

APPLICATION DE LA MÉTHODE ASYMPTOTIQUE NUMÉRIQUE AU CAS DES MODÈLES DE ZONE COHÉSIVE

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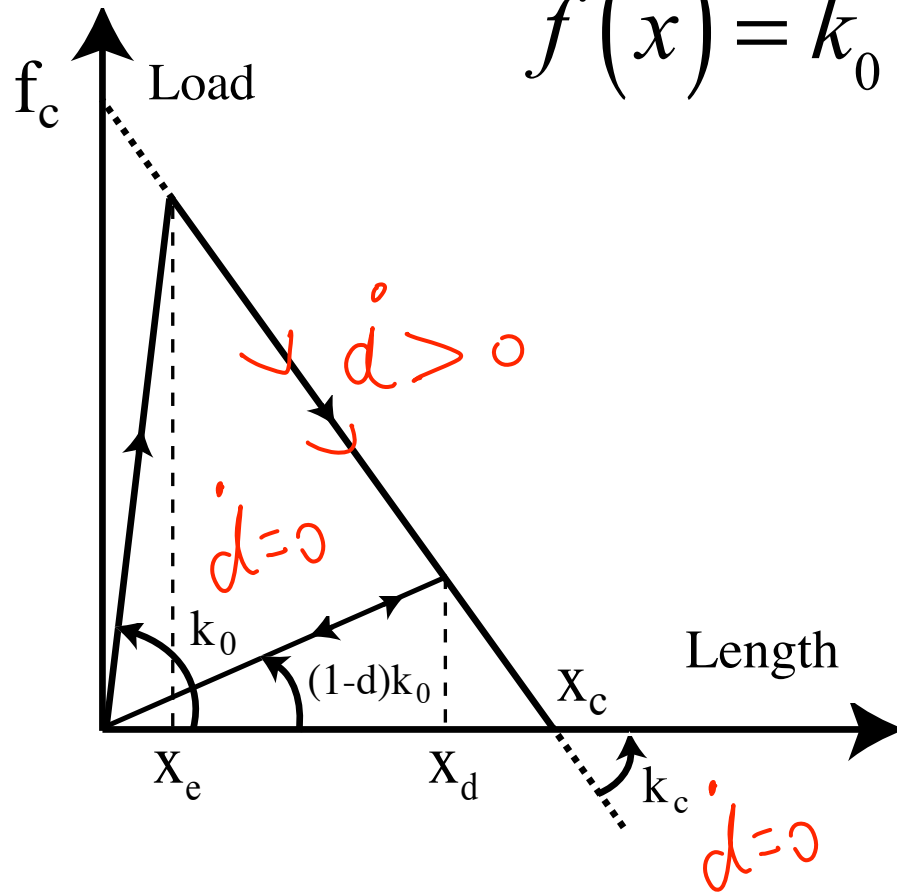
PLAN

- ✓ Modèle de zone cohésive
- ✓ Méthode Asymptotique Numérique
- ✓ Immersion dans le cadre thermodynamique
- ✓ Exemple éléments finis
- ✓ Couplage plasticité endommagement

MODÈLE DE ZONE COHÉSIVE

Approche phénoménologique

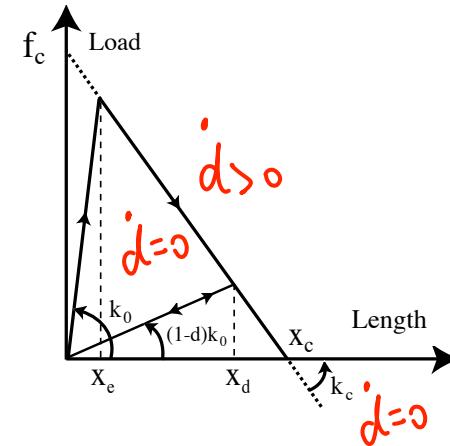
$$f(x) = k_0(1-d)x$$



TAUX D'ENDOMMAGEMENT

$$f(x) = k_0(1-d)x$$

$$F(x) = f(x) - (f_c - k_c x)$$



$$\dot{d} = \begin{cases} 0 & \text{si } F(x) < 0 \text{ ou } (F(x) = 0 \text{ et } \dot{x} \leq 0) \\ \frac{k_c x_c}{k_0 x^2} \dot{x} H\left(1 - \frac{x}{x_c}\right) & \text{si } F(x) = 0 \text{ et } \dot{x} > 0 \end{cases}$$

$$\dot{d} = \frac{k_c x_c}{k_0 x^2} \langle \dot{x} \rangle^+ H\left(1 - \frac{x}{x_c}\right)$$

PROBLÈME À RÉSOUDRE

LOI DE COMPORTEMENT

$$f = k_0(1-d)x$$

Loi d'évolution

$$\dot{d} = \frac{k_c}{k_0} \frac{x_c \langle \dot{x} \rangle^+}{x^2} H\left(1 - \frac{x}{x_c}\right) \Pi(F(x))$$

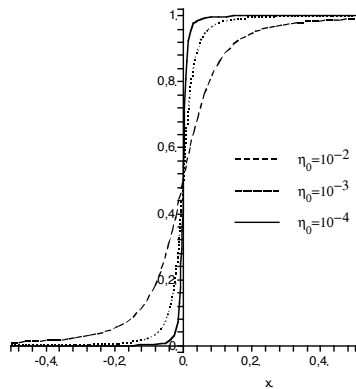
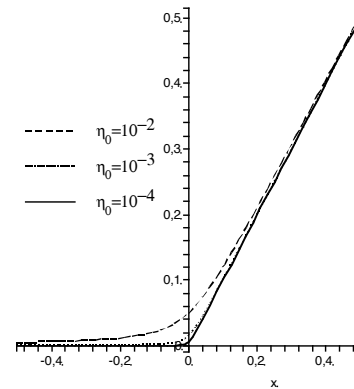
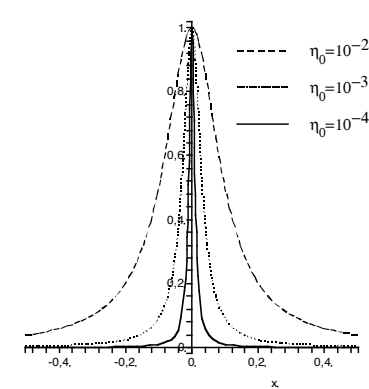
MAN



RÉGULARISATION

RÉGULARISATION DU PROBLÈME

$$\dot{d} = \frac{k_c}{k_0} \frac{x_c \langle \dot{x} \rangle^+}{x^2} H\left(1 - \frac{x}{x_c}\right) H(F(f, x)) H(-F(f, x))$$


 $H_{\eta_0}(x)$

 $P_{\eta_0}^+(x)$

 $\Pi_{\eta_0}(x)$

$$\dot{d} = \frac{k_c}{k_0} \frac{\dot{x}_0 P_{\eta_0}^+\left(\frac{\dot{x}}{\dot{x}_0}\right)}{x^2} x_c H_{\eta_0}\left(1 - \frac{x}{x_c}\right) \Pi_{\eta_0}(F(f, x))$$

