be a signature of the space localization of galaxies (as a digital print). Another interesting point is that gravitic field can open a way to explain dark energy, creating by the same time a link between dark matter and dark energy.

11. Acknowledgements

I thank Dr Eric Simon for his encouragement and useful correspondence on this topic.

Tab. 2: Numerical approximations for $v_{halo}(r)$ and $v(r)$ used to compute $k(r)$ of Fig.5.

	$v_{halo}(y)$	$v(y)$
Curves in Fig.4	$-\frac{5}{54}y^6+\frac{115}{18}y^5-\frac{54065}{378}y^4+\frac{64285}{42}y^3-\frac{1828390}{189}y^2+\frac{3041650}{63}y$	$.07526914441y^8-3.199934769y^7+54.98000686y^6-486.9847011y^5+2305.246690y^4-4872.331582y^3-3569.566352y^2+49571.78060y+50000$

Tab. 3: Numerical approximations for $v_{halo}(r)$ and $v(r)$ used to compute $k_{exp}(r)$ of Fig. 8.

	$v_{halo}(y)$	$v(y)$
NGC 5055	$\frac{3726000}{313}*y-\frac{11320}{313}*y^3, y<5$ $-\frac{2230000}{313}+\frac{5064000}{313}*y-\frac{267600}{313}*y^2+\frac{6520}{313}*y^3, y<10$ $\frac{2815000}{313}+\frac{3550500}{313}*y-\frac{116250}{313}*y^2+\frac{1475}{313}*y^3, y<20$ $\frac{8495000}{313}+\frac{2698500}{313}*y-\frac{73650}{313}*y^2+\frac{765}{313}*y^3, y<30$ $24830000/313+1065000/313*y-19200/313*y\char`^2+160/313*y\char`^3, otherwise$	$71068.3101*y+275726.759*y^3, y<.5$ $274745.7897-1577406.427*y+3296949.476*y^2-1922239.558*y^3, y<.6$ $-123337.8881+413011.9624*y-20414.5077*y^2-79259.5671*y^3, y<1$ $-373366.8873+1163098.961*y-770501.5063*y^2+170769.4325*y^3, y<1.5$ $202116.4129+12132.36060*y-3190.439287*y^2+255.8621300*y^3, y<5$ $262248.7924-23947.06710*y+4025.446258*y^2-225.1969059*y^3, y<8$ $-49697.3974+93032.75412*y-10597.03140*y^2+384.0729965*y^3, y<10$ $411968.2980-45466.95452*y+3252.939462*y^2-77.59269895*y^3, y<15$ $89680.1135+18990.68234*y-1044.236329*y^2+17.90009639*y^3, y<20$ $232804.0362-2477.906045*y+29.1930908*y^2+.96060556e-2*y^3, y<30$ $260115.2720-5209.02962*y+120.2305432*y\char`^2-1.001921193*y\char`^3, otherwise$
NGC 4258	$\frac{127220328}{13325}*y+\frac{1507418}{13325}*y^3, y<2$	$202541.4800*y-254148.0049*y^3, y<.1$ $-263.8226943+210456.1609*y-79146.80830*y^2+9674.68944*y^3, y<3$

