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# Individual vaccination as Nash equilibrium in a SIR model

the interplay between individual optimization and societal policies

Laetitia LAGUZET · Gabriel TURINICI

January 6, 2015 and 962 minutes

**Abstract** The vaccination against non recurrent epidemics is seldom compulsory but remains one of the most classical means to fight epidemic propagation. However recent debates concerning the innocuity of vaccines and their risk with respect to the risk of the epidemic itself lead to severe vaccination campaign failures and new mass behaviors appeared driven by individual self-interest. Prompted by this context we analyze, in a Susceptible-Infected-Recovered (SIR) model, whether egocentric individuals can reach an equilibrium with the rest of the society. Using techniques from the "Mean Field Games" theory, we show that an equilibrium exists and characterizes completely the individual best vaccination strategy. We also compare with a strategy based only on overall societal optimization and exhibit a situation with non-negative price of anarchy. Finally, we apply the theory to the 2009-2010 Influenza A (H1N1) vaccination campaign in France and show that a group of individuals stopped vaccinating at levels that indicated a very pessimistic perception of the risk of the vaccine.

Keywords: individual vaccination, mean field games, vaccine scares, SIR model, epidemic control, Nash equilibrium

MSC: 49, 62P10, 92B05

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## 1 Introduction

The vaccination, when available, is one of the most classical defense against an evolving epidemic. Theoretical works have been proposed to describe the optimal vaccination policy as function of the vaccine cost and epidemic dynamics and severity, see for instance (Hethcote and Waltman, 1973, Abakuks, 1974, Morton and Wickwire, 1974, Sethi and Staats, 1978, Diekmann and Heesterbeek, 1999, Laguzet and Turinici, 2014). The point of view of these initial studies is that of a benevolent planner that optimizes the overall societal welfare. However in most situations vaccination is not compulsory and people have the choice to vaccinate or not. For a number of childhood diseases (with no compulsory vaccination) a decay in the vaccination coverage has been observed in developed countries and debates over the danger of vaccines and their usefulness emerged. This suggested that the individual point of view has also to be taken into account: any individual may choose to vaccinate or not and the epidemic dynamics is influenced by all these individual choices. The individual defines a cost  $r_V$  incurred if he takes the vaccine and a cost  $r_I$  incurred if infected; the costs can be expressed in terms of money, medical side effects or general morbidity. For a discussion on this topic refer to Zeckhauser and Shepard (1976), Anand and Hanson (1997), Sassi (2006) and related literature on QALY/DALY measuring scales. The cost  $r_I$  is to be weighted by the probability to be infected; it is immediate that when the epidemic is near extinction there is very low incentive to vaccinate (because the probability to be infected is small). Thus people stop vaccinating before the epidemic stops, which can cause in fact the epidemic to start again. In theory the situation can be highly unstable oscillating between no vaccination, followed by an epidemic outburst, then massive vaccination followed by epidemic near extinction and so on. An important question is the existence of a stable equilibrium and the impact on the vaccination coverage.

In their seminal work, Bauch and Earn (2004) use a SIR model with vital dynamics to describe the propagation of a recurrent childhood disease; the individual choices converge to a sub-optimal vaccine coverage. The vaccination strategy of the individual is of all-or-nothing type and time-independent. Furthermore, Bauch (2005) proposes a learning process based on a "rule of thumb" to explain why and how the people vaccinate, resulting in a time-dependent vaccination strategy. The presence of a time-dependent optimal strategy is more realistic and an advance over the previous work but the model is dependent on the "rule of thumb" chosen *a priori*. This study was completed by Shim et al. (2012) where the depart from an 100% egocentric decision is described and altruism is seen to play an important role in vaccination decisions. Coelho and Codeço (2009) and Codeço et al. (2007) also model the vaccination behavior and apply the results to the yellow fever scare in Brazil. Buonomo et al. (2008) also introduce a feed-back mechanism but for a SEIR system.

In another work Reluga et al. (2006) studied the impact of the perceptions of the relative risk between vaccine and epidemic; they used a SIR model

with vital dynamics and also discussed the imitation dynamics. Galvani et al. (2007) consider a double SIR periodic model of influenza with vaccination and two age groups (more or less than 65 years). Vaccination is separated from dynamics and arrives once at the beginning of each season. The effect of the group dependent vaccination is analyzed. Cojocaru et al. (2007) and Cojocaru (2008), Chen (2006) also consider the mathematical questions related to the presence of several groups having distinct epidemic characteristics.

Further contributions for models with vital dynamics include d’Onofrio et al. (2007), d’Onofrio et al. (2008), Reluga and Galvani (2011).

A review on the relationship between human behavior and epidemic dynamics is proposed by Funk et al. (2010). In particular they discuss the relationship between the timescales of the vaccination and epidemic dynamics, which is also the object of several contributions by Vardavas et al. (2007), Breban et al. (2007). Chen (2009) discusses how the available information change the decisions made by an individual.

Even if realistic vaccination behavior is very likely to depend on the imitation and altruistic dynamics, there is a need to separate this part from the optimization of the cost-benefit ratio at the individual level.

Although most of the literature describes recurrent epidemics, non-recurrent epidemics also witnessed similar controversy as it was the case for the 2009-10 influenza A (H1N1) epidemic. In this case the vaccine is only relevant for that epidemic season and therefore recurrent epidemic models do not describe accurately the dynamics. We compare in Table 1 the difference between the target coverage of the vaccines, as defined by the sanitary authorities, and the effective rate of vaccination obtained. In many European countries one order of magnitude separates the two.

Country	Target coverage	Effective rate of vaccination
Germany	30 – 100%	8 – 10%
Spain	40%	4 – 27.1%
France	70 – 75%	7.9 – 8.5%
Italy	40%	1.4 – 4%

**Table 1** Vaccination Coverage expected and realized in different countries as percentage of population during the 2009-10 Influenza A (H1N1) epidemic. Sources: Guthmann et al. (2010), Mereckiene et al. (2012), Walter et al. (2011), Schwarzinger et al. (2010), Brien et al. (2012), (Door, 2010, page 157).

But today there is little theoretical guidance to explain this data from a model that only takes into account individual vaccination behavior for recurrent epidemics. In particular, while for recurrent epidemic it may be valid to suppose that some equilibrium is obtained, for an epidemic with a single propagation period the transient dynamics is important. Thus we expect the optimal vaccination strategy to be time-dependent. We also want to identify the vaccination dynamics and propose a model that only takes into account the

effect of the individual cost optimization; such a result can then help identify the impact of other effects: imitation, altruism.

We prove the existence of an equilibrium between individual vaccination and an epidemic propagation described by a deterministic SIR-model. We compare the theoretical result with the optimal policy at the societal level and describe the differences between the two. Finally we apply the theoretical result to the Influenza A (H1N1) 2009-10 epidemic season in France and observe that people were not homogeneous in their perception of the  $r_V/r_I$  quotient: while some saw the vaccine as harmless, a non-negligible portion of the population saw the vaccine as potentially dangerous.

### 1.1 The model

We consider an epidemic spreading in a non-immune population; the dynamics of the epidemic follows a SIR model (Anderson and May, 1992, Diekmann and Heesterbeek, 1999). The epidemic is supposed to take place at a rapid time frame (several months up to 1-2 years) when compared with the demographic dynamics (births, deaths) and therefore the model does not take into account any vital dynamics.

The costs incurred by an infected individual (either in terms of monetary value or of medical condition) are the same for any individual and are denoted by  $r_I$ . We also suppose that a vaccine giving lifelong immunity exists. Its cost is  $r_V$ , which takes into account not only the economic price but also all possible side-effects of the vaccine. The people in the Susceptible class can choose to vaccinate or not (people in other classes cannot vaccinate).

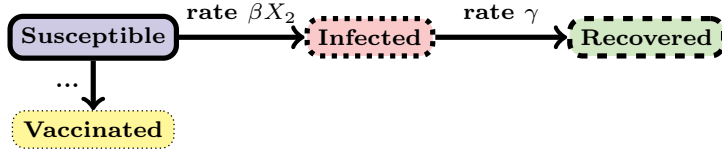
The mathematical equation describing the SIR model with vaccination is:

$$\begin{cases} dX_1(t) = -\beta X_1(t)X_2(t)dt - dU(t), & X_1(0^-) = X_{10}, \\ dX_2(t) = (\beta X_1(t)X_2(t) - \gamma X_2(t))dt, & X_2(0) = X_{20}, \\ dX_3(t) = \gamma X_2(t)dt, & X_3(0) = X_{30}, \\ X_4(t) = \int_0^t dU(\tau), & X_4(0^-) = 0. \end{cases} \quad (1)$$

Here  $X_1$  is the proportion of individuals in the Susceptible class,  $X_2$  is the proportion in the Infected class,  $X_3$  is the proportion of people in the Recovered class,  $X_4(t)$  is the proportion of people that are vaccinated by the time  $t$  and  $dU(t)$  the vaccination rate. The parameters  $\beta$ ,  $\gamma$  and the initial conditions  $X_{10}$ ,  $X_{20}$  and  $X_{30}$  are supposed known. Note that when vaccination is instantaneous the function  $X_4(t)$  may be discontinuous and its derivative  $dU(t)$  only exists as a measure. We refer to the Appendix A for the mathematical details.