MAN – CAS 1-D

Développement en série

$$U(t) - U_0 = \sum_{n=1}^N (t - t_0)^n U_n$$

$$U(t) = \begin{bmatrix} x(t) \\ d(t) \end{bmatrix}$$

Test de convergence

$$R = \left(\varepsilon \frac{\|U_1\|}{\|U_N\|} \right)^{\frac{1}{N-1}}$$

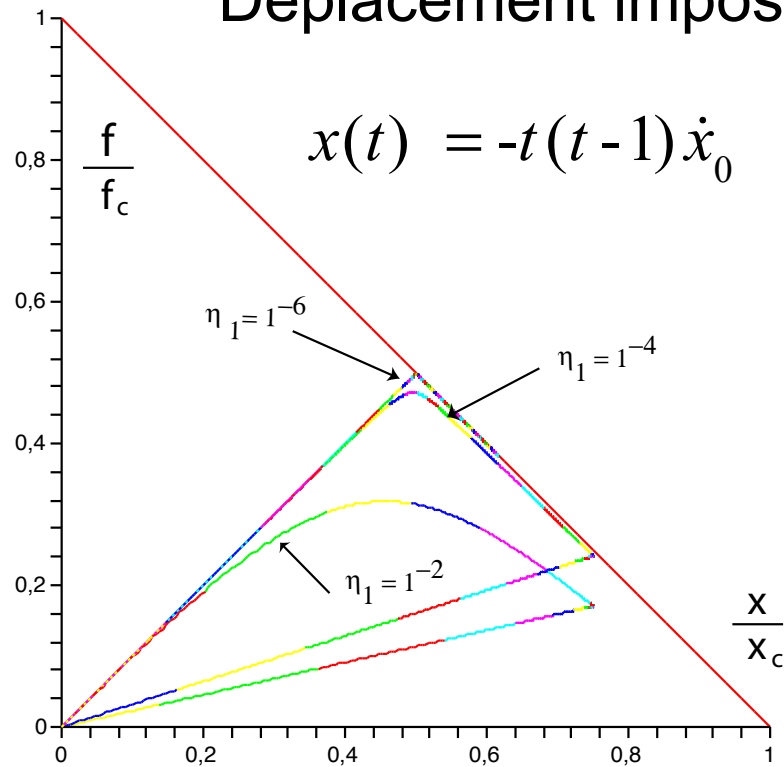
$$\varepsilon = 10^{-4}$$

Paramètres de la loi

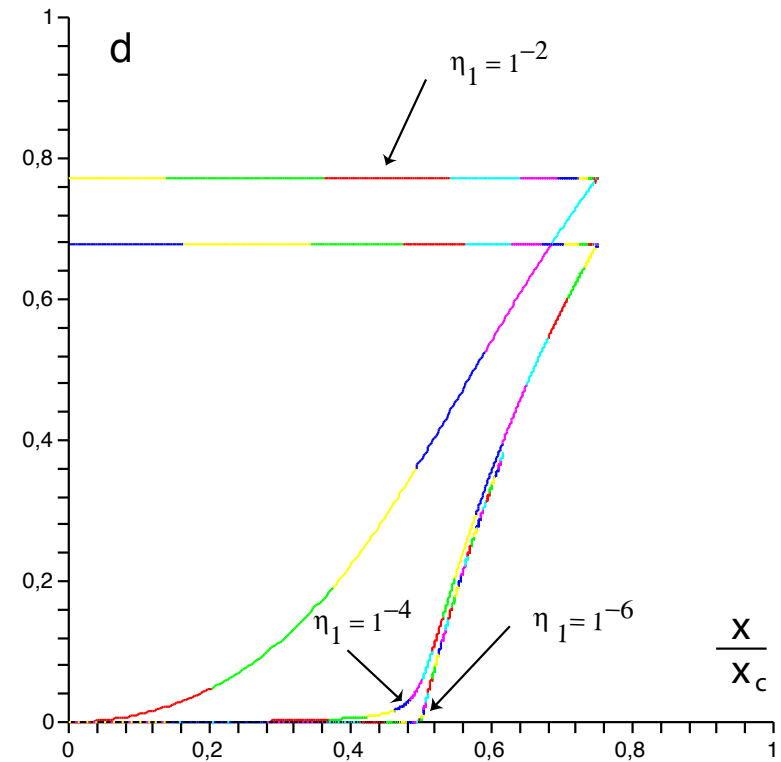
$$\{f_c = 1 \text{ N.mm}^{-1}, x_c = 1 \text{ mm}, k_0 = 1 \text{ N.mm}^{-2}\}$$

MAN – CAS 1-D – METH1

Déplacement imposé



Charge/élongation



Endommagement/élongation

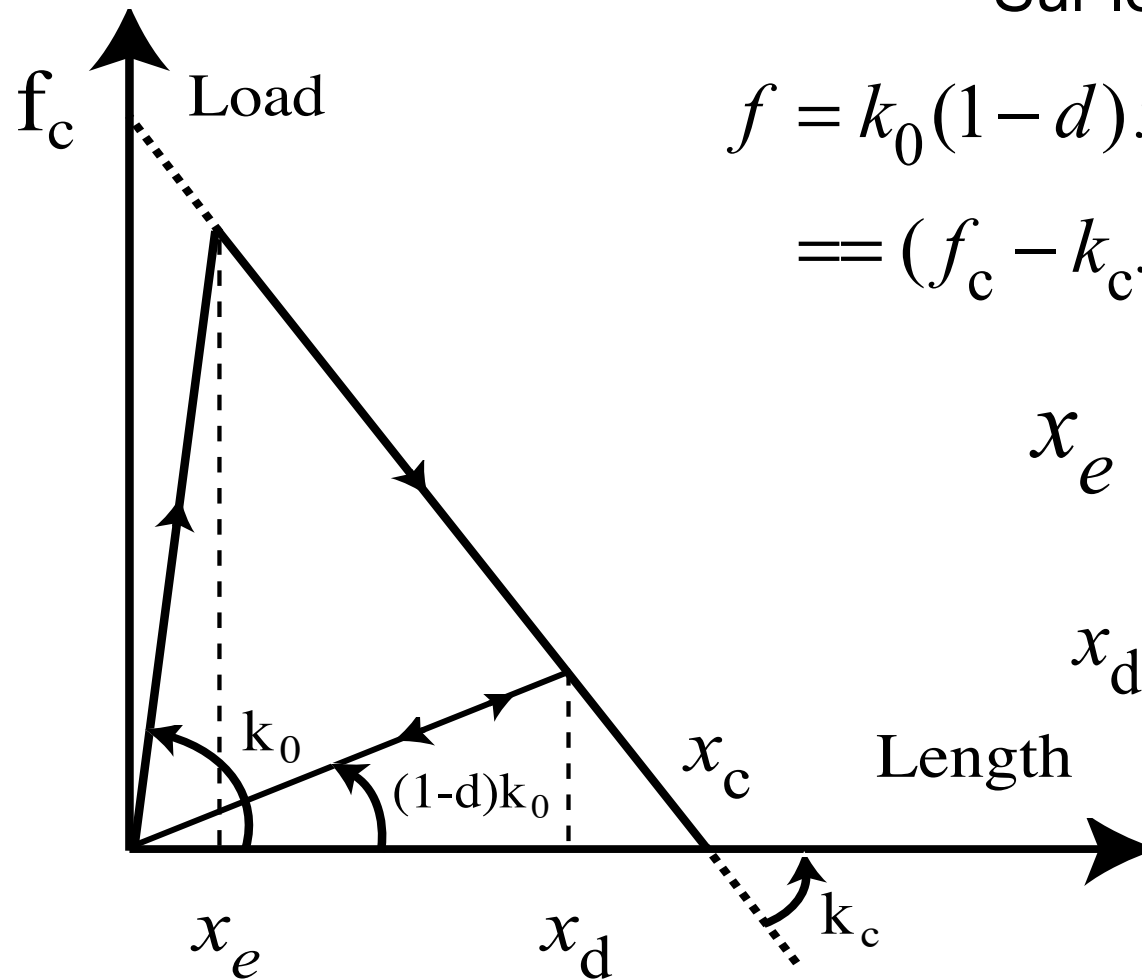
IMMERSION DANS LE CADRE DE LA THERMODYNAMIQUE

Hypothèses (TPI-MSG) :