	$\frac{32381064}{13325}+\frac{78648732}{13325}*y+\frac{24285798}{13325}*y^2-\frac{508043}{2665}*y^3, y<4$ $-\frac{1301896}{123}+\frac{41742716}{2665}*y-\frac{8230414}{13325}*y^2+\frac{508408}{39975}*y^3, y<10$ $-\frac{968}{13}+\frac{33339924}{2665}*y-\frac{4029018}{13325}*y^2+\frac{147114}{66625}*y^3, y<15$ $-\frac{2199256}{533}+\frac{35499492}{2665}*y-\frac{365298}{1025}*y^2+\frac{227098}{66625}*y^3, y<20$ $-\frac{22448600}{533}+\frac{10137300}{533}*y-\frac{8545626}{13325}*y^2+\frac{543494}{66625}*y^3, y<25$ $\frac{79122400}{533}-\frac{2051220}{533}*y+\frac{3642894}{13325}*y^2-\frac{20698}{5125}*y^3, y<30$ $-\frac{15028544}{533}+\frac{36819372}{2665}*y-\frac{4203018}{13325}*y^2+\frac{33362}{13325}*y^3, y<40$ $53347520/533+11178348/2665*y-199578/2665*y^\wedge 2+33263/66625*y^\wedge 3, otherwise$	$322666.1470-112473.8089*y+28496.51494*y^2-2285.679817*y^3, y<5$ $-135294.1316+162302.3576*y-26458.71827*y^2+1378.002402*y^3, y<7$ $409186.8580-71046.63794*y+6876.852493*y^2-209.4057277*y^3, y<10$ $324159.1553-45538.32711*y+4326.021405*y^2-124.3780247*y^3, y<13$ $-3937.00148+30176.17064*y-1498.170729*y^2+24.96023518*y^3, y<20$ $195373.7921+279.551521*y-3.3397692*y^2+.4638564710e-1*y^3, y<25$ $196243.5202+175.18414*y+.8349260*y^\wedge 2-.9276955555e-2*y^\wedge 3, otherwise$
NGC 5033	$\frac{4823500}{313}*y-\frac{17660}{313}*y^3, y<5$ $-\frac{3855000}{313}+\frac{7136500}{313}*y-\frac{462600}{313}*y^2+\frac{13180}{313}*y^3, y<10$ $\frac{8470000}{313}+\frac{3439000}{313}*y-\frac{92850}{313}*y^2+\frac{855}{313}*y^3, y<20$ $\frac{4590000}{313}+\frac{4021000}{313}*y-\frac{121950}{313}*y^2+\frac{1340}{313}*y^3, y<30$ $39555000/313+524500/313*y-5400/313*y^\wedge 2+45/313*y^\wedge 3, otherwise$	$127886.6993*y+208453.2027*y^3, y<.5$ $219738.9134-1190546.782*y+2636866.962*y^2-1549458.105*y^3, y<.6$ $-95482.75032+385561.5368*y+10019.76474*y^2-90098.55117*y^3, y<1$ $-356911.4272+1169847.569*y-774266.2682*y^2+171330.1266*y^3, y<1.5$ $220238.0172+15548.67987*y-4733.67545*y^2+322.8837593*y^3, y<5$ $229445.0169+10024.47988*y-3628.835446*y^2+249.2277590*y^3, y<8$ $615622.9469-134792.2439*y+14473.25503*y^2-505.0260109*y^3, y<10$ $61694.0144+31386.43587*y-2144.612949*y^2+48.90292179*y^3, y<15$ $272319.0307-10738.56739*y+663.7206015*y^2-13.50449044*y^3, y<20$ $95433.49906+15794.26233*y-662.9208847*y^2+8.606201007*y^3, y<30$ $428274.4118-17489.82894*y+446.5488241*y^\wedge 2-3.721240201*y^\wedge 3, otherwise$
NGC 2841	$\frac{11433125}{679}*y-\frac{22765}{679}*y^3, y<5$ $-\frac{3493750}{679}+\frac{13529375}{679}*y-\frac{419250}{679}*y^2+\frac{5185}{679}*y^3, y<10$ $-\frac{333750}{679}+\frac{12581375}{679}*y-\frac{46350}{97}*y^2+\frac{2025}{679}*y^3, y<15$ $-\frac{40327500}{679}+\frac{20580125}{679}*y-\frac{857700}{679}*y^2+\frac{13875}{679}*y^3, y<20$ $\frac{96312500}{679}+\frac{84125}{679}*y+\frac{167100}{679}*y^2-\frac{3205}{679}*y^3, y<25$ $\frac{62718750}{679}+\frac{4115375}{679}*y+\frac{5850}{679}*y^2-\frac{1055}{679}*y^3, y<30$	$243232.3588*y+169191.0307*y^3, y<.2$ $4060.584737+182323.5877*y+304543.8552*y^2-338382.0614*y^3, y<.6$ $-122346.7788+814360.4056*y-748850.8411*y^2+246837.2144*y^3, y<1$ $124174.0771+74797.83757*y-9288.27327*y^2+316.3585146*y^3, y<4$ $118154.4151+79312.58412*y-10416.95991*y^2+410.4157346*y^3, y<10$ $711596.8253-98720.13905*y+7386.312414*y^2-183.0266764*y^3, y<15$ $-300751.9836+103749.6228*y-6111.671710*y^2+116.9285264*y^3, y<20$ $1312175.526-138189.5036*y+5985.284610*y^2-84.68741227*y^3, y<25$ $-352020.0201+61513.96177*y-2002.854001*y^2+21.82110251*y^3, y<30$ $202090.4505+6102.91471*y-155.8190990*y^\wedge 2+1.298492492*y^\wedge 3, otherwise$

