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ADDENDUM TO : ORTHONORMAL BASES OF REGULAR WAVELETS IN SPACES OF HOMOGENEOUS TYPE

PASCAL AUSCHER AND TUOMAS HYTÖNEN*

Abstract. We bring a precision to our cited work concerning the notion of “Borel measures”, as the choice among different existing definitions impacts on the validity of the results.

We wish to bring a precision to our work [1]. The same remarks apply to the follow-up article [2]. Proposition 4.5 in [1] states that if \( \mu \) is a non-trivial Borel measure on a quasi-metric space \( X \), finite on bounded Borel sets, and \( 1 \leq p < \infty \), then Hölder-\( \eta \)-continuous functions of bounded support are dense in \( L^p(\mu) \), where \( \eta \) is the Hölder exponent of the splines constructed in [1].

Depending on the meaning of “Borel measure”, as different definitions can be found in the literature, the result is correct or wrong.

If \( \mu \) is a \( \sigma \)-additive measure on the Borel \( \sigma \)-algebra, then the result with the given proof is correct, as the measurable sets coincide with the Borel sets. In that case, all our results are valid as stated.

However, if \( \mu \) is an outer measure or a \( \sigma \)-additive measure on \( X \) for which the Borel sets are \( \mu \)-measurable, then for the first sentence of the proof to be valid one must add the condition that for every \( \mu \)-measurable set \( A \) (in the sense of Caratheodory for the outer measure case), there is a Borel set \( B \supseteq A \) such that \( \mu(A) = \mu(B) \), and the rest of the proof goes through. In [3] for example, Borel outer measures are called regular if this condition holds for all \( A \) (not necessarily \( \mu \)-measurable). With such a definition of a Borel measure, this regularity condition should be added to our statements. Without regularity, the correct conclusion of Proposition 4.5 is density in the space of \( L^p \) functions having a Borel measurable representative. Thus, our wavelet representations are valid for functions in this subspace and \( 1 < p < \infty \). This is enough for many purposes.

References


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