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Robust control of remotely operated vehicle in the vertical plane

Adel Khadhraoui*, Lotfi Beji*, Samir Otmane*, and Azgal Abichou†
*University of Evry, IBISC Laboratory, France, Evry 91020
Email: (adel.khadhraoui, lotfi.beji, samir.otmane)@ibisc.univ-evry.f
†Polytechnic School of Tunisia, LIM Laboratory, BP743, 2078 La Marsa, Tunisia
Email: Azgal.Abichou@ept.rnu.tn

***Abstract*—This paper addresses the problem of stabilizing and trajectory tracking control of a remotely operated vehicle (ROV-Observer) in the vertical plane. The underwater vehicle is not actuated in the pitch direction and cannot be locally asymptotically stabilized by continuous feedbacks which are functions of the state only. In the present note, exponential convergence is obtained by considering time-varying feedbacks which are only continuous. Furthermore, using the backstepping technique and the tracking error dynamics, the system states are stabilized by forcing the tracking errors to an arbitrarily small neighborhood of zero.**

I. Introduction

Autonomous and remotely operated marine vehicles are becoming a key component in several aspects of maritime industry and defense. The problem of stabilizing an underactuated underwater at a desired trajectory is an important issue in many offshore applications such as, for example, solving trajectory tracking, path-following, path-tracking and stabilization problems. Modern developments in the fields of control, sensing, and communications have contributed to make very complex and dedicated underwater robot systems a reality. This kind of vehicle could be used in highly hazardous and unknown environments. The autonomy and control of the robot is the key factor to its mission success. In spite of highly coupled and nonlinear the dynamics of underwater vehicle system in nature decoupled, linear control system strategy is widely used for practical applications. As autonomous underwater vehicle needs intelligent control system, it is necessary to develop control system that really takes into account the coupled and nonlinear characteristics of the system. In addition, most of the AUVs are underactuated [2], i.e., they have fewer actuated inputs than the degrees of freedom, imposing non-integrable acceleration constraints. The problem problem of designing a stabilizing feedback controller for under-actuated systems is challenging since the system is not able to be stabilized by a smooth static state feedback law [15].

For controlling the motion of underwater vehicles, various control strategies have been developed in years, such as PID, LQG, SMC, neural network, fuzzy logic, SM fuzzy logic controllers. An example for the application of PID control to the underwater vehicles is presented in [16]. In [17], a discontinuous state feedback control law was proposed by using σ-process to exponentially stabilize the underactuated ship at the origin. The authors in [18] developed a discontinuous time-varying feedback stabilizer for a nonholonomic system and applied to underactuated ships. As cited in [4], continuous time-invariant controller was developed to achieve global exponential position tracking for underactuated AUV. However, the orientation of the AUV was not controlled. In [19] the problem of stabilization of underactuated surface vessel with various disturbances such as waves and wind is treated. To achieve semi-global asymptotic tracking in the presence of uncertainty and unknown disturbances, a non linear controller for a fully actuated AUV using a robust integral of the design of the error feedback term with a neural network is proposed in [9]. Based on Lyapunov's direct method and passivity approach, two constructive tracking solutions were proposed in [5] for an underwater robot. In [3], a single controller was proposed to solve both stabilization and tracking simultaneously. A new type of control law is developed in [8] to steer an autonomous underwater vehicle (AUV) along a desired path, it overcomes stringent initial condition constraints that are present in a number of path-following control strategies described in the literature.

We study an ultraportable submarine vehicle, called ROV, where it is expected for observation and exploration in subsea historical sites. The ROV is procured by the Digital-Ocean 2 project from SUBSEA TECH society. In this paper, we propose two controls input to resolve the stabilizing and tracking problem of underactuated ROV moving in vertical plane. Our contribution is to design methodology to construct a nonlinear controller of underactuated ROV to stabilize the ROV kino-dynamic model around a desired position and attitude. Further, based on Lyapunov theory and backstepping techniques, the proposed controller achieves global stability and convergence of the position tracking error to a neighborhood of the origin that can be made arbitrarily small.

The paper is organized as follows: The kinematic and dynamic model of the ROV is addressed in section II. A set of continuous time-varying control laws which locally asymptotically and exponentially stabilizes the desired attitude is proposed in Section III. In section IV, we present the result for the tracking control problem. Finally, Simulation results are presented in section V.

II. ROV MODEL DESCRIPTION

The ROV has a close frame structure is equipped with two cameras which allow us the Tele-exploration in mixed-reality sites (see Fig.1). This vehicle is actuated with two reversible horizontal thrusters F_{1x} and F_{2x} for surge and yaw motion, and a reversible vertical thruster F_{3z} for heave motion. A 150 meters cable provides electric power to the thrusters and enables communication between the vehicle sensors and the surface equipment (see Fig.1).