	$-\frac{23748750}{97}+\frac{27011375}{679}*y-\frac{757350}{679}*y^2+\frac{7425}{679}*y^3, y < 35$ $30825000/97-5732875/679*y+178200/679*y\text{^}2-1485/679*y\text{^}3, otherwise$	
NGC 3198	$\frac{293412740}{6367}*y-\frac{9683185}{6367}*y^3, y < 2$ $-\frac{146442880}{6367}+\frac{513077060}{6367}*y-\frac{109832160}{6367}*y^2+\frac{8622175}{6367}*y^3, y < 4$ $\frac{1151080000}{19101}+\frac{115474900}{6367}*y-\frac{10431620}{6367}*y^2+\frac{1016390}{19101}*y^3, y < 10$ $\frac{753526000}{6367}+\frac{4525100}{6367}*y+\frac{663360}{6367}*y^2-\frac{31036}{6367}*y^3, y < 15$ $\frac{451261000}{6367}+\frac{64978100}{6367}*y-\frac{3366840}{6367}*y^2+\frac{58524}{6367}*y^3, y < 20$ $\frac{913981000}{6367}-\frac{4429900}{6367}*y+\frac{103560}{6367}*y^2+\frac{684}{6367}*y^3, y < 25$ $1085981000/6367-25069900/6367*y+929160/6367*y\text{^}2-10324/6367*y\text{^}3, otherwise$	$93824.86109*y-16552.19458*y^3, y < .7$ $-8178.94368+128877.4768*y-50075.16525*y^2+7293.12221*y^3, y < 2.5$ $166259.5811-80448.75314*y+33655.32672*y^2-3870.943384*y^3, y < 3.5$ $-68364.34133+120657.4660*y-23803.59303*y^2+1601.334687*y^3, y < 5$ $137821.3623-3053.95617*y+938.691490*y^2-48.150949*y^3, y < 10$ $47486.56233+24046.48382*y-1771.352509*y^2+42.18385099*y^3, y < 15$ $218829.5039-10222.10438*y+513.2200344*y^2-8.58442774*y^3, y < 20$ $148923.5942+263.780383*y-11.07416018*y^2+.1538077657*y^3, y < 25$ $151807.4887-82.28696*y+2.7685335*y\text{^}2-.3076148333e-1*y\text{^}3, otherwise$
NGC 7331	$\frac{4794600}{443}*y-\frac{3952}{443}*y^3, y < 5$ $-\frac{306000}{443}+\frac{4978200}{443}*y-\frac{36720}{443}*y^2-\frac{1504}{443}*y^3, y < 10$ $-\frac{3078000}{443}+\frac{5809800}{443}*y-\frac{119880}{443}*y^2+\frac{1268}{443}*y^3, y < 20$ $-39074000/4873+64690200/4873*y-1357800/4873*y\text{^}2+14600/4873*y\text{^}3, otherwise$	$212182.6079*y-128730.4317*y^3, y < .5$ $-36547.82379+431469.5507*y-438573.8856*y^2+163652.1587*y^3, y < 1$ $172982.5391-197121.5382*y+190017.2033*y^2-45878.20428*y^3, y < 1.5$ $13425.08720+121993.3659*y-22726.06625*y^2+1398.077914*y^3, y < 5$ $151016.1452+39438.73131*y-6215.139392*y^2+297.3494651*y^3, y < 8$ $358311.0499-38296.85847*y+3501.809375*y^2-107.5234025*y^3, y < 10$ $253301.4346-6793.973966*y+351.5209292*y^2-2.51378785*y^3, y < 15$ $332142.0734-22562.10177*y+1402.729450*y^2-25.87397720*y^3, y < 20$ $85247.19501+14472.12994*y-448.9821363*y^2+4.987882573*y^3, y < 30$ $219854.5902+1011.39042*y-.2908189*y\text{^}2+.2423490834e-2*y\text{^}3, otherwise$
NGC 2903	$21379/143640*y\text{^}5-850483/71820*y\text{^}4+1486603/4104*y\text{^}3-78143515/14364*y\text{^}2+52108475/1197*y$	$415459.9988*y-1545999.876*y^3, y < .1$ $-1730.507376+467375.2200*y-519152.2128*y^2+184507.5001*y^3, y < 1$ $185740.5740-95038.02464*y+43261.03254*y^2-2963.58188*y^3, y < 2$ $305413.2309-274547.0112*y+133015.5265*y^2-17922.66429*y^3, y < 3$ $-326611.8774+357478.0971*y-77659.50959*y^2+5485.673053*y^3, y < 5$ $485961.3992-130065.8686*y+19849.28352*y^2-1014.913154*y^3, y < 7$ $-147538.6900+141434.1694*y-18936.43614*y^2+832.0258747*y^3, y < 8$ $376318.2422-55012.18008*y+5619.357539*y^2-191.1321953*y^3, y < 10$ $179676.8668+3980.232569*y-279.8837259*y^2+5.509180172*y^3, y < 15$ $163516.5331+7212.29966*y-495.3548729*y^2+10.29742797*y^3, y < 20$ $354776.6712-21476.72119*y+939.0961703*y\text{^}2-13.61008942*y\text{^}3, otherwise$

