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# COMPARISON OF THE EFFECT OF HAMMER STRIKING IRREGULARITIES AND MISTUNING ON THE DOUBLE DECAY OF PIANO TONES

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**Abstract:** A characteristic of the piano tones is their double decay (the initial sound followed by the “aftersound”), related to the existence of two polarizations. The goal of this study is to understand and to compare more quantitatively than the previous work, the influence of parameters which are under the control of the piano tuners: the mistuning and irregularities in striking action of the hammer. We simulated on a computer the vibration of an unison group of strings, which includes the dominant damping at the bridge as well as the vertical coupling between strings. Excitation irregularities were simulated by varying the initial displacement and velocities of the strings. The study analyses the ratio of the aftersound to the immediate sound, the compound decay, and the amount of destructive interference. This interference can be seen where the slope of the decay curve changes, and is most pronounced at the bridge, where the strings are dynamically coupled. This comparison leads to the conclusion that if necessary, the piano tuner can to a certain extent correct for a poorly adjusted hammer by modifying the mistuning. But all hammers properly adjusted are essential for a constant and uniform level for all notes, which is necessary for a very good piano.

## MODELLING AND SIMULATION OF A UNISON GROUP OF STRINGS

Pianos are built, for the 60 higher notes, with three strings for each key. In our simulation, we restrict ourselves to a set of two strings, which we call a « doublet ». To obtain numerical results, we choose to simulate the vibration of a doublet associated to the E<sub>4</sub> flat of the piano, which is the key which was used in the experimental part of our study.

### Modelling of the bridge/soundboard system.

We choose to characterize the dynamic behaviour of the bridge/soundboard system by its complex impedance  $Z_{br}$  in the direction normal to the soundboard. If we apply a sinusoidal force to the bridge,  $Z_{br}$  is the complex quotient of this force and the velocity of the bridge:

$$Z_{br} = \frac{\text{Force}}{\text{Velocity}} = \frac{F \cdot e^{j\alpha x}}{V \cdot e^{j(\alpha x - \varphi)}}$$

Some numerical values of  $Z_{br}$  have been measured by Wogram<sup>1</sup>, which enable us to estimate and adjust our simulation parameters.

### Modelling of the strings

We assume that the decay of the movement of the strings in a free vibration mode is mostly due to the deformations of the bridge, transmitted to the soundboard, which is taken into account in the resistive part of the bridge impedance (its real part). Thus all the other dissipations (air/string friction, thermoelasticity, viscoelasticity) are negligible compared to the energy loss by soundboard radiations.

Considering only the linear part of the equation of a string motion, we obtain the familiar equation:

$$y(x, t) = \sum_{n=0}^{+\infty} A_n \cdot \sin(k_n x) \cdot \cos(\omega_n t + \varphi_n)$$

where  $y$  is the transverse displacement of an arbitrary cross-section of the string function of its longitudinal coordinate  $x$  and time  $t$ . It is the sum of an infinity of undamped oscillators of angular frequency  $\omega_n$  named the normal modes of the string, whose coefficients ( $A_n, \varphi_n$ ) are set by the initial

conditions. To simplify the model, like Weinreich<sup>2</sup>, we decide to take only one normal mode into account, in our case the fundamental, although the model is equally valid for any higher-order mode. As a result, we decide to represent each string by a *1st order mass/string undamped harmonic oscillator*.

### Equations of motion of the complete system

We obtain for the complete system of two strings coupled on the bridge the model in the figure 1. Each mass/spring system  $((M_1, K_1)$  and  $(M_2, K_2))$  represents a string, whose positions are named  $y_1$  and  $y_2$ ;  $Z_{br}$  is the complex impedance of the bridge/soundboard system, with  $y_{br}$  the displacement of the bridge.

