



HAL
open science

Computation and Modelisation of the Sound Radiation of an Upright Piano Using Modal Formalism and Integral Equations

Philippe Dérogis, René Causse

► **To cite this version:**

Philippe Dérogis, René Causse. Computation and Modelisation of the Sound Radiation of an Upright Piano Using Modal Formalism and Integral Equations. ICA: International Congress on Acoustics, 1995, Trondheim, Norway. pp.409-412. hal-01105448

HAL Id: hal-01105448

<https://hal.science/hal-01105448>

Submitted on 20 Jan 2015

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

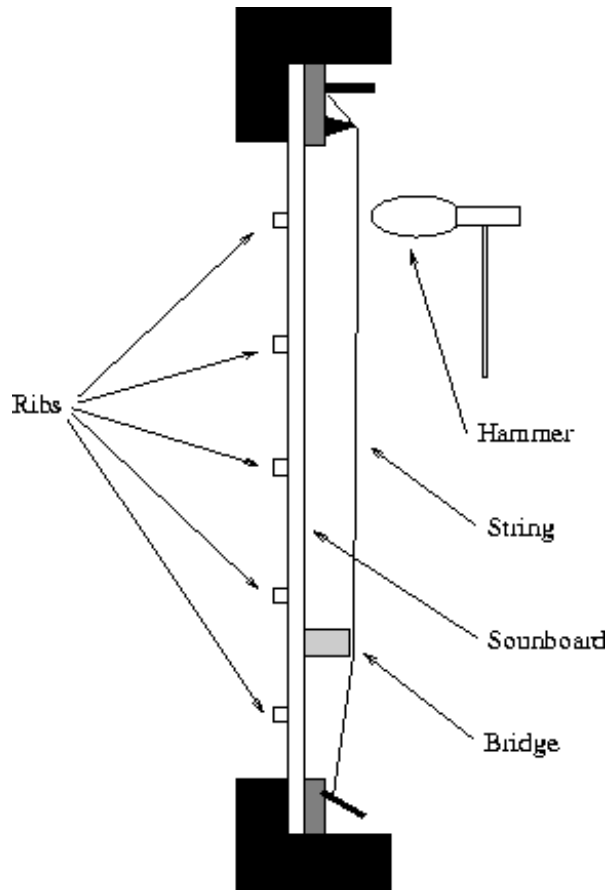
Computation and Modeling of The Sound Radiation of An Upright Piano Using Modal Formalism and Integral Equations

Philippe Dérogis, René Causse

ICA 95, Trondheim (Norvège), 1995

Introduction

Upright Piano



Modal Analysis

Modal Formalism for general viscous damping

$$M \ddot{x} + C \dot{x} + K x = p \quad (1)$$

$$\begin{bmatrix} C & M \\ M & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} + \begin{bmatrix} K & 0 \\ 0 & -M \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} p \\ 0 \end{bmatrix} \quad (2)$$

$$A r + B \dot{r} = s \quad (3)$$

Eigenvalue Problem

$$A y_k = -\lambda_k B y_k \quad (4)$$

where y_k can be written as :

$$y_k = \begin{bmatrix} z_k \\ \lambda_k z_k \end{bmatrix} \quad (5)$$

where $\{\lambda_k, z_k\}$ are respectively the eigen values and the eigen vectors of equation(1)

Modal parameters extraction

The response a_{rs} at the point r to a sinusoidal excitation at the point s can be expressed using the eigen values and the eigen vectors of equation (1) :

$$a_{rs} = \sum_{k=1}^N \left(\frac{A_{rs}(k)}{j\omega - \lambda_k} + \frac{\overline{A_{rs}(k)}}{j\omega - \overline{\lambda_k}} \right) \quad (6)$$

where : $A_{rs}(k) = \frac{z_r(k) z_s(k)}{\rho_k}$

Computation of the displacement resulting from any excitation

The displacement $x(t)$ resulting from an excitation $s e^{j\omega t}$ can be computed using the formula :