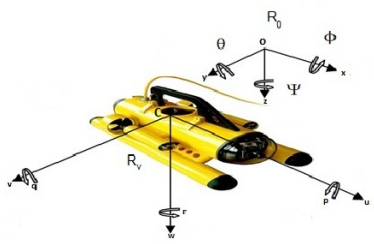

Fig. 1. Body-fixed frame and earth-fixed reference frame for the ROV

A. ROV kinematics and dynamics

In this subsection, the kinematic and dynamic equations of the motion of a ROV moving on the vertical plane are described. Further details on the ROV's dynamical modeling are given in [1] . Using an inertial reference frame R and a body-fixed frame R_v (Fig. 1). the kinematic equations of motion of the center of mass G for the ROV on the vertical XZ plane can be written as:

$$\dot{x} = c\theta u + s\theta w \qquad \dot{z} = -s\theta u + c\theta w \qquad \dot{\theta} = q \tag{1}$$

where $c\theta = \cos\theta$, $s\theta = \sin\theta$, x and z represent the inertial coordinates of the center of the mass G and u, w are the surge and heave velocities respectively, defined in the body-fixed frame. The orientation of the vehicle is described by the angle θ and q is its pitch(angular) velocity. The dynamic of the ROV is expressed by the following differential equations:

$$\begin{cases} m_x\dot{u} &= -m_z wq - d_u u - (F_W - F_B)s\theta + \tau_u \\ m_z\dot{w} &= m_x uq - d_w w + (F_W - F_B)c\theta + \tau_w \\ J_y\dot{q} &= -(m_x - m_z)wu - d_q q + z_g F_B s\theta \end{cases} \tag{2}$$

where

- $m_x = m - X_{\dot{u}}$, $m_z = m - Z_{\dot{w}}$, $J_y = I_{yy} - M_{\dot{q}}$
  with - m represent the mass of the ROV
  - $(X_{\dot{u}}, Z_{\dot{w}})$ are the added mass in the surge and heave directions, respectively.
  - $M_{\dot{q}}$ are the added inertia in the pitch direction.
- $\tau_u = F_{1x} + F_{2x}$, $\tau_w = F_{3z}$ are the imputes.
- (F_B, F_W) are the buoyancy and gravity magnitudes.
- $d_u = X_u - X_{uu} \mid u \mid, d_w = Z_w - Z_{ww} \mid w \mid$
  and $d_q = M_q - M_{qq} \mid q \mid$ are the drag parameters.

B. Coordinate transformations

We introduce the following coordinate transformation to the vehicle:

$$z_1 = xc\theta - zs\theta,\ z_2 = xs\theta + zc\theta,\ z_3 = \theta$$

Using the above change of coordinates, the vehicle dynamics in (1)-(2) can be rewritten as in the form of equations

$$\dot{z}_1 = u - z_2 q \qquad \dot{z}_2 = w + z_1 q \qquad \dot{z}_3 = q \tag{3}$$

It is assumed that the angle z_3 is small enough in a neighborhood of zero, then the dynamics of the ROV can be written

$$\begin{cases} \dot{u} &= -\frac{m_z}{m_x} wq - \frac{d_u}{m_x} u - \frac{(F_W - F_B)}{m_x} z_3 + \frac{1}{m_x}\tau_u \\ \dot{w} &= \frac{m_x}{m_z} uq - \frac{d_w}{m_z} w + \frac{1}{m_z}\overline{\tau}_w \\ \dot{q} &= \frac{m_z - m_x}{J_y} wu - \frac{d_q}{J_y} q + \frac{z_g F_B}{J_y} z_3 \end{cases} \tag{4}$$

where $\overline{\tau}_w = \tau_w + (F_W - F_B)$.
In the next section, we will consider the system (3)-(4) and study its controllability and stabilizability proprieties.

III. CONTROLLABILITY AND STABILITY ANALYSIS

Before tackling the stabilization problem, it is essential to study the controllability of the model (3)-(4).

A. Locally Controllability

In the following we show that the system (3)-(4) is Small Time Locally Controllable (STLC) in the neighborhood of the equilibrium. First, the equation (3)-(4) define a non linear control system of the form

$$\dot{\xi} = g_0(\xi) + \sum_{i=1}^{2} g_i \tau_i \tag{5}$$

where $\xi = (z_1, z_2, z_3, u, w, q)^T \in S \times \mathbf{R}^5$ is the state, g_0 and g_i are the drift and control vector

$$g_0(\xi) = \begin{pmatrix} u - z_2 q \\ w + z_1 q \\ q \\ -\frac{m_z}{m_x} wq - \frac{d_u}{m_x} u - \frac{(F_W - F_B)}{m_x} z_3 \\ \frac{m_x}{m_z} uq - \frac{d_w}{m_z} w \\ \frac{m_z - m_x}{J_y} wu - \frac{d_q}{J_y} q + \frac{z_g F_B}{J_y} z_3 \end{pmatrix} \tag{6}$$

$$g_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{m_x} \\ 0 \\ 0 \end{pmatrix},\ g_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{m_z} \\ 0 \end{pmatrix} \tag{7}$$

Note that the set of equilibrium solutions corresponding to $\tau_i = 0$ is given by the equilibrium manifold

$$M^e = \{\xi \in M | z_3 = u = w = q = 0\}$$

It is easily verified that the linearization of equations (3)-(4) about the equilibrium has an uncontrollable eigenvalue at the origin. This implies that a non linear analysis is necessary to

characterize the controllability and stabilizability proprieties of the system [11]. As a result, from [15], we cannot stabilize the system (3)-(4) by a continuous pure-state feedback. However Coron's theorem [13] proves that a time periodic continuous feedback is sufficient to stabilize the system to a point.

Proposition 3.1: The ROV dynamics described by equation (3)-(4) is small-time locally controllable at the origin.

Proof: Consider the system (5). Sins the vector fields

$$g_1, g_2, [g_0, g_1], [g_0, g_2], [g_2, [g_0, g_1]], [[g_0, g_2], [g_0, g_1]]$$

span a six-dimensional space at any point $\xi \in S \times \mathbf{R}^5$, the strong accessibility Lie rank condition is satisfied at any point and the system is small-time locally controllable at the origin ([10], [11], [14]). ■

B. stabilization of the ROV

The control objective is to find a feedback law that asymptotically stabilizes the origins of the ROV in the vertical plane. We will show first, that it is not possible to stabilize the ROV using a feedback law that is continuous and function of the state only. This follows from results given by [15]. Thus, we propose a continuous periodic time-varying feedback law that stabilizes the ROV.