- (i) Energie de déformation est dissipée uniquement par l'endommagement ou stockée élastiquement.
- (ii) Pas de couplage thermomécanique
- (iii) Cas des chargements isothermes

Choix des variables d'état

Sur le seuil



$$f = k_0(1-d)x = (f_c - k_c x)$$

$$\implies (f_c - k_c x_d) = k_c(x_c - x_d)$$

$$x_e \leq x_d \leq x_c$$

$$x_d = \frac{k_c}{k_0(1-d) + k_c} x_c$$

$$0 \leq d \leq 1$$

Choix des variables d'état

Variables d'état

$$(x, x_d)$$

Énergie libre

$$\psi(x, x_d) = \frac{k_c}{2} \langle x_c - x_d \rangle^+ \frac{x^2}{x_d}$$

Équations d'état

$$f_x = k_c \langle x_c - x_d \rangle^+ \frac{x}{x_d}$$

$$f_{x_d} = -\frac{k_c}{2} x_c \left(\frac{x}{x_d} \right)^2 H(x_c - x_d)$$

SEUIL ET LOI DE NORMALITÉ (TPI-MSG)

Force thermodynamique

$$X_d = -f_{x_d}$$

Fonction seuil d'endommagement

$$G(X_d) = X_d - \frac{f_c}{2} \leq 0$$

Loi de normalité

$$\dot{x}_d = \begin{cases} 0 & \text{if } G < 0 & (a) \\ \lambda \frac{dG}{dX_d} = \lambda = \dot{x}, \lambda \geq 0 & \text{if } G(X_d) = 0 & (b) \end{cases}$$

COMPARAISON

LOI DE COMPORTEMENT

$$f = k_0(1-d)x$$

$$f_x = k_c \langle x_c - x_d \rangle^+ \frac{x}{x_d}$$

$$f_{x_d} = -\frac{k_c}{2} x_c \left(\frac{x}{x_d} \right)^2 H(x_c - x_d)$$

SEUIL

$$F(x) = f(x) - (f_c - k_c x)$$

$$G(X_d) = X_d - \frac{f_c}{2} \leq 0$$

Loi d'évolution

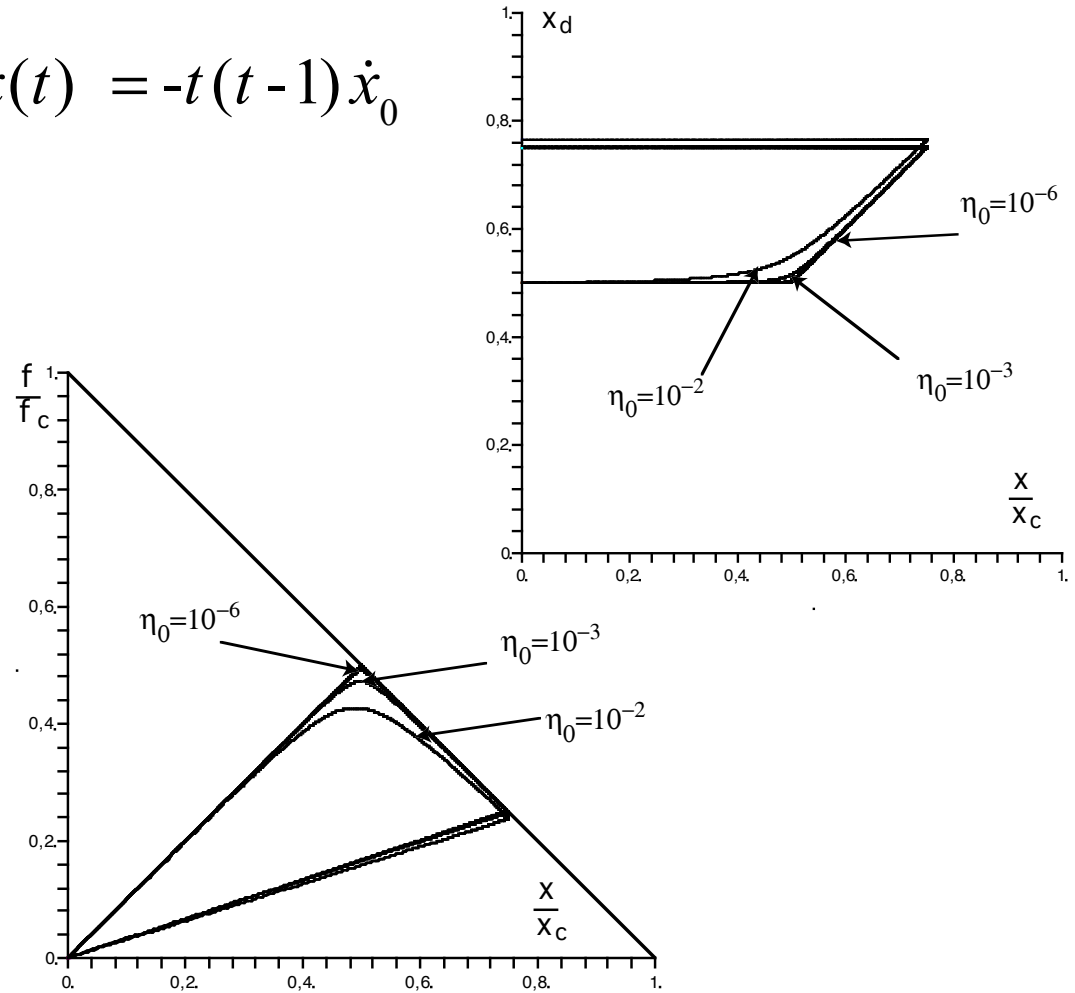
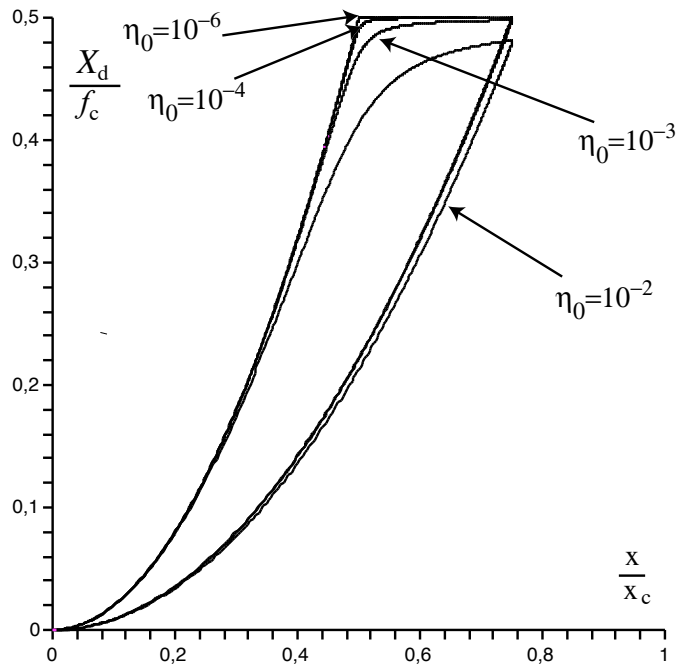
$$\dot{d} = \frac{k_c}{k_0} \frac{\dot{x}_0 \left\langle \frac{\dot{x}}{\dot{x}_0} \right\rangle^+}{x^2} x_c H\left(1 - \frac{x}{x_c}\right) \Pi(F(f, x))$$