NGC 3031	$26743.83559*y-439.013694*y^3, y < 2.5$ $-11157.53400+40132.87638*y-5355.61631*y^2+275.0684805*y^3, y < 5$ $13520.54625+25326.02821*y-2394.246683*y^2+77.6438387*y^3, y < 10$ $83547.94313+4317.808584*y-293.4246740*y^2+7.616438449*y^3, y < 15$ $123123.2679-3597.256355*y+234.2463215*y^2-4.109583667*y^3, y < 20$ $83671.29132+2320.53991*y-61.6434925*y\char`^2+.8219132331*y\char`^3, otherwise$	$189876.7738*y+253080.6547*y^3, y < .2$ $4823.935713+117517.7382*y+361795.1784*y^2-349911.3094*y^3, y < .6$ $-115914.8568+721211.7004*y-644361.4254*y^2+209064.5817*y^3, y < 1$ $91465.97151+99069.21553*y-22218.94053*y^2+1683.753411*y^3, y < 4$ $190434.0526+24843.14000*y-3662.419121*y^2+137.3764595*y^3, y < 10$ $331893.2604-17594.62234*y+581.357113*y^2-4.08274834*y^3, y < 15$ $463698.3927-43955.64878*y+2338.758875*y^2-43.13612084*y^3, y < 20$ $-14408.3742+27760.36621*y-1247.041875*y\char`^2+16.62722500*y^3, otherwise$
NGC 2403	$-2*y\char`^4+356/3*y\char`^3-2650*y\char`^2+83800/3*y$	$-282354289225649567/12952950506496*y\char`^5-22573477007046319/269853135552*y\char`^3+133187390454658231/25905901012992*y\char`^6+101042261599165/3747960216*y\char`^2+3929289082412429/67463283888*y\char`^4-5292189132515423/6476475253248*y\char`^7+53566280982301/138164805402624*y\char`^10-162623638202939/23027467567104*y\char`^9+4686121933585051/51811802025984*y\char`^8-2190475429/414494416207872*y\char`^13+150173866691/414494416207872*y\char`^12+44202063850/503217*y+549071/15942092931072*y\char`^14-6069909731069/414494416207872*y\char`^11$
NGC 247	$-12500/49*y\char`^2+82500/7*y$	$-575/1482624*y\char`^10+169175/5189184*y\char`^9-2065225/1729728*y\char`^8+10776925/432432*y\char`^7-1125775775/3459456*y\char`^6+431509825/157248*y\char`^5-38626919725/2594592*y\char`^4+65166877975/1297296*y\char`^3-10701265525/108108*y\char`^2+15901450/143*y$
NGC 4236	$-55/756*y\char`^6+110/63*y\char`^5-1480/189*y\char`^4-1685/42*y\char`^3-477865/756*y\char`^2+1975475/126*y$	$-10/693*y\char`^10+78425/99792*y\char`^9-604075/33264*y\char`^8+3859925/16632*y\char`^7-1420195/792*y\char`^6+40909925/4752*y\char`^5-31346025/1232*y\char`^4+277117825/6237*y\char`^3-367945145/8316*y\char`^2+29263225/693*y$
NGC 4736	$\frac{46336750}{691}*y-\frac{4876750}{691}*y^3, y < 1$ $-\frac{8530500}{691}+\frac{71928250}{691}*y-\frac{25591500}{691}*y^2+\frac{3653750}{691}*y^3, y < 2$ $\frac{15685500}{691}+\frac{35604250}{691}*y-\frac{7429500}{691}*y^2+\frac{626750}{691}*y^3, y < 3$ $\frac{27136875}{691}+\frac{24152875}{691}*y-\frac{3612375}{691}*y^2+\frac{202625}{691}*y^3, y < 5$	$550647.7605*y-5064776.046*y^3, y < .1$ $-10388.65628+862307.4487*y-3116596.883*y^2+5323880.231*y^3, y < .2$ $35233.72684+177971.7020*y+305081.8504*y^2-378917.659*y^3, y < .6$ $-115393.8023+931109.3480*y-950147.5600*y^2+318432.0135*y^3, y < 1$ $203629.0993-25959.35679*y+6921.144988*y^2-590.8875212*y^3, y < 5$ $84986.1054+45226.43958*y-7316.014285*y^2+358.2564302*y^3, y < 8$ $398938.2925-72505.63059*y+7400.494489*y^2-254.9314355*y^3, y < 10$