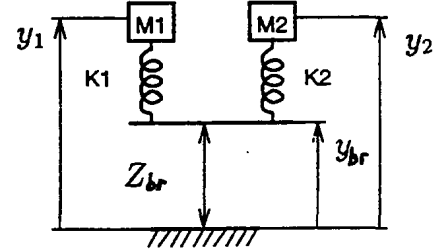


Figure 1: model of the « doublet »

For this system of three degrees of freedom we need three equations of motion. The dynamic equilibrium of the two masses  $M_1$  and  $M_2$  gives the first two of them. The third one is obtained from the definition of  $Z_{br}$ . The intensity of the force applied by the two strings on the bridge is:

$$F_{str/br} = K_1(y_1 - y_{br}) + K_2(y_2 - y_{br}).$$

We then obtain the system:

$$\begin{cases} M_1 \ddot{y}_1 + K_1 y_1 - K_1 y_{br} = 0 \\ M_2 \ddot{y}_2 + K_2 y_2 - K_2 y_{br} = 0 \\ V_{br} \cdot Z_{br} = K_1(y_1 - y_{br}) + K_2(y_2 - y_{br}) \end{cases}$$

We research a complex solution for  $y(t)$  of the form  $y = p.e^{\lambda t}$ , with  $\lambda = -\alpha + j\omega$ , where  $\alpha$  is the damping rate and  $\omega$  the angular frequency. In our system, the value of  $\omega$  is very close to the fundamental angular frequency of E<sub>4</sub> flat ( $\omega_0 = 1954 \text{ rad.s}^{-1}$ ). Thus,  $0 < \alpha < 10$  (familiar values), so  $\omega \gg \alpha$  which enables us to approximate  $\lambda$  by  $j\omega_0$  so that the velocity of the bridge is:

$$V_{br} = \frac{dy_{br}}{dt} = \lambda y_{br} \approx j\omega_0 y_{br}.$$

With this approximation, we evaluate easily  $y$  in function of  $y_1$  and  $y_2$ :

$$y_{br} = \frac{K_1 y_1 + K_2 y_2}{K_1 + K_2 + j\omega_0 Z_{ch}}$$

The problem is then reduced to a set of two equations representing a familiar undamped system of 2 degrees of freedom of the form  $\mathbf{M}\ddot{\vec{Y}} + \mathbf{K}\vec{Y} = 0$ , with  $\vec{Y} = (y_1, y_2)^t$ ,

$$\mathbf{M} = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} \text{ and } \mathbf{K} = \frac{1}{K_1 + K_2 + j\omega_0 Z_{ch}} \begin{pmatrix} K_1(K_2 + j\omega_0 Z_{ch}) & -K_1 K_2 \\ -K_1 K_2 & K_2(K_1 + j\omega_0 Z_{ch}) \end{pmatrix}$$

To resolve it, we seek solutions of the form  $\vec{Y} = \vec{P}.e^{\lambda t}$ , where  $\lambda$  and  $\vec{P}$  are respectively the eigenvalues and the eigenvectors of the  $\mathbf{M}^{-1}\mathbf{K}$  matrix. Then,  $y_1$  and  $y_2$  comes from the real part of  $\vec{Y}$ . They are the sum of two damped sinusoids which form the two normal modes of the dynamic system constituted by the two strings coupled on the bridge:

$$\begin{aligned} y_1 &= A|P_{11}|e^{-\alpha_1 t} \cos(\omega_1 t + \varphi_1 + \psi_{11}) + B|P_{12}|e^{-\alpha_2 t} \cos(\omega_2 t + \varphi_2 + \psi_{12}) \\ y_2 &= A|P_{21}|e^{-\alpha_1 t} \cos(\omega_1 t + \varphi_1 + \psi_{21}) + B|P_{22}|e^{-\alpha_2 t} \cos(\omega_2 t + \varphi_2 + \psi_{22}) \end{aligned}$$

where  $A$ ,  $B$ ,  $\varphi_1$  and  $\varphi_2$  come from the initial conditions,  $(\alpha_i, \omega_i)$  are the rate of decay and the angular frequency of the system's  $i^{\text{th}}$  normal mode of vibration ( $i \in \{1, 2\}$ ),  $\vec{P}_i = (|P_{1i}|e^{j\psi_{1i}}, |P_{2i}|e^{j\psi_{2i}})'$  is the eigenvector related to the normal mode  $i$ .