Equations (1) only represents the overall, societal dynamics. At the microscopic level, the individual dynamics is modeled by a continuous time Markov chain with individual jumping between the Susceptible, Infected, Recovered and Vaccinated classes, as illustrated in Figure 1.

In this model all individuals are identical (but each may be in a specific state). Let  $M_t$  be the state of one such individual at time  $t$ . The jump from



**Fig. 1** Individual dynamics: continuous time Markov jumps between Susceptible, Infected, Recovered and Vaccinated classes.

the Susceptible class to the Infected class depends on the proportion of people already infected; the time of the jump from the Infected to the Recovered class is a exponential random variable of mean  $1/\gamma$ . We write in terms of probabilities:

$$\mathbb{P}\left(M_{t+\Delta t} = \text{Infected} \mid M_t = \text{Susceptible}\right) = \beta X_2(t) \Delta t + o(\Delta t) \quad (2)$$

$$\mathbb{P}\left(M_{t+\Delta t} = \text{Recovered} \mid M_t = \text{Infected}\right) = \gamma \Delta t + o(\Delta t). \quad (3)$$

The jumps from the Susceptible to the Vaccinated class are dependent on the willingness of the individual to vaccinate expressed through the vaccination parameter  $p_V(t)$ :

$$\mathbb{P}\left(M_{t+\Delta t} = \text{Vaccinated} \mid M_t = \text{Susceptible}\right) = p_V(t+\Delta t) - p_V(t) + o(\Delta t). \quad (4)$$

Note the different writing which is required by the fact that  $p_V(t)$  may not be differentiable in general.

We suppose that each individual is aware of the propagation dynamics at the societal level (1) but cannot influence it. His only concern is whether it is useful for himself to vaccinate or not and when to do it. This decision is called a *strategy*. A naive interpretation of the strategy is to believe that it represents the instant  $t^*$  in the future when vaccination will occur (with 100% certainty); in game theory terms this is called a *pure strategy*. However it turns out (see discussion in Appendix B) that it is better to represent the individual decision as a cumulative probability of vaccination  $\varphi_V$  with  $\varphi_V(t)$  representing the probability to vaccinate in the interval  $[0, t]$  **in the absence of any infection**. The mathematical definition of  $\varphi_V$  is:

$$d\varphi_V(t) = (1 - \varphi_V(t)) dp_V(t), \quad \varphi_V(0^-) = 0. \quad (5)$$

Similarly, a second useful mathematical object is the cumulative probability  $\varphi_I^{X_0, U}(t)$  of infection in  $[0, t]$  **in absence of vaccination** defined as:

$$\frac{d}{dt} \varphi_I^{X_0, U}(t) = \beta X_2(t) (1 - \varphi_I^{X_0, U}(t)), \quad \varphi_I^{X_0, U}(0) = 0. \quad (6)$$

Given the epidemic dynamics (fully determined by  $X(0^-)$  and  $U$ ) the individual can associate a cost to each individual strategy  $\varphi_V$ . The cost functional is the sum, for each time  $t$ , of the cost of vaccination (weighted by the probability  $d\varphi_V(t)$  that vaccination occurs at that time) and the cost of infection

(weighted by the probability  $d\varphi_I^{X_0,U}(t)$  that infection occurs at that time, see Appendix A). This leads to the following cost functional:

$$J_{indi}(\varphi_V; X_0, U) = \int_0^\infty r_I(1-\varphi_V(t))d\varphi_I^{X_0,U}(t) + r_V(1-\varphi_I^{X_0,U}(t))d\varphi_V(t). \quad (7)$$

Note that the probability for the individual to be in the Susceptible class at time  $t$  is  $(1-\varphi_V(t))(1-\varphi_I^{X_0,U}(t))$ . This explains the terms  $1-\varphi_V(t)$  (and  $1-\varphi_I^{X_0,U}(t)$  hidden inside  $d\varphi_I^{X_0,U}(t)$ ), and the corresponding ones for the second part of the integral. An equivalent form of the cost functional is:

$$J_{indi}(\varphi_V; X_0, U) = r_I\varphi_I^{X_0,U}(\infty) + \int_0^\infty [r_V - r_I\varphi_I^{X_0,U}(\infty) + (r_I - r_V)\varphi_I^{X_0,U}(t)]d\varphi_V(t). \quad (8)$$

Denoting:

$$\Omega = \{X = (X_1, X_2) \in \mathbb{R}^2 \mid X_1, X_2 > 0, X_1 + X_2 < 1\}, \quad (9)$$

we work under the constraints  $X \in \overline{\Omega}$ .

## 1.2 Questions regarding the equilibrium

When everybody is vaccinating this collective behavior stops the epidemic. For a given individual that still has to make its own choice, the perceived benefit of vaccination is low because the risk associated with the epidemic is very low. Why should he vaccinate then ? The individual will therefore **not** act as everybody else.

On the contrary, when nobody vaccinates and the individual foresees a severe epidemic (because of lack of vaccination for instance), the individual is lead to vaccinate, therefore will **not** act as everybody else.

In both situations there is an incoherence between the overall, societal, dynamics and the individual self-interest. A legitimate question is whether an equilibrium scenario exists where the best vaccination policy of the individual and the vaccination policy of everybody else agrees.

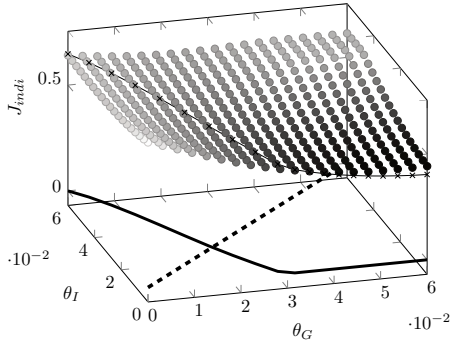
To illustrate this question consider the Figure 2. Vaccination is represented by a parameter indicated the time during which vaccination takes place. A low time indicates low vaccination level and a large time a high vaccination level. We plot a three-dimensional surface of the cost  $J_{indi}(\theta_I; X_0, \theta_G)$  incurred by an individual which has the vaccination time  $\theta_I$  in a circumstance when the societal vaccination time is  $\theta_G$ . Note that, with an abuse of notation, we write  $\theta_G$  instead of  $dU = \mathbb{1}_{[0,\theta_G]}u_{max}dt$  and  $\theta_I$  instead of the solution of  $d\varphi_V(t) = (1-\varphi_V(t))\mathbb{1}_{[0,\theta_I]}u_{max}/X_1(t)dt$ , see Appendices A and B.

For any societal vaccination strategy represented by the parameter  $\theta_G$  the individual minimizes  $\theta_I \mapsto J_{indi}(\theta_I; X_0, \theta_G)$ . The optimal value  $\theta_I^{opt}$  of  $\theta_I$  as a function of  $\theta_G$  is represented as the solid bottom curve in the Figure 2. The existence and uniqueness of the function  $\theta_I^{opt}(\theta_G)$  (for general societal

vaccination strategies  $dU$ , not necessarily given in terms of vaccination time) is illustrated in Section 2.1 and proved in Appendix C.

As expected, for low  $\theta_G$  the curve  $\theta_I^{opt}(\theta_G)$  takes a large value while for large values of  $\theta_G$  the curve  $\theta_I^{opt}(\theta_G)$  takes small values. The equilibrium is when  $\theta_I^{opt}(\theta_G) = \theta_G$ . We therefore ask for the existence of a common point of the solid and dashed curves. The existence and uniqueness of an equilibrium is illustrated in Section 2.2 and proved in Appendix D.

Such questions have been given a firm mathematical ground since the introduction of the "Mean Field Games" theory in the pioneering works of Lasry and Lions (2006a,b).