$$x(t) = \sum_{k=1}^N \left(\frac{1}{j\omega - \lambda_k} \frac{z_k z_k^t}{\rho_k} + \frac{1}{j\omega - \overline{\lambda_k}} \frac{\overline{z_k z_k^t}}{\rho_k} \right) s e^{j\omega t} \quad (7)$$

where : $\rho_k = z_k^t C z_k + 2\lambda_k z_k^t M z_k$

Mesasurements and Results

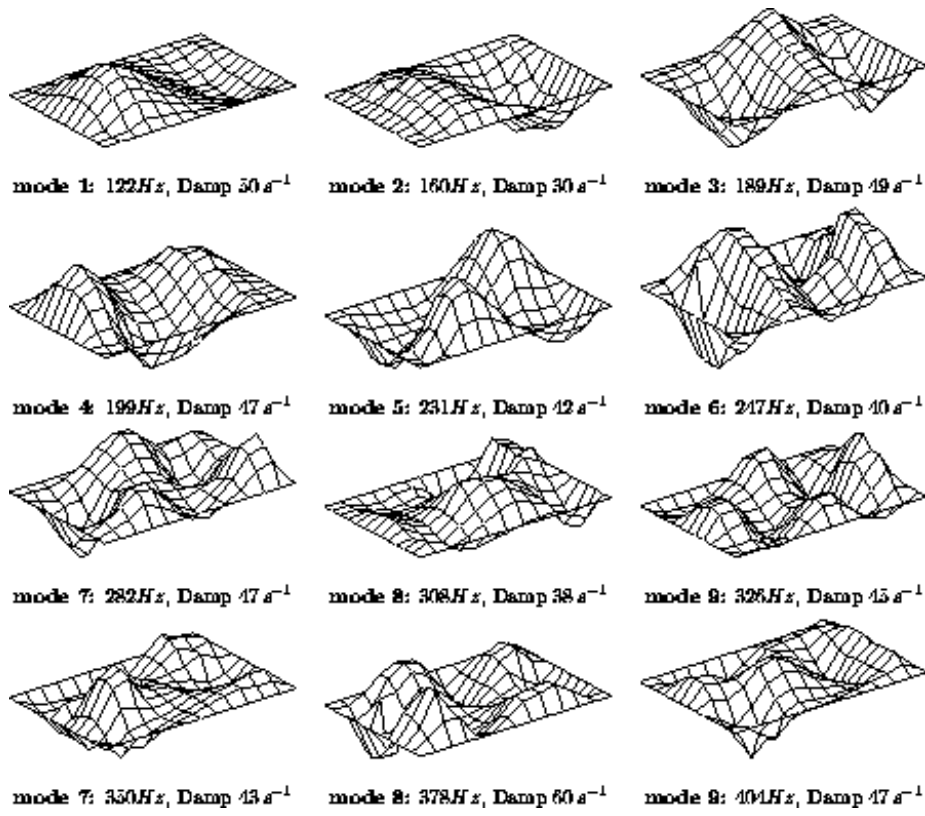


Figure 1: Modes resulting from the curve-fitting of measurements

Sound Radiation

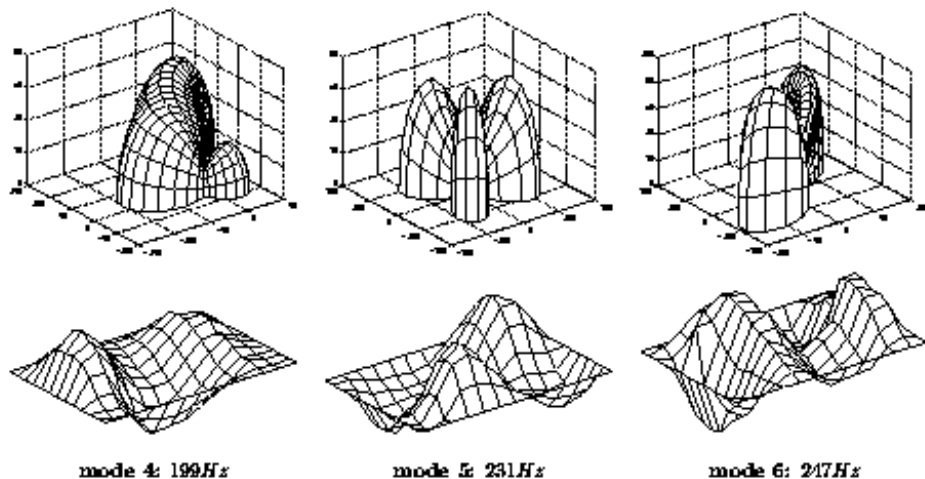
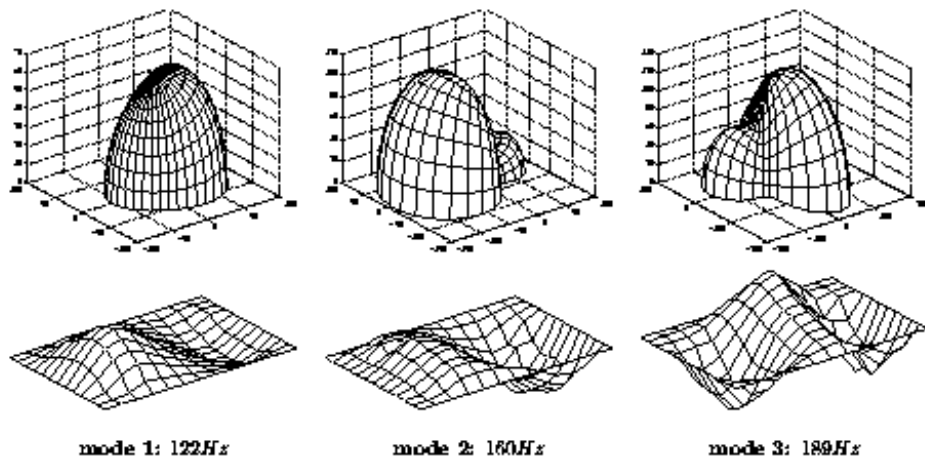
Sound radiation of a baffled plate

The sound pressure $p(r)$ and the acoustic velocity $\vec{v}(r)$ resulting from the vibrations of a baffled plate moving with normal acceleration γ_n are given by :

$$p(r) = \rho_0 \int_{r_0 \in \text{Plate}} \gamma_n(r_0) \frac{e^{-jk\|r-r_0\|}}{2\pi\|r-r_0\|} dr_0 \quad (8)$$

$$\vec{v}(r) = \rho_0 \int_{r_0 \in \text{Plate}} \gamma_n(r_0) \text{grad} \left(\frac{e^{-jk\|r-r_0\|}}{2\pi\|r-r_0\|} \right) dr_0 \quad (9)$$