$$\dot{x}_d = \langle \dot{x} \rangle^+ H(G(X_d))$$

MAN – CAS 1-D – METH2

Déplacement imposé

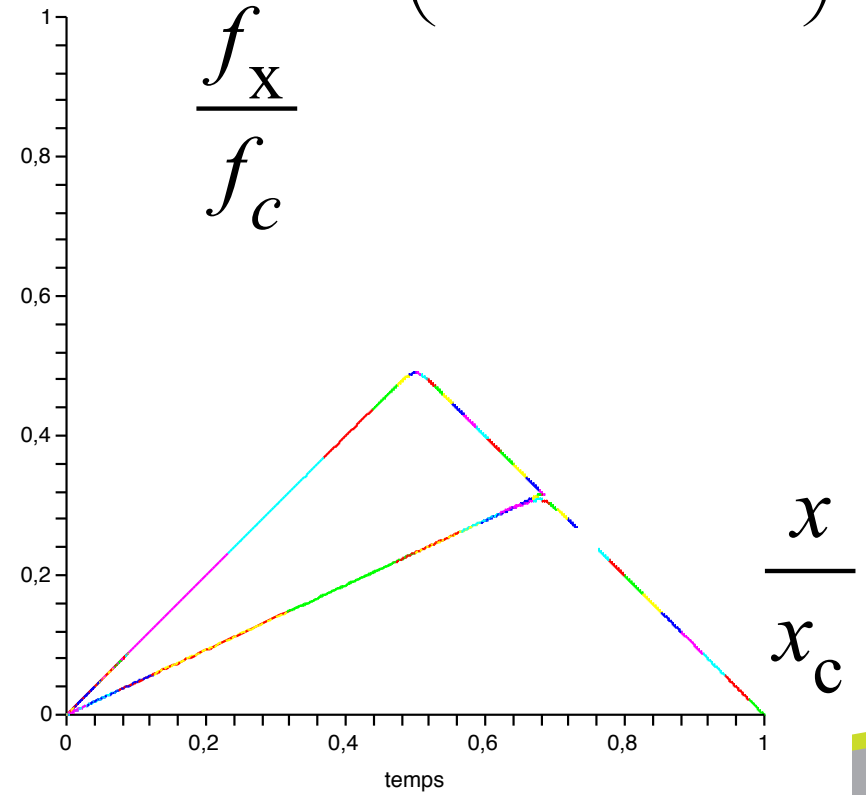
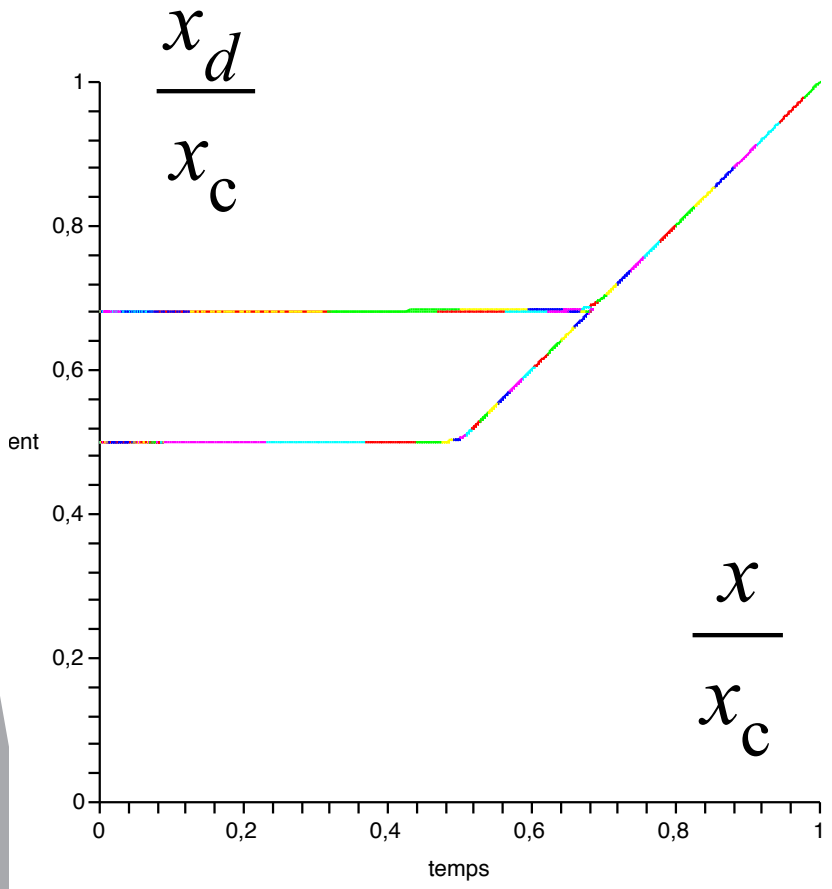
$$x(t) = -t(t-1)\dot{x}_0$$



MAN – CAS 1-D – METH2

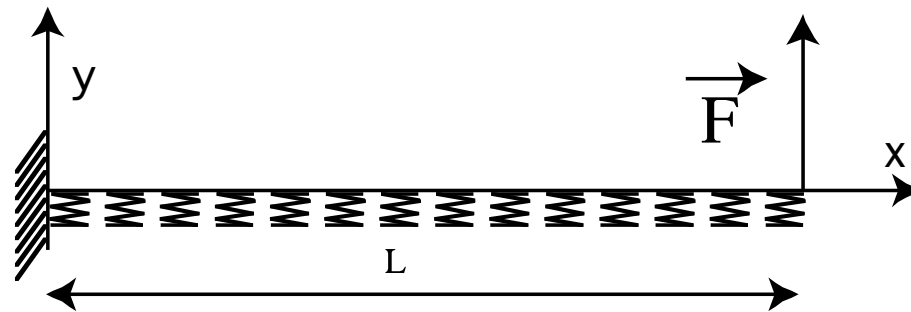
Déplacement imposé

$$x(t) = \dot{x}_0 t \left(\frac{\cos(3\pi \frac{t}{tc}) + 1}{2} \right)$$



EXEMPLE ÉLÉMENTS FINIS

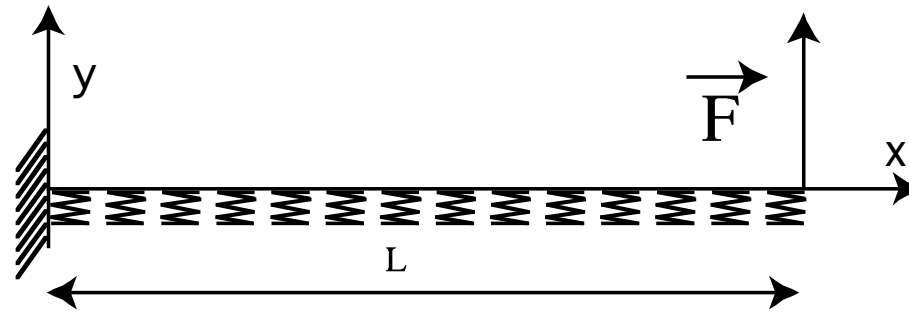
Modèle de Poutre sur Fondation



Formulation variationnelle

$$P_{ot}(s, x_d) = -\int_0^L \frac{1}{2} EI (v''(s))^2 ds - \int_0^L \frac{k_c}{2} \langle x_c - x_d(s) \rangle^+ \frac{v(s)^2}{x_d(s)} ds + Fv(L)$$