### Simulation program - Introduction of the mistuning and irregularities in hammer striking

To introduce the mistuning in the model, we choose a dimensionless parameter called  $\varepsilon$ . If  $\omega_{01}$  and  $\omega_{02}$  are respectively the angular frequencies of the fundamental modes of the two strings,  $\varepsilon$  is defined by:

$$\omega_{02} = (1 + \varepsilon)\omega_{01}$$

Knowing that the fundamental angular frequency of a typical mass  $M$ /string  $K$  system is  $\omega_0 = \sqrt{K/M}$ , we tune the two oscillators of our model by setting  $M_1 = M_2 = \mu$  (the mass per unit length of the string, which is the same for both),  $K_1 = \mu\omega_{01}^2$  and  $K_2 = \mu\omega_{02}^2 = \mu(1 + \varepsilon)^2\omega_{01}^2$ .

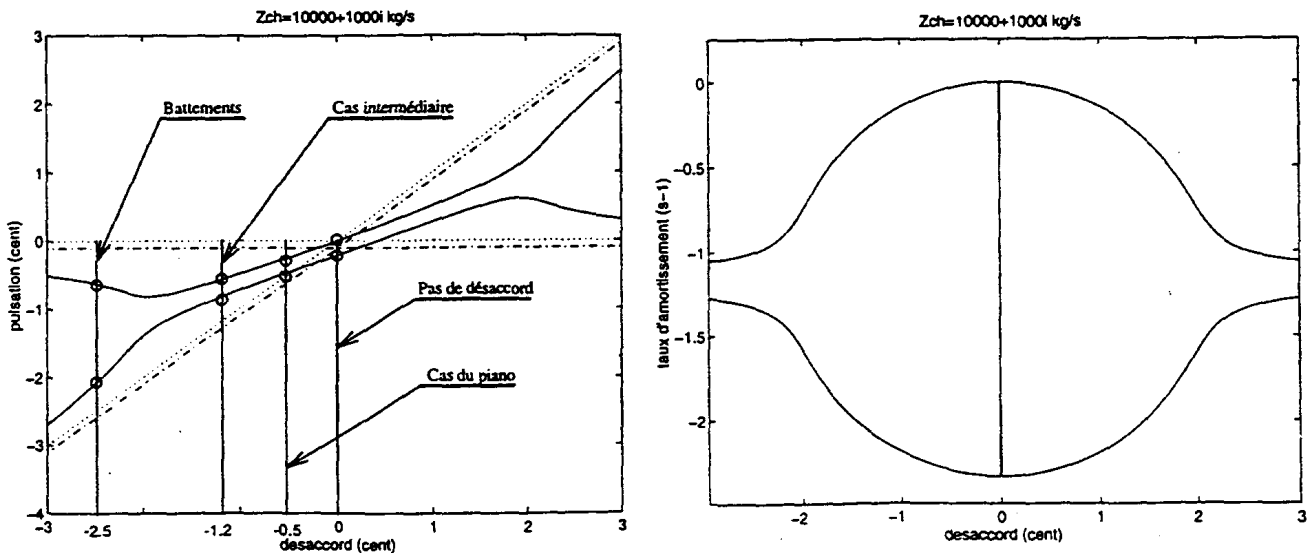
The easiest way to introduce the hammer striking irregularities in the model is to give different initial conditions to the two strings. In this study, we choose to adjust only the two initial positions,  $(y_{01}$  et  $y_{02})$  and to set the initial velocities of the strings to zero. Because the system is linear, it is enough to fix  $y_{01}$  and adjust the value of  $y_{02} \in [0, y_{01}]$ .