Sound pressure radiated by the first mode of the soundboard



Active Intensity

Normal Active Intensity

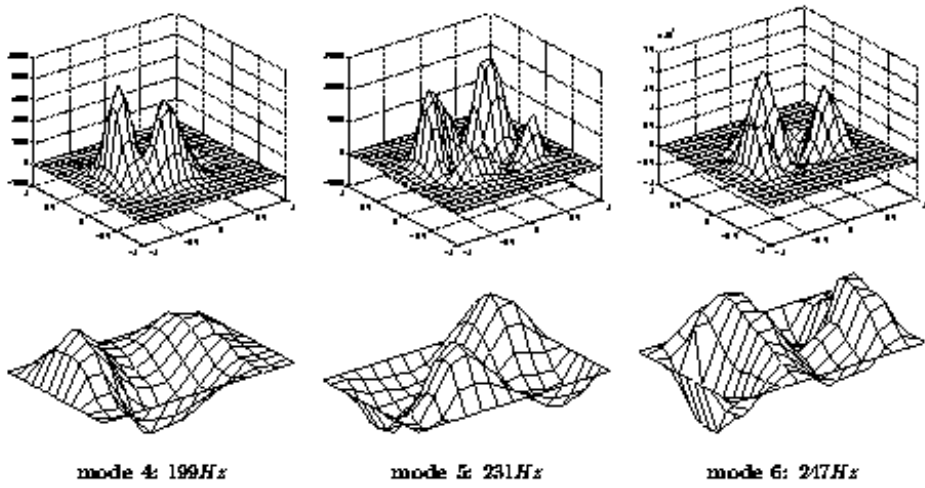
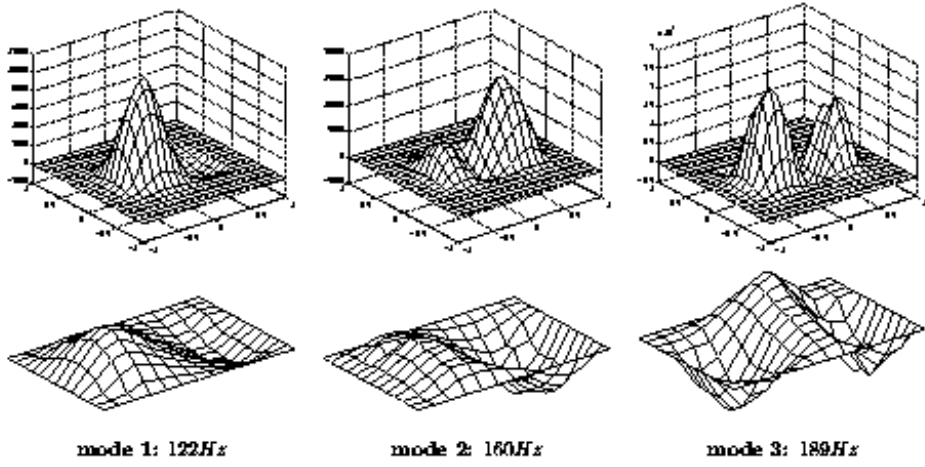
The active intensity is the power radiated per unit surface.
It is given by the formula :

$$\vec{I}(\mathbf{r}) = \frac{1}{2} \Re(p(\mathbf{r}) \vec{v}(\mathbf{r})^*) \quad (10)$$

It is interesting to calculate the z component of the active intensity near the soundboard in order to know which regions produce the acoustic power, in particular :

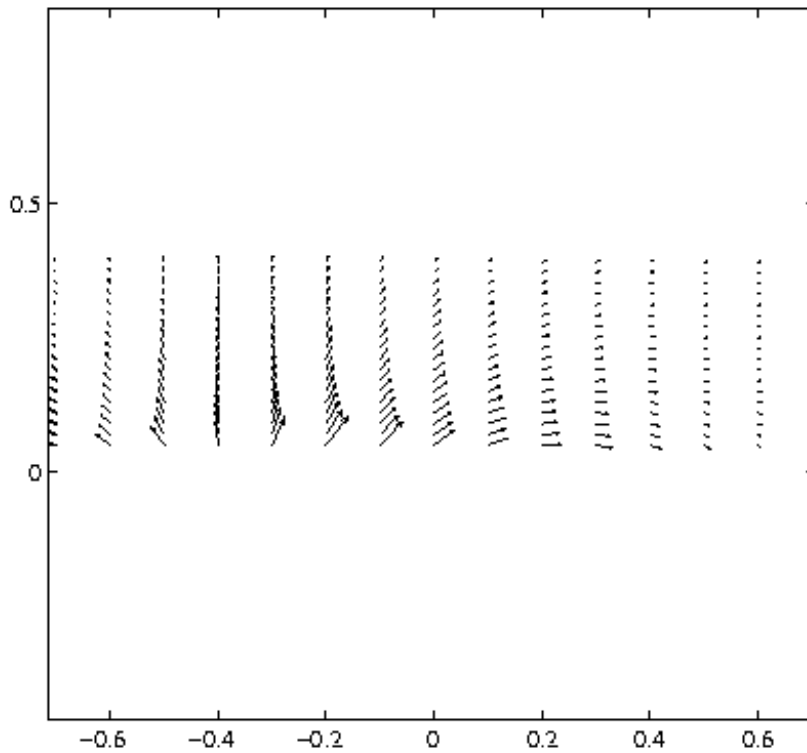
- If it is positive, the energy travels from the soundboard to the acoustic field
- If it is negative, the energy travels from the acoustic field to the soundboard.