EXEMPLE ÉLÉMENTS FINIS

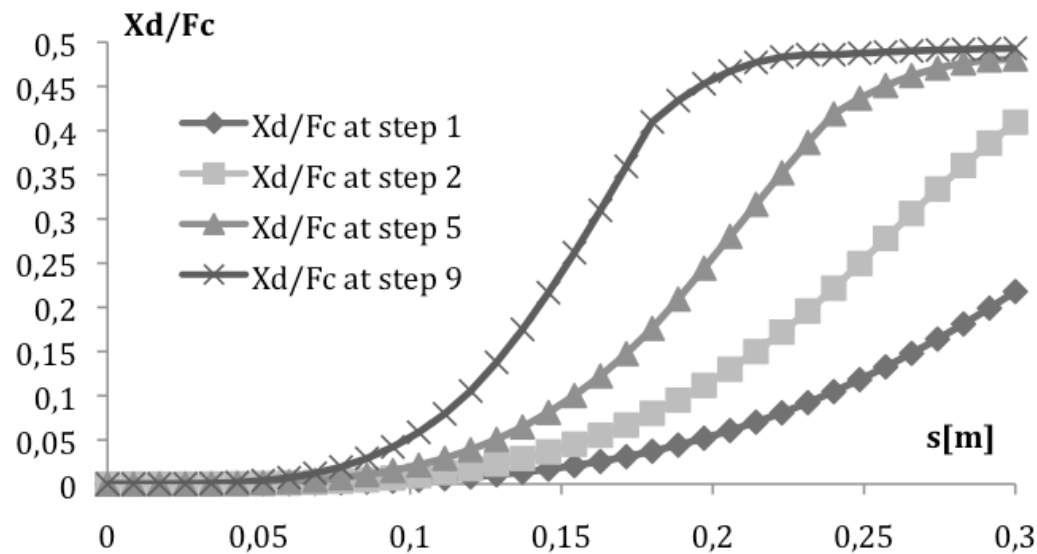
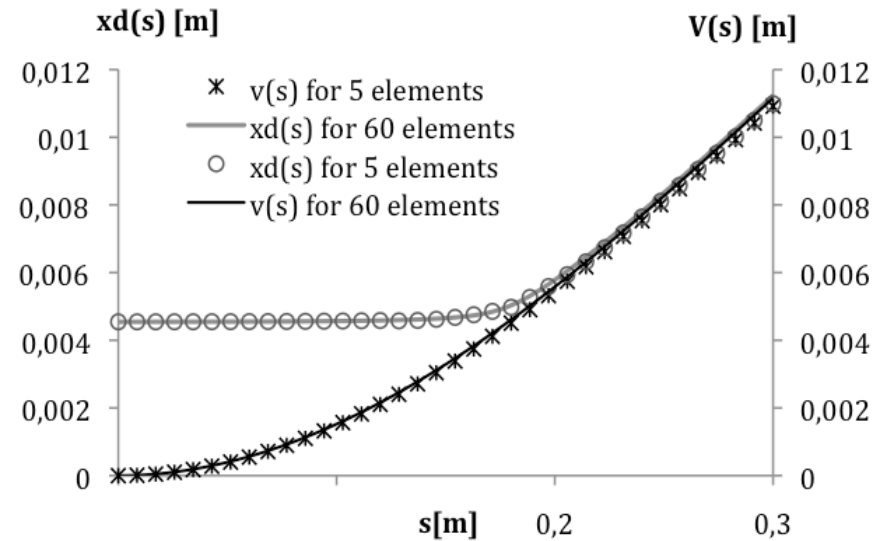
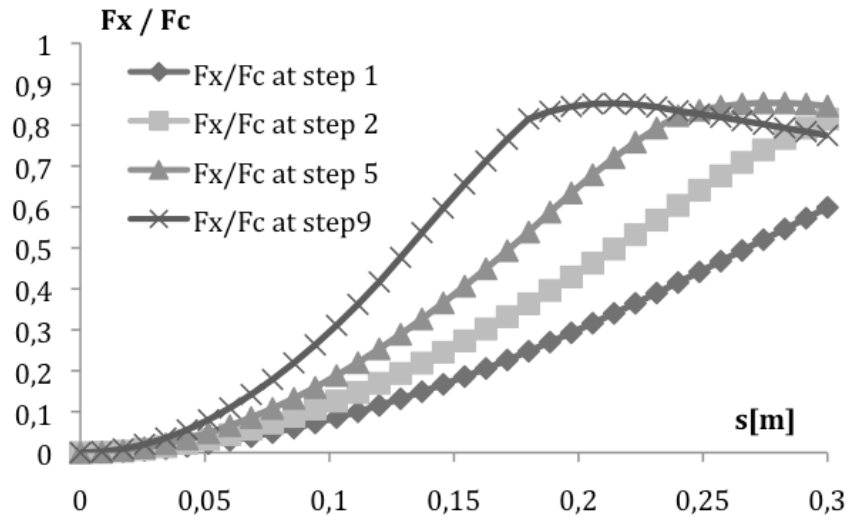


Formulation faible : **+** MaN

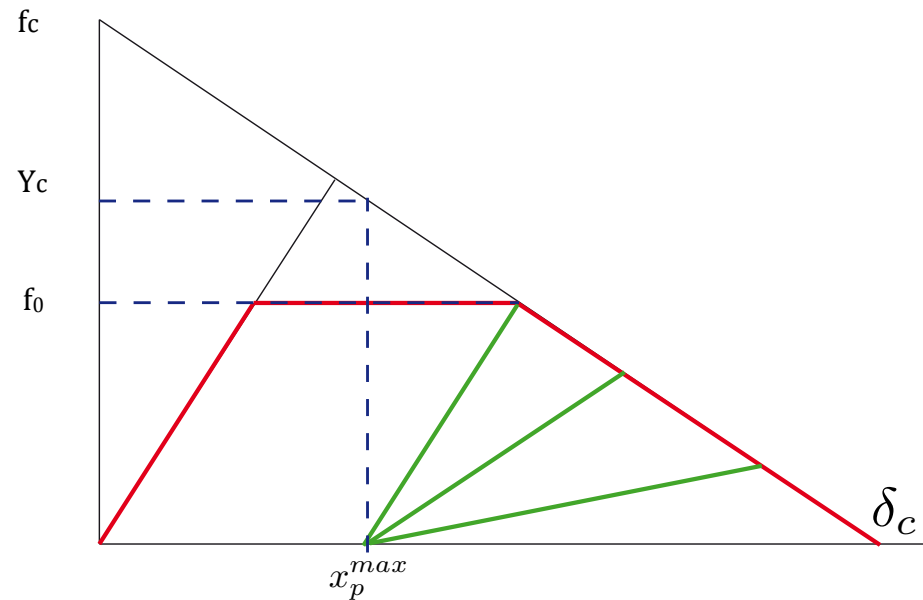
$$\left\{ \begin{array}{l}
 \int_0^L EI v''(s) \delta v''(s) - k_c v(s) \delta v(s) ds + \int_0^L k_c ed(s) v(s) \delta v(s) ds = F(t) \delta v(L) \\
 ed(s) = \frac{x_c}{x_d(s)} \\
 wd(s) = \frac{v(s)}{x_d(s)} - 1 \\
 td(s) = (wd(s))^2 \\
 \dot{x}_d(s) = \dot{v}(s) \frac{\eta_0}{td(s) + \eta_0}
 \end{array} \right. + \left\{ \begin{array}{l}
 x_d(s) = \sum_{k=0}^N x d_k(s) t^k \quad ; \quad v(s) = \sum_{k=0}^N v_k(s) t^k \\
 ed(s) = \sum_{k=0}^N ed_k(s) t^k \quad ; \quad wd(s) = \sum_{k=0}^N wd_k(s) t^k \\
 td(s) = \sum_{k=0}^N t d_k(s) t^k \quad ; \quad F(t) = \sum_{k=0}^N F_k t^k
 \end{array} \right.$$