## SIMULATION RESULTS

As mentioned before, the following results correspond to a « doublet » tuned on the  $E_4$  flat of a grand piano, with  $\mu = 6,17 \cdot 10^{-3} \text{ kg.m}^{-1}$ ,  $\omega_0 = 1954 \text{ rad.s}^{-1}$  et  $Z_{ch} = 10000 + 1000j \text{ kg.s}^{-1}$  (from Wogram<sup>1</sup>).

### Angular frequencies and decay rates of the two normal modes of the « doublet »

The simulation enables firstly to demonstrate the dependance of the angular frequencies and the damping rates on mistuning, and to confirm the results of Weinreich<sup>2</sup>.



**Figure 2:** angular frequencies and damping rates of the two normal modes of vibration of the « doublet », as a function of the mistuning  $\varepsilon$ . The dotted curves gives the behaviour in absence of coupling, the dotted-dashed ones represents the strings on two rigid ends.

Considering the left diagram of the figure 2, two distinct areas appear:

- for  $\varepsilon$  between -2 and 2 cent, the angular frequencies of the two modes are separated by approximately 0,4 cent. This is the area where the tuner has some freedom for fixing the amount of mistuning of the note. Indeed 0,4 cent represents a beat period of 14 seconds at  $1954 \text{ rad.s}^{-1}$ , a duration which is comparable to the one of the piano sound, which then is not perceived as a beat by the listener. In addition, Marandas<sup>3</sup>, professional tuner, mesured 0,5 cent as an average value for the mistuning. On the other hand, he found that it exists an area where the note is out of tune, without the presence of any audible beats. This area is probably located between 1 and 2 cent.

- for  $|\varepsilon| > 2$  cent, the difference between the two angular frequencies becomes significant to produce perceptible beats.

Concerning the damping rates shown on the right diagram of figure 2, one may observe that if the mistuning  $\varepsilon$  increases, then the difference between the two rates decreases, that is to say that the two decays tend to be similar.

### Qualitative interpretation of the motion of a doublet - case of a mistuning equal to zero

The obvious case of a zero mistuning enables us to have a intuitive understanding of the double decay phenomenon. (Figure 3) The two strings are left without initial velocity, with a small striking irregularity ( $y_{01} = 1 \text{ mm}, y_{02} = 0,8 \text{ mm}$ ). At the beginning of the motion, the hammer forces the two strings to vibrate in phase, with different amplitudes. As their mechanical energy dissipation has the same rate, their amplitude decreases rapidly until the smallest of the two amplitudes (that is to say string 2) reaches zero. After that, the bridge goes on moving under the influence of string 1, and drags the second one in opposite phase. This shows us the superposition of two normal modes of vibration:

- the first mode, *symmetric*, is preponderant during the immediate sound. It has an important decay rate
- the second mode, *antisymmetric*, dominates the aftersound. It has a small decay rate.