Computation of the normal Active Intensity at 5 centimeters of the soundboard



An example of a loop of active intensity

Active intensity field in a plane orthogonal to the plate for the mode 1 : 122Hz



Radiation efficiency

Power radiated

The total power radiated by a source can be computed using :

$$W_a = \oint_S \vec{I} \cdot \vec{n} dS \quad (11)$$

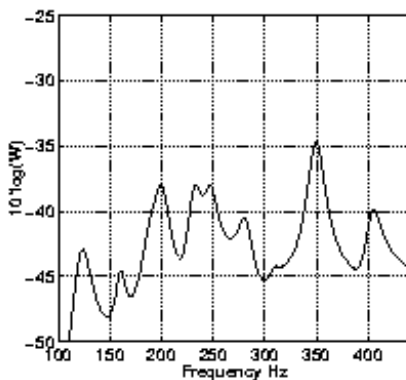
Power radiated by the soundboard versus frequency for several driving point

- Computation of the displacement of the soundboard using modal formalism
- Computation of the acoustic field
- Computation of the radiated power

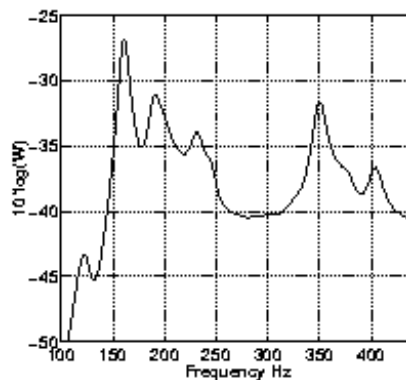
Results

Radiated power for several points excited by a 1N force

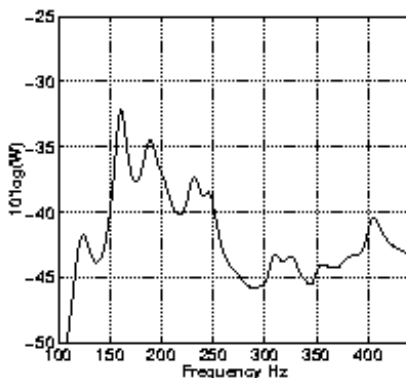
Point #1: Bass bridge A1 : 55Hz



Point #2: Main bridge A2 : 110Hz



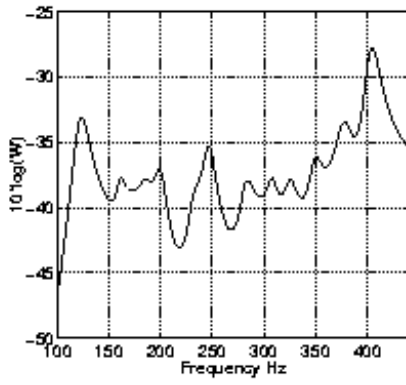