Theorem 3.2: Consider the following functions

$$\begin{aligned} u_d &= -k_1 z_1 - \tfrac{k_2 z_3 + k_3 q}{\sqrt{|z_3|+|q|}} \sin(t/\epsilon) \\ w_d &= -k_4 z_2 + 2\sqrt{|\ z_3\ | + |\ q\ |}\sin(t/\epsilon) \end{aligned} \tag{8}$$

where k_1, k_2, k_3 and $k_4 > 0$. Furthermore, consider the following continuous periodic time-varying feedback law:

$$\begin{aligned} \tau_u &= -k_u m_x (u - u_d) + d_u u \\ \overline{\tau}_w &= -k_w m_z (w - w_d) + d_w w \end{aligned} \tag{9}$$

Then there exists ϵ_0 such that for any $\epsilon \in (0, \epsilon_0)$ and for positive and sufficiently large parameters k_u and k_w the feedback law (9) locally exponentially stabilizes the origin of the system (4)-(3), with respect to the dilation

$$\delta^r_\lambda(\nu, \eta, t) = (\lambda u, \lambda w, \lambda^2 q, \lambda z_1, \lambda z_2, \lambda^2 z_3, t)$$

Proof: The ROV model with τ_u and τ_w given by (1) can be written

$$\begin{pmatrix} \dot{\nu} \\ \dot{\eta} \end{pmatrix} = f(\nu, \eta, t) + g(\nu, \eta, t) \tag{10}$$

where $\nu = [u,\ w,\ q]^T,\ \eta = [z_1,\ z_2,\ z_3]^T$

$$\begin{aligned} f(\nu, \eta, t) &= [-k_u(u + k_1 z_1 + \tfrac{k_2 z_3 + k_3 q}{\sqrt{|z_3|+|q|}} \sin(t/\epsilon)), \\ &\quad -k_w(w + k_4 z_2 - 2\sqrt{|\ z_3\ | + |\ q\ |}\sin(t/\epsilon)), \\ &\quad \tfrac{m_z - m_x}{J_y} wu - \tfrac{d_q}{J_y} q + \tfrac{z_g F_B}{J_y} z_3, u, w, q]^T \end{aligned} \tag{11}$$

and the remaining terms

$$g(\nu, \eta, t) = [-\tfrac{m_z}{m_x} wq, \tfrac{m_x}{m_z} uq, 0, -z_2 q, z_1 q, 0]^T \tag{12}$$

Consider the dilation δ^r_λ, the functions τ_u and $\overline{\tau}_w$ are homogeneous of degree strictly positive with respect to the dilation, and continuous for $(\eta, \nu) \neq 0$, they are also continuous at zero. The vector field f is thus continuous. It is furthermore time-periodic, $f(0, 0, t) = 0$ and f is homogeneous of degree zero with respect to the dilation $\delta^r_\lambda(\nu, \eta, t)$. Furthermore, the vector field g is continuous and defines a sum of homogeneous vector fields of degree strictly positive with respect to $\delta^r_\lambda(\nu, \eta, t)$. It is sufficient to show that the origin $(\eta, \nu) = (0, 0)$ of the system

$$\begin{pmatrix} \dot{\nu} \\ \dot{\eta} \end{pmatrix} = f(\nu, \eta, t). \tag{13}$$

is locally asymptotically stable.

To this purpose, let us consider the following reduced system obtained from (13), by tacking $u = u_d$ and $w = w_d$ as new control variables. We have obtained the following resulting system:

$$\begin{pmatrix} \dot{q} \\ \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{pmatrix} = \begin{pmatrix} \frac{m_z - m_x}{J_y} w_d u_d - \frac{d_q}{J_y} q + \frac{z_g F_B}{J_y} z_3 \\ u_d \\ w_d \\ q \end{pmatrix} \tag{14}$$

Let (8) defines the control inputs to (14). Due to the periodic time-variant control, the resulting system is a periodic time-varying system, which can be written in the form,

$$\begin{aligned} \begin{pmatrix} \dot{q} \\ \dot{\eta} \end{pmatrix} &= h(q, \eta, t/\epsilon) \\ &= \begin{pmatrix} \frac{m_z - m_x}{J_y}(-k_1 z_1 + \frac{k_2 z_3 + k_3 q}{\sqrt{|z_3|+|q|}} \sin(t/\epsilon)) \\ (-k_4 z_2 - 2\sqrt{|\ z_3\ | + |\ q\ |}\sin(t/\epsilon)) \\ -\frac{d_q}{J_y} q + \frac{z_g F_B}{J_y} z_3 \\ -k_1 z_1 + \frac{k_2 z_3 + k_3 q}{\sqrt{|z_3|+|q|}} \sin(t/\epsilon) \\ -k_4 z_2 - 2\sqrt{|\ z_2\ | + |\ q\ |}\sin(t/\epsilon) \\ q \end{pmatrix} \end{aligned} \tag{15}$$