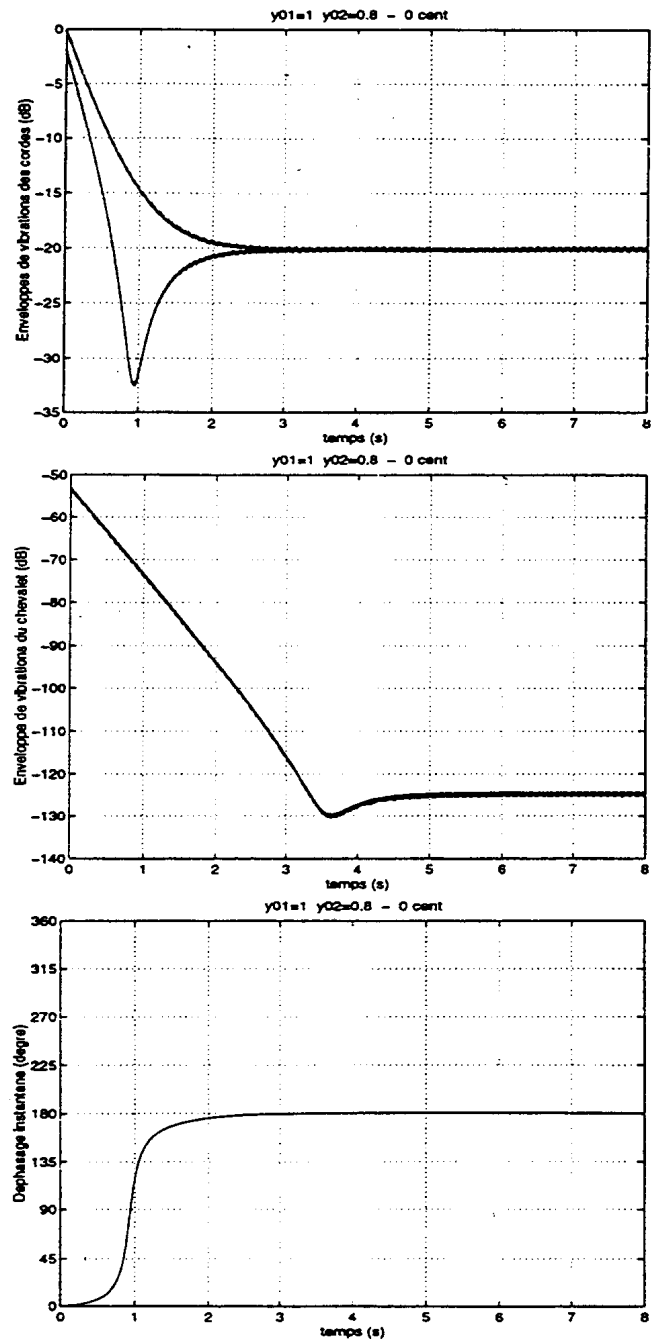


Figure 3: Vibration amplitude of the two strings ( $y_1$  and  $y_2$ , top diagram), of the bridge ( $y_{br}$ , center) and phase relation between the two strings (bottom), when  $\varepsilon$  equals zero

## Comparative effect of the mistuning and the hammer striking irregularities

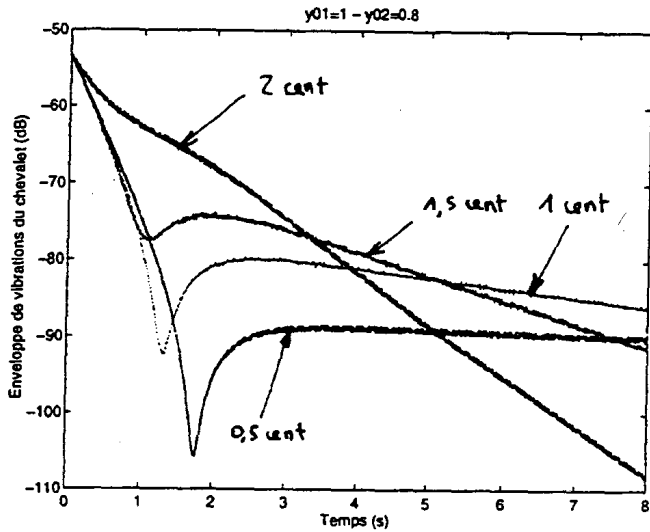


Figure 4: Vibration amplitude of the bridge for various values of the mistuning ( $y_{01} = 1$  mm;  $y_{02} = 0,8$  mm)