**Fig. 2**  $J_{indi}(\theta_I; 0.62501, 0.15001, \theta_G)$  for parameters  $\beta = 73$ ,  $\gamma = 36.5$ ,  $u_{max} = 10$ ,  $r_I = 1$  and  $r_V = 0.5$ . The solid bottom curve represent the optimal individual choice  $\theta_I^{opt}(\theta_G)$  and the dashed bottom curve is  $\theta_G = \theta_I$ .

## 2 Illustration of the theoretical results

### 2.1 Individual optimal strategy for arbitrary epidemic propagation

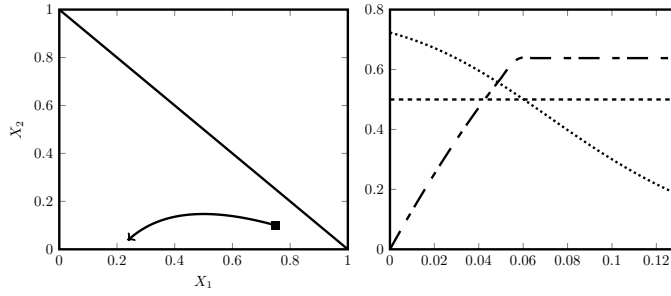
If the societal vaccination strategy  $dU$  is given, there exists an individual strategy  $\varphi_V$  that minimizes the individual cost (7). The strategy is unique (except degenerate cases). The technical details concerning the existence, uniqueness and other properties of the optimal individual strategy are proved in the Appendix C. Here we only illustrate these results in a particular case.

The theoretical results tell us that, in order to find the optimal decision at time  $t$ , an individual has to calculate the quantity  $\frac{\varphi_I^{X_0,U}(\infty) - \varphi_I^{X_0,U}(t)}{1 - \varphi_I^{X_0,U}(t)}$  and

compare it with  $r_V/r_I$ . While  $\frac{\varphi_I^{X_0,U}(\infty) - \varphi_I^{X_0,U}(t)}{1 - \varphi_I^{X_0,U}(t)} \geq r_V/r_I$  it is optimal for the individual to vaccinate, otherwise he should not vaccinate. Note that  $\frac{\varphi_I^{X_0,U}(\infty) - \varphi_I^{X_0,U}(t)}{1 - \varphi_I^{X_0,U}(t)}$  can only decrease during the course of an epidemic.

The Figure 3 gives an example of such a situation with  $U = 0$ . Until the time 0.06,  $\frac{\varphi_I^{X_0,U}(\infty) - \varphi_I^{X_0,U}(t)}{1 - \varphi_I^{X_0,U}(t)} \geq r_V/r_I$  and the individual vaccinates. After this time individual vaccination stops. The probability to be vaccinated is constant equal to 0.07% after time 0.06.





**Fig. 3** The parameters used are  $X_0 = (0.75, 0.1)$ ,  $\beta = 73$ ,  $\gamma = 36.5$ ,  $r_I = 1$ ,  $r_V = 0.5$ ,  $u_{max} = 10$  and constraint in Equation (20)  $f(t) = \frac{u_{max}}{X_1(t)}$ . **Left:** The trajectory  $(X_1(t), X_2(t))$  of the system (1) with  $U = 0$ . **Right:** The decreasing dotted curve is  $t \mapsto \frac{\varphi_I^{X_0, U}(\infty) - \varphi_I^{X_0, U}(t)}{1 - \varphi_I^{X_0, U}(t)}$ , the constant densely dashed line is the level  $r_V/r_I$  and the increasing dashed-dotted curve is the probability of vaccination over time.

## 2.2 Equilibrium strategies

In classical settings that disregard individual choices (see (Abakuks, 1974, Laguzet and Turinici, 2014)) the cost for the society defined as:

$$J_{soc}(X_0, U) = \int_0^\infty r_I X_1(t) \frac{d\varphi_I^{X_0, U}(t)}{1 - \varphi_I^{X_0, U}(t)} + r_V dU(t), \quad (10)$$

is the only quantity of interest and is minimized. An optimal societal strategy can be found, which will be denoted from now on (OS). The strategy (OS) partitions the domain  $\Omega$  into a region of vaccination and a region without vaccination. The curve that delimits the two regions is

$$\{X = (X_1, X_2) \in \Omega \mid \partial_{X_1} \zeta(X_1, X_2) = r_V/r_I\}. \quad (11)$$

*Remark 1* We recall that for any  $X \in \Omega$  one can introduce the number  $\zeta(X)$  of infected people (in absence of vaccination) for a trajectory starting in  $X$ . In particular  $\zeta$  is the solution of the following equation (see (Abakuks, 1974, 1972, Laguzet and Turinici, 2014)):

$$1 - \frac{\zeta(X)}{X_1} = e^{-\frac{\beta}{\gamma}(X_2 + \zeta(X))}. \quad (12)$$

In our setting, the situation is not a simple optimization. Any individual optimizes its cost but the coherence of the model **requires** that the aggregation of individual optimal policies  $\varphi_V$  result in the global vaccination policy  $dU$ . Therefore the situation is more adequately described by an equilibrium in the form of a fixed point property: a global vaccination  $dU$  gives optimal individual responses  $\varphi_V$  which sum up to form a societal response  $dU'$ . The equilibrium is reached only when  $dU = dU'$ .

The first good news is that the results in Appendix D show that an equilibrium always exists, i.e., even if the individuals are only driven by self-interest

some vaccination level is conserved (and does not drop to zero). This is completely coherent with other results from the literature, see Bauch and Earn (2004) and related works.

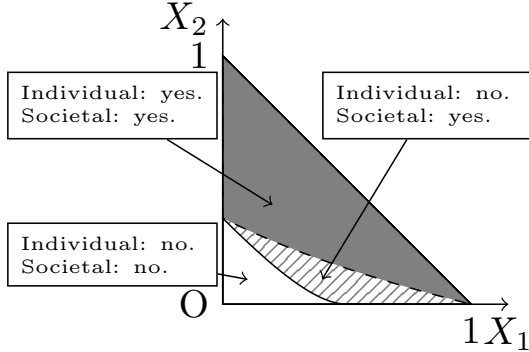
The couple of optimal individual strategy and resulting societal strategy in equilibrium will be denoted from now on (EIS). The equilibrium (EIS) is also described as a partition of  $\Omega$  into a region of vaccination and a region without vaccination. Individuals will vaccinate in the first region and stop vaccination upon reaching the boundary of the second region. The curve that delimits the two regions is of equation:

$$\{X = (X_1, X_2) \in \Omega \mid \zeta(X_1, X_2)/X_1 = r_V/r_I\}. \quad (13)$$

The Proposition 1 shows that the vaccination region of the (OS) strategy includes strictly the vaccination region of the (EIS) strategy.

The Figure 4 presents the main result of our paper and the comparison of the three regions in  $\Omega$ . The response of an individual to a given state  $X$  of the epidemic dynamics depends on the location of  $X$  in  $\Omega$ . In the gray region the individual (in the (EIS) strategy) will vaccinate and also the (OG) strategy is to vaccinate. In the hashed region the individual (in the (EIS) strategy) will not vaccinate but the (OG) strategy is to vaccinate. In the white region the individual (in the (EIS) strategy) will not vaccinate and the (OS) strategy is to not vaccinate.

Therefore only the hashed region, delimited by the curves  $\partial_{X_1}\zeta(X_1, X_2) = r_V/r_I$  and  $\zeta(X_1, X_2)/X_1 = r_V/r_I$ , is conflictual, in the sense that the individual does not have incentive to vaccinate but the societal best decision is to vaccinate.

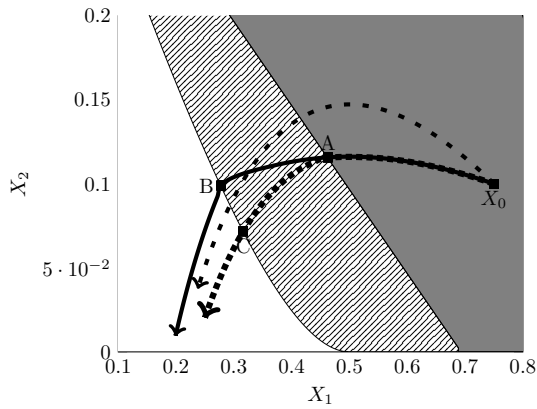


**Fig. 4** The representation of the domain  $\Omega$  with the three regions. Gray region: vaccination in the (OG) strategy and in the (EIS) strategy. Hashed region: vaccination in the (OG) strategy but not in the (EIS) strategy. White region: no vaccination in the (OG) strategy and no vaccination in the (EIS) strategy.