We approximate this system by an averaged-system which is in particular autonomous (see [12]). The averaged system of (15) is defined as

$$\begin{pmatrix} \dot{q} \\ \dot{\eta} \end{pmatrix} = h_0(q, \eta) \tag{16}$$

where $h_0(q, \eta) = (1/T_t) \int_0^{T_t} h(q, \eta, t/\epsilon) dt$ (T_t is the period). The averaged system of (14) with controls given by (8) lead to

$$\begin{pmatrix} \dot{q} \\ \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{pmatrix} = \begin{pmatrix} k_1 k_4 \frac{m_z - m_x}{J_y} z_1 z_2 + k' z_3 + k'' q \\ -k_1 z_1 \\ -k_4 z_2 \\ q \end{pmatrix} \tag{17}$$

where $k' = -k_2 + \frac{z_g F_B}{J_y}$ and $k'' = -k_3 - \frac{d_q}{J_y}$.
The linear part from (17) is given by

$$\begin{pmatrix} \dot{q} \\ \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 0 & k'' & k' \\ -k_1 & 0 & 0 & 0 \\ 0 & -k_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{=A} \begin{pmatrix} q \\ z_1 \\ z_2 \\ z_3 \end{pmatrix} \tag{18}$$

This permits to apply the Hurwitz's criterion [7] and deduce the $(k_i)_{i=1,2,3,4}$ gains. Then (q, z_1, z_2, z_3) tends asymptotically to zero. Now, the dynamic of u and w in (13) are given by

$$\begin{aligned} \dot{u} &= -k_u(u - u_d) \\ \dot{w} &= -k_w(w - w_d) \end{aligned} \tag{19}$$

The functions u_d and w_d in (8) are continuous, time-periodic, differentiable with respect to t, of class C^1 on $(\mathbf{R}^2 \times \mathbf{R}^2 - \{0,0\}) \times \mathbf{R}$ and homogeneous of degree strictly positive with respect to the dilation $\delta^r_\lambda(\nu, \eta, t)$ in (3.2).
Thus, adequate parameters k_u and k_w can be defined such that the actual velocities u and w track the desired velocities u_d and w_d. Consequently, for positive and sufficiently large values of k_u and k_w, the origin of (10) is locally asymptotically stable. Thus $(\eta, \nu) = (0, 0)$ is a locally exponentially stable equilibrium of (5), with respect to the dilation $\delta^r_\lambda(\nu, \eta, t)$ in (3.2). ■

C. Robustness study

To study the robustness problem of the proposed control input, we consider the additive terms $\widetilde{g}$. Note that the disturbance terms can be result from modeling errors or uncertainties parameters. Generally, we do not know $\widetilde{g}$, moreover we know some information like the boundary of the additive term $\widetilde{g}$. Further, we consider a time varying disturbance is vanishing additive perturbation ($\widetilde{g}(t, 0) = 0$). Let us consider

$$\dot{\xi} = f(t, \xi) + g(t, \xi) + \widetilde{g}(t, \xi) \tag{20}$$

where $\xi = (\nu, \eta)^T$, f, g is defined in the previous section, and $\widetilde{g}(t, \xi)$ is continuous function in t and locally Lipshitz in ξ. Suppose that

$$\|\widetilde{g}(t, \xi)\| \leq \lambda \|\xi\| \tag{21}$$

for all $t \geq 0$ and $\xi \in \Omega$ (Ω is an open neighborhood containing the origin) where k is non-negative constant. Using the converse Lyapunov theorem [7], there exist a continuous Lyapunov function $V(t, \xi)$ that satisfies

i) $c_1\|\xi\|^2 \leq V(t, \xi) \leq c_2\|\xi\|^2$
ii) The derivative of $V(t, \xi)$ along the nominal equation $\dot{\xi} = f(t, \xi) + g(t, \xi)$ satisfies

$$\dot{V}(t, \xi) = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial \xi} f(t, \xi) + \frac{\partial V}{\partial \xi} g(t, \xi) \leq -c_3\|\xi\|^2$$

iii) $\|\frac{\partial V}{\partial \xi}\| \leq c_4\|\xi\|$

where c_1, c_2, c_3 and c_4 are positives constants. The idea consist to use V as a candidate Lyapunov function of the additive perturbed system (20) and the time derivative of V with respect to system (20)

$$\begin{aligned} \dot{V}(t, \xi) &= \frac{\partial V}{\partial t} + \frac{\partial V}{\partial \xi} f(t, \xi) + \frac{\partial V}{\partial \xi} g(t, \xi) + \frac{\partial V}{\partial \xi} \widetilde{g}(t, \xi) \\ &\leq -c_3\|\xi\|^2 + \|\frac{\partial V}{\partial \xi}\| \|\widetilde{g}(t, \xi)\| \\ &\leq -c_3\|\xi\|^2 + \lambda c_4 \|\xi\|^2 \\ &\leq -(c_3 - \lambda c_4)\|\xi\|^2 \end{aligned}$$

Then if we choose $\lambda < \frac{c_3}{c_4}$, the perturbed system is exponentially stable.

IV. Robust tracking control

The section is divided into subsection; in the first one, we will write the error dynamic model that present the difference between the ROV model and the reference model, and in the second subsection we will use the backstepping method that provides a non linear control law that realizes the control objective.