Point #3: Main bridge E2 : 164Hz



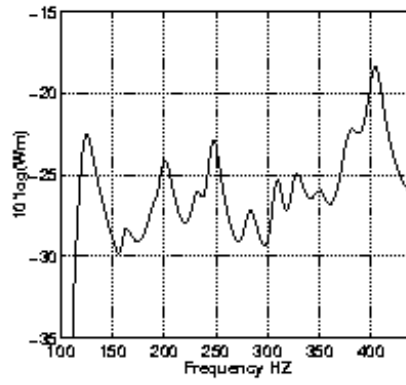
Radiation efficiency

Radiation efficiency of the soundboard

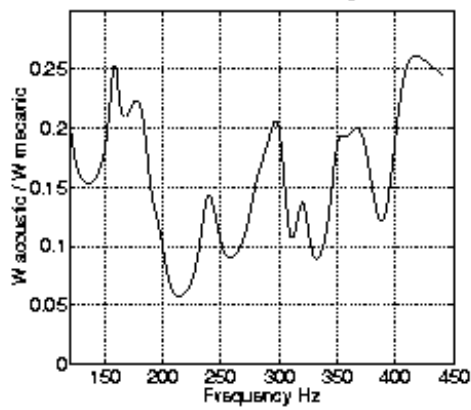
Acoustic power



Mechanical power

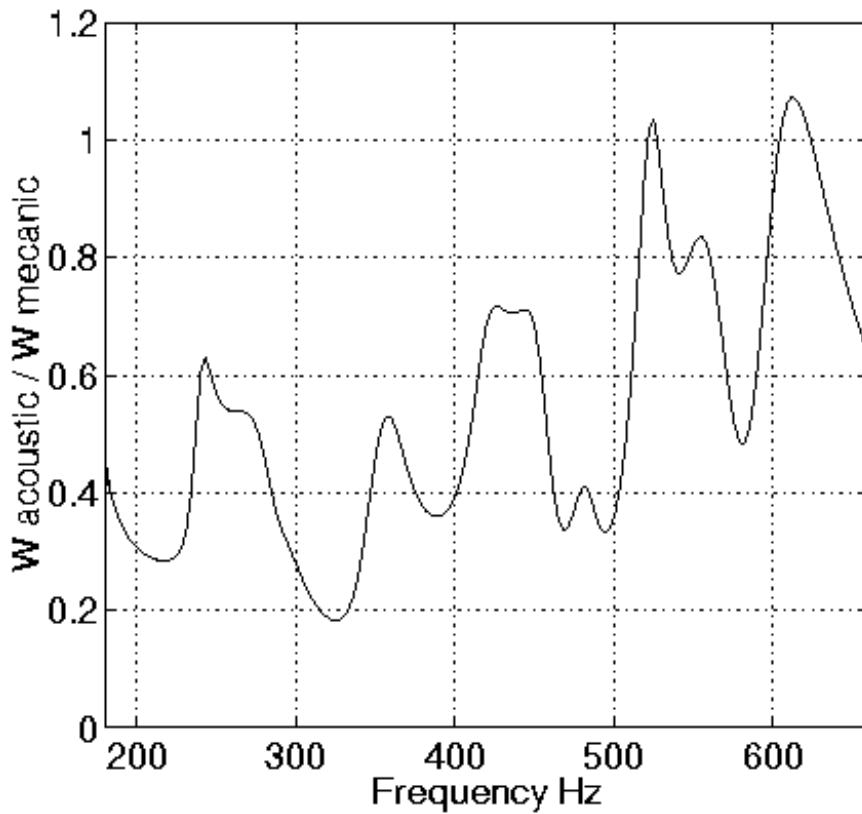


Radiation efficiency



Estimated radiation efficiency

Radiation efficiency of a soundboard having eigen values (frequency and damping) a factor 1.5 higher.



Conclusion

- The first modes of the soundboard look like the ones of an isotropic supported plate
- The pressure radiated by the modes depends strongly of the position of the observer
- The frequencies of the first modes are well below the coincidence frequency of these modes
- There are loops of active Intensity
- The acoustic power depends on the location of the excitation point
- The radiation efficiency is about 15-20%