We illustrate in Figure 5 the societal trajectories for three possible strategies in equilibrium, that is, satisfying the constraint (19) (but not all optimal at the individual level). The dashed curve with no label corresponds to no vaccination and the individual cost is 0.65. The second dashed curve corresponds to the (EIS) strategy: between point  $X_0$  and  $A$ , the individual vaccinates

( $d\varphi_V = u_{max}/X_1(t)(1 - \varphi_V(t))$ ) and after point A, there is no vaccination ( $d\varphi_V = 0$ ). The individual cost is 0.55. The third curve corresponds to the trajectory with vaccination coherent with the (OS) societal strategy: vaccination occurs between point  $X_0$  and B. The individual cost is 0.53.

A very counter-intuitive fact is that the individual cost for the (OS) strategy is **lower** than the cost of the (EIS) strategy. Is this not in contradiction with the fact that the (EIS) strategy is the optimal strategy for an individual? How can be something better than the optimal? In fact the equilibrium is a Nash equilibrium which is not globally optimal. Although everybody would be better off adopting the (OS) strategy this choice is not stable at the individual level. A new individual added to such a population has the incentive to vaccinate less, driving the equilibrium towards the (EIS) stable point. See also Appendix E for the relation between the two costs. In game theory such a circumstance is termed "price of anarchy".



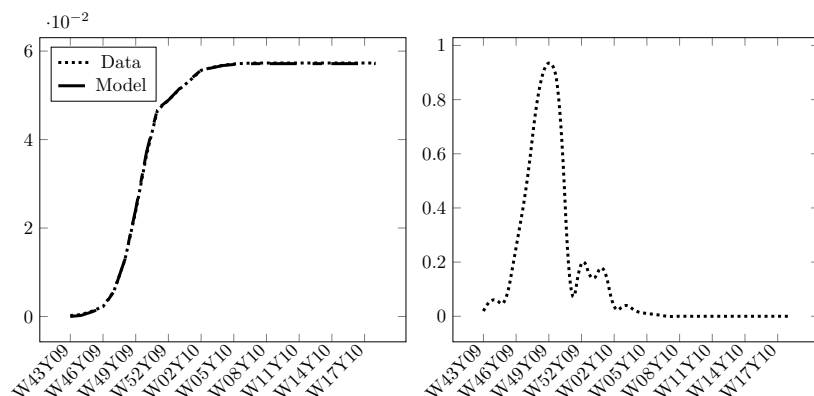
**Fig. 5** The dashed curve (without the point A) is with no vaccination ( $dU = 0$ ) and has individual cost 0.65; the solid curve is with (OS) strategy and has individual cost 0.53; the dashed curve (with point A) is with the (EIS) strategy and has individual cost 0.55. The parameters used for the three trajectories are  $\beta = 73$ ,  $\gamma = 36.5$ ,  $u_{max} = 10$ ,  $r_V = 0.5$ ,  $r_I = 1$ ,  $X_{10} = 0.75$ ,  $X_{20} = 0.1$ .

### 3 An application to the Influenza A (H1N1) 2009/2010 epidemic in France

#### 3.1 Epidemic context

The 2009/2010 Influenza A (H1N1) epidemic in France is a recent example of vaccination campaign that displays a large difference between the target, planned, vaccination coverage and the effective coverage obtained at the end of the campaign (see also (Schwarzinger et al., 2010)).

The 2009/2010 worldwide H1N1 epidemic spread through 213 countries and has been attributed 18156 deaths by 15/06/2010 (when epidemic was declared over by the WHO). In France the first cases appeared in May 2009 and 1334 severe forms were declared out of, approximately, 7.7 to 14.7 Millions people infected. The vaccination campaign in France was costly (around 500M



**Fig. 6** **Left:** Fit of the cumulative vaccination in percents weighted by the effectiveness coefficient. The curve  $U$  (labeled "Model") and the curve in Guthmann et al. (2010) (labeled "Data") cannot be distinguished. **Right:** Instantaneous vaccination  $dU$ , weighted by the effectiveness coefficient.

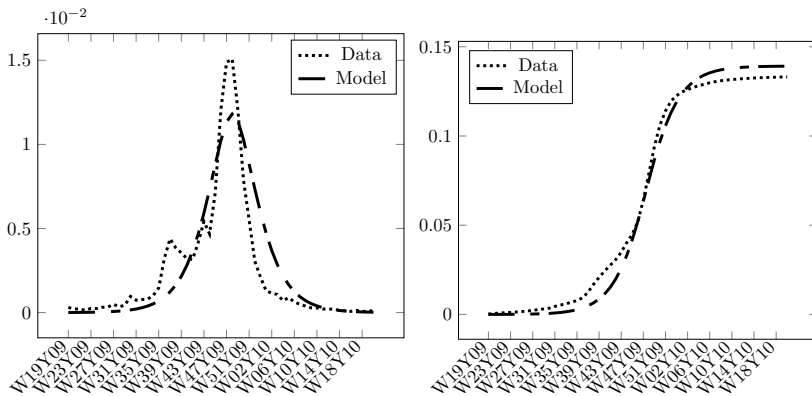
EUR, although cost estimation vary) for a low efficiency of 8% coverage (to be compared with 24% in the US or 74% in Canada) although the target was set to 75%. During the French vaccination campaign some undesired neurological side-effects of the adjuvant present in the vaccines were under debate and known to the public.

The campaign was designed in waves (people were called to the vaccine centers according to their risk status) and all vaccination centers were not 100% operational at once. Moreover the government mobilized additional medical personnel (military, medicine interns,...) latter during the epidemic propagation. The joint combination of these effects is that the maximum vaccination capacity was non-constant increasing from zero to some peak obtained by the end of 2009.

### 3.2 Data sources

The information concerning the vaccination capacities were available in the general news but no quantitative estimation is, up to our knowledge, available. However the actual cumulative vaccination curve was reported by Guthmann et al. (2010) (up to the maximum coverage of 7.9%). The vaccine effectiveness coefficient was estimated by Valenciano et al. (2011) to be 71.9%. We set  $U$  in the model (1) to fit this data, see Figure 6.

In what concerns the size of the Infected class, the French "Sentinel" network (cf. Flahault et al. (2006)) reports the estimation of the number of infections per week starting from the beginning of the epidemic. Using the same data Valleron and Guidet (2010) compared the epidemic with seasonal epidemic dynamics. Finally Bone et al. (2012) measure seroconversion and obtain results on the number of people immunized at the end of the epidemics. All



**Fig. 7** Comparison between the Infected class as obtained from the Sentinel network and the one from the model in (1). **Left:** Infected class as function of time. The peak of the epidemic is well reproduced. **Right:** cumulative number of Infections. The overall number of infection (at the end of the epidemic) is well reproduced.

this data was used to estimate the curve  $X_2(t)$  in the model (1). The data obtained by the Sentinel network takes into account the number of consultations in medical offices. However some infected persons are asymptomatic (but contribute to the propagation) and thus only some percentage of the infected individuals will consult a medical doctor. This percentage is difficult to evaluate; we follow the European Centre for Disease Prevention and Control (ECDC) (2009) who estimated this rate at 50%; thus to estimate the number of infections we set  $X_2(t)$  two times larger than the curve reported by the Sentinel network.

The propagation parameters  $\beta$  and  $\gamma$  were chosen consistent with ranges from the literature (see Boëlle et al. (2011)) although large confidence intervals were present, see for instance (Carrat et al., 2010). We took  $\mathcal{R}_0 = \beta/\gamma = 1.35$ . We fit  $\gamma$  in a very limited range,  $X_{10}$  in the range 0.8 to 0.95 (compatible with estimation concerning already immunized persons) and  $X_{20}$ . The parameters were optimized in order to reproduce the cumulative number of infections  $\int_0^\infty \beta X_1(t) X_2(t) dt$  and the peak of the epidemic. We obtained  $\gamma = 365/3.2$ ,  $X_{10} = 0.84$  and  $X_{20} = 2 \times 10^{-6}$ . The overall fit obtained is described in Figures 6 and 7. Given the large uncertainties surrounding the propagation parameters and the number of infections the fit is considered very satisfactory.

### 3.3 Methods

Once the overall dynamics (1) is given, there is still the quotient  $r_V/r_I$  to be estimated. Such an endeavor is very difficult because this is depending on the individuals' perception and very few data is available on the heterogeneity of this parameter within the population. Rather than using very uncertain data we preferred to see what the model says about the quotient  $r_V/r_I$ .