A. Error dynamics formulation

The aim here is to track the following reference variables: $x_r, z_r, \theta_r, u_r, w_r, q_r$. To this end, we define the following tracking errors: $x_e = x - x_r$, $z_e = z - z_r$, $\theta_e = \theta - \theta_r$, $u_e = u - u_{1r}$, $w_e = w - w_r$, $q_e = q - q_r$ and $X_e = (x_e\,,\, z_e)^T$. According to (1) and the definition of the tracking errors we obtain the error dynamics as the kinematic ones:

$$\begin{aligned} \dot{X}_e =& \underbrace{\begin{pmatrix} c\theta & s\theta \\ -s\theta & c\theta \end{pmatrix}}_{R_\theta} \underbrace{\begin{pmatrix} u_e \\ w_e \end{pmatrix}}_{U_e} \\ &+ \underbrace{\begin{pmatrix} c\theta_c\theta_r & s\theta - s\theta_r \\ -s\theta + s\theta_r & c\theta - c\theta_r \end{pmatrix}}_{R_{\theta_r}} \underbrace{\begin{pmatrix} u_r \\ w_r \end{pmatrix}}_{U_r} \\ \dot{\theta}_e =& q_e \end{aligned} \tag{22}$$

and the dynamic ones:

$$\begin{aligned} m_x\dot{u}_e =& - m_z(w_e q_e + w_r q_e + q r w_e + w_r q_r) - d_u u_e \\ &-(F_W - F_B)s(\theta_e + \theta_r) + \tau_u - \dot{u}_r + \chi_u \end{aligned}$$

$$\begin{aligned} m_z\dot{w}_e =& m_x(u_e q_e + u_r q_e + q_r u_e + u_r q_r) - d_w w_e \\ &+(F_W - F_B)c(\theta_e + \theta_r) + \tau_w - \dot{w}_r + \chi_w \end{aligned} \tag{23}$$

$$\begin{aligned} J_y\dot{q}_e =& (m_z - m_x)(u_e w_e + u_r w_e + w_r u_e + u_r w_r) - d_q q_e \\ &+z_g F_B s(\theta_e + \theta_r) - \dot{q}_r + \chi_q \end{aligned}$$

where χ_u, χ_w and χ_q show the three dimension disturbances, assumed to be bounded.

B. Control design

The tracking control objective has been converted to a stabilizing problem given by the system (22)-(23). Thus, we make the following assumption.

Assumption 4.1: 1) Each of the time-varying terms has a constant upper bound, for example $(0 \leq \| q_r \| \leq q_{r,max})$
2) The pitch and heave velocity has lower and upper bounds, $\| q \| \leq u_{max}$ and $\| w \| \leq w_{max}$.
3) The uncontrolled velocity errors q_e has upper bounds.

Proposition 4.2: With the proposed feedback controllers

$$\begin{aligned}\tau_u =& m_x\{\overline{\tau}_u + m_z(w_e q_e + w_r q_e + q r w_e + w_r q_r) + d_u u_e \\ &+(F_W - F_B)s(\theta_e + \theta_r) + \dot{u}_r - \chi_u\} \qquad (24)\\ \tau_w =& m_z\{\overline{\tau}_w - m_x(u_e q_e + u_r q_e + q_r u_e + u_r q_r) + d_w w_e \\ &-(F_W - F_B)c(\theta_e + \theta_r) + \dot{w}_r - \chi_w\}\end{aligned}$$

with

$$\begin{aligned}\overline{\tau}_u =& -c_{1u}(u_e - \alpha_u) - c_{2u}(u_e - \alpha_u)^3 \\ &+\dot{\alpha}_u - x_e c\theta + z_e s\theta \qquad (25)\\ \overline{\tau}_w =& -c_{1w}(w_e - \alpha_w) - c_{2w}(w_e - \alpha_w)^3 \\ &+\dot{\alpha}_q - x_e s\theta - z_e c\theta\end{aligned}$$

where α_w and α_q will be specified later. Then, we prove that the proposed control law (24) guaranteed the convergence of the error tracking in the neighborhood of the origin.

Proof:

step 1) In order to stabilize the vector position $(X_e, \theta_e)^T$, we assume u_e, w_e and q_e as virtual controls. We start by defining the following Lyapunov function candidate:

$$V_1 = \frac{1}{2}X_e^T X_e + \frac{1}{2}\theta_e^2 \qquad (26)$$

The time derivative of V_1 can be writhen as

$$\dot{V}_1 = X_e^T(R_\theta U_e + R_{\theta_r} U_r) + \theta_e q_e \qquad (27)$$

Then, the first part of the desired expressions for the virtual controls is chosen as

$$U_e = (\alpha_{u1}, \alpha_{w1})^T = -R_\theta^T K X_e,\ \alpha_q = -k'\theta_e \qquad (28)$$

where $K = diag(k_1{+}k_2, k_1{+}k_2)$ and (k_1, k_2, k') are positives constants. The time derivative of V_1 becomes

$$\dot{V}_1 = -X_e^T K X_e - k'\theta_e^2 + X_e^T \delta \qquad (29)$$

where $\delta = [\delta_1,\ \delta_2]^T \equiv R_{\theta_r} U_r$ a bounded terms.

step 2) Since the components of the vector $(U_e, q_e)^T$ are not true controls, we need to introduce new error variables ϖ_u, ϖ_w and ϖ_q defined as:

$$\varpi = [\varpi_u, \varpi_w, \varpi_q]^T = [u_e - \alpha_{1u}, w_e - \alpha_{1w}, q_e - \alpha_q]^T$$

Then, the inertial position (X_e, θ_e) and the error ϖ_q equations are rewritten as

$$\begin{aligned}\dot{X}_e =& -KX_e + R_\theta \varpi + \delta \qquad (30)\\ \dot{\theta}_e =& -k'\theta_e \qquad (31)\\ \dot{\varpi}_q =& -\frac{m_x - m_z}{J_y}(u_e w_e + u_r w_e + w_r u_e + u_r w_r) \\ &+(k' - \frac{d_q}{J_y})q_e + \frac{z_g F_B}{J_y}s(\theta_e + \theta_r) - \dot{q}_r + \chi_q \qquad (32)\end{aligned}$$