EXEMPLE ÉLÉMENTS FINIS

Résultats



COUPLAGE PLASTICITÉ-ENDOMMAGEMENT



$$\Psi(x, x_d, x_p) = \frac{1}{2} * \left(\frac{x_c - x_p^{max}}{x_d - x_p^{max}} - 1 \right) * k_c * (x - x_p)^2$$

$$\Psi_{AC}(x, x_d, x_p) = \frac{1}{2} \left\langle \frac{x_c}{x_d} - 1 \right\rangle^+ k_c (x - x_p)^2$$

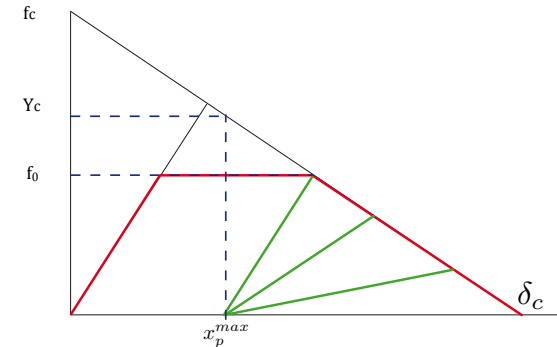
COUPLAGE PLASTICITÉ-ENDOMMAGEMENT

Variables d'état

$$(x, x_d, x_p)$$

Énergie libre

$$\Psi_{AC}(x, x_d, x_p) = \frac{1}{2} \left\langle \frac{x_c}{x_d} - 1 \right\rangle^+ k_c (x - x_p)^2$$



Forces thermodynamiques

$$Y_x = \frac{\partial \Psi(x, x_d, x_p)}{\partial x} = \left\langle \frac{x_c}{x_d} - 1 \right\rangle^+ k_c (x - x_p)$$

$$Y_{x_p} = \frac{\partial \Psi}{\partial x_p} = - \left\langle \frac{x_c}{x_d} - 1 \right\rangle^+ k_c (x - x_p)$$

$$Y_{x_d} = \frac{\partial \Psi}{\partial x_d} = - \frac{1}{2} f_c \left(\frac{x - x_p}{x_d} \right)^2 H\left(\frac{x_c}{x_d} - 1\right) x_c$$

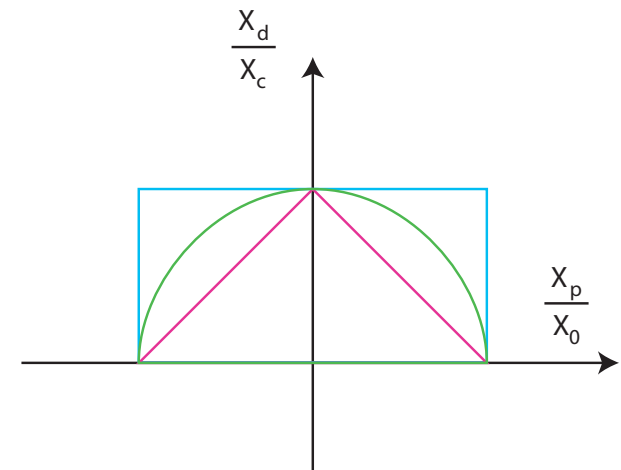
COUPLAGE PLASTICITÉ-ENDOMMAGEMENT

La fonction seuil

$$f_0(X_{\dot{x}_d}, X_{\dot{x}_p}; x_p) = \sup\left(\frac{X_{\dot{x}_d}}{X_c^p}, \frac{X_{\dot{x}_p}}{X_0}\right) - 1 \leq 0$$

$$f_1(X_{\dot{x}_p}, X_{\dot{x}_d}; x_p) = \left| \frac{X_{\dot{x}_p}}{X_0} \right| + \left| \frac{X_{\dot{x}_d}}{X_c^p} \right| - 1$$

$$f_2(X_{\dot{x}_p}, X_{\dot{x}_d}; x_p) = \left(\frac{X_{\dot{x}_p}}{X_0}\right)^2 + \left(\frac{X_{\dot{x}_d}}{X_c^p}\right)^2 - 1$$

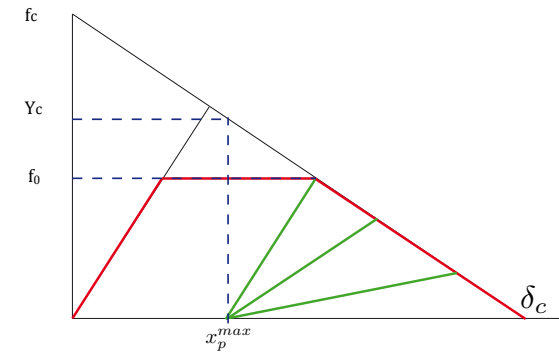


$$X_c^p = \frac{1}{2} f_c \left(1 - \frac{x_p}{x_c}\right)^2$$

COUPLAGE PLASTICITÉ-ENDOMMAGEMENT

Plasticité puis endommagement

$$f_0(X_{\dot{x}_d}, X_{\dot{x}_p}; x_p) = \sup\left(\frac{X_{\dot{x}_d}}{X_c^p}, \frac{X_{\dot{x}_p}}{X_0}\right) - 1 \leq 0$$



Plasticité couplée avec endommagement

$$f_1(X_{\dot{x}_p}, X_{\dot{x}_d}; x_p) = \left| \frac{X_{\dot{x}_p}}{X_0} \right| + \left| \frac{X_{\dot{x}_d}}{X_c^p} \right| - 1$$

$$f_2(X_{\dot{x}_p}, X_{\dot{x}_d}; x_p) = \left(\frac{X_{\dot{x}_p}}{X_0} \right)^2 + \left(\frac{X_{\dot{x}_d}}{X_c^p} \right)^2 - 1$$

COUPLAGE PLASTICITÉ-ENDOMMAGEMENT

La fonction seuil

$$f_0(X_{\dot{x}_d}, X_{\dot{x}_p}; x_p) = \sup\left(\frac{X_{\dot{x}_d}}{X_c^p}, \frac{X_{\dot{x}_p}}{X_0}\right) - 1 \leq 0$$

Prise en compte de la loi de normalité

$$\dot{x}_d = \lambda \frac{\partial f_0(X_{\dot{x}_p}, X_{\dot{x}_d}; x_p)}{\partial X_{\dot{x}_d}}$$

$$\dot{x}_p = \lambda \frac{\partial f_0(X_{\dot{x}_p}, X_{\dot{x}_d}; x_p)}{\partial X_{\dot{x}_p}}$$

la relation de fermeture

$$f_0(X_{\dot{x}_p}, X_{\dot{x}_d}; x_p) = 0 \quad \frac{df_0}{dt}(X_{\dot{x}_p}, X_{\dot{x}_d}; x_p) = 0$$