On the vaccination intensity Figure 6 (left) one can note that vaccination had a sharp increase up to week 50 – 51 of 2009 followed by a sharp decrease. The decrease was not due to the capacity of vaccination centers. We interpret it as reflecting a perception of a low infection risk coupled with high vaccination risk, i.e., a high quotient  $r_V/r_I$ . We can compute the precise value of the quotient  $r_V/r_I$  compatible with our model. In Figure 8 we plot three risk indicators: first of them is  $\frac{\varphi_I^{X_0,U}(\infty) - \varphi_I^{X_0,U}(t)}{1 - \varphi_I^{X_0,U}(t)}$ , the second  $\frac{\zeta(X(t))}{X_1(t)}$ ; the third is the quotient between the number of infections from  $t$  to  $\infty$  divided by the number of susceptible people in  $t$  (which is the initial value  $X_{10}$  minus the total infections and vaccinations between 0 and  $t$ ). Note that this last criterion is model free and can be computed from the vaccination coverage report and the Sentinel network data (weighted by the asymptomatic infected individuals rate).

We looked therefore at the level of the three criterions that were attained at the time when vaccination decreased. According to our model this level is an indication of the quotient  $r_V/r_I$ .

### 3.4 Results

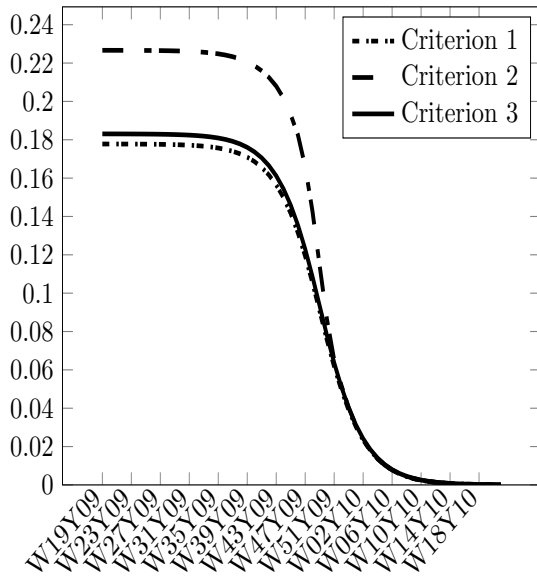
An individual that stops vaccinating at the peak of the curve in Figure 6 (left) will have  $r_V/r_I$  in the range 5% – 10%. This is a huge value meaning that 1 out of 10 vaccinated people will develop side-effects as severe as the influenza A H1N1 itself. Such situations reflect probably a communication failure around the epidemic and the vaccines (infection risk, severity, vaccine side effects, need for a mass vaccination campaign, ...) rather than specific medical data about the risks involved. See also (Brien et al., 2012) for a discussion on the determinants of individual vaccination and (Basu et al., 2008) for a related discussion concerning the controversial Human papillomavirus (HPV) vaccines.

Note also that vaccination did not stop completely at week 50 – 51 of 2009 but continued at detectable levels up to week 10 of 2010. An individual that stops vaccinating at week 10/2010 time has  $r_V/r_I < 1\%$  (probably much less as the model cannot be more precise with available data). This is a more common accepted figure.

Therefore at least two groups with very heterogeneous perceptions of the vaccination risk were present: a first group very worried about vaccine risk and another group less pessimistic.

## 4 Conclusions

In the context of individual vaccination (as opposed to global, compulsory vaccination) we develop an equilibrium model that allows to quantify the relationship between the individual perception of vaccine side-effects and of the epidemic morbidity.



**Fig. 8** Using the dynamics in (1), fitted with our data, we plot three criterions:  $\frac{\varphi_I^{X_0,U}(\infty) - \varphi_I^{X_0,U}(t)}{1 - \varphi_I^{X_0,U}(t)}$ ,  $\frac{\zeta(X(t))}{X_1(t)}$  and the quotient between the infections in  $[t, \infty[$  and the susceptibles in  $t$  (see text). By the time the first group of people stopped vaccination (week 50 – 51 of 2009) there was still 5 – 10% chance for an individual to contract the Influenza A. The individual that do not vaccinate at this point estimates the side effects of the vaccine to occur with frequency of about 5 – 10%, which is considered a very large and pessimistic value. Individuals that stopped last (week 10 of 2010) had this figure down to less than 1% (probably much less given data uncertainties).

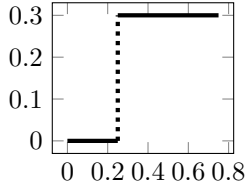
The first important result is that, even when individuals are only driven by self interest, such an equilibrium exists. Secondly we are able to characterize the individual decision in a very simple manner: the individual will vaccinate or not depending on his estimations of the number of Infected and Susceptible persons. The set  $\Omega$  of possible values of (Susceptible, Infected) couples is divided in two regions by the curve  $\frac{\zeta(X)}{X_1} = \frac{r_V}{r_I}$ . In one region vaccination is desired by the individual with maximum intensity (the perceived infection probability exceeds  $r_V/r_I$ ), while in the second region vaccination is not worthwhile at the individual level.

The equilibrium is a Nash equilibrium with an infinity of players (as pioneered by Lasry and Lions (2006a,b)), which means in particular that it can have (and has) a non-null "price of anarchy". This means that self-interest, although stable from both individual-societal points of view, is not the best solution that can be obtained.

Finally, we apply the results to the Influenza A 2009/2010 vaccination campaign in France and show, under the assumptions of the model, that at least two distinct groups existed, one very pessimistic about vaccine side-effects and the other rather optimistic.

## A On the societal and individual SIR model with vaccination

The mathematical description of the SIR model with vaccination has to take into account the possibility of instantaneous vaccination, which means that a non-negligible proportion of the total population can be vaccinated instantaneously at some time  $t$ . Recall that  $X_4(t)$  represents the proportion of people vaccinated by the time  $t$ . If, for instance, nobody vaccinates before time  $t^* = 0.25$  years (3 months) and 30% of the population vaccinates at time



**Fig. 9** Illustration of instantaneous vaccination of 30% percent of the population at time  $t^* = 0.25$ . The function  $X_4(t) = \int_0^{t^*} dU(\tau)$  is plotted. It has a discontinuity at time  $t^*$  and is equal to  $H(\cdot - t^*)$  with  $H(\cdot)$  the Heaviside function.

$t^*$  this means that  $X_4(t)$  is discontinuous at this point (see Figure 9 for an illustration). In particular it is not derivable and neither will be  $X_1(t)$ ; as such it is not possible to use in Equation (1) the derivative  $dX_1(t)/dt$ . This explains why the equation is only written in the differential form:  $dX_1(t) = -\beta X_1(t)X_2(t)dt - dU(t)$ . In this writing all objects have a well defined mathematical meaning:  $dU(t)$  is a positive measure which, for our example will be the Dirac mass  $0.3\delta_{t^*}$ . We refer to Bressan and Rampazzo (1991), Dal Maso and Rampazzo (1991), Miller (1996), Silva and Vinter (1997) for the mathematical properties of the solutions to such evolution equations. In the particular situation when  $U(t)$  has a jump at 0 we will have  $X_1(0^+) = X_1(0^-) + (U(0^+) - U(0^-))$ .

In this dynamics all individuals are the same. Each individual is following a continuous time Markov dynamics jumping between the states Susceptible, Infected, Recovered and Vaccinated.

In particular if one neglects the Infection the model has only two classes, Susceptible and Vaccinated. Consider the following identity:

$$\begin{aligned} \mathbb{P}(M_{t+\Delta t} = \text{Susceptible}) = \\ \left(1 - \mathbb{P}(M_{t+\Delta t} = \text{Vaccinated} \mid M_t = \text{Susceptible})\right) \mathbb{P}(M_t = \text{Susceptible}). \end{aligned}$$

We obtain, for  $\Delta t \rightarrow 0$  the differential equation for  $\varphi_V$  written in (5). This equation implies that we can choose to work with the variable  $\varphi_V(t)$  instead of  $p_V(t)$ . The interpretation of  $\varphi_V(t)$  as probability to vaccinate in  $[0, t]$  remains only an approximation when the Infected class exists.

For a given individual in the Susceptible class at time  $t$ , the probability to be infected during the time interval  $[t, t + \Delta t]$  is  $\beta X_2(t)\Delta t + o(\Delta t)$ . Define the probability of infection in the absence of vaccination, denoted  $\varphi_I^{X_0, U}(t)$ , that satisfies the Equation 6. Since  $X_2(t)$  is continuous  $\varphi_I^{X_0, U}$  is differentiable everywhere.