	$\frac{47360000}{691}+\frac{12019000}{691}*y-\frac{1185600}{691}*y^2+\frac{40840}{691}*y^3, y<10$ $90840000/691-1025000/691*y+118800/691*y\text{^}2-2640/691*y\text{^}3, otherwise$	$127510.2856+8922.77147*y-742.345718*y\text{^}2+16.49657151*y\text{^}3, otherwise$
NGC 300	$.8201058201*y\text{^}6-23.25396825*y\text{^}5+212.4074073*y\text{^}4-316.9047616*y\text{^}3-5996.084656*y\text{^}2+38123.01587*y$	$-115/189*y\text{^}6+2965/126*y\text{^}5-142595/378*y\text{^}4+137455/42*y\text{^}3-911945/54*y\text{^}2+476725/9*y$
NGC 2259	$325/12852*y\text{^}7-415/378*y\text{^}6+15785/918*y\text{^}5-388660/3213*y\text{^}4+4719565/12852*y\text{^}3-4030265/6426*y\text{^}2+13242700/1071*y$	$-.1005701747e-1*y\text{^}10+.6559331859*y\text{^}9-18.29303697*y\text{^}8+285.4915926*y\text{^}7-2735.333803*y\text{^}6+16578.82267*y\text{^}5-63092.40395*y\text{^}4+145111.6377*y\text{^}3-190899.9559*y\text{^}2+147769.3889*y$
NGC 3109	$14000/3*y$	$.1172414586e-1*y\text{^}9-.6726765728*y\text{^}8+16.43331861*y\text{^}7-223.1214090*y\text{^}6+1841.805727*y\text{^}5-9488.700957*y\text{^}4+30135.90051*y\text{^}3-56649.97586*y\text{^}2+62368.31962*y$
NGC 224	$\frac{241000}{39}*y-\frac{280}{39}*y^3, y<5$ $-\frac{31000}{13}+\frac{296800}{39}*y-\frac{3720}{13}*y^2+\frac{464}{39}*y^3, y<10$ $\frac{233000}{13}+\frac{59200}{39}*y+\frac{4200}{13}*y^2-\frac{328}{39}*y^3, y<15$ $-\frac{388000}{13}+\frac{431800}{39}*y-\frac{4080}{13}*y^2+\frac{224}{39}*y^3, y<20$ $\frac{60000}{13}+\frac{230200}{39}*y-\frac{720}{13}*y^2+\frac{56}{39}*y^3, y<25$ $1060000/13-129800/39*y+4080/13*y\text{^}2-136/39*y\text{^}3, otherwise$	$\frac{9329878000}{46689}*y-\frac{925858000}{46689}*y^3, y<1$ $-\frac{1262020000}{46689}+\frac{13115938000}{46689}*y-\frac{1262020000}{15563}*y^2+\frac{112054000}{15563}*y^3, y<4$ $\frac{21329980000}{46689}-\frac{3828062000}{46689}*y+\frac{149980000}{15563}*y^2-\frac{16838000}{46689}*y^3, y<10$ $\frac{31020000}{15563}+\frac{2543014000}{46689}*y-\frac{62389200}{15563}*y^2+\frac{4398920}{46689}*y^3, y<15$ $\frac{4883730000}{15563}-\frac{368612000}{46689}*y+\frac{2313600}{15563}*y^2+\frac{85400}{46689}*y^3, y<20$ $\frac{7792530000}{15563}-\frac{1677572000}{46689}*y+\frac{24129600}{15563}*y^2-\frac{1005400}{46689}*y^3, y<25$ $1508780000/15563+584578000/46689*y-6032400/15563*y\text{^}2+201080/46689*y\text{^}3, otherwise$