Before starting, we perform some manipulations on the pitch error dynamic equations. Then $\dot{\varpi}_q$ can be expanded to:

$$\dot{\varpi}_q = -\frac{m_x - m_z}{J_y} w u_e - \frac{m_x - m_z}{J_y} u_r w_e + \varrho_q \qquad (33)$$

where

$$\varrho_q = (k' - \frac{d_q}{J_y})q_e + \frac{z_g F_B}{J_y}s(\theta_e + \theta_r) + u_r w_r - \dot{q}_r + \chi_q \qquad (34)$$

We now consider the stabilization of the subsystems that are controlled by the assumed virtual controls u_e and w_e

$$\dot{\varpi}_q = -\frac{m_x - m_z}{J_y} w u_e - \frac{m_x - m_z}{J_y} u_r w_e + \varrho_q \qquad (35)$$

Here, we choose

$$\begin{aligned}u_e =& c_{1q}\frac{J_y}{m_x - m_z} w \varpi_q \equiv \alpha_{u2} \qquad (36)\\ w_e =& c_{2q}\frac{J_y}{m_x - m_z} u_r \varpi_q \equiv \alpha_{w2} \qquad (37)\end{aligned}$$

where c_{1q} and c_{2q} are positive constants. Then, taking into account (29) it is

$$\begin{aligned}\alpha_u =& \alpha_{u1} + \alpha_{u2} \qquad (38)\\ \alpha_w =& \alpha_{w1} + \alpha_{w2} \qquad (39)\end{aligned}$$

So far, the controlled subsystem of the kinematics and the errors ϖ_q is transformed as

$$\begin{aligned}\dot{X}_e =& -KX_e + R_\theta \varpi + \delta \qquad (40)\\ \dot{\theta}_e =& -k'\theta_e \qquad (41)\\ \dot{\varpi}_q =& -c_{1q} w^2 \varpi_q - c_{2q} u_r^2 \varpi_q + \varrho_q \qquad (42)\end{aligned}$$

In order to stabilize the above subsystem,we choose

$$V_2 = V_1 + \frac{1}{2}\varpi_q^2 \qquad (43)$$

Taking into account (29), its time derivative becomes

$$\begin{aligned}\dot{V}_2 =& -X_e^T K X_e - k'\theta_e^2 + X_e^T \delta \\ &-(c_{1q} w^2 + c_{2q} u_r^2)\varpi_q^2 + \varrho_q \varpi_q \qquad (44)\end{aligned}$$

step 3) In the following we would force ϖ_u and ϖ_w to zero, so we consider the following Lyapunv function:

$$V_3 = V_2 + \frac{1}{2}\varpi_u^2 + \frac{1}{2}\varpi_w^2 \qquad (45)$$

The time derivative of V_2 can be expressed as:

$$\begin{aligned}\dot{V}_3 =& - X_e^T K X_e - k'\theta_e^2 + X_e^T\delta \\ &+\varpi_u(\overline{\tau}_u - \dot{\alpha}_u + x_e c\theta - z_e s\theta) \\ &+\varpi_w(\overline{\tau}_w - \dot{\alpha}_w + x_e s\theta + z_e c\theta) \\ &-(c_{1q}w^2 + c_{2q}u_r^2)\varpi_q^2 + \varrho_q\varpi_q \end{aligned} \tag{46}$$

Using $\overline{\tau}_u$ and $\overline{\tau}_u$ given by the proposition, equation (46) becomes:

$$\begin{aligned}\dot{V}_3 =& - X_e^T K X_e - k'\theta_e^2 - c_{1u}\varpi_u^2 - c_{2u}\varpi_u^4 \\ &-c_{1w}\varpi_w^2 - c_{2w}\varpi_w^4 - (c_{1q}w^2 + c_{2q}u_r^2)\varpi_q^2 \\ &+X_e^T\delta + \varrho_q\varpi_q \end{aligned} \tag{47}$$

Note that although $\dot{V}_3$ is not necessarily always negative, this will be sufficient to achieve practical stability.

Now, the last term with uncertain sign is examined using a worst case analysis, i.e., considering that all of them are positive. In the following analysis we use Youngs inequality, the quantities $(\varepsilon_i)_{i=1,2,3,4}$, are positive constants. For the first three of the uncertain terms we have

$$(k' - \frac{M_q}{J_y})q_e\varpi_q \leq \frac{1}{4\epsilon_1}[k' - \frac{M_q}{J_y}]^2|q_e|^2 + \epsilon_1|\varpi_q|^2 \tag{48}$$

$$-\frac{M_{qq}}{J_y}q|q|\varpi_q \leq \frac{M_{qq}}{J_y}q^2\varpi_q \leq \frac{1}{4\epsilon_2}|\varpi_q|^2 + \epsilon_2\frac{M_{qq}}{J_y}^2|q|^4 \tag{49}$$

$$(\frac{z_g F_B}{J_y}s(\theta_e + \theta_r) + u_r w_r - \dot{q}_r)\varpi_q \leq \frac{1}{4\epsilon_3}|\varpi_q|^2 + \epsilon_3\xi^2 \tag{50}$$

where we have $\xi \geq |\frac{z_g F_B}{J_y}s(\theta_e + \theta_r) + u_r w_r - \dot{q}_r|$

$$\chi_q\varpi_q \leq \frac{1}{4\epsilon_4}|\chi_q|^2 + \epsilon_4|\varpi_q|^2 \tag{51}$$

$$\begin{aligned}-X_e^T K X_e + X_e^T\delta =& - k_1 x_e^2 - k_1 z_e^2 - k_2(x_e - \frac{\delta_1}{2k_2})^2 \\ &-k_2(z_e - \frac{\delta_2}{2k_2})^2 + \frac{\| \delta \|^2}{4k_2} \end{aligned} \tag{52}$$