In particular, for an individual in the Susceptible class at time  $\tau$  that does not vaccinate any more from that time on, the probability of infection after time  $\tau$  is:

$$\frac{\varphi_I^{X_0, U}(\infty) - \varphi_I^{X_0, U}(\tau)}{1 - \varphi_I^{X_0, U}(\tau)}. \quad (14)$$

We can prove, by direct computations, that  $dU = 0$  on  $[\tau, \infty[$  implies:

$$\frac{\varphi_I^{X_0, U}(\infty) - \varphi_I^{X_0, U}(\tau)}{1 - \varphi_I^{X_0, U}(\tau)} = \frac{\zeta(X_1(\tau), X_2(\tau))}{X_1(\tau)}. \quad (15)$$

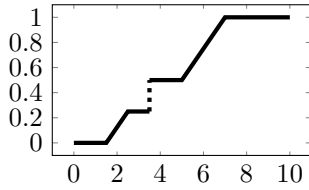
As a summary we recall that:

$$\begin{aligned} \mathbb{P}(M_{t+\Delta t} = \text{Infected} \mid M_t = \text{Susceptible}) &= \frac{\varphi_I^{X_0, U}(t + \Delta t) - \varphi_I^{X_0, U}(t)}{1 - \varphi_I^{X_0, U}(t)} + o(\Delta t) \\ &= \frac{d}{dt} \varphi_I^{X_0, U}(t) \Delta t + o(\Delta t), \end{aligned} \quad (16)$$

$$\mathbb{P}(M_{t+\Delta t} = \text{Recovered} \mid M_t = \text{Infected}) = \gamma \Delta t + o(\Delta t), \quad (17)$$

$$\mathbb{P}(M_{t+\Delta t} = \text{Vaccinated} \mid M_t = \text{Susceptible}) = \frac{\varphi_V(t + \Delta t) - \varphi_V(t)}{1 - \varphi_V(t)} + o(\Delta t). \quad (18)$$





**Fig. 10** The individual cumulative probability of vaccination  $\varphi_V(t)$  is an increasing, right continuous with left limits (càdlàg) function with  $\varphi_V(0) = 0$ ,  $\varphi_V(\infty) \leq 1$ .

We have therefore a full specification of the Markov chain in terms of  $\varphi_I^{X_0, U}$  and  $\varphi_V$ .

Recall that the global dynamics is an aggregation of individual dynamics. Therefore:

- $X_2$  in Equation (1) is the same as in (6)
- a compatibility relation has to exist between  $dU(t)$  (societal vaccination) and  $\varphi_V$  (individual vaccination).

To make explicit this last compatibility requirement one has to investigate the relationship between the Markov chain of one individual and the evolution dynamics of the population; such an endeavor is beyond the scope of this work but has been explored in several papers starting with Kurtz (1970), see also Sandholm (2010, Theorem 10.2.3 pages 373-374) for an application to population dynamics. Further results under specific technical assumptions are given in Benaim et al. (2005). The conclusion is that the societal dynamics is compatible with the individual dynamics when  $dp_V = \frac{1}{X_1(t)} dU$  or in terms of  $\varphi_V$ :

$$dU(t) = \frac{d\varphi_V(t)}{1 - \varphi_V(t)} X_1(t). \quad (19)$$

## B Individual strategies

The simplest individual strategy is to vaccinate or not at some given time; this strategy is naive because the individual faces some uncertainty: he is not sure whether he will be still in class  $X_1$  by time  $t^*$ . In addition the process of vaccination may depend on random factors that the individual does not control. Moreover it was shown in the general context of *timing games* (see the reference (Fudenberg and Tirole, 1991, page 118)) that representing the individual decision as a cumulative probability of vaccination  $\varphi_V$  allows to obtain a better theoretical description of the equilibrium. Here  $\varphi_V(t)$  is the probability to be vaccinated in the interval  $[0, t]$  (in absence of any infection). A strategy of this type is called a *mixed strategy*. Note that  $\varphi_V$  is not necessary a continuous function (the discussion is very much similar to the one in Appendix A). In particular when the decision of the individual is a pure strategy to vaccinate with certainty at time  $t^*$  then  $\varphi_V$  is the Heaviside function  $H(\cdot - t^*)$ . See the illustration in Figure 10.

It may also be necessary to impose some constraints. Suppose that global vaccination (at the society level) can only happen with the maximal rate of  $u_{max}$  percent of population in a unit time. Then, with the notations in Equation (1):  $U(t + \Delta t) - U(t) \leq u_{max} \Delta t$ . Suppose now that all individuals want to vaccinate at the same time with the same  $\varphi_V$  then the constraint above, coupled with (19), implies the individual constraint:  $d\varphi_V(t) \leq \frac{u_{max}}{X_1(t)} (1 - \varphi_V(t)) dt$ . In general we consider a function  $f(t)$  instead of  $\frac{u_{max}}{X_1(t)}$ ; when the function  $f$  is locally bounded (in  $L_{loc}^\infty$ ) it follows that  $\varphi_V(t)$  is derivable and one can write the constraints in the form:

$$\frac{d\varphi_V(t)}{dt} \leq f(t)(1 - \varphi_V(t)). \quad (20)$$

## C Proof of the properties of the individual optimal strategy

**Theorem 1 (Case  $\varphi_V$  bounded)** *Let  $U$  be a given societal policy in (1), increasing,  $U(t) \leq 1$ ,  $\forall t \geq 0$  and  $f_V : \mathbb{R}_+ \rightarrow [0, +\infty[$  be a locally bounded function (that is  $f \in$*

$L_{loc}^\infty([0, \infty[))$ . Suppose that the set of admissible individual strategies is composed of all  $\varphi_V$  such that:

$$\frac{d\varphi_V(t)}{dt} \leq f_V(t)(1 - \varphi_V(t)). \quad (21)$$

Then the individual optimal strategy  $\varphi_V^*$  that minimizes the cost in Equation (7) (with the system (1)) is:

- a/ If  $r_I \leq r_V$ :  $\varphi_V^*(t) = 0$ .
- b/ If  $r_I > r_V$  and  $\varphi_I^{X_0, U}(\infty) \leq r_V/r_I$  then  $\varphi_V^*(t) = 0$ .
- c/ If  $r_I > r_V$  and  $\varphi_I^{X_0, U}(\infty) > r_V/r_I$  then  $\varphi_V^*(t)$  is the solution of

$$\frac{d\varphi_V^*(t)}{dt} = f_V(t)\mathbb{1}_{[0, \theta_I]}(1 - \varphi_V^*(t)). \quad (22)$$

The parameter  $\theta_I$  is the unique solution of the equation:

$$\frac{\varphi_I^{X_0, U}(\infty) - \varphi_I^{X_0, U}(\theta_I)}{1 - \varphi_I^{X_0, U}(\theta_I)} = \frac{r_V}{r_I}. \quad (23)$$

*Proof* We use individual cost in the form in Equation (8). It may be noted that:

$$r_V - r_I \varphi_I^{X_0, U}(\infty) + (r_I - r_V) \varphi_I^{X_0, U}(t) = r_V(1 - \varphi_I^{X_0, U}(\infty)) + (r_V - r_I)(\varphi_I^{X_0, U}(\infty) - \varphi_I^{X_0, U}(t)). \quad (24)$$

**Case a/** If  $r_I \leq r_V$ , since  $\varphi_I^{X_0, U}$  is an increasing function, the quantity in (24) is the sum of positive terms. The minimum attainable value is therefore zero and it obtained when  $d\varphi_V(t) = 0 \forall t$ . Or  $\varphi_V(0^-) = 0$  thus  $\varphi_V(t) = 0 \forall t \geq 0$ .

**Cases b/ and c/** If  $r_I \geq r_V$  let us compute

$$\frac{d}{dt} \left[ r_V - r_I \varphi_I^{X_0, U}(\infty) + (r_I - r_V) \varphi_I^{X_0, U}(t) \right] = (r_I - r_V) \frac{d\varphi_I^{X_0, U}(t)}{dt} \geq 0, \quad (25)$$

(recall that  $\varphi_I^{X_0, U}(t)$  is increasing) and moreover  $r_V - r_I \varphi_I^{X_0, U}(\infty) + (r_I - r_V) \varphi_I^{X_0, U}(0) = r_V - r_I \varphi_I^{X_0, U}(\infty)$  is positive as soon as  $\varphi_I^{X_0, U}(\infty) \leq \frac{r_V}{r_I}$ . To minimize the cost, vaccination should not occur when the term to integrate against  $d\varphi_V(t)$  is positive therefore there is no vaccination if  $\varphi_I^{X_0, U}(\infty) \leq \frac{r_V}{r_I}$ .