References

AMS Collaboration, "High Statistics Measurement of the Positron Fraction in Primary Cosmic Rays of 0.5–500 GeV with the Alpha Magnetic Spectrometer on the International Space Station", Phys. Rev. Lett. 113, 121101, 2014

ANGLOHER G., et al., "Results on low mass WIMPs using an upgraded CRESST-II detector", arXiv:1407.3146, 2014

BASDEVANT J. L., “Mécanique quantique”, ISBN 2-7298-8614-1, 1986 July

BEKENSTEIN J. D., "Relativistic gravitation theory for the modified Newtonian dynamics paradigm", Physical Review D 70 (8): 083509, 2004

BRUNI M., et al., "Computing general relativistic effects from Newtonian N-body simulations: Frame dragging in the post-Friedmann approach", arXiv:1306.1562v1, 2013

CARIGNAN C., PUCHE D., “HI studies of the Sculptor group galaxies II NGC 7793”, Astron. J. 100, 394 (1990)

CARMELI M., "Is galaxy dark matter a property of spacetime?", Int. J. Theor. Phys. 37 (10): 2621-2625, 1998

CARRICK J. D., COOPERSTOCK F. I., "General relativistic dynamics applied to the rotation curves of galaxies", arXiv:1101.3224, 2010

CLIFFORD M. WILL, “The confrontation between general relativity and experiment”, arXiv:1403.7377v1, 2014

CLOWE D. et al., "A direct empirical proof of the existence of dark matter", arXiv:0608407, 2006

COOPERSTOCK F. I., TIEU S., "General relativity resolves galactic rotation without exotic dark matter", arXiv:0507619, 2005

DAVIS J. H., "Fitting the annual modulation in DAMA with neutrons from muons and neutrinos", arXiv:1407.1052, 2014

DE BLOK W. J. G., "The core-cusp problem", arXiv:0910.3538, 2009

DISNEY M. J., "The Hidden Universe", London, Dent, 1984

EXIRIFARD Q., "Phenomenological covariant approach to gravity", General Relativity and Gravitation 43 (1): 93–106, 2010

HARRIS J., CHADWICK P.M., "Photon-axion mixing within the jets of active galactic nuclei and prospects for detection", Journal of Cosmology and Astroparticle Physics, 10: 018, 2014

HARTNETT J. G., "Spiral galaxy rotation curves determined from Carmelian general relativity", arXiv:0511756v3, 2006

HARVEY D. et al., "The non gravitational interactions of dark matter in colliding galaxy clusters", arXiv:1503.07675, 2015