After tedious but straightforward algebraic manipulations of the various terms in (48)-(52), and taking into account the above assumptions, we end up with the following form of the derivative of V_3

$$\begin{aligned}\dot{V}_3 \leq& - k_1 x_e^2 - k_1 z_e^2 - k_2(x_e - \frac{\delta_1}{2k_2})^2 - k_2(z_e - \frac{\delta_2}{2k_2})^2 - k'\theta_e^2 \\ &-c_{1u}\varpi_u^2 - c_{2u}\varpi_u^4 - [c_{1q}w^2 + c_{2q}u_r^2 - \epsilon_1 - \epsilon_4 - \frac{1}{4\epsilon_2} - \frac{1}{4\epsilon_3}]\varpi_q^2 \\ &-c_{1w}\varpi_w^2 - c_{2w}\varpi_w^4 + \frac{1}{4\epsilon_1}[k' - \frac{M_q}{J_y}]^2 q_{e,max}^2 \\ &+\epsilon_2\frac{M_{qq}}{J_y}^2 q_{max}^4 + \epsilon_3\xi^2 + \frac{\delta_{max}^2}{4k_2} + \frac{1}{4\epsilon_4}|\chi_q|^2 \end{aligned} \tag{53}$$

In Eq. (48) the term $[c_{1q}w^2 + c_{2q}u_r^2 - \epsilon_1 - \epsilon_4 - \frac{1}{4\epsilon_2} - \frac{1}{4\epsilon_3}]$ must be positive. Setting $\varepsilon = \epsilon_1 + \epsilon_4 + \frac{1}{4\epsilon_2} + \frac{1}{4\epsilon_3} > 0$, it must be $c_{1q}w^2 + c_{2q}u_r^2 > \varepsilon$, which holds for large and positive c_{1q}, c_{2q} and small ε.

Choosing the various gains in order for the coefficients of the states of the error dynamics to be negative we rewrite Eq. (53) as follows:

$$\dot{V}_3 \leq - k_1 x_e^2 - k_1 z_e^2 - k'\theta_e^2 - c_{1u}\varpi_u^2 - c_{1w}\varpi_w^2 - k_3\varpi_q^2 + \mu \tag{54}$$

where

- $k_3 = c_{1q}w_{max}^2 + c_{2q}u_{r,max}^2 - \varepsilon$ (55)
- $\mu = \frac{1}{4\epsilon_1}[k' - \frac{M_q}{J_y}]^2 q_{e,max}^2 + \epsilon_2\frac{M_{qq}}{J_y}^2 q_{max}^4 + \epsilon_3\xi^2 + \frac{\delta_{max}^2}{4k} + \frac{1}{4\epsilon_4}\chi_q^2$ (56)

Taking $\sigma = min\{k_1, k^{'}, k_3, c_{1u}, c_{1w}\}$. Then

$$\dot{V}_3 \leq - 2\sigma V_3 + \mu \tag{57}$$

By using the comparison lemma [7], the previous equation leads to:

$$V_3(t) \leq V_3(0)e^{-2\sigma t} + \frac{\mu}{2\sigma} \tag{58}$$

Now, if we define

$$\xi = [x_e, z_e, \theta_e, \varpi_u, \varpi_w, \varpi_q]^T \tag{59}$$

then, consideringn equation (45) it is $2V_3 = \|\xi\|^2$ we conclude

$$\|\xi(t)\| \leq \|\xi_2(0)\|e^{-\sigma t} + \sqrt{\frac{\mu}{\sigma}} \tag{60}$$

Eq. (60) means that the states of the error dynamics remain in a small, bounded set around zero, which can be reduced using an appropriate combination of the controller gains. At this result we arrived using (25) along with (20). ■

V. Simulation results

In this section, we give a numerical simulation to illustrate our theoretical results. Before starting, we will present the system parameter values (IS units) used for simulations. The added masses and hydrodynamic coefficients calculated from the CAD-geometry are presented in Table I.

TABLE I
RIGID BODY AND HYDRODYNAMIC PARAMETERS OF THE ROV

Parameter	Symbol	Value
mass (kg)	m	10.84
moment of inertia (kg/m^2)	I_{yy}	0.216
Added mass in surge	$X_{\dot{u}}$	-1.0810
Added mass in heave	$Z_{\dot{w}}$	-0.3.848
Added inertia in pitch	$M_{\dot{q}}$	-0.0075
Surge linear drag	X_u	2.1613
heave linear drag	Z_w	2.4674
Surge linear drag	M_q	0.053
Surge quadratic drag	X_{uu}	4.4674
heave quadratic drag	Z_{ww}	5.989
Quadratic pitch drag	M_{qq}	0.1011
Center of gravity G	(x_G, y_G, z_G)	$(0, 0, -0.16)$
Center of buoyancy C	(x_B, y_B, z_B)	$(0, 0, 0)$

- The feedback laws given by (9) make the origin of (1)-(2) exponentially stable for small enough values of ε and large enough values of k_u and k_w : The control parameters were chosen as

$$k_1 = k_4 = 10,\ k_2 = 2,\ k_3 = k_u = k_w = 3,\ \varepsilon = 0.01$$

The initial vector of states of the ROV is taken as:

$$[u, w, q, \theta, x, z]^T(0) = [0.3, 0, 0, 0.2, 0.5, -0.5]^T$$

Figure 2 shows the time evolution of the ROV state variables and we can see that the inertial position converge in a small neighborhood of zero. The linear and angular velocities convergence are depicted in figures 3.
Figure 4 sketches the control forces τ_u, τ_w needed for stabilization.
In figures 3, we can see that the inertial position and the Euler angles converge in a small neighborhood of zero which can be considered as a practical stability results