If  $\varphi_I^{X_0, U}(\infty) > \frac{r_V}{r_I}$ , vaccination occurs for all  $t$  such that  $r_V - r_I \varphi_I^{X_0, U}(\infty) + (r_I - r_V) \varphi_I^{X_0, U}(t) \leq 0$ , or equivalently  $\frac{\varphi_I^{X_0, U}(\infty) - \varphi_I^{X_0, U}(t)}{1 - \varphi_I^{X_0, U}(t)} \geq \frac{r_V}{r_I}$ .

Moreover,

$$\frac{d}{dt} \left[ \frac{\varphi_I^{X_0, U}(\infty) - \varphi_I^{X_0, U}(t)}{1 - \varphi_I^{X_0, U}(t)} \right] = - \frac{[1 - \varphi_I^{X_0, U}(\infty)] d\varphi_I^{X_0, U}(t)}{[1 - \varphi_I^{X_0, U}(t)]^2} < 0. \quad (26)$$

Then  $t \mapsto \frac{\varphi_I^{X_0, U}(\infty) - \varphi_I^{X_0, U}(t)}{1 - \varphi_I^{X_0, U}(t)}$  is a decreasing, continuous function from  $\varphi_I^{X_0, U}(\infty)$  to zero.

To minimize the cost, the vaccination should appear right at the beginning and last until the boundary of the domain  $\Omega$  is attained by the dynamics (1) or until time  $\theta_I$ , the unique solution of  $\frac{\varphi_I^{X_0, U}(\infty) - \varphi_I^{X_0, U}(\theta_I)}{1 - \varphi_I^{X_0, U}(\theta_I)} = \frac{r_V}{r_I}$ . Therefore in order to minimize the integral one has to set  $d\varphi_V = 0$  on  $[\theta_I, \infty[$  and  $d\varphi_V > 0$  on  $[0, \theta_I[$  with maximal values coming first. Taking into account the constraint (21) we obtain  $d\varphi_V = f_V(t)(1 - \varphi_V(t))$  on  $[0, \theta_I]$  and  $d\varphi_V = 0$  on  $[\theta_I, \infty[$ .

The next results applies when the individual vaccination can be unbounded, i.e.,  $d\varphi_V$  can contain Dirac masses.

**Theorem 2 (Case  $\varphi_V$  unbounded)** *The individual strategy  $\varphi_V^*$  that minimizes the cost (7) with the system (1):*

- a/ if  $\frac{\varphi_I^{X_0,U}(\infty) - \varphi_I^{X_0,U}(0)}{1 - \varphi_I^{X_0,U}(0)} < \frac{r_V}{r_I}$  then  $\varphi_V^* \equiv 0$ ,
- b/ if  $\frac{\varphi_I^{X_0,U}(\infty) - \varphi_I^{X_0,U}(0)}{1 - \varphi_I^{X_0,U}(0)} > \frac{r_V}{r_I}$  then  $\varphi_V^* = \delta_0$ ,
- c/ if  $\frac{\varphi_I^{X_0,U}(\infty) - \varphi_I^{X_0,U}(0)}{1 - \varphi_I^{X_0,U}(0)} = \frac{r_V}{r_I}$  then  $\varphi_V^* = \alpha \delta_0$  with  $\alpha$  arbitrary in  $[0, 1]$ .

*Proof Case a/* If  $\int_0^\infty r_V - r_I \varphi_I^{X_0,U}(\infty) + (r_I - r_V) \varphi_I^{X_0,U}(t) d\varphi_V(t)$  is positive, then the minimum is greater than  $r_I \varphi_I^{X_0,U}(\infty)$ . This value is attained only if  $\int_0^\infty r_V - r_I \varphi_I^{X_0,U}(\infty) + (r_I - r_V) \varphi_I^{X_0,U}(t) d\varphi_V(t) = 0$  so  $\varphi_V(t) = 0$  for all  $t$ .

*Case b/* The strict monotony of the integrand  $r_V - r_I \varphi_I^{X_0,U}(\infty) + (r_I - r_V) \varphi_I^{X_0,U}(t)$  allows to write:

$$\begin{aligned} & \int_0^\infty r_V - r_I \varphi_I^{X_0,U}(\infty) + (r_I - r_V) \varphi_I^{X_0,U}(t) d\varphi_V(t) \\ & \geq \int_0^\infty \left[ r_V - r_I \varphi_I^{X_0,U}(\infty) + (r_I - r_V) \varphi_I^{X_0,U}(t) \right]_{t=0} d\varphi_V(t) \\ & = \left[ r_V - r_I \varphi_I^{X_0,U}(\infty) + (r_I - r_V) \varphi_I^{X_0,U}(0) \right] \int_0^\infty d\varphi_V(t) \\ & \geq \left[ r_V - r_I \varphi_I^{X_0,U}(\infty) + (r_I - r_V) \varphi_I^{X_0,U}(0) \right], \end{aligned}$$

where we used  $\varphi_V(\infty) \leq 1$  and  $\left[ r_V - r_I \varphi_I^{X_0,U}(\infty) + (r_I - r_V) \varphi_I^{X_0,U}(0) \right] \leq 0$ . This gives a lower bound for the minimum. The bound is attained when both inequalities become equalities that is  $\varphi_V(t)$  is the Dirac mass in 0.

*Case c/* The difference with the previous case is that the last term  $\left[ r_V - r_I \varphi_I^{X_0,U}(\infty) + (r_I - r_V) \varphi_I^{X_0,U}(0) \right]$  is 0. Therefore the last inequality is always satisfied. We obtain the conclusion.

## D Equilibrium strategy

**Theorem 3 (Case  $u_{max}$  bounded)** *Consider  $u_{max} < \infty$  and admissible individual strategies that satisfy:*

$$\frac{d}{dt} \varphi_V(t) \leq \frac{u_{max}}{X_1(t)} (1 - \varphi_V(t)). \quad (27)$$

*Then:*

- a/ If  $r_V \geq r_I$  or  $r_V < r_I$  and  $\zeta(X_0) \leq \frac{r_V}{r_I} X_{10}$  then the unique equilibrium is  $U = 0 = \varphi_V$ .
- b/ Otherwise consider the dynamics  $(Y_1^\infty, Y_2^\infty)$  starting from  $X_0$  with  $dU = u_{max} dt$  at all times until  $X_{10} = 0$ . Let  $\theta^{u_{max}}(X_0)$  be the first time when this dynamics touches the ensemble of curves:

$$\left\{ (X_1, X_2) \in \Omega \left| \frac{\zeta(X_1, X_2)}{X_2} = \frac{r_V}{r_I} \right. \right\} \cup \left\{ (X_1, X_2) \in \Omega \mid X_1 = 0 \right\}. \quad (28)$$

*Then the unique equilibrium is characterized by:*

$$dU(t) = u_{max} \mathbb{1}_{[0, \theta^{u_{max}}(X_0)]}(t) dt, \quad (29)$$

$$d\varphi_V(t) = \frac{u_{max}}{X_1(t)} (1 - \varphi_V(t)) \mathbb{1}_{[0, \theta^{u_{max}}(X_0)]}(t). \quad (30)$$

*Proof Case a/* If  $r_V > r_I$  then any optimal individual strategy is  $\varphi_V = 0$  and the compatibility relation (19) imply  $U = 0$ . Therefore the only possible equilibrium in this case is  $U = 0 = \varphi_V$ . It is easy to see that this is effectively an equilibrium.

If  $r_V < r_I$  and  $\zeta(X_0) \leq \frac{r_V}{r_I} X_{10}$ , note that  $\zeta(X_0)$  does not depend on  $U$  therefore any optimal individual strategy is  $\varphi_V = 0$ . As before we obtain the unique equilibrium  $U = 0 = \varphi_V$ .

**Case b/** Since  $\zeta(X_0) > \frac{r_V}{r_I} X_{10}$  any optimal individual strategy must satisfy (22) with  $f_V(t) = u_{max}/X_1(t)$ . Therefore, from the compatibility relation (19) we obtain  $dU(t) = u_{max} \mathbb{1}_{[0, \theta]}(t)$  for some  $\theta \geq 0$ . We have to find  $\theta$  such that  $\theta$  is solution of (23) for  $dU(t) = u_{max} \mathbb{1}_{[0, \theta]}(t)$ . This is a fixed point equation.