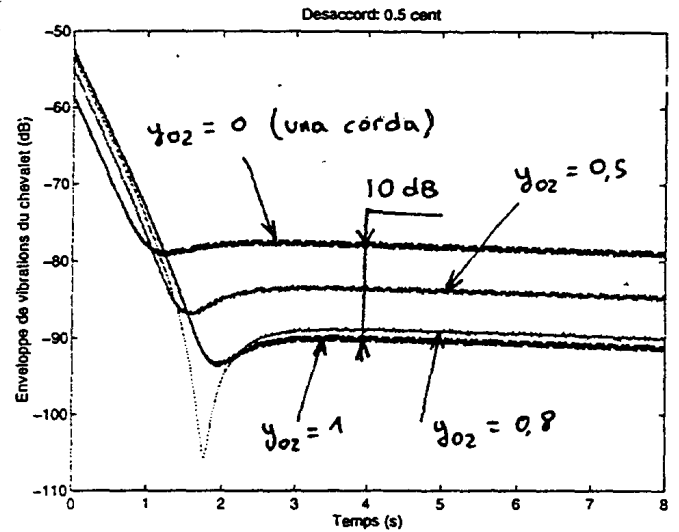


Figure 5: Vibration amplitude of the bridge for various values of the striking irregularities ( $\epsilon = 0,5$  cent)

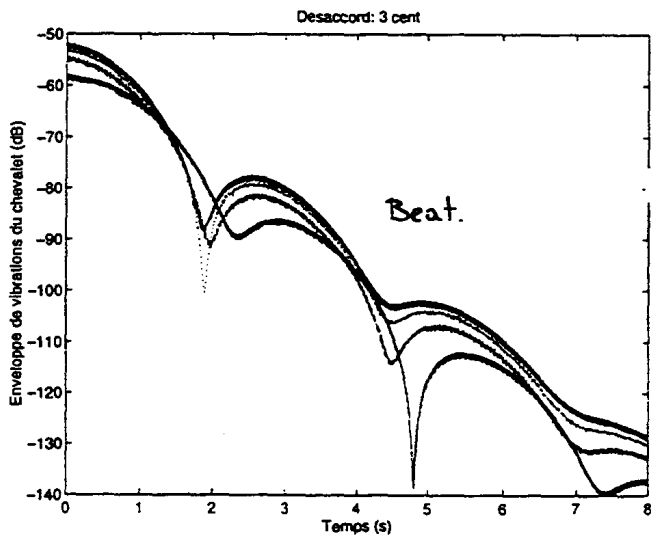


Figure 6: Vibration amplitude of the bridge for various values of the striking irregularities ( $\epsilon = 3$  cent)

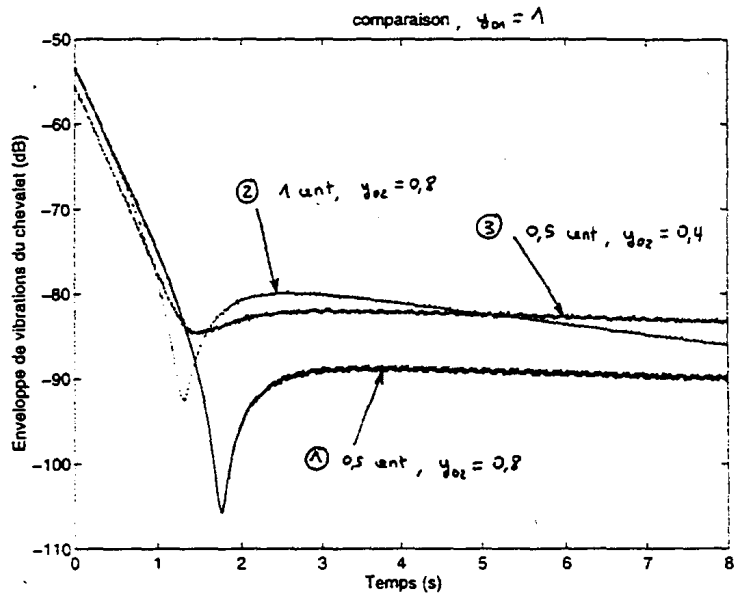


Figure 7: Comparison: vibration amplitude of the bridge for a note with striking irregularities compensate or not by the mistuning.