COUPLAGE PLASTICITÉ-ENDOMMAGEMENT

$$\Psi_{AC}(x, x_d, x_p) = \frac{1}{2} \left\langle \frac{x_c}{x_d} - 1 \right\rangle^+ k_c (x - x_p)^2$$

Les lois d'évolution

$$\begin{aligned} \frac{X_{\dot{x}_d}}{X_c^p} < 1 \quad \text{et} \quad \frac{X_{\dot{x}_p}}{X_0} = 1 \quad \text{et} \quad & \begin{cases} \dot{x}_p = \dot{x} \\ \dot{x}_d = 0 \end{cases} \\ \frac{X_{\dot{x}_d}}{X_c^p} = 1 \quad \text{et} \quad \frac{X_{\dot{x}_p}}{X_0} < 1 \quad \text{et} \quad & \begin{cases} \dot{x}_p = 0 \\ \dot{x}_d = \frac{x_c}{x_c - x_p} \dot{x} \end{cases} \end{aligned}$$

Les lois d'état

$$Y_x = \frac{\partial \Psi(x, x_d, x_p)}{\partial x} = \left\langle \frac{x_c}{x_d} - 1 \right\rangle^+ k_c (x - x_p)$$

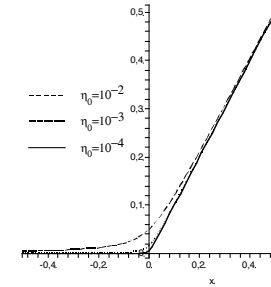
$$Y_{x_p} = \frac{\partial \Psi}{\partial x_p} = - \left\langle \frac{x_c}{x_d} - 1 \right\rangle^+ k_c (x - x_p)$$

$$Y_{x_d} = \frac{\partial \Psi}{\partial x_d} = - \frac{1}{2} f_c \left(\frac{x - x_p}{x_d} \right)^2 H\left(\frac{x_c}{x_d} - 1\right) x_c$$

COUPLAGE PLASTICITÉ-ENDOMMAGEMENT + MAN

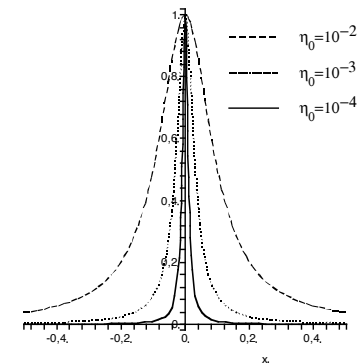
On régularise directement le potentiel

$$\Psi_{reg}(x, x_d, x_p) = \frac{1}{2} P_{\eta_0}^+ \left(\frac{x_c}{x_d} - 1 \right) k_c (x - x_p)^2$$



Par dérivation on obtient les lois d'état régularisées

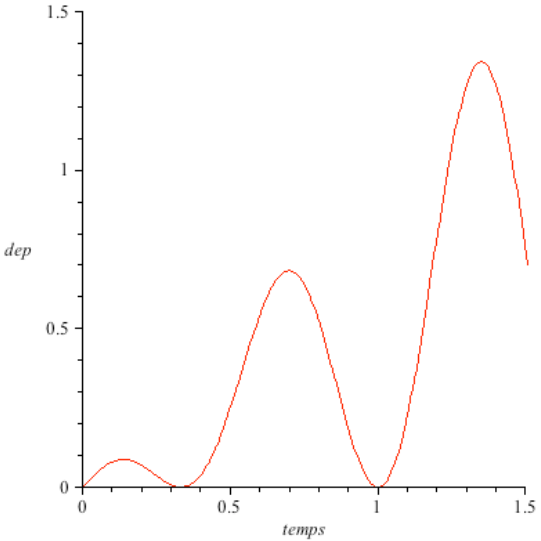
$$\begin{aligned}
 Y_x &= \frac{\partial \Psi_{reg}}{\partial x}(x, x_d, x_p) \\
 Y_{x_p} &= \frac{\partial \Psi_{reg}}{\partial x_p}(x, x_d, x_p) \\
 Y_{x_d} &= \frac{\partial \Psi_{reg}}{\partial x_d}(x, x_d, x_p)
 \end{aligned}$$



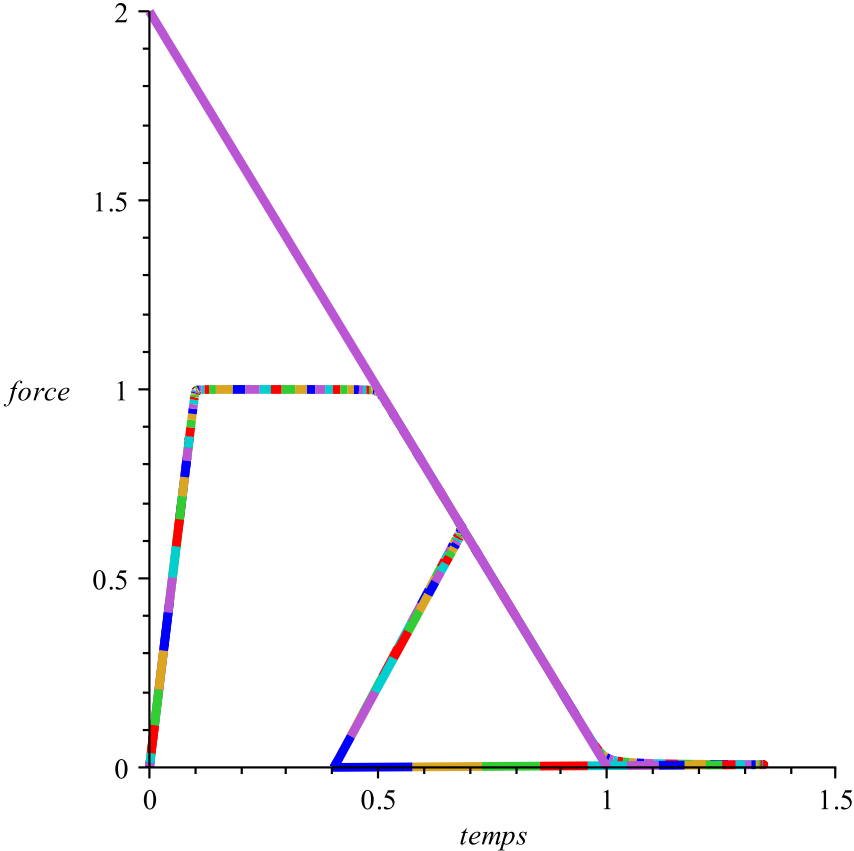
On régularise les lois d'évolution

$$\begin{cases} \dot{x}_p^{reg} = P_{\eta_0}^+(\dot{x}) G_{\eta_0} \left(\frac{X \dot{x}_p}{X_0} - 1 \right) \\ \dot{x}_d = 0 \end{cases} \quad \begin{cases} \dot{x}_p = 0 \\ \dot{x}_d^{reg} = \frac{x_c}{x_c - x_p} P_{\eta_0}^+(\dot{x}) G_{\eta_0} \left(\frac{\dot{x}_d}{X_c^p} - 1 \right) \end{cases}$$