HOBSON et al. “Relativité générale”, ISBN 978-2-8041-0126-8, 2009 December

HUI L., "Unitarity Bounds and the Cuspy Halo Problem", Phys. Rev. Lett. 86: 3467–3470, 2001
HUTSEMEKERS D., "Evidence for very large-scale coherent orientations of quasar polarization vectors", Astronomy and Astrophysics, 1998
HUTSEMEKERS D. et al., "Mapping extreme-scale alignments of quasar polarization vectors", arXiv:0507274v1, 2005
IBATA N. et al., “Velocity anti-correlation of diametrically opposed galaxy satellites in the low-redshift Universe”, Nature, July 2014
JAUZAC M. et al., “Hubble Frontier Fields : A High Precision Strong Lensing Analysis of Galaxy Cluster MACSJ0416.1-2403 using ~200 Multiple Images”, MNRAS, 443, 1549, 2014
KENT S. M., “Dark matter in spiral galaxies. I - Galaxies with optical rotation curves”, The astronomical journal, Volume 91, Number 6, 1986 June
KENT S. M., “Dark matter in spiral galaxies. II - Galaxies with H I rotation curves”, The astronomical journal, Volume 93, Number 4, 1987 April
KLYPIN A., et al., "Where are the Missing Galactic Satellites?", Astrophysical Journal 522 (1): 89–92, 1999
LE CORRE S., "Dark energy, a new proof of the predictive power of General Relativity", ensl-01122689v1, 2015 March
LETELIER P. S., "Rotation curves, dark matter and general relativity", International Astronomical Union, doi:10.1017/S1743921307005650, 2006
MASHHOON B., “Gravitoelectromagnetism: A brief review”, arXiv:0311030v2, 2008 April 17
MATEO M. L., "Dwarf Galaxies of the Local Group", Annual Review of Astronomy and Astrophysics 36 (1): 435–506, 1998
MCKEE M., "Gravity theory dispenses with dark matter", New Scientist, 2008 July 26
MILGROM M., "A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis", The Astrophysical Journal 270: 365–370, 1983
MOFFAT J. W., "Scalar-Tensor-Vector Gravity Theory", Journal of Cosmology and Astroparticle Physics, 03: 004, 2006
MOORE B., et al., "Dark Matter Substructure within Galactic Halos", The Astrophysical Journal Letters 524 (1): L19–L22, 1999
NEGI P. S., "Rotation curves of spiral galaxies: A general-relativistic model", arXiv:0405572v1, 2004
NEYMAN J. , PAGE T., SCOTT E., The Astronomical Journal, Volume 66,633, 1961
PLANCK Collaboration, "Planck 2013 results. XVI. Cosmological parameters", arXiv:1303.5076v3, 2014 March 21
RODRIGUES D. C., et al, "Galaxy rotation curves from general relativity with infrared renormalization Group effects", arXiv:1102.2188v1, 2011
RUBIN V. et al. "Rotational Properties of 21 Sc Galaxies with a Large Range of Luminosities and Radii from NGC 4605 (R = 4kpc) to UGC 2885 (R = 122kpc)", The Astrophysical Journal, 238:471, 1980
SANDERS R. H., "A tensor-vector-scalar framework for modified dynamics and cosmic dark matter", arXiv:0502222v2, 2005
SOFUE Y., RUBIN V., “Rotation curves of spiral galaxies”, Annu. Rev. Astron. Astrophys. 39: 137-174, 2001
TAYLOR A. N., et al., "Gravitational Lens Magnification and the Mass of Abell 1689", The Astrophysical Journal 501 (2): 539, 1998
TULLY B. R., et al., "Two planes of satellites in the Centaurus A Group", arXiv:1503.05599v1, 2015
VAN ALBADA T. S., et al., "Distribution of dark matter in the spiral galaxy NGC 3198", The Astrophysical Journal 295: 305-313, 1985
WRIGHT A. E., DISNEY M. J., "Universal gravity: was Newton right? ", Proc. ASA 8 (4): 334-338, 1990
WU X. et al. "A comparison of different cluster mass estimates: consistency or discrepancy?", Monthly Notices of the Royal Astronomical Society 301 (3): 861–871, 1998
ZARITSKY D. et al. "More Satellites of Spiral Galaxies", The astrophysical journal, 478:39–48, 1997 March 20
ZWICKY F., "On the Masses of Nebulae and of Clusters of Nebulae", The Astrophysical journal, 86:217, 1937