For this study, the suitable quantity is the vibration amplitude of the bridge (shown for different values of the parameters in figures 4,5,6 and 7), because it is the one in our model which is the nearest to the sound pressure level signal, and then to the sound which is listened when one tunes the piano.

We can notice, with the help of the figures 4,5,6 and 7, that:

- *Mistuning* is responsible essentially of the rate of the two decays. A small mistuning leads to a pronounced double decay. At the opposite if  $\epsilon$  increases, the change of slopes tends to diminish (soften), to reach first a curve with a single slope, and then beats. (Figures 4 and 6)

- The aftersound level is mainly linked with the *striking irregularities*. The more different are the initial conditions between the two strings, the more the aftersound level is increased in comparison with the one of the immediate sound. (It is in this case the antisymmetric mode which is mainly excited)(figure 5)
- The effect of the *una-corda* pedal appears clearly in the simulation. This pedal mechanically shifts the action sideways so that only two (or three) strings are struck by the hammer. It allows the player to enhance the aftersound level, increasing the dynamic range of the instrument when played *pianissimo*. For exemple, figure 5, we see that the *una-corda* pedal leads to a 10 dB aftersound gain. We could compare its effect to an extreme striking irregularity where is excited the antisymmetric mode.
- When tuning a piano, a tuner is supposed to regulate the sound level from key to key, in order to have, in a given nuance of playing, two notes in the same intensity. When he doesn't want to touch the mechanism of the action to adjust the irregularities, it is well known that he can increase the mistuning of one key to compensate its irregularities. That is shown on figure 7, where the aftersound of curve 1 ( $\varepsilon = 0,5 \text{ cent}$ ,  $y_{02} = 0,8 \text{ mm}$ ) has been enhanced to the position of curve 2 by increasing the mistuning to 1 cent. Curve 2 has almost the same evolution as curve 3, which corresponds to a small mistuning ( $\varepsilon = 0,5 \text{ cent}$ ) but with large striking irregularities ( $y_{01} = 1 \text{ mm}$ ,  $y_{02} = 0,4 \text{ mm}$ ). So we can think that the sound corresponding to curve 2 has, for the first 8 seconds, almost the same level as the sound of curve 3. However, this operation is not equivalent to a proper adjustment of the regularity of the action of the whole keyboard. This adjustment would lead to a more uniform temporal evolution of the sound, especially for the end of the decay. (Note in particular that the aftersound in curve 2 decreases faster than the one in curve 3, which can be noticeable after 7 seconds.) In addition, even if the piano sounds tuned, it is probably difficult for the player to execute pronounced nuances; since this case tends to be similar to the case where the *una-corda* pedal is down.

## CONCLUSION

With this study, we can understand more precisely than before that an excellent tuner can adjust the unisons so as to compensate for hammer irregularities, making the total aftersound uniform from note to note. This is a first step in understanding how a tuner carries out when tuning a piano. For that, one can firstly define what tuners call the « quality of a unison », which means finding objective and quantitative criterias to decide that a piano is tuned. Our simulation needs to be improved by taking into account some of the higher-order normal modes and the existence of a third string, with hammer striking irregularities modelled with realistic hammer deformation laws. Psychoacoustics tests based on the resulting sounds, with the help of piano tuners, would enable to identify what characteristics of the sound are significant to tune the piano.

While working on this subject (Rousseau<sup>4</sup>), we did additional experiments which show us that there is also double decays of vibration for an isolated string. Weinreich<sup>1</sup> explains this by considering that the bridge behaviour isn't isotropic (bridge impedance in a direction parallel to the soundboard is assumed to be different than the one in the normal direction), but there is probably also non-linear coupling intrinsic to the string which need to be explored.

## ACKNOWLEDGMENT

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