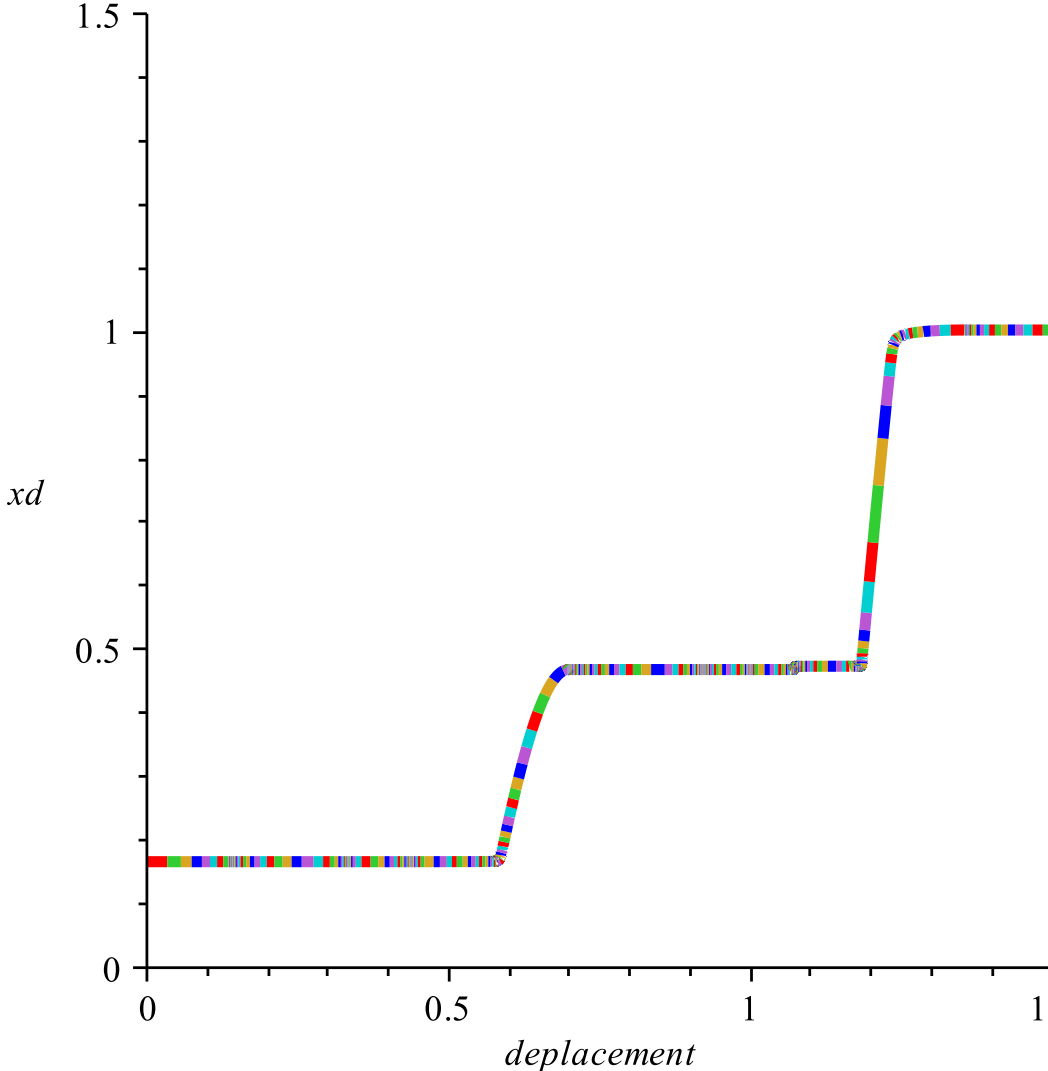
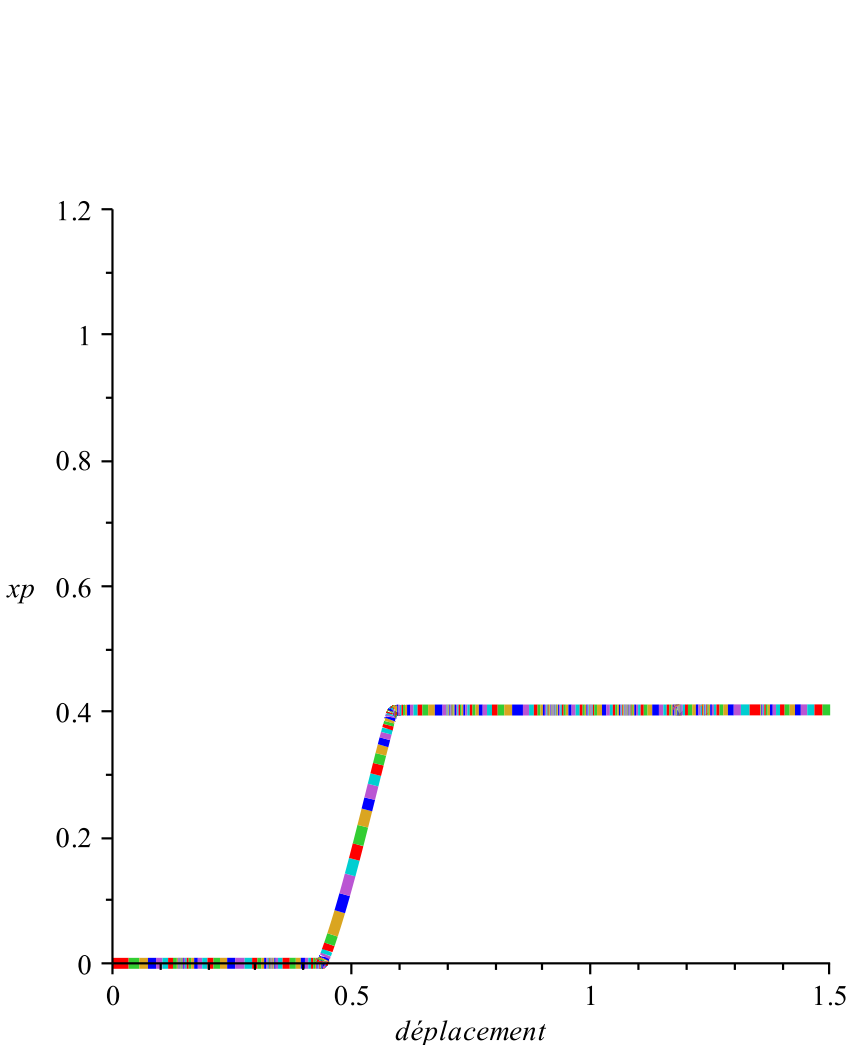
COUPLAGE PLASTICITÉ-ENDOMMAGEMENT + MAN



Chargement



COUPLAGE PLASTICITÉ-ENDOMMAGEMENT + MAN



COUPLAGE PLASTICITÉ-ENDOMMAGEMENT + MAN

Couplage fort de la plasticité et de l'endommagement

$$f_1 (X_{\dot{x}_p}, X_{\dot{x}_d}; x_p) = 0 \Rightarrow |X_{\dot{x}_p}| = X_0 \left(1 - \left| \frac{X_{\dot{x}_d}}{X_c^p} \right| \right)$$

$$f_2 (X_{\dot{x}_p}, X_{\dot{x}_d}; x_p) = 0 \Rightarrow X_{\dot{x}_p} = X_0 \sqrt{\left(1 - \left(\frac{X_{\dot{x}_d}}{X_c^p}\right)^2\right)}$$

CONCLUSION

- ✓ Application de la MaN au cas des Z.C
- ✓ Immersion du modèle ZC Triangle dans le cadre de TPI
- ✓ Faisabilité (cas EF)
- ✓ Faisabilité Couplage-Plasticité et ZC Plas-Endo

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Pages 476-484. L. Daridon, B. Wattrisse, A. Chrysochoos, M.
Potier-Ferry