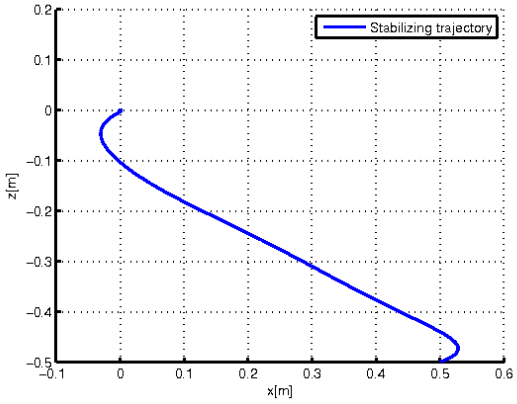

Fig. 2. Trajectory of the ROV in the vertical plane

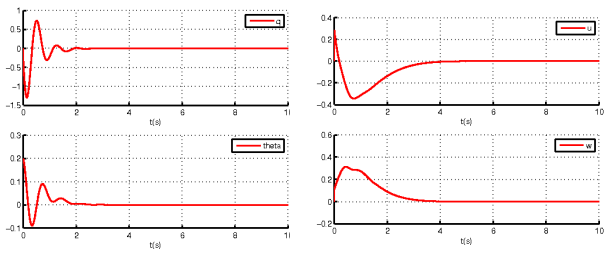

Fig. 3. Sketch of the different states in the vertical plane

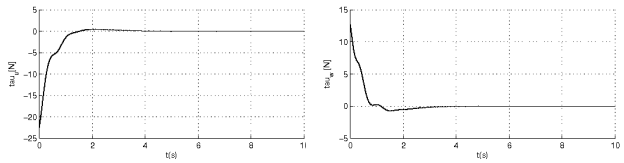

Fig. 4. Sketch of Feedback stabilizing controller in the vertical plane

- The reference trajectory in the vertical plane is described by the following equations

$$x_r = 10\sin(0.3t),\ \ z_r = 0.3t,\ \ \theta_r = 0.1\,rad$$

The initial vector of states of the ROV is taken as:

$$[u, w, q, \theta, x, z]^T(0) = [0, 0, 0, 0.1, -1, 1]^T$$

The simulations results were obtained with gain chosen as: $k_1 = 0.5$, $k_2 = 0.4$, $k' = 0.5$, $c_{1u} = c_{2u} = 3$, $c_{1w} = c_{2w} = 3$, $c_{1q} = c_{2q} = 1$.
In figure 5, the reference and the resulting trajectory of the CM of the vehicle in the inertial XZ plane are displayed. The convergence of the errors in linear, angular velocities and Euler angles are depicted in figure 6. Figure 7 sketches the control forces τ_u and τ_w needed for tracking in the vertical plane.

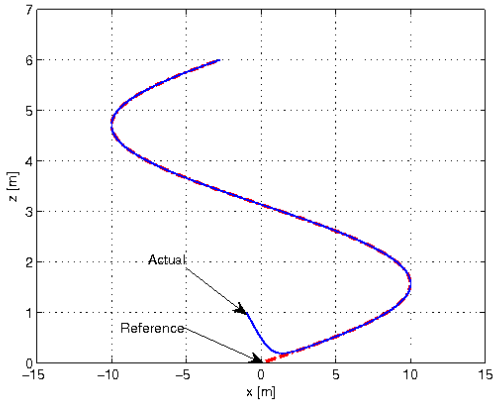

Fig. 5. Actual and reference trajectory in vertical plane

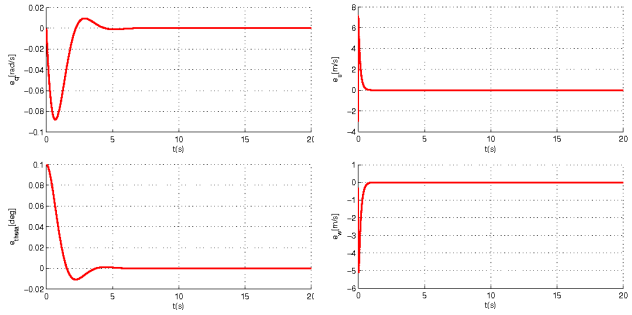

Fig. 6. Sketch of the different tracking error in the vertical plane

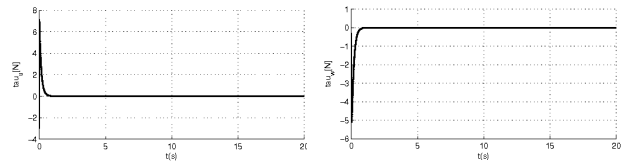

Fig. 7. Feedback tracking controller in the vertical plane

VI. Conclusion

In this paper, the design control problem for underactuated ROV on the vertical plane was addressed. In the first part, We prove that the presented system in the paper is not stabilizable by continuous pure state feedback law. The exponential stabilization problem of the origin by means of smooth time-variant feedback law has been proposed for the kino-dynamic model. The averaging theory is used to prove the stabilizing results. Secondly, given a reference trajectory to be followed by the ROV and using these reference values, the dynamic of the ROV was transformed to the error one. Backstepping techniques is used to stabilize the above system and force the tracking error to a neighborhood about zero that can be made arbitrarily small. Computer simulations are showed very good tracking performance and robustness of the proposed method in the presence of parametric uncertainty or of trajectories that are described by time-varying velocities.

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