Let  $(X_1, X_2)$  be the solution of the system (1) with  $dU(t) = u_{max} \mathbb{1}_{[0, \theta]}(t)$ . Since  $dU = 0$  on  $[\theta, \infty]$  by Equation (15), any  $\theta$  solution of (23) is also solution of

$$\frac{\zeta(X_1(\theta), X_2(\theta))}{X_1(\theta)} = \frac{r_V}{r_I}. \quad (31)$$

But  $(X_1, X_2) = (Y_1^\infty, Y_2^\infty)$  for  $t \leq \theta$ , thus:

$$\frac{\zeta(X_1(\theta), X_2(\theta))}{X_1(\theta)} = \frac{\zeta(Y_1^\infty(\theta), Y_2^\infty(\theta))}{Y_1^\infty(\theta)}. \quad (32)$$

Therefore, any  $\theta$  that represents an equilibrium is also solution of the equation:

$$\frac{\zeta(Y_1^\infty(\theta), Y_2^\infty(\theta))}{Y_1^\infty(\theta)} = \frac{r_V}{r_I}. \quad (33)$$

But,

$$\begin{aligned} \frac{d}{dt} \left[ \frac{\zeta(Y_1^\infty(t), Y_2^\infty(t))}{Y_1^\infty(t)} \right] &= \frac{1}{(Y_1^\infty(t))^2} \left[ \left( \frac{\partial \zeta}{\partial X_1} \frac{dY_1^\infty(t)}{dt} + \frac{\partial \zeta}{\partial X_2} \frac{dY_2^\infty(t)}{dt} \right) Y_1^\infty(t) - \zeta \frac{dY_1^\infty(t)}{dt} \right] \\ &= \frac{(\zeta - Y_1^\infty(t))(\zeta u_{max} + \beta Y_1^\infty(t) Y_2^\infty(t) (\zeta - Y_1^\infty(t) + \gamma/\beta))}{(Y_1^\infty(t))^2 (1 + \beta/\gamma (\zeta - Y_1^\infty(t)))} < 0, \end{aligned}$$

because  $\zeta < X_1$  and  $\zeta > X_1 - \gamma/\beta$  (see Laguzet and Turinici (2014) for details and the expressions of the partial derivative of  $\zeta$ ).

Thus if (33) has a solution then this solution is unique. Therefore in any case at most one equilibrium exists. Under the hypothesis  $\zeta(X_0) < \frac{r_V}{r_I} X_{10}$  Equation (33) does have a solution so  $\theta$  is also solution of Equation (31) and this is the equilibrium.

**Theorem 4 (Case  $u_{max}$  unbounded)** When  $u_{max} = \infty$  the equilibrium is:

- a/ If  $r_V \geq r_I$  or  $r_V < r_I$  and  $\zeta(X_0) \leq \frac{r_V}{r_I} X_{10}$  then the unique equilibrium is  $dU = 0$  and  $d\varphi_V = 0$  (no vaccination).
- b/ If for any  $\alpha \in [0, X_{10}] : \zeta(X_0 - \alpha(1, 0)) > \frac{r_V}{r_I} (X_{10} - \alpha)$  then the unique equilibrium is  $dU = \delta_0 X_{10}$  and  $d\varphi_V = \delta_0$  (total instantaneous vaccination).
- c/ Otherwise let  $\alpha \in [0, 1]$  be the (unique) solution of

$$\zeta(X_0 - \alpha(1, 0)) = \frac{r_V}{r_I} (X_{10} - \alpha). \quad (34)$$

In this case the unique equilibrium is  $dU = \alpha \delta_0$  and  $d\varphi_V = \alpha/X_{10} \delta_0$  (partial instantaneous vaccination).

*Proof* Previous results indicate that in the unbounded case the individual optimal strategies are of the form  $\varphi_V = \eta \delta_0$  thus  $dU = \eta X_{10} \delta_0$ . Moreover, let  $\alpha = \eta X_{10}$  and recall that  $X_1(0) = X_1(0^-) - \alpha = X_{10} - \alpha$ ; moreover:

$$\begin{aligned} \frac{d}{d\alpha} \left[ \frac{\zeta(X_{10} - \alpha, X_{20})}{X_{10} - \alpha} \right] &= \frac{1}{(X_{10} - \alpha)^2} \left[ \frac{\partial \zeta}{\partial X_1} (-1)(X_{10} - \alpha) - \zeta(-1) \right] \\ &= \frac{\zeta(\zeta - X_{10} + \alpha)}{(X_{10} - \alpha)^2 (\gamma/\beta + \zeta - X_{10} + \alpha)} < 0, \end{aligned}$$

so the function  $\alpha \rightarrow \frac{\zeta(X_{10}-\alpha, X_{20})}{X_{10}-\alpha}$  is decreasing. Similar arguments as in the proof of the Theorem 3 apply and allow to reach the conclusion.

**Proposition 1** *The vaccination region of the (OS) strategy contains the vaccination region of the (EIS) strategy.*

*Proof* In the (OS) strategy, the vaccination only stops if  $\partial_{X_1}\zeta \leq \frac{r_V}{r_I}$  (see Laguzet and Turinici (2014) for details). Or  $\partial_{X_1}\zeta = \frac{\zeta}{X_1} \frac{1}{1+\frac{\beta}{\gamma}(\zeta-X_1)}$  and  $\zeta - X_1 \leq 0$  so  $1+\frac{\beta}{\gamma}(\zeta-X_1) \leq 1$ .

Then  $\frac{\zeta}{X_1} \leq \partial_{X_1}\zeta \leq \frac{r_V}{r_I}$  and the conclusion follows.

## E Relation between global and individual cost.

**Lemma 1** *Let  $\varphi_V$  and  $U$  satisfy Equation (19). Then the individual cost is the average of the global cost, that is:*

$$\frac{1}{X_1(0)} J_{soc}(X_0, U) = J_{indi}(\varphi_V; X_0, U). \quad (35)$$

*Proof* We can write:

$$\int_0^\infty r_I(1-\varphi_V(t))d\varphi_I^{X_0, U}(t) = \int_0^\infty r_I(1-\varphi_V(t))(1-\varphi_I^{X_0, U}(t)) \left[ -\frac{dX_1(t)}{X_1(t)} - \frac{dU(t)}{X_1(t)} \right] \quad (36)$$

because from (1) for  $X_1(t) \neq 0$ :

$$\frac{d\varphi_I^{X_0, U}(t)}{1-\varphi_I^{X_0, U}(t)} = \beta X_2(t) = -\frac{dX_1(t)}{X_1(t)} - \frac{dU(t)}{X_1(t)}. \quad (37)$$

Furthermore (19) implies:

$$\int_0^\infty r_V(1-\varphi_I^{X_0, U}(t))d\varphi_V(t) = \int_0^\infty r_V(1-\varphi_V(t))(1-\varphi_I^{X_0, U}(t)) \left[ \frac{dU(t)}{X_1(t)} \right]. \quad (38)$$

By summing the Equations (36) and (38) we get:

$$\begin{aligned} J_{indi}(\varphi_V; X_0, U) &= \int_0^\infty (1-\varphi_V(t))(1-\varphi_I^{X_0, U}(t)) \left[ -r_I \frac{dX_1(t)}{X_1(t)} - r_I \frac{dU(t)}{X_1(t)} + r_V \frac{dU(t)}{X_1(t)} \right] \\ &= \int_0^\infty \frac{X_1(t)}{X_1(0)} \left[ -r_I \frac{dX_1(t)}{X_1(t)} - r_I \frac{dU(t)}{X_1(t)} + r_V \frac{dU(t)}{X_1(t)} \right] \\ &= \frac{1}{X_1(0)} \int_0^\infty [-r_I dX_1(t) - r_I dU(t) + r_V dU(t)], \end{aligned} \quad (39)$$

where we used

$$(1-\varphi_V(t))(1-\varphi_I^{X_0, U}(t)) = \frac{X_1(t)}{X_1(0)}. \quad (40)$$

Using Equation (6) and the definition of  $J_{soc}(X_0, U)$  in Equation (10), we obtain the result.

**Corollary 1** *The average cost per person with the (OS) strategy is lower than the average cost per person with the (EIS) strategy.*

*Proof* Denote by  $U^G$  the (OG) strategy and  $\varphi_V^G$  its individual counterpart. Also denote by  $\varphi_V^I$  the individual strategy in the (EIS) equilibrium and  $U^I$  its global counterpart. By the definition of the optimality of  $U^G$ :  $J_{soc}(X_0, U^G) \leq J_{soc}(X_0, U^I)$ . From Lemma 1 after simplification by  $X_1(0)$  we obtain:  $J_{indi}(\varphi_V^G; X_0, U^G) \leq J_{indi}(\varphi_V^G; X_0, U^I)$  which is the conclusion.

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